PARTON MODELS FOR WEAK AND ELECTROMAGNETIC INTERACTIONS

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INTRODUCTION

The aim of these lectures is to give a review of the situation concerning inclusive reactions induced by charged and neutral leptons in the light of the quark parton model. They will be divided into three parts.

PART A is devoted to theoretical generalities. We first describe the kinematics in order to introduce the notation. The consequences of a scaling à la Bjorken are presented for differential and total cross sections. A general formulation of the parton model is presented and structure functions are computed when the interacting partons are identified with the basic quarks of a symmetry group of strong interactions. As a byproduct the Adler and Gross-Llewellyn Smith sum rules are written in the quark parton language.

PART B is a study of three inclusive processes
- electroproduction
- neutrino and antineutrino induced reactions with charge changing current
- neutrino and antineutrino induced reactions with charge conserving current.

We begin with a review of the main experimental facts. The scaling of electroproduction structure functions and of neutrino and antineutrino total cross sections is exhibited. Then the particular quark parton model based on SU(3) symmetry is described and various experimental data are analysed in this framework. It is shown how the model is consistent with experiments. In particular, the naive Weinberg model for hadronic neutral current fits nicely the Gargamelle bubble chamber data and a value of the mixing angle is computed from experiment and turns out to be compatible with the range of values permitted by purely leptonic processes.

PART C is more speculative in the sense that a comparison with experiments is not yet possible and belongs to the future. Always in the framework of the SU(3) quark parton model we study two applications of the previous techniques

(i) The polarization effects in electroproduction, the target namely a nucleon, and the incident beam being polarized.

(ii) The weak effects in electroproduction due to the possible exchange of a neutral vector boson interfering with the usual one-photon exchange contribution.

Only simple cases have been considered and straightforward extensions can be made, in case (i) to weak reactions, in case (ii) to a polarized nucleon target.

Finally the last section of this part is concerned with a schematic description of a modern approach of scaling, using the renormalization group techniques.

PART A THEORETICAL GENERALITIES

I CURRENTS

1) In unified gauge theories of electromagnetic and weak processes the interaction between leptons and hadrons is mediated by vector bosons; one of them, the photon, is massless, the other ones being expected very heavy.

To each physical vector boson field corresponds a current with one leptonic part and one hadronic part. In models based on the $SU(2) \otimes U(1)$ gauge group there are four vector boson fields

i) the electromagnetic field
ii) the two charged boson fields
iii) the neutral boson field

The corresponding Lagrangians involving only known leptons have the following structure,

a - electromagnetic Lagrangian associated to the photon field

$$\mathcal{L}_Q = e\left[i \sum_{\ell} \bar{\ell}\gamma_\mu \ell + J_\mu^Q \right] A^\mu \tag{1}$$

b - weak charged Lagrangian associated to the charged boson fields

$$\mathcal{L}_C = g_C\left[i \sum \bar{\ell}\gamma_\mu(1+\gamma_5)\nu_\ell + J_\mu^{W^-} \right] W^{\mu +} \text{ Hermitian conjugate} \tag{2}$$

c - weak neutral Lagrangian associated to the neutral boson field

$$\mathcal{L}_N = g_N\Big[i\sum_\ell\big(\bar{\nu}_\ell\gamma_\mu(1+\gamma_5)\nu_\ell + \bar{\ell}\gamma_\mu(a-b\gamma_5)\ell\big) + c\,J_\mu^Z\Big]Z^\mu \quad (3)$$

where all the coupling constants - but e - are model dependent.

2) Let us call q_μ the energy momentum four vector of the intermediate boson. At actual available energies, $\sqrt{q^2}$ remains small as compared to the vector boson masses expected to be as large as 40 GeV or more. Therefore the local Fermi effective interaction becomes a good approximation.

The Fermi constant G measured with the μ life time is defined by

$$\frac{G}{\sqrt{2}} = \frac{g_C^2}{m_W^2} \quad (4)$$

The normalization C of the neutral hadronic current is defined by the convention

$$\frac{G}{\sqrt{2}} = \frac{g_N^2}{m_Z^2} \quad (5) .$$

II KINEMATICS

1) Inclusive reactions induced by leptons are described to lowest order by the diagram of Fig. 1 where the kinematical notations are indicated.

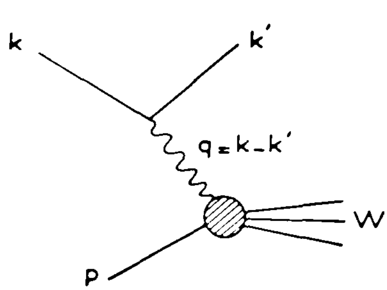

Fig. 1 One vector boson exchange in inelastic lepton scattering.

We introduce, as usual, the scalar variables q^2, W^2 and ν defined by

$$q^2 = (k-k')^2 \qquad W^2 = -(p+q)^2 \qquad \nu M = -\,p\cdot q$$

with the relation $W^2 = M^2 + 2M\nu - q^2$, M being the nucleon mass. In the laboratory frame the lepton variables are

- E incident lepton energy
- E' final lepton energy
- θ scattering angle between leptons.

They can expressed in terms of the invariant q^2 and ν by

$$q^2 = 4EE' \sin^2 \frac{\theta}{2} \qquad \nu = E - E'$$

Let us recall that the physical region is restricted by the inequality

$$o \leq q^2 \leq 2M\nu$$

The elastic case $W^2 = M^2$ corresponds to $q^2 = 2M\nu$.

2) As a consequence of the one vector boson exchange B the transition matrix element is factorized into the product of two matrix elements of the B current, one for leptons and one for hadrons

$$T^B = \frac{C^B}{q^2+m_B^2} \left[\bar{u}(k')\gamma^\mu(a-b\gamma_5)u(k)\right]\langle\Gamma|J_\mu^B(o)|p\rangle \tag{6}$$

where u(k) and u(k') are lepton free Dirac spinors and C^B is a product of coupling constants.

The cross-section is therefore written as the product of a leptonic tensor $m^{\mu\nu}$ by a hadronic tensor $M_{\mu\nu}$. We shall work, in what follows, in the zero mass limit for charged leptons and only a longitudinal polarization survives. We call η the helicity of the incident lepton

$\eta = +1$	Right-hand	R
$\eta = -1$	Left-hand	L

The leptonic tensor is easily computed in this approximation and the result is

$$m^{\mu\nu} = (a^2+b^2)(t^{\mu\nu}+ \eta s^{\mu\nu})+2ab(s^{\mu\nu}+ \eta t^{\mu\nu}) \tag{7}$$

where

$$t^{\mu\nu} = \frac{1}{2} \left[k^\mu k'^\nu + k'^\mu k^\nu + \frac{1}{2} q^2 g^{\mu\nu}\right] \tag{8}$$

$$s^{\mu\nu} = \frac{i}{2} \varepsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma \tag{9}$$

Let us specify the leptonic tensor in the various cases considered a - electromagnetic current: incident charged lepton a = 1 b = 0

$$m^{\mu\nu} = t^{\mu\nu} + \eta\ s^{\mu\nu} \qquad (10)$$

b - weak currents: incident left-hand neutrinos and right-hand antineutrinos a = 1 b = -1

$$m^{\mu\nu} = 4\ (\ t^{\mu\nu} \mp s^{\mu\nu}\ ) \qquad (11)$$

c - interference between electromagnetic and weak neutral currents: incident charged leptons

$$m^{\mu\nu} = a(t^{\mu\nu} + \eta\ s^{\mu\nu}) + b(s^{\mu\nu} + \eta\ t^{\mu\nu}) \qquad (12)$$

3) The hadronic tensor $M_{\mu\nu}$ is generally defined by

$$M^{\alpha\beta}_{\mu\nu} = M \sum_{\Gamma} \int_{\Gamma} (2\pi)^3 \delta_4(p+q-p_\Gamma) \langle \Gamma | J^{\alpha}_{\mu}(0) | p \rangle \langle \Gamma | J^{\beta}_{\nu}(0) | p \rangle^{*} \qquad (13)$$

where $\int_\Gamma$ means a phase space integration and a summation over polarization for all the particles belonging to Γ.

We shall use four hadronic tensors in the various processes studied

$M^{QQ}_{\mu\nu}$	for electroproduction with one photon exchange
$M^{WW}_{\mu\nu}, M^{ZZ}_{\mu\nu}$	for neutrino and antineutrino scattering
$\frac{1}{2}(M^{QZ}_{\mu\nu} + M^{ZQ}_{\mu\nu})$	for one photon, one neutral boson interference in electroproduction.

The hadronic tensor $M^{\alpha\beta}_{\mu\nu}$ can also be written as the Fourier transform of the one particle matrix element of the product of two current operators. From equation (13) we get

$$M^{\alpha\beta}_{\mu\nu} = \frac{M}{2\pi} \int e^{-iq\cdot x} \langle p | J^{\beta}_{\nu}(x)\, J^{\alpha}_{\mu}(0) | p \rangle\, d_4 x$$

This equality shows that the hadronic tensor is proportional to the imaginary part of a forward Compton scattering amplitude as appears in Fig. 2.

In the polarization space the hadronic tensor is repesented by a 8 x 8 matrix which is Hermitian by construction when $\alpha = \beta$. The total helicity is a conserved quantity for the forward Compton amplitude and

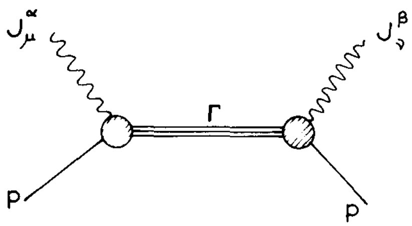

Fig. 2 Compton like amplitude for the hadronic tensor.

the 8 x 8 matrix is reducible into:

two 1 x 1 matrices for total helicity $\pm \frac{3}{2}$

two 3 x 3 matrices for total helicity $\pm \frac{1}{2}$

It follows that the total number of structure functions is 20 and it reduces to 6 when the target is unpolarized.

Furthermore the structure functions associated to a scalar polarization of the current give contributions to the cross section proportional to the lepton mass. In the zero lepton mass limit it is then sufficient to restrict to vector polarizations for the current and the 3 x 3 matrices associated to the total helicity $\pm \frac{1}{2}$ reduce to 2 x 2 matrices. In this approximation the total number of structure functions is 10 for a polarized spin $\frac{1}{2}$ target and 3 for an unpolarized one. With respect to space and time reflections these ten structure functions are classified as shown in Table 1.

	Parity conserving	Parity violating
Time Reversal invariant	2 + 2	1 + 3
Time Reversal violating	0 + 1	0 + 1

Table 1

In this table the first number refers to the unpolarized part $T_{\mu\nu}$ of the hadronic tensor and the second number to the polarized part $S_{\mu\nu}$

$$M_{\mu\nu} = T_{\mu\nu} + S_{\mu\nu}$$

The diagonal elements of the hadronic matrix for $\alpha = \beta = B$ can be interpreted as the total cross sections $\sigma^B_{\lambda s}(q^2,W^2)$ for the process

$$B + p \rightarrow \text{HADRONS}$$

where λ is the helicity of the virtual boson B ($\lambda = \pm 1,0$) and s the helicity of the spin $\frac{1}{2}$ target ($s = \pm \frac{1}{2}$) .

This is the case for the three unpolarized structure functions and three polarized structure functions the other ones being transverse longitudinal correlations for total helicity $\pm \frac{1}{2}$. Because of the semi definite positive character of the hadronic tensor for $\alpha = \beta = B$ the structure functions $\sigma^B_{\lambda s}(q^2, W^2)$ are positive in the physical region $q^2 \geq 0 \quad W^2 \geq M^2$ and the correlations are bounded by total cross sections using the Schwartz inequality in the 2 x 2 matrices.

4) We now consider the case of an unpolarized target. Three structure functions $\sigma^B_\lambda(q^2, W^2)$ will describe the hadronic tensor and the double differential cross section for inelastic scattering of polarized leptons off unpolarized targets has the general structure

$$\frac{d^2\sigma^B_\eta}{dq^2 dW^2} = \frac{A^B(q^2)}{8\pi^2} \frac{q^2}{M\sqrt{\nu^2+q^2}} \frac{E'}{E} \Big\{ \cos^2\frac{\theta}{2}\left[\sigma^B_T(q^2,W^2) + \sigma^B_L(q^2,W^2)\right] +$$

$$+ 2\sin^2\frac{\theta}{2}\,\frac{\nu^2+q^2}{q^2}\,\sigma^B_T(q^2,W^2) -$$

$$- \eta\sin^2\frac{\theta}{2}\,\frac{(E+E')\sqrt{\nu^2+q^2}}{q^2}\left[\sigma^B_{-1}(q^2,W^2) - \sigma^B_{+1}(q^2,W^2)\right] \tag{14}$$

where

$$A^Q(q^2) = \frac{e^4}{q^4}$$

$$A^W(q^2) = \frac{4\,g_c^2}{(q^2+m_W^2)^2} = \frac{2\,G^2}{(1+q^2/m_W^2)^2}$$

$$A^Z(q^2) = \frac{4\,c^2 g_N^2}{(q^2+m_Z^2)^2} = \frac{2\,G^2}{(1+q^2/m_Z^2)^2}$$

In the particular case of electromagnetic interactions, parity is conserved and $\sigma^Q_{-1} = \sigma^Q_{+1}$. The last term in equation (14) disappears and the cross section is independent of the lepton helicity η. Analogous expressions can be written with a polarized target. The particular case of electroproduction will be discussed in part C.

III SCALING

1) Let us define new dimensionless variables

$$\xi = \frac{q^2}{2M\nu} \qquad\qquad \rho = \frac{p\cdot k'}{p\cdot k} = \frac{E'}{E}$$

and new structure functions

$$\sigma_\lambda^{\mathcal{B}}(q^2, W^2) = \frac{\pi}{M\sqrt{\nu^2+q^2}}\, F_\lambda^{\mathcal{B}}(q^2, \xi)$$

$$F_\lambda(q^2,\xi) = \left|\text{Forward Compton Amplitude}\right|^2$$

The double differential cross sections (14) are equivalently written as

$$\frac{d^2\sigma_\eta^{\mathcal{B}}}{d\rho\, d\xi} = \frac{A^{\mathcal{B}}(q^2)}{2\pi} M E \xi \Big\{(1-\rho^2)\, F_T^{\mathcal{B}}(q^2,\xi) + \tag{16}$$

$$+ \frac{\nu^2}{\nu^2+q^2}\left(2\rho - \frac{q^2}{2E^2}\right)\left[F_T^{\mathcal{B}}(q^2,\xi) + F_L^{\mathcal{B}}(q^2,\xi)\right] - \eta\,\frac{1-\rho^2}{2}\,\frac{\nu}{\sqrt{\nu^2+q^2}}\left[F_{-1}^{\mathcal{B}}(q^2,\xi) + F_{+1}^{\mathcal{B}}(q^2,\xi)\right]\Big\}$$

2) When the variables ρ and ξ are fixed the high energy limit $E \to \infty$ implies the Bjorken limit LIM for the structure functions where LIM means q^2, $W^2 \to \infty$ with ξ fixed

$$\mathrm{LIM}\; F_\lambda^{\mathcal{B}}(q^2,\xi) = F_\lambda^{\mathcal{B}}(\xi)$$

For the differential cross sections (16) we simply obtain

$$\lim_{\substack{E\to\infty \\ \rho,\xi \text{ fixed}}} \frac{d^2\sigma_\eta^{\mathcal{B}}}{d\rho\, d\xi} = \frac{A^{\mathcal{B}}(q^2)}{2\pi} M E \xi \Big\{(1+\rho^2)\, F_T^{\mathcal{B}}(\xi) +$$

$$+ 2\rho\, F_L^{\mathcal{B}}(\xi) - \eta\,\frac{1-\rho^2}{2}\left[F_{-1}^{\mathcal{B}}(\xi) - F_{+1}^{\mathcal{B}}(\xi)\right] \tag{17}$$

For electroproduction

$$A^{Q}(q^2) = \frac{e^4}{4M^2E^2(1-\rho)^2\xi^2}$$

and the fixed ρ, ξ differential cross section tends to zero like 1/E at high energy.

For neutrino and antineutrino induced processes we assume that there exists in the q^2, W^2 plane a region where scaling takes place and where the Fermi theory is still valid. Then

$$A^W = A^Z = 2\,G^2$$

and the differential cross sections (17) increase linearly with the incident energy E and their ρ dependence becomes simply a second order polynomial

$$\lim_{\substack{E\to\infty \\ \rho,\xi \text{ fixed}}} \frac{d^2\sigma^{\nu}}{d\rho\, d\xi} = \frac{G^2 M E}{\pi}\, \xi \left[\rho^2 F_+^{\nu}(\xi) + F_-^{\nu}(\xi) + 2\rho\, F_0^{\nu}(\xi)\right]$$

$$\lim_{\substack{E\to\infty \\ \rho,\xi \text{ fixed}}} \frac{d^2\sigma^{\bar\nu}}{d\rho\, d\xi} = \frac{G^2 M E}{\pi}\, \xi \left[\rho^2 F_-^{\bar\nu}(\xi) + F_+^{\bar\nu}(\xi) + 2\rho\, F_0^{\bar\nu}(\xi)\right] \tag{18}$$

3) Because of the zero mass of the photon the electroproduction total cross section is infrared divergent. The situation is different for weak processes the bosons W and Z being massive.

In order to compute total cross sections we integrate over the variables ρ and ξ in the square 0 - 1. The assumption generally made is that in such an integration we can use everywhere the form of $d^2\sigma/d\rho\, d\xi$ obtained in the scaling region. The result is simply a linear rising with energy of the total neutrino and antineutrino cross sections

$$\lim_{E\to\infty} \sigma_{TOT} = \frac{G^2 M E}{\pi} A \tag{19}$$

where the coefficients A are given by

$$A^{\nu} = \frac{1}{3} I_+^{\nu} + I_-^{\nu} + I_0^{\nu}$$

$$A^{\bar\nu} = \frac{1}{3} I_-^{\bar\nu} + I_+^{\bar\nu} + I_0^{\bar\nu} \tag{20}$$

and the integrals I_λ by

$$I_\lambda = \int_0^1 \xi\, F_\lambda(\xi)\, d\xi$$

4) The final lepton energy distributions can be computed with the same assumptions and the result is

$$\lim_{E\to\infty} \frac{d\sigma^{\nu}}{d\rho} = \frac{G^2 M E}{\pi}\left[\rho^2 I_+^{\nu} + I_-^{\nu} + 2\rho\, I_0^{\nu}\right]$$

$$\lim_{E\to\infty} \frac{d\sigma^{\bar\nu}}{d\rho} = \frac{G^2 M E}{\pi}\left[\rho^2 I_-^{\bar\nu} + I_+^{\bar\nu} + 2\rho\, I_0^{\bar\nu}\right] \tag{21}$$

5) The structure functions $\sigma_\lambda(q^2, W^2)$ being total cross sections, they are positive functions of q^2 and W^2 in the physical region $q^2 \geq 0$,

$W^2 \geq M^2$. Therefore the scaling function $F_\lambda(\xi)$ and the first moment integrals I_λ are also positive

$$F_\lambda(\xi) \geq 0 \qquad \text{for} \qquad 0 \leq \xi \leq 1$$

$$I_\lambda \geq 0$$

IV PARTON MODELS

1) The hadrons are assumed to be composite systems of elementary constituents called partons. The structure functions are Lorentz invariant quantities so that they can be computed in any frame of reference.

A simple description of the hadron occurs in the $P \to \infty$ system where the hadron momentum P becomes very large as compared to the hadron mass. Then the partons appear to be quasi-free particles and the impulse approximation can be used for the interaction of the current with the hadron. The partons have an instantaneous interaction with the current which is point-like and only the parton charge associated to the current is seen. After interaction the partons gain a transverse momentum q^2 and they remain quasi-free on mass shell.

The main condition for the impulse approximation to be valid is that the time of interaction of the current with the parton must be small as compared with the typical life time of metastable states in the hadron. In other words the effective mass W of the final hadronic system must be large as compared with a typical resonance energy W_R so that the scattering must be deeply inelastic.

2) As pointed out in section II the structure functions $\sigma_\lambda^B(q^2,W^2)$ are directly proportional to the imaginary part, in the forward direction, of a Compton type amplitude $B + p \to B + p$.
In the parton model we make an incoherent summation of the various parton contributions as shown in Fig. 3, and the partons are assumed to interact in a point like manner with the current J^B.

Let us call $D_j(\xi)$ the distribution function of the parton of type j in the hadron, its momentum being $\xi\vec{P}$. The normalization integral of these distributions

$$\int_0^1 D_j(\xi)\, d\xi = \langle N_j \rangle \tag{22}$$

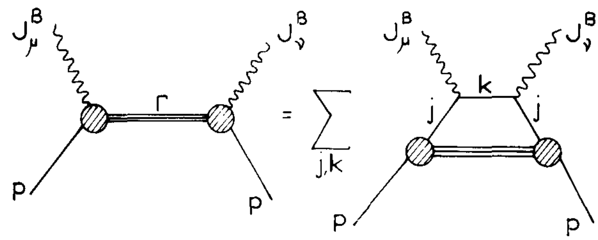

Fig. 3
Parton models for the hadronic tensor.

gives the average value of the number of type j parton in the hadron. One important property of the D_j's which are essentially probability distributions is their positivity.

3) The point-like matrix element of the current J^B of helicity λ between the partons j and k is called charge $a_{jk}^{B\lambda}$. From Fig. 3 the scaling functions are simply written as

$$F_\lambda^B(\xi) = \sum_{j,k} \left[a_{jk}^{B\lambda} \right]^2 D_j(\xi) \tag{23}$$

We assume the partons to have only spin 0 and $\frac{1}{2}$. At high energy it is straightforward to check that

F_+ contains only right-hand partons and antipartons
F_- contains only left-hand partons and antipartons
F_o contains only spin zero partons and antipartons.

As a first experimental result we shall discuss in the next lesson the transverse scaling functions dominate over the longitudinal ones both in electromagnetic and weak interactions. It is then legitimate to associate the interacting partons with spin $\frac{1}{2}$ quarks. Nevertheless we shall reserve the possibility of existence of integer spin gluons which have zero charges and which only carry energy momentum. The physical role attributed to gluons is then to bind the quarks into the hadron.

4) In weak interactions the partons may have different right-hand and left-hand couplings with the currents. We call a_{jk}^{BR} and a_{jk}^{BL} the corresponding charges. The indices j, k being from now positive we associate the distributions $D_j(\xi)$ to quarks and the distributions $D_{-j}(\xi)$ to antiquarks. Taking into account the symmetry relation

$$(a_{jk}^B)^2 = (a_{-k,-j}^B)^2$$

we easily compute the scaling functions associated to the current J^B

$$2\, F_+^B(\xi) = \sum_{j,k} \left[(a_{jk}^{BL})^2 D_{-k}(\xi) + (a_{jk}^{BR})^2 D_j(\xi) \right]$$

$$2\, F_-^B(\xi) = \sum_{j,k} \left[(a_{j,k}^{BL})^2 D_j(\xi) + (a_{jk}^{BR})^2 D_{-k}(\xi) \right] \tag{24}$$

For self Hermitian currents like J^Q and J^Z the charges are diagonal in the parton space: $a_{jk} = a_j \delta_{jk}$. Moreover, for the electromagnetic current, parity is conserved and we simply have

$$a_j^{QR} = a_j^{QL} = Q_j$$

where Q_j is the electric charge of the parton j. The electroproduction scaling function is then written as

$$2\, F_T^Q(\xi) = \sum_j Q_j^2 \left[D_j(\xi) + D_{-j}(\xi) \right] \tag{25}$$

For a non Hermitian current like the weak charged current the scaling functions associated to its Hermitian conjugate $J^{\overline{B}}$ are easily deduced from equations (24) using the symmetry relation

$$(a_{jk}^B)^2 = (a_{kj}^{\overline{B}})^2$$

and the result is

$$2\, F_+^{\overline{B}}(\xi) = \sum_{j,k} \left[(a_{jk}^{BL})^2 D_{-j}(\xi) + (a_{jk}^{BR})^2 D_k(\xi) \right]$$

$$2\, F_-^{\overline{B}}(\xi) = \sum_{j,k} \left[(a_{jk}^{BL})^2 D_k(\xi) + (a_{jk}^{BR})^2 D_{-j}(\xi) \right] \tag{26}$$

5) Let us integrate the scaling functions with respect to ξ. Using the definition of the $\langle N_j \rangle$'s previously given we obtain two sum rules already derived from current algebra. Defining

$$2\, F_T = F_- + F_+ \qquad\qquad F_3 = F_- - F_+$$

we get

- a - the Adler sum rule for the weak charged current

$$ {}_0\int^1 2\left[F_T^{\overline{W}}(\xi) - F_T^{W}(\xi)\right]d\xi = $$

$$ = \frac{1}{2}\sum_{j,k}\left[(a_{jk}^{WL})^2 + (a_{jk}^{WR})^2\right]\left[\langle N_k - N_{-k}\rangle - \langle N_j - N_{-j}\rangle\right] \tag{27} $$

- b - The Gross-Llewellyn Smith sum rules for the weak currents

$$ {}_0\int^1 \left[F_3^{W}(\xi) + F_3^{\overline{W}}(\xi)\right] d\xi = $$

$$ = \frac{1}{2}\sum_{j,k}\left[(a_{jk}^{WL})^2 - (a_{jk}^{WR})^2\right]\left[\langle N_k - N_{-k}\rangle + \langle N_j - N_{-j}\rangle\right] \tag{28} $$

$$ \int_0^1 F_3^{Z}(\xi)\,d\xi = \frac{1}{2}\sum_{j}\left[(a_j^{ZL})^2 - (a_j^{ZR})^2\right]\langle N_j - N_{-j}\rangle \tag{29} $$

The differences $\langle N_j - N_{-j}\rangle$ are linear combinations of conserved charges like the baryonic charge, the electric charge, the hypercharge, etc... These combinations and therefore the right-hand sides of the sum rules depend on the algebra of the quark model.

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PART B QUARK PARTON MODEL

I EXPERIMENTAL DATA ON ELECTROPRODUCTION

1) A systematic study of the electron deep inelastic scattering on hydrogen and deuterium is made at SLAC and DESY. The region of the q^2, W^2 plane where measurements have been performed is represented in Fig. 4. The two electroproduction structure functions on protons have been separated in the shaded region of Fig. 4 where data at three or more angles are available.

The results are generally presented in terms of the two quantities

$$F_2(q^2,\xi) = \frac{1}{\pi}\frac{\nu q^2}{\sqrt{\nu^2+q^2}}\left[\sigma_T(q^2,W^2)+\sigma_L(q^2,W^2)\right]$$

$$R(q^2,\xi) = \frac{\sigma_L(q^2,W^2)}{\sigma_T(q^2,W^2)}$$

If scaling holds

$$F_2(q^2,\xi) \longrightarrow F_2(\xi) = 2\xi\left[F_T(\xi)+F_L(\xi)\right]$$

$$R(q^2,\xi) \longrightarrow F_L(\xi)/F_T(\xi) \qquad (30)$$

2) The function $F_2^{ep}(q^2,\xi)$ for a proton target has been plotted in Fig. 5 versus ξ as usual. The results are compatible with a unique curve $F_2^{ep}(\xi)$ as suggested by the Bjorken scaling law (30) for values of q^2 larger than 1 GeV^2 and of W larger than 2.6 GeV. A typical example of scaling at $\xi = 0.25$ is shown in Fig. 6.

The ratio R^p is always smaller than 0.4 and its determination is considerably less accurate than that of $F_2{}^{ep}$. Various forms have been proposed for R^p and two possible fits having reasonable χ^2 values are

a - R constant with $R^p = 0.168 \pm 0.014$
b - $R^p = c\,\frac{M^2}{q^2}$ with $c = 0.35 \pm 0.05$

More sophisticated expressions between the forms a- and b- will obviously fit the data but a scaling of the quantity $\frac{\nu}{M} R^p$ as predicted by the U(3) quark model light cone algebra is certainly consistent with experiment.

Let us notice that a fit with a form $R^p = a\,\frac{q^2}{M^2}$ has a larger χ^2 value than fits of type a- and b-.

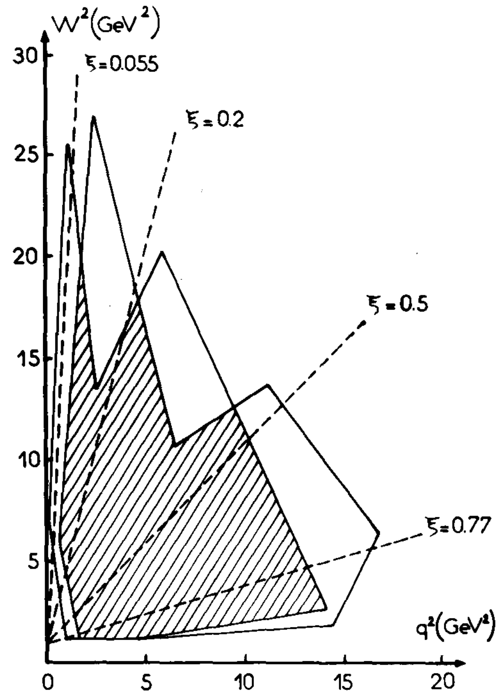

Fig. 4
Region of the q^2, W^2 plane studied at SLAC.

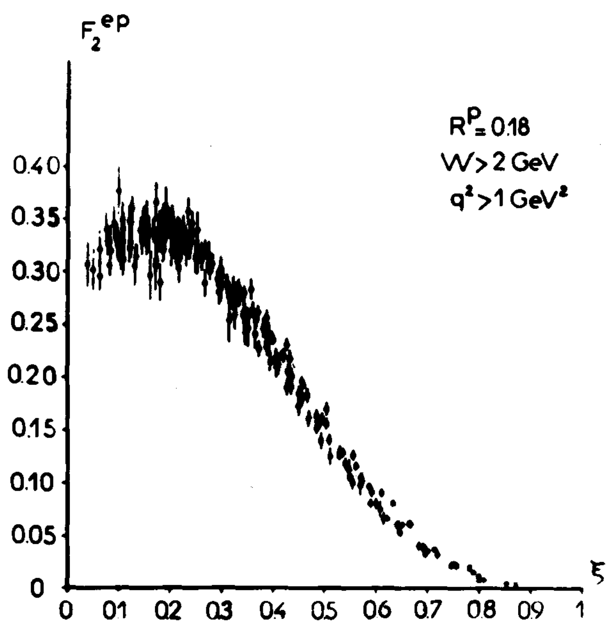

Fig. 5
$F_2^{ep}(\xi,q^2)$ versus ξ.

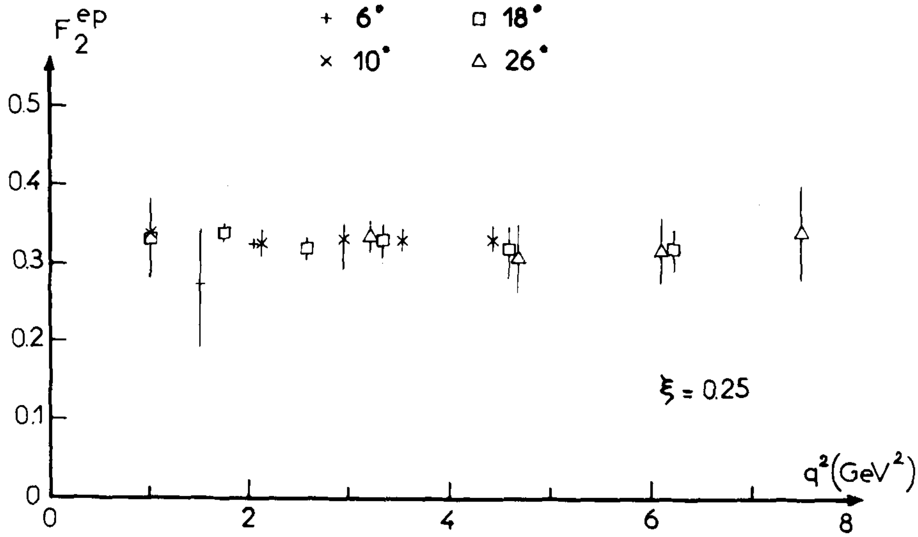

Fig. 6 $F_2^{ep}(\xi, q^2)$ versus q^2; $\xi = 0.25$

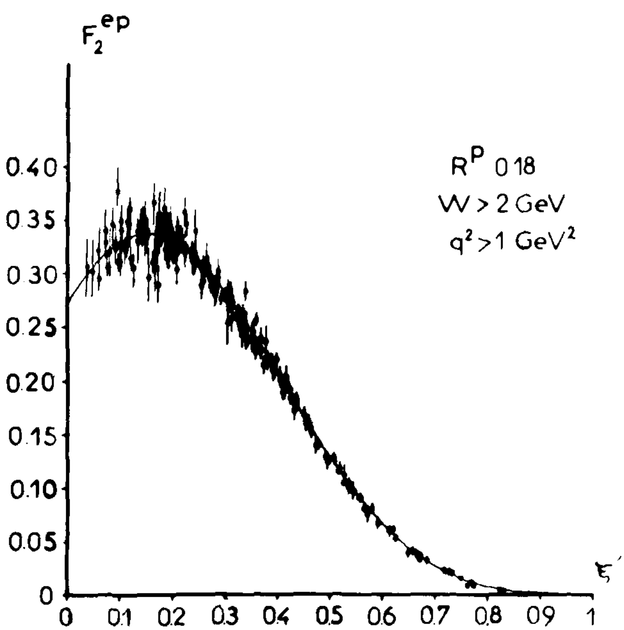

Fig. 7

$F_2^{ep}(\xi', q^2)$ versus ξ'

3) Other scaling variables have been proposed in order to extend the region of the q^2, W^2 plane where the experimental data scale. Two well-known examples are the Bloom-Gilman variable

$$\xi' = \frac{q^2}{q^2 + W^2} \qquad \text{or} \qquad \omega' = 1 + \frac{W^2}{q^2} = \frac{1}{\xi} + \frac{M^2}{q^2}$$

and the Rittenberg-Rubinstein variable

$$\omega_W = \frac{2M\nu + M^2}{q^2 + a^2} = \frac{W^2 + q^2}{a^2 + q^2}$$

Fig. 7 represents a plot of F_2^{ep} versus ξ'. The dispersion in q^2 is somewhat less important than in Fig. 5.
Let us remark on the other hand that for resonances the quasi-elastic form factors exhibit similar shapes when plotted in the variable q^2/W^2. Therefore the variable ξ' has the advantage of nicely averaging the resonance contributions in a local way.

4) Experiments performed with a deuterium target have the same features: scaling for F_2^{ed} and smallness for R^d. A plot of F_2^{ed} versus ξ' is given in Fig. 8 and the shape of F_2^{ed} looks similar to that of F_2^{ep}. Moreover within errors $R^d = R^p$ as shown in Fig. 9.
After application of deuteron nuclear physics corrections due to the Fermi motion the neutron scaling functions are extracted by difference. But these corrections, very small for $\xi' < 0.65$, become more and more important when ξ' increase and also the uncertainty on these corrections and on the neutron scaling function F_2^{en}.

II EXPERIMENTAL DATA ON WEAK PROCESSES WITH CHARGED CURRENTS

1) The inclusive neutrino and antineutrino experiments with production of a charged final lepton or antilepton cannot be used to obtain an individual information about the structure functions. Most features observed in electroproduction like the scaling and the smallness of the longitudinal contribution in the deep inelastic region have not been directly checked but rather assumed in the analysis of data.

The experimental results are compatible with a linear rising with energy of the total cross sections and the ratio of antineutrino

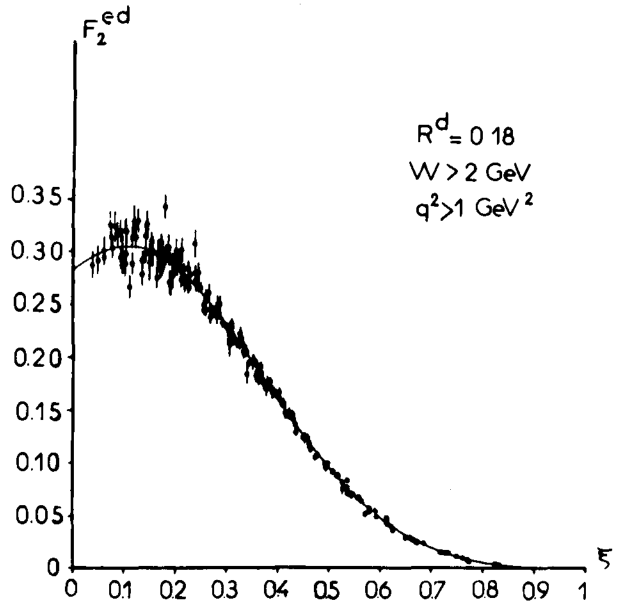

Fig. 8

$F_2^{ed}(\xi', q^2)$ versus ξ'.

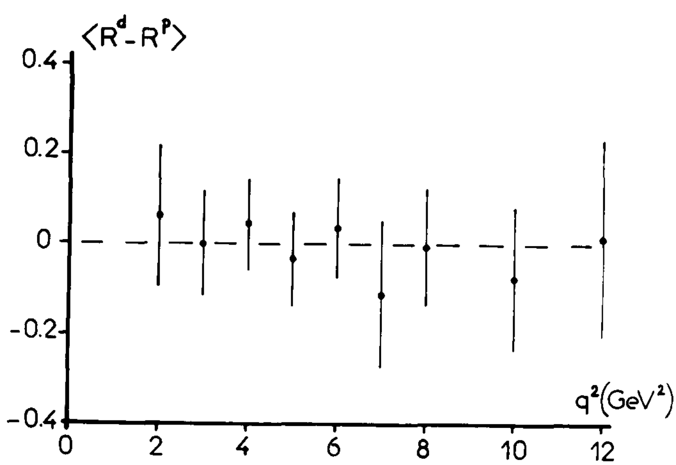

Fig. 9

$R^d - R^p$.

to neutrino total cross section is consistent with a constant value. The data is shown in Table 2 where the total cross sections are written

$$\sigma_{TOT}^{\nu,\bar{\nu}} = \alpha_{\nu,\bar{\nu}} E$$

with α given in units $10^{-38}cm^2$ GeV^{-1} per nucleon.

	Beam	α	$\sigma^{\bar{\nu}}$ σ^{ν}	Energy Range
CERN Propane	ν_μ	0.8 ± 0.2		1-12 GeV
CERN Gargamelle FREON	ν_μ	0.76 ± 0.08	0.38 ± 0.02	1-10 GeV
	$\bar{\nu}_\mu$	0.28 ± 0.03		
	ν_e	0.93 ± 0.17	0.40 ± 0.12	1 GeV
	$\bar{\nu}_e$	0.37 ± 0.09		
NAL Caltech IRON	ν_μ	0.83 ± 0.11	0.33 ± 0.08	40 GeV
	$\bar{\nu}_\mu$	0.28 ± 0.05		110 GeV
NAL Harvard Pennsylvania Wisconsin	ν_μ	0.70 ± 0.18	0.41 ± 0.11	10-200 GeV
	$\bar{\nu}_\mu$	0.28 ± 0.09		
AVERAGE		0.78 ± 0.07		
		0.28 ± 0.025		

Table 2

The CERN Gargamelle data is shown in Figs. 10 and 11. The values of q^2 and W^2 brought into play are low and a large part of the experimental points in the q^2 - W^2 plane lies outside the scaling region obtained at SLAC $q^2 \gtrsim 1$ GeV^2, $W \gtrsim 2.6$ GeV. Nevertheless linearity can be achieved for the total cross sections with reasonable χ^2 values.

Finite energy corrections are certainly important and they should be taken into account in a more refined analysis. Unfortunately these corrections depend on the choice of the scaling function and of the scaling variable and they cannot be uniquely predicted.

The N.A.L. high energy data for the neutrino total cross section

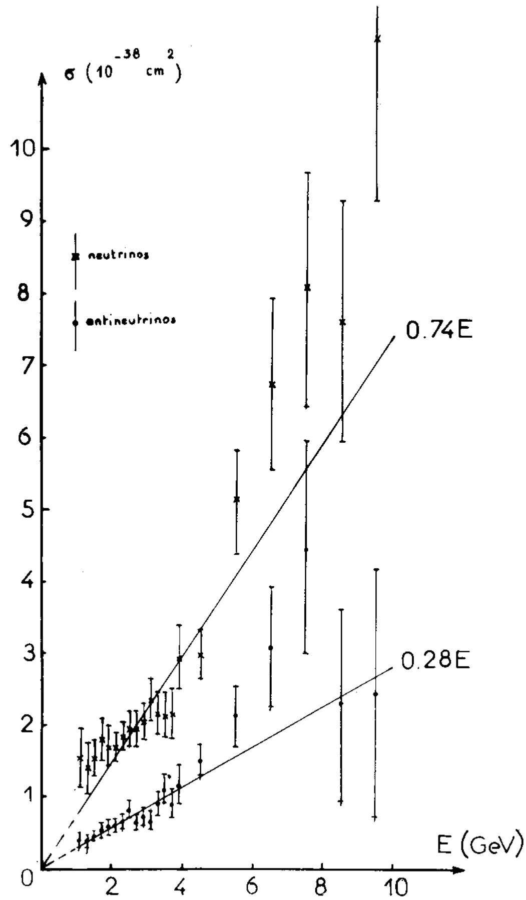

Fig. 10

Neutrino and antineutrino total cross sections versus E (Gargamelle).

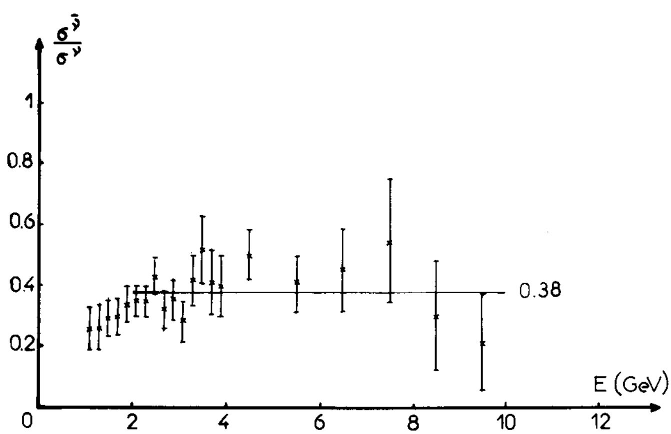

Fig. 11

Ratio of antineutrino to neutrino total cross sections versus E (Gargamelle).

and the antineutrino to neutrino ratio of total cross sections is consistent with the low energy one as shown in Fig. 12. In this case

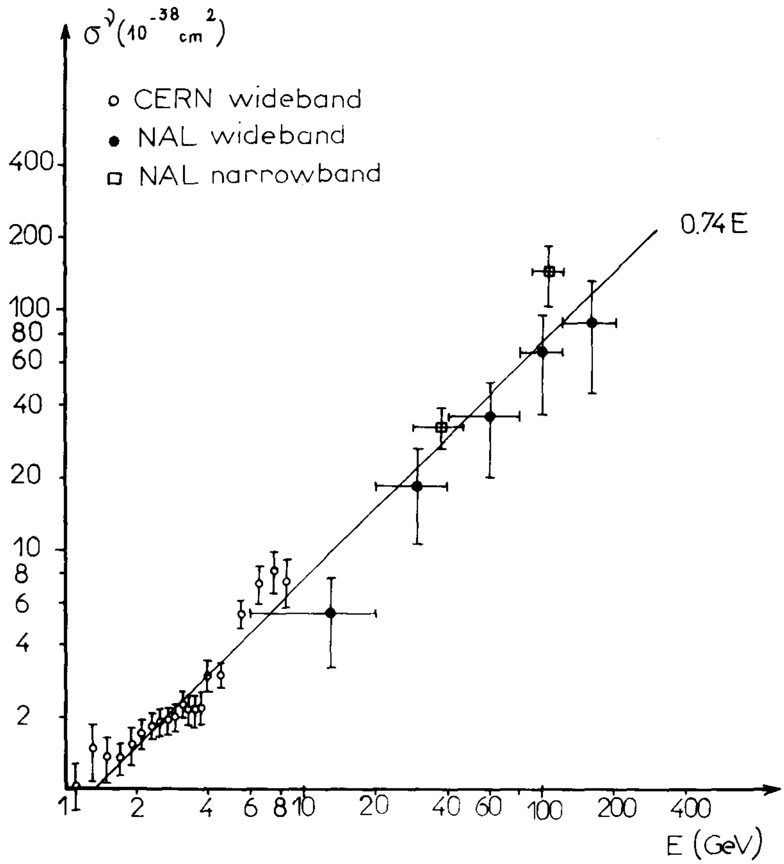

Fig. 12

Neutrino total cross section versus E (Gargamelle and NAL)

finite energy corrections are negligible but another type of phenomena may distort the linearity: the non locality due to an intermediate vector boson.

This effect cannot be measured with the present accuracy of the data and only a lower limit around 12 GeV can be put on the W meson mass.

2) We shall analyse the CERN-Gargamelle data in the units of G^2ME/π and the experimental value for the coefficients A^{ν} and $A^{\bar{\nu}}$ is

$$A^{\nu} = 0.483 \pm 0.051 \qquad A^{\bar{\nu}} = 0.178 \pm 0.019$$

In order to obtain the same quantities for an isoscalar target we must correct for the unequal number of protons and neutrons in the CF_3Br chamber where $N_n/N_p \simeq 1.19$. Anticipating the results of the quark parton model analyses we use

$$A^{\nu n}/A^{\nu p} \simeq 1.8 \qquad A^{\bar{\nu}p}/A^{\bar{\nu}n} \simeq 2$$

and we get

$$A^{\nu N} \simeq 0.977 \ A^{\nu} \simeq 0.471 \pm 0.050$$
$$A^{\bar{\nu}N} \simeq 1.03 \ A^{\bar{\nu}} \simeq 0.183 \pm 0.020 \qquad (31)$$

3) Other quantities have been measured in these experiments:

a- energy distributions of the final lepton or antilepton
b- averaged value of the final lepton or antilepton energy
c- averaged value of q^2
d- fixed ξ differential cross sections.

The points a-, b-, and d- will be studied in section V from both experimental and theoretical points of view.
For the averaged values of q^2 the CERN Gargamelle data has been fitted with a two parameter linear function of energy and the result for events with E>2 GeV is

$$< q^2 >_{\nu} = (0.21 \pm 0.02)E + (0.22 \pm 0.06)$$
$$< q^2 >_{\bar{\nu}} = (0.14 \pm 0.03)E + (0.11 \pm 0.08)$$

The data is presented in Figs. 13 and 14. Let us remark that the dimensionless quantity $v = q^2/2ME$ is known from the final lepton or antilepton parameters

$$V = \frac{2E' \sin^2 \frac{\theta}{2}}{M}$$

and therefore is independent of the incident spectrum. Theoretically the average values of q^2 involve the second moment of the scaling functions.

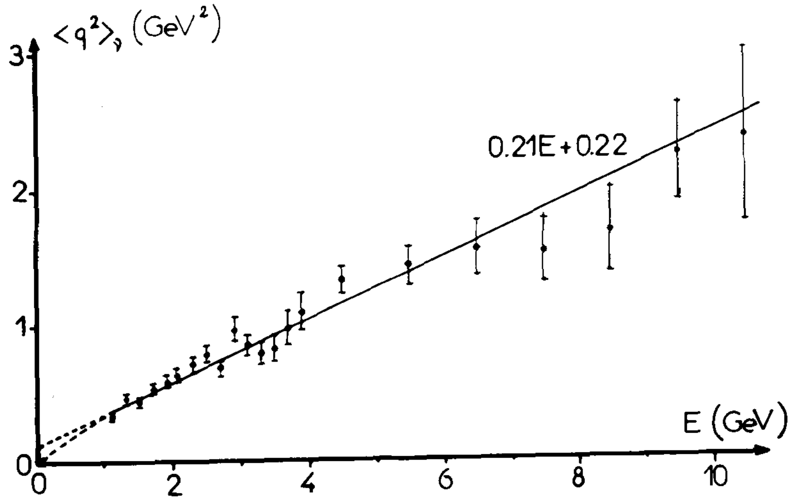

Fig. 13

Averaged q^2 for neutrinos versus E (Gargamelle).

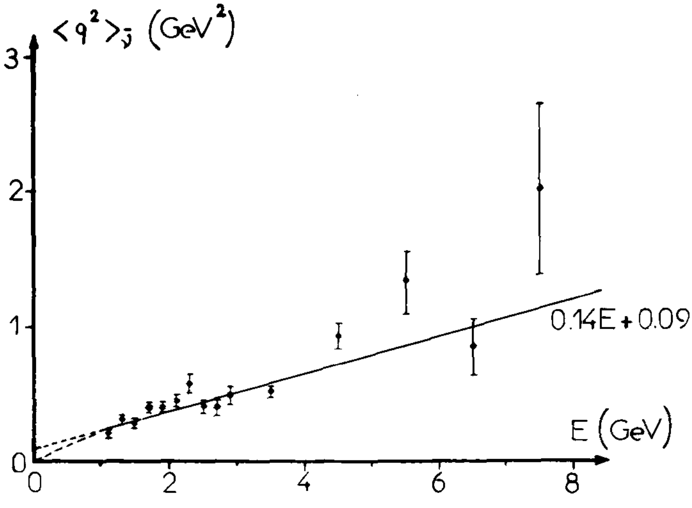

Fig. 14

Averaged q^2 for antineutrinos versus E (Gargamelle).

III EXPERIMENTAL DATA ON WEAK PROCESSES WITH NEUTRAL CURRENTS

1) The systematic research of neutral currents has been carried out at CERN with neutrino and antineutrino beam entering the Gargamelle chamber. The neutrino and antineutrino can scatter either off atomic electrons or off nucleons. Neutral currents have been studied in both cases and positive results found.

Here, we only discuss the hadronic case where the main characteristic of neutral current events is the absence in the final state of a chargedlepton or antilepton trace. Such events have been observed and after a careful study of possible background sources they have been attributed for a large part to neutral currents.
The actual results for relative rates of neutral current events to charged currents events for interaction with hadron energy release larger than 1 GeV are as follows

$$\left(\frac{NC}{CC}\right)_{\nu} = 0.217 \pm 0.026 \tag{32}$$

$$\left(\frac{NC}{CC}\right)_{\bar{\nu}} = 0.43 \pm .12 \tag{33}$$

We notice that these quantities refer to numbers of events with identical cuts and not to total cross sections. In the same situation the ratio of antineutrino to neutrino for charged currents has been found to be

$$\frac{(CC)_{\bar{\nu}}}{(CC)_{\nu}} = 0.26 \pm 0.03 \tag{34}$$

2) Two different counter experiments have been performed at NAL by

- the Harvard-Pennsylvania-Wisconsin group (HPW)
- the Caltech group.

Data obtained by the HPW group has been presented in successive fluctuating steps but positive evidence for hadronic neutral currents is now claimed. The results are given in terms of two ratios R^{ν} and $R^{\bar{\nu}}$, comparing neutral current events with charged current ones. The mean incident energy is 50 GeV and only events carrying a total hadronic energy larger than 4 GeV have been retained. The average of successive experiments gives

$$R^{\nu} = 0.11 \pm 0.05$$

$$R^{\bar{\nu}} = 0.32 \pm 0.09$$

Another positive evidence is also claimed by the Caltech group with the following numerical estimates

$$R^{\nu} = 0.22$$
$$R^{\bar{\nu}} = 0.33$$

but the errors have not yet been computed.

3) It is clear that the results that come from the three experiments are consistent with each other. But we must keep in mind that there are no ratios of total cross sections but only of the number of events restricted with different cuts.

IV U(3) SYMMETRY GROUP

1) In the quark parton model based on SU(3) symmetry the interacting partons are 3 quarks and 3 antiquarks whose quantum numbers are given in Table 3

	p	n	λ
j	1	2	3
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
Y	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
B	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Table 3

The model is described by six distribution functions $D_j(\xi)$ and $D_{-j}(\xi)$ with $j = 1, 2, 3$ which are positive functions of ξ in the physical range $0 \leq \xi \leq 1$.

The conservation of the baryonic charge B, the electric charge Q and the hypercharge Y implies constraints on the mean values of quarks and antiquarks. From Table 3 we have

$$\langle N_1 - N_{-1} \rangle = B + Q \qquad \langle N_2 - N_{-2} \rangle = B + Y - Q \tag{35}$$
$$\langle N_3 - N_{-3} \rangle = B - Y$$

and for a given hadron only the mean numbers of antiquarks are free parameters.
The electromagnetic scaling function is immediately computed from Table 3:

$$2\, F_T^Q(\xi) = \frac{4}{9}\left[\mathcal{D}_1(\xi) + \mathcal{D}_{-1}(\xi)\right] + \frac{1}{9}\left[\mathcal{D}_2(\xi) + \mathcal{D}_{-2}(\xi) + \mathcal{D}_3(\xi) + \mathcal{D}_{-3}(\xi)\right] \quad (36)$$

2) The weak charged current is the Cabibbo current which in the quark language is simply written as

$$J_\mu^W = i\,\bar{p}\,\gamma_\mu(1+\gamma_5)\left[n\cos\theta_c + \lambda\sin\theta_c\right] \quad (37)$$

where θ_c is the Cabibbo angle.
The weak charges are purely left handed and they are given from equation (37) by

$$a_{21}^{WL} = 2\cos\theta_c \qquad\qquad a_{31}^{WL} = 2\sin\theta_c \quad (38)$$

It is convenient to separate in the scaling functions the contributions coming from the $\Delta Y=0$ and $\Delta Y=\pm 1$ transitions

$$F_\lambda = \cos^2\theta_c \; G_\lambda + \sin^2\theta_c \; H_\lambda$$

and the result is the following

$$G_+^W(\xi) = 2\,\mathcal{D}_{-1}(\xi) \qquad\qquad H_+^W(\xi) = 2\,\mathcal{D}_{-1}(\xi)$$

$$G_-^W(\xi) = 2\,\mathcal{D}_2(\xi) \qquad\qquad H_-^W(\xi) = 2\,\mathcal{D}_3(\xi)$$

$$G_+^{\overline{W}}(\xi) = 2\,\mathcal{D}_{-2}(\xi) \qquad\qquad H_+^{\overline{W}}(\xi) = 2\,\mathcal{D}_{-3}(\xi)$$

$$G_-^{\overline{W}}(\xi) = 2\,\mathcal{D}_1(\xi) \qquad\qquad H_-^{\overline{W}}(\xi) = 2\,\mathcal{D}_1(\xi)$$

We easily see that the splitting in states of definite helicity allows to isolate each quark and antiquark distribution function.
From electroproduction, neutrino and antineutrino processes on a given target one can measure nine structure functions. The number of different types of quarks and antiquarks being six in this model we have at our

disposal only six distribution functions $D_j(\xi)$ so that the SU(3) quark parton model predicts three relations that one can write as

$$H_+^W(\xi) = G_+^W(\xi) \qquad\qquad H_-^{\overline{W}}(\xi) = G_-^{\overline{W}}(\xi) \tag{40}$$

$$F_T^Q(\xi) = \frac{1}{9}\left[G_+^W(\xi) + G_-^{\overline{W}}(\xi)\right] + \frac{1}{36}\left[G_-^W(\xi) + G_+^{\overline{W}}(\xi) + H_-^W(\xi) + H_+^{\overline{W}}(\xi)\right]$$

The relations between scaling functions are strict tests of the quark parton model.

The Adler and Gross-Llewellyn Smith sum rules (27) and (28) are obtained using equations (35) and (38)

$$\int_0^1 \left[F_T^{\overline{W}}(\xi) - F_T^W(\xi)\right] d\xi = (2B - Y)\cos^2\theta_c + (Q + Y)\sin^2\theta_c$$

$$\frac{1}{2}\int_0^1 \left[F_3^W(\xi) + F_3^{\overline{W}}(\xi)\right] d\xi = (2B + Y)\cos^2\theta_c + (2B - Y + Q)\sin^2\theta_c$$

In this quark parton model based on SU(3) $\otimes$ SU(3) algebra the sum rules have their original form.

3) When the target is a nucleon, charge symmetry relates the neutron and proton distributions

$$D_{\pm 1}^n(\xi) = D_{\pm 2}^p(\xi) \qquad D_{\pm 2}^n(\xi) = D_{\pm 1}^p(\xi) \qquad D_{\pm 3}^n(\xi) = D_{\pm 3}^p(\xi)$$

Only proton distributions will be used in what follows.
All the scaling functions on a neutron target are known from the scaling functions on a proton target. As an illustration let us give some examples of such relations: for neutrinos, antineutrinos

$$\begin{aligned} G_\pm^{\nu n}(\xi) &= G_\pm^{\bar\nu p}(\xi) \qquad & G_\pm^{\bar\nu n}(\xi) &= G_\pm^{\nu p}(\xi) \\ H_-^{\nu n}(\xi) &= H_-^{\nu p}(\xi) \qquad & H_+^{\bar\nu n}(\xi) &= H_+^{\nu p}(\xi) \end{aligned} \tag{41}$$

and between electromagnetic and weak scaling functions

$$F_T^{ep}(\xi) - F_T^{en}(\xi) = \frac{1}{12}\left[G_+^{\nu p}(\xi) - G_-^{\nu p}(\xi) - G_+^{\nu n}(\xi) + G_-^{\nu n}(\xi)\right] \tag{42}$$

The situation is particularly simple for an isoscalar target N averaged over proton and neutron N = $\frac{p+n}{2}$. From equations (40) and (41) we get

$$G_{\pm}^{\nu N} = G_{\pm}^{\bar{\nu} N} = G_{\pm}^{N}$$
$$F_T^{eN} = \frac{5}{18} G_T^N + \frac{1}{36}\left[H_-^{\nu N} + H_+^{\bar{\nu} N}\right] \tag{43}$$

The equalities (42) and (43) are unambigous consequences of the quark parton model and positivity implies the simple inequality due to Llewellyn Smith

$$F_T^{eN} \geq \frac{5}{18} G_T^N \tag{44}$$

V QUARK PARTON MODEL FOR ELECTROPRODUCTION

1) The proton and neutron scaling functions are given by

$$2F_T^{ep}(\xi) = \frac{4}{9}\left[D_1 + D_{-1}\right] + \frac{1}{9}\left[D_2 + D_{-2}\right] + \frac{1}{9}\left[D_3 + D_{-3}\right]$$
$$2F_T^{en}(\xi) = \frac{1}{9}\left[D_1 + D_{-1}\right] + \frac{4}{9}\left[D_2 + D_{-2}\right] + \frac{1}{9}\left[D_3 + D_{-3}\right] \tag{45}$$

As a consequence of charge symmetry and of the positivity of the D_j's we obtain the inequalities

$$\frac{1}{4} \leq \frac{F_T^{en}(\xi)}{F_T^{ep}(\xi)} \leq 4 \tag{46}$$

or in terms of structure functions in the scaling region

$$\frac{1}{4} \leq \frac{\sigma^{en}(q^2, W^2)}{\sigma^{ep}(q^2, W^2)} \leq 4 \tag{47}$$

The experimental data shown in Fig. 15 are consistent with the theoretical bounds. Let us notice that the lower bound of 1/4 may eventually be reached at ξ = 1.

2) We now integrate the scaling function over ξ assuming these integrals to be convergent

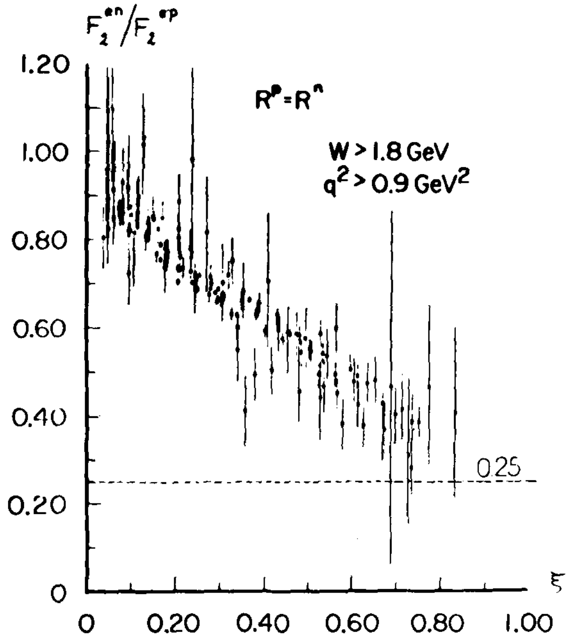

Fig. 15

σ_n/σ_p versus ξ'.

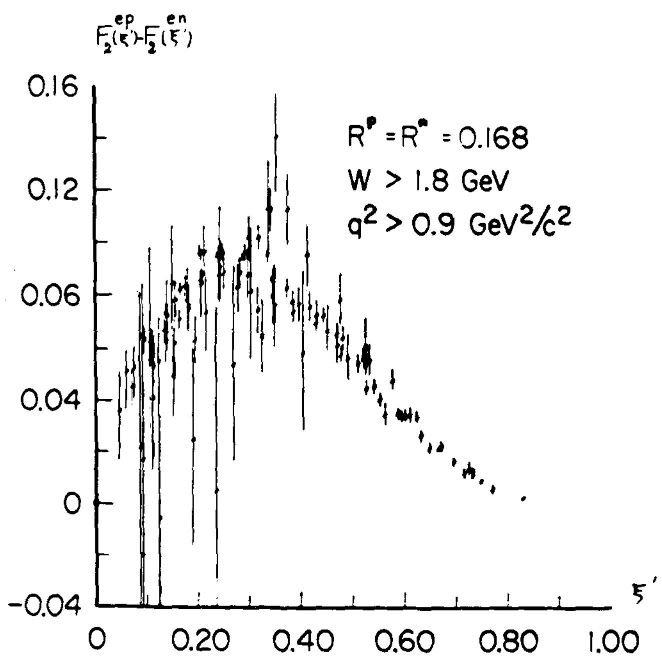

Fig. 16

$F_2^{ep} - F_2^{en}$ versus ξ'.

$$K^{e} = \int_0^1 2\, F_T^Q(\xi)\, d\xi$$

Using the equations (45) and the charge conservation relations (35) written for the proton we get

$$K^{ep} = 1 + \frac{8}{9}\langle N_{-1}\rangle + \frac{2}{9}\langle N_{-2} + N_{-3}\rangle$$

$$K^{en} = \frac{2}{3} + \frac{8}{9}\langle N_{-2}\rangle + \frac{2}{9}\langle N_{-1} + N_{-3}\rangle$$

The positivity of the mean number of antiquarks implies lower bounds for K^{ep} and K^{en}

$$K^{ep} \geq 1 \qquad\qquad K^{en} \geq \frac{2}{3} \tag{48}$$

The experimental situation is not very accurate. In fact the integrals K^{ep} and K^{en} look very dependent on the limits of integration. The most recent evaluation is

$$K^{ep} \simeq 0.81 \pm 0.04$$

$$K^{en} \simeq 0.65 \pm 0.03$$

the lower limit of integration being $\xi_m = 0.04$.
Obviously the lower bounds (48) are not violated. Unfortunately our information is unsufficient to decide whether the integrals K^e are convergent or not, or, in the parton language, whether the averaged number of partons is finite or not. The behaviour of the scaling function near $\xi = 0$ is obviously crucial to answer that question.
An interesting quantity expected to be convergent is the difference $K^{ep}-K^{en}$

$$K^{ep} - K^{en} = \frac{1}{3} + \frac{2}{3}\langle N_{-1} - N_{-2}\rangle$$

The structure function difference has been plotted in Fig. 16 and the experimental evaluation of $K^{ep}-K^{en}$ with $\xi_m = 0.05$ is

$$K^{ep}-K^{en} \simeq 0.18 \pm 0.04$$

Data below $\xi = 0.05$ are certainly crucial in evaluating this difference. In particular, the Gottfried sum rule which holds in parton models where $\langle N_{-1}\rangle = \langle N_{-2}\rangle$ predicts 1/3 for that difference and cannot be ruled out from experiment.

3) Let us now study the first moment of the quark and antiquark distributions

$$d_j = \int_0^1 \xi\, D_j(\xi)\, d\xi$$

These quantities are positive $d_j \geq 0$ and using energy momentum conservation we obtain

$$\sum_j \left(d_j + d_{-j}\right) = 1 - \varepsilon$$

where the parameter ε measures, in an averaged sense, the amount of gluons in the hadron

$$0 \leq \varepsilon \leq 1$$

By positivity, a non vanishing value for ε implies the existence of gluons in this model.
The first moment integrals for electroproduction are defined by

$$I^e = \int_0^1 \xi\, 2 F_T^Q(\xi)\, d\xi$$

For proton and neutron, using equations (45) we get

$$I^{ep} = \frac{1}{3}\left(d_1 + d_{-1}\right) + \frac{1}{9}\left(1 - \varepsilon\right)$$

$$I^{en} = \frac{1}{3}\left(d_2 + d_{-2}\right) + \frac{1}{9}\left(1 - \varepsilon\right)$$

Averaging over proton and neutron

$$I^{eN} = \frac{5}{18}\left(1 - \varepsilon\right) - \frac{1}{6}\left(d_3 + d_{-3}\right)$$

we deduce, by positivity, an absolute upper bound for the magnitude of electromagnetic scaling functions

$$I^{eN} \leq \frac{5}{18} \tag{49}$$

When the integral I^{en} is known from experiment the gluon parameter ε is restricted by

$$0 \leq \varepsilon \leq 1 - \frac{18}{5} I^{eN} \tag{50}$$

The most recent experimental evaluation of I^{eN} gives

$$I^{eN} \simeq 0.15 \pm 0.01 \tag{51}$$

The absolute bound (49) is satisfied and the limits for ε are

$$0 \leq \varepsilon \leq 0.46 \pm 0.04 \tag{52}$$

The proton-neutron difference is known with a poor accuracy and the result is

$$I^{ep} - I^{en} = 0.04 \pm 0.02 \tag{53}$$

VI QUARK PARTON MODEL FOR WEAK PROCESSES WITH CHARGED CURRENTS

A detailed analysis of weak processes can be done with the quark parton model starting from the set of expressions (39) for the scaling functions. Unfortunately our experimental information being extremely limited we had better concentrate over specific points where experimental data are available.

1) The first of these points is the study of total cross sections. The interesting quantities are the constants A^{ν} and $A^{\bar{\nu}}$ which govern the linear rising of the total cross sections in the local Fermi interaction. Using the Cabibbo current the separation between strangeness conserving and strangeness changing transitions is achieved by putting

$$A^{\nu,\bar{\nu}} = \cos^2\theta_C \, B^{\nu,\bar{\nu}} + \sin^2\theta_C \, C^{\nu,\bar{\nu}}$$

From equation (20) these constants involve the first moments of the quark and antiquark distributions. The result is

$$\begin{aligned}
B^{\nu p} &= \tfrac{2}{3} d_{-1} + 2 d_2 & \qquad C^{\nu p} &= \tfrac{2}{3} d_{-1} + 2 d_3 \\
B^{\nu n} &= \tfrac{2}{3} d_{-2} + 2 d_1 & \qquad C^{\nu n} &= \tfrac{2}{3} d_{-2} + 2 d_3 \\
B^{\bar{\nu} p} &= \tfrac{2}{3} d_1 + 2 d_{-2} & \qquad C^{\bar{\nu} p} &= \tfrac{2}{3} d_1 + 2 d_{-3} \\
B^{\bar{\nu} n} &= \tfrac{2}{3} d_2 + 2 d_{-1} & \qquad C^{\bar{\nu} n} &= \tfrac{2}{3} d_2 + 2 d_{-3}
\end{aligned} \tag{54}$$

In order to compare these expressions with the Gargamelle results we first average over proton and neutron

$$B^{\nu N} = \frac{1}{3}\left(d_{-1}+d_{-2}\right)+\left(d_1+d_2\right) \qquad C^{\nu N} = \frac{1}{3}\left(d_{-1}+d_{-2}\right)+2d_3$$

$$B^{\bar{\nu} N} = \frac{1}{3}\left(d_1+d_2\right)+\left(d_{-1}+d_{-2}\right) \qquad C^{\bar{\nu} N} = \frac{1}{3}\left(d_1+d_2\right)+2d_{-3} \tag{55}$$

Let us recall that the quark parton model for the electroproduction integral I^{eN} is

$$I^{eN} = \frac{5}{18}\left(d_1+d_2+d_{-1}+d_{-2}\right)+\frac{1}{9}\left(d_3+d_{-3}\right)$$

We solve these linear expressions and we get the theoretical expressions

$$d_1+d_2+d_{-1}+d_{-2} = \frac{3}{4\left[1-\frac{9}{2}\sin^2\theta_c\right]}\left[A^{\nu N}+A^{\bar{\nu}N}-18\sin^2\theta_c\, I^{eN}\right] \tag{56}$$

$$d_3+d_{-3} = \frac{15}{8\left[1-\frac{9}{2}\sin^2\theta_c\right]}\left[\frac{24}{5}\left(1-\frac{3}{4}\sin^2\theta_c\right)I^{eN}-\left(A^{\nu N}+A^{\bar{\nu}N}\right)\right] \tag{57}$$

and the numerical results with $\sin\theta_c = 0.23$ are

$$d_1+d_2+d_{-1}+d_{-2} = 0.505 \pm 0.054 \tag{58}$$

$$d_3+d_{-3} = 0.091 \pm 0.176 \tag{59}$$

For the gluon parameter ε the explicit expression is

$$\varepsilon = 1 - \frac{9}{8\left[1-\frac{9}{2}\sin^2\theta_c\right]}\left[8\left(1-\frac{9}{4}\sin^2\theta_c\right)I^{eN}-\left(A^{\nu N}+A^{\bar{\nu}N}\right)\right] \tag{60}$$

and from experiment we obtain

$$\varepsilon = 0.40 \pm 0.13 \tag{61}$$

The quark parton model is consistent with electroproduction, neutrino and antineutrino data if and only if gluons are present.
It is now possible to have a first estimate of $\Delta S = 0$ and $\Delta S = \pm 1$ contributions using the expressions (54)

$$\frac{C^{\nu N}+C^{\bar{\nu}N}}{B^{\nu N}+B^{\bar{\nu}N}} = \frac{1}{4}+\frac{3}{2}\,\frac{d_3+d_{-3}}{d_1+d_2+d_{-1}+d_{-2}} \geq \frac{1}{4} \tag{62}$$

Taking into account the Cabibbo angle we get for total cross sections

$$\frac{(\sigma^{\nu N}+\sigma^{\bar{\nu} N})(|\Delta S|=1)}{(\sigma^{\nu N}+\sigma^{\bar{\nu} N})(\Delta S=0)} = \left(3 \begin{array}{l}+2\\-1.4\end{array}\right)\% \qquad (63)$$

We now try to use the experimental information on the difference of neutrino and antineutrino total cross sections which in the quark parton model is written as

$$A^{\nu N}-A^{\bar{\nu} N} = \frac{2}{3}\left(1-\frac{3}{2}\sin^2\theta_c\right)\left[d_1+d_2-d_{-1}-d_{-2}\right]+2\sin^2\theta_c\left(d_3-d_{-3}\right) \qquad (64)$$

Taking into account the smallness of $\sin^2 \theta_c$ and of the strange quark and antiquark moments from (59) we assume that the second term in the right-hand side of equation (64) is negligible as compared to the first one. We are now in position to separate the non strange quarks and non strange antiquarks contributions. The result is

$$d_1+d_2 = 0.487 \pm 0.052 \qquad (65)$$

$$d_{-1}+d_{-2} = 0.018 \pm 0.052 \qquad (66)$$

It clearly shows that the quark contributions dominate strongly over the antiquark ones and this feature will simplify the description of the nucleon in terms of quarks and antiquarks.
Let us emphasize that a ratio of antineutrino to neutrino cross-sections for the $\Delta S = 0$ part of 1/3 would imply the absence of antiquarks in the nucleon. The experimental value of that ratio is close to 1/3: 0.38 ± 0.02 and we immediately recover the previous result.
It is possible to compute the $\Delta S = 0$ part of the total cross section and to compare this contribution with the experimental value for all the events

$$B^{\nu N}\cos^2\theta_c = 0.467 \pm 0.051 \qquad A^{\nu N}_{exp} = 0.471 \pm 0.050$$

$$B^{\bar{\nu} N}\cos^2\theta_c = 0.170 \pm 0.051 \qquad A^{\bar{\nu} N}_{exp} = 0.183 \pm 0.020$$

By comparing these numbers we expect a very small $|\Delta S| = 1$ cross section induced by neutrinos and a measurable $|\Delta S| = 1$ cross section induced by antineutrinos

$$C^{\nu N} \ll B^{\nu N} \qquad\qquad C^{\bar{\nu} N} \simeq B^{\bar{\nu} N}$$

but a quantitative prediction is not possible because of the large experimental errors beside the antineutrino lower bound

$$C^{\bar{\nu}N} > \frac{1}{3}\left(d_1 + d_2\right)$$

which gives from equation (65)

$$C^{\bar{\nu}N} > 0.162 \pm 0.017$$

2) The proton and neutron total cross sections have not been separated in the CERN-Gargamelle experiment. Nevertheless in specific quark parton models and using the electroproduction result for $I^{ep}-I^{en}$ it is possible to make predictions. We shall give here two examples.

In the equipartition quark parton model the charge conservation relations (35) are assumed to be satisfied also by the first moment integrals and we have for a proton target

$$d_1 - d_{-1} = 2\left\langle \frac{1}{N}\right\rangle \qquad d_2 - d_{-2} = \left\langle \frac{1}{N}\right\rangle \qquad d_3 - d_{-3} = 0$$

where the parameter $<\frac{1}{N}>$ is interpreted as the averaged inverse number of partons in the nucleon. From equations (65) and (66) we compute a large value for that quantity

$$< \frac{1}{N} > \; = \; 0.156 \pm 0.029 \qquad (67)$$

so that the first moments of the scaling functions can be described with a small number of partons. We obtain the predictions

$$\begin{aligned} A^{\nu p} &= 0.334 \pm 0.060 \qquad & A^{\bar{\nu}p} &= 0.246 \pm 0.032 \\ A^{\nu n} &= 0.607 \pm 0.060 \qquad & A^{\bar{\nu}n} &= 0.120 \pm 0.032 \end{aligned} \qquad (68)$$

and for the neutron to proton ratios

$$\frac{A^{\nu n}}{A^{\nu p}} \simeq 1.8 \pm 0.3 \qquad\qquad \frac{A^{\bar{\nu}p}}{A^{\bar{\nu}n}} \quad 2 \pm 0.4$$

In the two component quark parton model we have also 4 independent parameters:

2 for the diffractive part represented by non strange valence quarks;

2 for the diffractive part associated to isoscalar quark-antiquarks seas.

The first moments are then written as

$$d_1 = V_1 + p \qquad d_2 = V_2 + p \qquad d_3 = q$$

$$d_{-1} = p \qquad d_{-2} = p \qquad d_{-3} = q$$

and from equations (65) and (66) we obtain

$$v_1 = 0.294 \pm 0.053 \qquad v_2 = 0.174 \pm 0.053 \tag{69}$$

Let us remark that the ratio v_1/v_2 is compatible with the value of 2 suggested by the naive quark model.
Again the proton and neutron total cross sections are separated

$$\begin{aligned} A^{\nu p} &= 0.357 \pm 0.080 & A^{\bar{\nu} p} &= 0.223 \pm 0.028 \\ A^{\nu n} &= 0.584 \pm 0.080 & A^{\bar{\nu} n} &= 0.143 \pm 0.028 \end{aligned} \tag{70}$$

and for the neutron-proton ratios we predict

$$\frac{A^{\nu n}}{A^{\nu p}} = 1.63 \pm 0.16 \qquad \frac{A^{\bar{\nu} p}}{A^{\bar{\nu} n}} = 1.56 \pm 0.14 \quad .$$

The predictions of these two models are qualitatively the same.
In particular the neutron proton ratio of total cross sections induced by neutrinos is compatible with the value 1.8 ± 0.3 obtained in a previous CERN propane experiment.

3) The second point we wish to discuss here is the energy distribution of the final charged lepton or antilepton. We define a normalized distribution by

$$f(\rho) = \frac{1}{\sigma_{TOT}} \frac{d\sigma}{d\rho}$$

In the scaling region, from equations (19) and (21) they are independent of the incident energy E and given by

$$\begin{aligned} f^{\nu}(\rho) &= \frac{\rho^2 I_+^{\nu} + I_-^{\nu}}{\frac{1}{3} I_+^{\nu} + I_-^{\nu}} \\ f^{\bar{\nu}}(\rho) &= \frac{\rho^2 I_-^{\bar{\nu}} + I_+^{\bar{\nu}}}{\frac{1}{3} I_-^{\bar{\nu}} + I_+^{\bar{\nu}}} \end{aligned} \tag{71}$$

For an isoscalar target the first moment integrals $I_{\pm}$ have the following expressions

$$I_+^{\nu N} = d_{-1} + d_{-2} \qquad I_-^{\nu N} = \cos^2\theta_C\,(d_1+d_2) + 2\sin^2\theta_C\, d_3$$

$$I_+^{\bar\nu N} = \cos^2\theta_C\,(d_{-1}+d_{-2}) + 2\sin^2\theta_C\, d_{-3} \qquad I_-^{\bar\nu N} = d_1 + d_2$$

Using the numerical results of the previous analysis of total cross sections we obtain

$$\begin{aligned} I_+^{\nu N} &= 0.018 \pm 0.052 & I_-^{\nu N} &= 0.465 \pm 0.050 \\ I_+^{\bar\nu N} &= 0.021 \pm 0.050 & I_-^{\bar\nu N} &= 0.487 \pm 0.052 \end{aligned} \qquad (72)$$

Let us recall that in parton models with only left-hand (right-hand) couplings of partons (antipartons) with the weak current, the dominance of the helicity $\lambda = -1$ contribution over the helicity $\lambda = +1$ contribution is equivalent, at high energy, to the dominance of parton distributions over antiparton ones.

It is then convenient to rewrite the normalized energy distributions in the form

$$\begin{aligned} f^{\nu N}(\rho) &= 1 + (3\rho^2-1)\,\frac{1}{3}\,\frac{I_+^{\nu N}}{A^{\nu N}} \\ f^{\bar\nu N}(\rho) &= 3\rho^2 - (3\rho^2-1)\,\frac{I_+^{\bar\nu N}}{A^{\bar\nu N}} \end{aligned} \qquad (73)$$

The expected distribution for neutrino is essentially flat and the antineutrino one is very close to $3\rho^2$. The deviations from pure helicity $\lambda = -1$ shape are governed by the two coefficients

$$\frac{1}{3}\,\frac{I_+^{\nu N}}{A^{\nu N}} = 0.013\,{}^{+\,0.037}_{-\,0.013} \qquad \frac{I_+^{\bar\nu N}}{A^{\bar\nu N}} = 0.11\,{}^{+\,0.28}_{-\,0.11} \qquad (74)$$

The predictions are shown in Figs. 17 and 18. The experimental data at high energy are in qualitative agreement with these results but they are not accurate enough to allow a quantitative comparison.

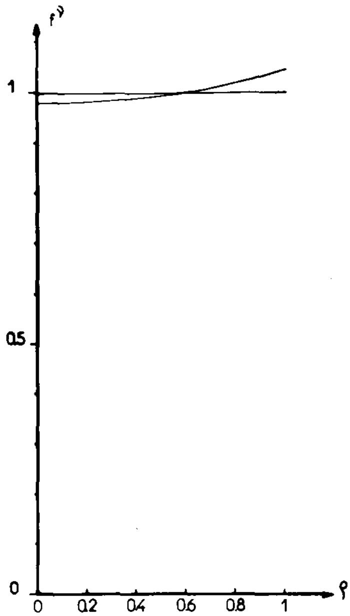

Fig. 17: Neutrino scattering: ℓ^- energy distribution

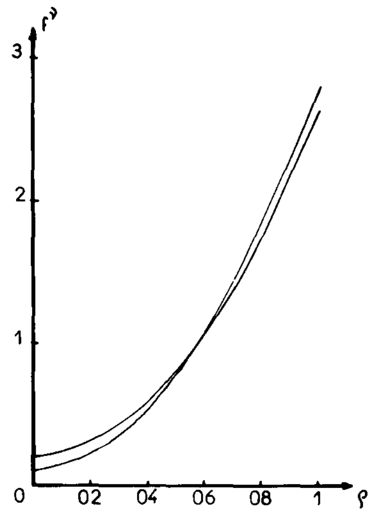

Fig. 18: Antineutrino scattering: ℓ^+ energy distribution

4) It is now straightforward to compute the averaged values of the final lepton and antilepton energies

$$\langle \rho \rangle = \int_0^1 \rho \, f(\rho) \, d\rho$$

By positivity of the I_λ's these quantities are bounded

$$\frac{1}{2} \leq \langle \rho \rangle \leq \frac{3}{4}$$

and from equations (73) and (74) we predict

$$\begin{aligned} \langle \rho \rangle_\nu &= 0.503 \begin{smallmatrix} +\,0.009 \\ -\,0.003 \end{smallmatrix} \\ \langle \rho \rangle_{\bar\nu} &= 0.722 \begin{smallmatrix} +\,0.028 \\ -\,0.071 \end{smallmatrix} \end{aligned} \qquad (75)$$

The results of the Gargamelle experiment

$$< \rho >_{\nu\ exp} = 0.54 \pm 0.04$$

$$< \rho >_{\bar{\nu}\ exp} = 0.72 \pm 0.05$$

are in good agreement with the quark parton model values based on total cross sections.

5) The weak scaling functions have not been experimentally separated and the only quantity we can discuss is the fixed ξ distribution which is written in the scaling limit as

$$\frac{d\sigma}{d\xi} \longrightarrow \frac{G^2 M E}{\pi} A(\xi)$$

The functions $A(\xi)$ for neutrino and antineutrino processes can be written as linear combinations of the quark and antiquark distributions $D(\xi)$. In the U(3) quark parton model and for an isoscalar target we get from equation (55)

$$A^{\nu N}(\xi) = \xi \left\{ \frac{1}{3}\left[D_{-1}(\xi) + D_{-2}(\xi)\right] + \cos^2\theta_C \left[D_1(\xi) + D_2(\xi)\right] + 2 \sin^2\theta_C\, D_3(\xi) \right\} \qquad (76)$$

$$A^{\bar{\nu} N}(\xi) = \xi \left\{ \frac{1}{3}\left[D_1(\xi) + D_2(\xi)\right] + \cos^2\theta_C \left[D_{-1}(\xi) + D_{-2}(\xi)\right] + 2 \sin^2\theta_C\, D_{-3}(\xi) \right\} \qquad (77)$$

Adding now the decomposition of the corresponding electroproduction function $F_2 = 2\xi F_T$

$$F_2^{eN}(\xi) = \xi \left\{ \frac{5}{18}\left[D_1(\xi) + D_2(\xi) + D_{-1}(\xi) + D_{-2}(\xi)\right] + \frac{1}{9}\left[D_3(\xi) + D_{-3}(\xi)\right] \right\} \qquad (78)$$

the system of equations (76), (77) and (78) can be solved as in the first paragraph of this section. With the present accuracy of experimental data the most interesting relation is that involving strange quark and antiquark distributions

$$\left[1 - \frac{9}{2}\sin^2\theta_C\right] \xi \left[D_3(\xi) + D_{-3}(\xi)\right] =$$

$$= 9\left(1 - \frac{3}{4}\sin^2\theta_C\right) F_2^{eN}(\xi) - \frac{15}{8}\left[A^{\nu N}(\xi) + A^{\bar{\nu} N}(\xi)\right] \qquad (79)$$

Because of the positivity of the distribution functions the right-hand side of equation (79) must be positive for all values of ξ. This result which involves electromagnetic and weak functions is a non trivial and unambiguous test of the quark parton model. The comparison with experiment is shown in Fig. 19 where the variable ξ' is used for convenience. Positivity is satisfied within experimental errors and the quantity $\xi' \,|D_3(\xi') + D_{-3}(\xi')\,|$ is consistent with zero for $\xi' > 0.3$. This last result is expected in a two component model where the diffractive contributions are important only for small values of ξ'.

An analogous result is obtained by comparing the difference $A^{\nu N}(\xi) - A^{\nu N}(\xi)$ with $F_2^{eN}(\xi)$ the coefficients being adjusted in order to eliminate the non strange quark distributions

$$\frac{12}{5}\left(1-\frac{3}{2}\sin^2\theta_c\right) F_2^{eN}(\xi) - \left[A^{\nu N}(\xi) - A^{\bar\nu N}(\xi)\right] =$$

$$= \frac{2}{3}\left(1-\frac{3}{2}\sin^2\theta_c\right)\xi\left[D_{-1}(\xi)+D_{-2}(\xi)\right] + \frac{4}{15}\left(1-9\sin^2\theta_c\right)\xi D_3(\xi) + \frac{4}{15}\left(1+\frac{3}{2}\sin^2\theta_c\right)\xi D_{-3}(\xi) \qquad (80)$$

The experimental situation is exhibited in Fig. 20. Positivity is consistent with experiment and the diffractive component of the right-hand side of eq. (80) is only sizeable for values of ξ' smaller than 0.4.

The Gross-Llewellyn Smith sum rule is easily translated into this language and from eqs. (76) and (77) we get

$$\int_0^1 \frac{d\xi}{\xi}\left[A^{\nu N}(\xi) - A^{\bar\nu N}(\xi)\right] = 2 - 3\sin^2\theta_c = 1.84$$

Moreover, taking into account the different numbers of neutron and proton in freon, the theoretical prediction becomes

$$\int_0^1 \frac{d\xi}{\xi}\left[A^{\nu}(\xi) - A^{\bar\nu}(\xi)\right] = 2.114 - 3.08\sin^2\theta_c = 1.96$$

Using the CERN-Gargamelle data an estimate of the integral has been done and the result 1.97 ± 0.20 is in excellent agreement with the theoretical prediction.

VII QUARK PARTON MODEL FOR WEAK PROCESSES WITH NEUTRAL CURRENTS

1) The simple quark parton model based on U(3) symmetry relates nicely electroproduction, neutrino and antineutrino data as shown in the previous section. Moreover the production of strange particles reduced by the Cabibbo angle remains small. It is then appealing to use for the neutral hadronic current a naive model proposed by Weinberg where strange

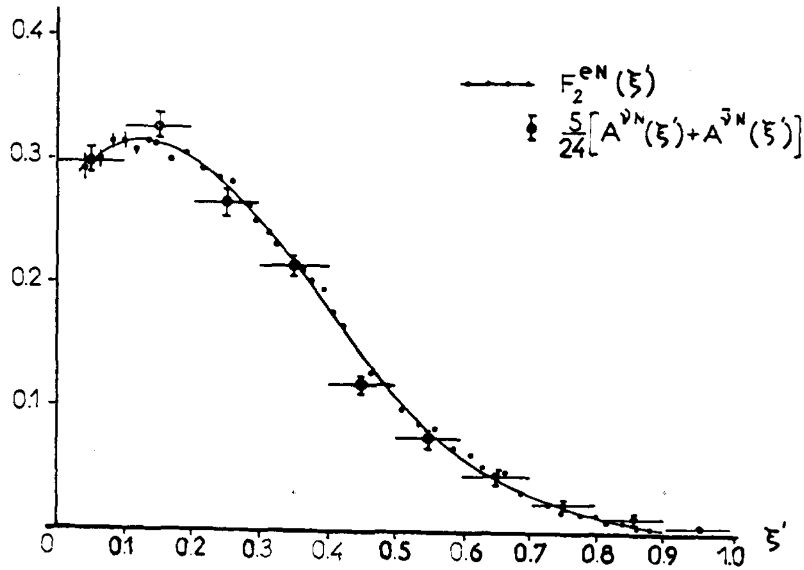

Fig. 19
Sum of neutrino and antineutrino cross sections compared to electroproduction.

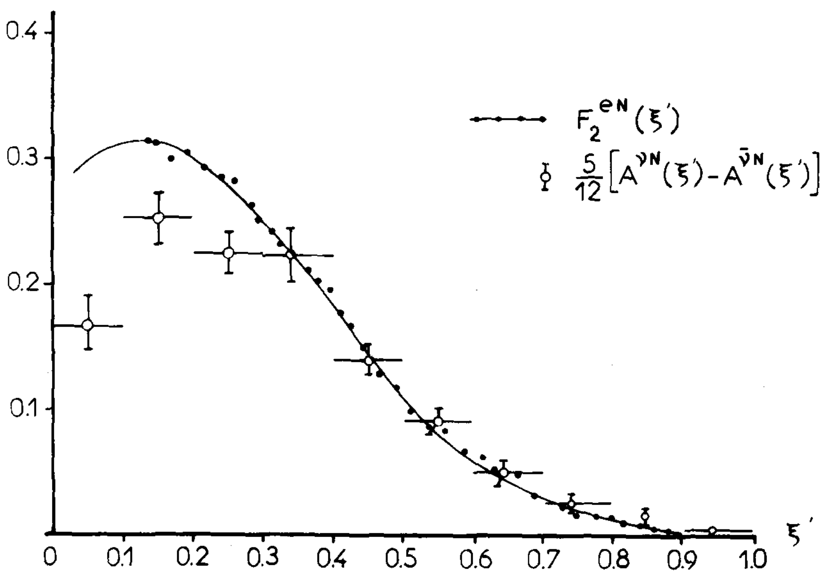

Fig. 20
Difference of neutrino and antineutrino cross sections compared to electroproduction.

particles and more exotic ones are ignored. The connection between the weak isotopic spin of gauge theories and the strong isotopic spin is made as follows

WEAK LEFT-HAND SU(2) $\rightarrow$ STRONG SU(2) $\otimes$ SU(2)

$$2 J_k^\mu = V_k^\mu + A_k^\mu$$

where as usual V means vector and A axial vector.

The neutral and charged weak current $\Delta S = 0$ have very simple expressions

$$J_Z^\mu = (V_3^\mu + A_3^\mu) - 2 \sin^2\theta_W (V_3^\mu + V_S^\mu)$$

$$J_W^\mu = (V_1^\mu + A_1^\mu) + i (V_2^\mu + A_2^\mu)$$

the electromagnetic current being decomposed as usual into an isoscalar and isovector component

$$J_Q^\mu = V_3^\mu + V_S^\mu$$

The Weinberg mixing angle θ_W is a free parameter in the theory and we put $x = \sin^2 \theta_W$

2) The number of independent quark and antiquark distributions being six the scaling functions

$F_\pm^Z$ for $\nu_\ell(\bar\nu_\ell) + p \rightarrow \nu_\ell(\bar\nu_\ell)$ + HADRONS ($\Delta S = 0$)

can be written as linear combinations of the scaling functions

$G_\pm^\nu$ for $\nu_\ell + p \longrightarrow \ell^-$ + HADRONS ($\Delta S = 0$)

$G_\pm^{\bar\nu}$ for $\bar\nu_\ell + p \longrightarrow \ell^+$ + HADRONS ($\Delta S = 0$)

F_T^Q for $\ell^\mp + p \longrightarrow \ell^\mp$ + HADRONS

For an arbitrary target we get

$$F_+^Z(\xi) = \left(\frac{1}{4} - \frac{2x}{3}\right) G_+^\nu(\xi) + \left(\frac{1}{4} - \frac{x}{3}\right) G_+^{\bar\nu}(\xi) + 4x^2 F_T^Q(\xi)$$

$$F_-^Z(\xi) = \left(\frac{1}{4} - \frac{x}{3}\right) G_-^\nu(\xi) + \left(\frac{1}{4} - \frac{2x}{3}\right) G_-^{\bar\nu}(\xi) + 4x^2 F_T^Q(\xi)$$

In the particular case of an isoscalar target these relations become simpler

$$F_{\pm}^{ZN}(\xi) = \left(\frac{1}{2} - x\right) G_{\pm}^{N}(\xi) + 4x^2 F_T^{eN}(\xi) \tag{81}$$

Differential cross section relations can easily be obtained in the scaling region. Using equation (81) we get

$$d\sigma_{NC}^{\nu N} = \left(\frac{1}{2} - x\right) d\sigma_{CC}^{\nu N} + 4x^2 X\, d\sigma^{eN}$$

$$d\sigma_{NC}^{\bar\nu N} = \left(\frac{1}{2} - x\right) d\sigma_{CC}^{\bar\nu N} + 4x^2 X\, d\sigma^{eN} \tag{82}$$

where $X = 2(G^2/e^4)q^4$. The indices NC and CC mean neutral current and charged current respectively.
In fact the relation between the differences $d\sigma^{\nu N}-d\sigma^{\bar\nu N}$ for neutral and charged current reactions is simply due to an isotopic spin rotation and it is a trivial consequence of the simple structure assumed for the weak currents.
Integrating the differential cross sections we obtain in the scaling limit

$$\int d^2\sigma^{\nu,\bar\nu} \rightarrow \frac{G^2 ME}{\pi} B^{\nu,\bar\nu}$$

$$\int X\, d^2\sigma^{e} \rightarrow \frac{G^2 ME}{\pi} \frac{2}{3} I^{e}$$

and from equations (82)

$$\begin{aligned} B_{NC}^{\nu N} &= \left(\frac{1}{2} - x\right) B_{CC}^{\nu N} + \frac{8}{3} x^2 I^{eN} \\ B_{NC}^{\bar\nu N} &= \left(\frac{1}{2} - x\right) B_{CC}^{\bar\nu N} + \frac{8}{3} x^2 I^{eN} \end{aligned} \tag{83}$$

The ratio R^{ν} and $R^{\bar\nu}$ of neutrino and antineutrino total cross sections

$$R^{\nu} = \frac{\sigma_{NC}^{\nu N}}{\sigma_{CC}^{\nu N}} \qquad\qquad R^{\bar\nu} = \frac{\sigma_{NC}^{\bar\nu N}}{\sigma_{CC}^{\bar\nu N}}$$

become quadratic functions of x

$$\begin{aligned} R^{\nu} &= \frac{1}{2} - x + \frac{8}{3} x^2 \frac{I^{eN}}{B_{CC}^{\nu N}} \\ R^{\bar\nu} &= \frac{1}{2} - x + \frac{8}{3} x^2 \frac{I^{eN}}{B_{CC}^{\bar\nu N}} \end{aligned} \tag{84}$$

The corresponding parabola have been represented in Figs. 21 and 22

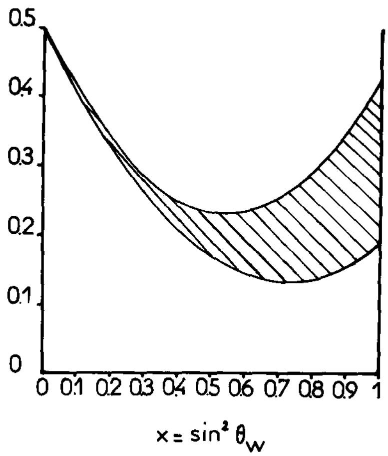

Fig. 21

Quark parton model prediction for the ratio of neutrino cross sections versus x.

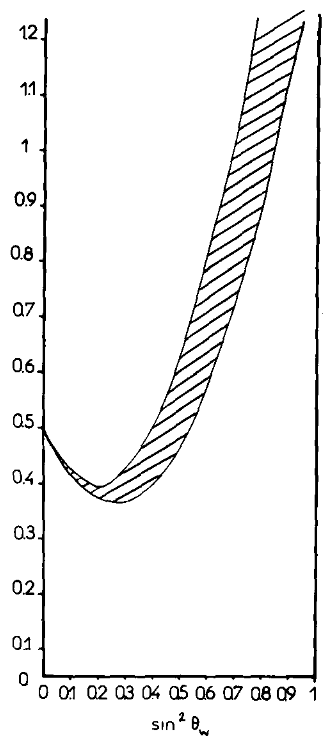

Fig. 22

Quark parton model prediction for the ratio of antineutrino cross sections versus x.

using the experimental data

$$B_{CC}^{\nu N} = 0.493 \pm 0.52 \qquad B_{CC}^{\bar{\nu}N} = 0.180 \pm 0.020$$

$$I^{eN} = 0.15 \pm 0.01$$

Lower bounds for the ratios R^{ν} and $R^{\bar{\nu}}$ are easily computed and in the one standard deviation limit we get

$$R^{\nu} > 0.14 \qquad R^{\bar{\nu}} > 0.37 \tag{85}$$

By eliminating x between the two equations (84) we obtain a relation between R^{ν} and $R^{\bar{\nu}}$

$$x = \frac{1}{2} + \frac{r_c R^{\bar{\nu}} - R^{\nu}}{1 - r_c} = \left\{ \frac{3}{8} \frac{1}{1-r_c} \frac{B_{cc}^{\bar{\nu}N}}{I^{eN}} \left[R^{\bar{\nu}} - R^{\nu} \right] \right\}^{1/2} \qquad (86)$$

where $r_c = B_{CC}^{\bar{\nu}N}/B_{CC}^{\nu N}$. The corresponding parabola in the R^{ν}, $R^{\bar{\nu}}$ plane has been drawn in Fig. 23 including the one standard deviation errors.

Let us write for completeness the Gross-Llewellyn Smith sum rule in this model

$$\int_0^1 F_3^{Z} d\xi = \left(\frac{1}{2} - x\right)\left(B + \frac{Y}{2}\right) - \frac{2x}{3} I_3$$

In particular for an isoscalar nucleon target we obtain

$$\int_0^1 F_3^{ZN} d\xi = 3\left(\frac{1}{2} - x\right) \qquad (87)$$

3) The Gargamelle data is now analysed in the framework of this simple quark parton model. We use as a first approximation*) numbers (32), (33) and (34) as ratios of total cross sections. The lower bounds (85) are satisfied and the Weinberg angle is computed from the two expressions of equation (86)**)

$$\sin^2 \theta_W = 0.36 \pm 0.06 \qquad (88)$$

$$\sin^2 \theta_W = 0.36 \pm 0.11 \qquad (89)$$

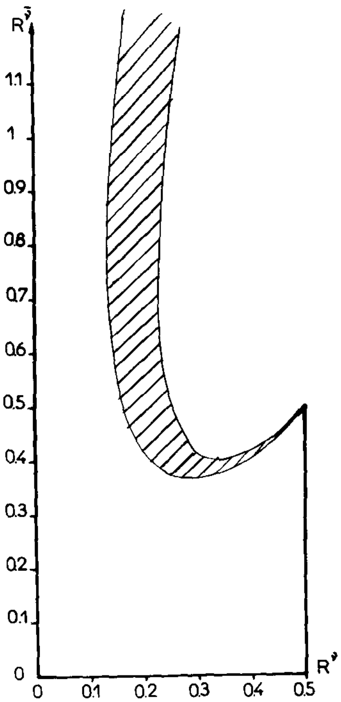

Fig. 23

Quark parton model prediction in the R^{ν}, $R^{\bar{\nu}}$ plane.

*) Without a good knowledge of the energy distributions it is not possible to compute the error made in replacing the ratio of total cross sections by that of the number of events. We expect the correction to be minimized by using, for the three ratios r_c, R^{ν} and $R^{\bar{\nu}}$ of eq. (86) the number of events with identical cuts.

**) Because of the existence of energy cut, the second expression of x in the right-hand side of eq. (86) can be computed in different ways and the central value may vary between 0.30 and 0.40. Such an uncertainty must be kept in mind when comparing the results (88) and (89).

The consistency of these results measures the consistency of the quark parton model with experiment. Moreover if the Weinberg current is replaced by a somewhat more general form

$$J_Z^\mu = \alpha\left(V_3^\mu + A_3^\mu \right) - 2x\left(V_3^\mu + V_S^\mu \right)$$

we easily check that a value of α close to unity can be found

$$\alpha = 1 \pm 0.10$$

the parameter x being given by equation (89).

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N. B. The experimental data shown in the figures correspond to the situation in the spring 1974.

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PART C FURTHER APPLICATIONS. BREAKING OF SCALING

I POLARIZATION EFFECTS IN ELECTROPRODUCTION

1) In the one photon exchange approximation polarization effects occur in electroproduction when the target is polarized. If time reversal invariance is assumed we have two new structure functions as seen in Table 1.
The polarized cross section is then generally written as

$$\frac{d^2\sigma_\eta}{dq^2\,dW^2} = \left(\frac{d^2\sigma}{dq^2\,dW^2}\right)_{UNP} \left[1 + \eta\, N\cdot\Delta\right] \tag{90}$$

where N is a unit space like polarization vector of the spin 1/2 target orthogonal to p

$$N^2 = 1 \qquad\qquad N\cdot p = 0$$

In the laboratory frame N has only space components. The asymmetry vector Δ has no component orthogonal to the scattering plane if time reversal invariance holds. Then, with a polarization vector N in the scattering plane we can measure two independent asymmetries and therefore determine the two structure functions for polarization. Because of parity conservation in electromagnetic interactions there exist only three independent total cross sections for the photoabsorption reaction $\gamma + p \to$ HADRONS

$$\sigma^{Q}_{\lambda\, s}(q^2, W^2) = \sigma^{Q}_{-\lambda\, -s}(q^2, W^2)$$

Two of them have been associated to scattering on an unpolarized target and the third one describes the polarization effect in the $\vec{q}_{lab}$ direction,

$$\Delta_q = -\frac{\sqrt{1-\varepsilon^2}}{2}\;\frac{\sigma_{1\,1/2} - \sigma_{1\,-1/2}}{\sigma_T + \varepsilon\,\sigma_L} \tag{91}$$

where the kinematical parameter ε is defined as usual by

$$\varepsilon^{-1} = 1 + 2\,\frac{\nu^2+q^2}{q^2}\,tg^2\frac{\theta}{2}$$

The second structure function for polarization is a transverse longitudinal correlation associated to total helicity $\pm\frac{1}{2}$.

2) An easily measurable quantity is the parallel-antiparallel asymmetry $\Delta_{\parallel}$ corresponding to a nucleon polarization vector $\vec{N}$ collinear to the incident beam momentum $\vec{k}$.
At high energy $\Delta_{\parallel} = \Delta_q$ and moreover if scaling holds for the structure functions we get scaling for the asymmetry $\Delta_{\parallel}$

$$\Delta_{\parallel} \longrightarrow - \frac{1-\rho^2}{2} \frac{F^Q_{1\,1/2}(\xi) - F^Q_{1\,-1/2}(\xi)}{(1+\rho^2) F^Q_T(\xi) + 2\rho\, F^Q_L(\xi)} \tag{92}$$

where we have used the high energy limit

$$\varepsilon \longrightarrow \frac{2\rho}{1+\rho^2}$$

From the positivity constraints on total cross sections and the relation $2\,F_T = F_{1\,1/2} + F_{1-1/2}$ we get an upper bound for the asymmetry

$$|\Delta_{\parallel}| \leq \frac{1-\rho^2}{(1+\rho^2) + 2\rho R(\xi)}$$

3) Let us define the polarization scaling function

$$2\,F^Q_{\parallel}(\xi) = F^Q_{1\,1/2}(\xi) - F^Q_{1\,-1/2}(\xi) \tag{93}$$

In the quark parton model we have an expression for $F^Q_{\parallel}(\xi)$ analogous to the equation (25) for $F^Q_T(\xi)$

$$2\,F^Q_{\parallel}(\xi) = \sum_{j,\sigma} Q_j^2\, \sigma \left[D_{j\sigma}(\xi) + D_{-j\,\sigma}(\xi) \right] \tag{94}$$

The $D_{j\sigma}$ are the distribution functions of the parton of type j in the hadron with momentum $\xi\vec{p}$ and spin parallel ($\sigma = +1$) or antiparallel ($\sigma = -1$) to the nucleon spin. We then have the obvious relation

$$D_j(\xi) = D_{j\,+1}(\xi) + D_{j\,-1}(\xi)$$

4) The distributions $D_{j\sigma}(\xi)$ are not known but we can have some information on their normalization integrals

$$\int_0^1 D_{j\sigma}(\xi)\, d\xi = \langle N_{j\sigma} \rangle \tag{95}$$

Let us define the quantity Z^Q by

$$Z^Q = \int_0^1 2\, F_{\parallel}^Q(\xi)\, d\xi$$

From equations (94) and (95) we obtain

$$Z^Q = \sum_{j,\sigma} Q_j^2 \sigma \left[\langle N_{j\sigma} \rangle + \langle N_{-j\sigma} \rangle \right] \tag{96}$$

or in an equivalent language

$$Z^Q = \sum_{j,\sigma} \left[\langle N_{j\sigma} \rangle \langle j\sigma | \sigma_3 Q^2 | j\sigma \rangle + \langle N_{-j\sigma} \rangle \langle -j\sigma | \sigma_3 Q^2 | -j\sigma \rangle \right] \tag{97}$$

It is clear, from equation (97) that Z^Q is the average value in the hadron of the operator $\sigma_3 Q^2$.

We now introduce the symmetric coefficients of the Lie algebra defined by the anticommutators

$$\{ X^a , X^b \}_+ = d^{ab}{}_c\, X^c$$

Equation (97) can be written in the form of the Bjorken sum rule

$$Z^Q = \frac{1}{2}\, d^{QQ}{}_c\; g_A^c \tag{98}$$

the axial vector coupling constant being defined in the quark parton model by

$$g_A^c = \sum_{j,\sigma} \left[\langle N_{j\sigma} \rangle \langle j\sigma | \sigma_3 X^c | j\sigma \rangle - \langle N_{-j\sigma} \rangle \langle -j\sigma | \sigma_3 X^c | -j\sigma \rangle \right] \tag{99}$$

In the U(3) algebra the symmetric coefficients computed in the 3 dimensional representation are given by

$$d^{QQ}{}_B = \frac{4}{3} \qquad\qquad d^{QQ}{}_Q = \frac{2}{3}$$

The nucleon belonging to an octet representation we have three reduced matrix elements

$$\begin{aligned} Z^{ep} &= \frac{2}{3} f_1 + \frac{1}{9} f_s + \frac{1}{3} f_a \\ Z^{en} &= \frac{2}{3} f_1 - \frac{2}{9} f_s \end{aligned} \tag{100}$$

The octet coefficients are known from an analysis of neutron and hyperon β decays with the Cabibbo current. From experiment we get

$$f_a + f_s = -\frac{G_A}{G_V} = 1.23 \qquad\qquad \frac{f_s}{f_a + f_s} \simeq 0.6$$

The difference $Z^{ep} - Z^{en}$ is therefore known from neutron decay only and we get the famous Bjorken relation

$$Z^{ep} - Z^{en} = \frac{1}{3}\left(-\frac{G_A}{G_V}\right) = 0.41 \tag{101}$$

The baryonic constant f_1 is not experimentally known. It can be computed in a model where the gluon spin is assumed to be uncorrelated in average with the nucleon one. Using equation (99) and spin conservation the result is simply $f_1 = 1/3$. In such a model we have the predictions

$$Z^{ep} \simeq 0.47 \qquad\qquad Z^{en} \simeq 0.06 \tag{102}$$

5) It is interesting to compare these predictions obtained in the $q^2 \to \infty$ limit with those of photoproduction at $q^2 = 0$. In this case the Drell-Hearn, Gerasimow sum rule derived on general grounds is written as

$$\frac{1}{\pi}\int_{M^2}^{\infty} \frac{\sigma_{1\,1/2}(0,W^2) - \sigma_{1\,-1/2}(0,W^2)}{W^2 - M^2}\, dW^2 = \frac{\mu^2}{2M^2} \tag{103}$$

In average the difference $\sigma_{1\ 1/2} - \sigma_{1\ -1/2}$ is expected to be positive. On the other hand phenomenological analysis of photoproduction agrees with this qualitative statement and quantitatively the sum rule (103) seems to be in good shape.

On the basis of the quark parton model the situation looks radically different for large q^2. If one uses elementary arguments as those presented before we expect a positive asymmetry for the proton in the deep inelastic region and therefore a change of sign of the asymmetry $\Delta_{\parallel}^{ep}$ between the photoproduction region $q^2 = 0$ and the scaling region $q^2 > 1$ GeV^2. There is no experimental evidence, at least for $q^2 \lesssim 0.6$ GeV^2 for such a change of sign in the proton asymmetry and if one believes, following Bloom and Gilman, that the resonance region is well averaged using convenient variables by the deep elastic one we get a difficulty.

Of course if one accepts to leave to gluons a dominant role in polarization effects it is easy to obtain a negative asymmetry for the proton. In this case, from the Bjorken relation, the neutron asymmetry would become very large and negative.

An experiment is in progress at SLAC with a polarized electron beam. The target is a polarized proton target with proton polarization longitudinal to the direction of the incident electron beam. The parallel-antiparallel asymmetry $\Delta_{\parallel}$ will be measured either reversing the electron polarization or the proton polarization.

II PARITY-VIOLATING EFFECTS IN ELECTROPRODUCTION

1) In inelastic scattering of charged leptons the one-photon exchange amplitude dominated the cross section at currently available energies. However an additional contribution due to neutral intermediate vector boson exchange may give peculiar effects we wish now to study. We compute the differential cross section in the inclusive case where only the final lepton is detected retaining the 1γ exchange term and the 1γ - $1Z$ interference contribution. The target is assumed to be unpolarized and we call η the longitudinal polarization of the incident beam. Details about the kinematics have been given in part A and the result has the following structure

$$\frac{d\sigma^{\mp}}{d\sigma_0} = 1 + \frac{\sqrt{2}\,G}{e^2}\,q^2\left\{(a \pm \eta b)\,\phi_1(q^2,W^2,E) - (a\eta \pm b)\,\phi_2(q^2,W^2,E)\right\} \tag{104}$$

The upper (lower) sign refers to lepton ℓ^- (antilepton ℓ^+) scattering and $d\sigma_0$ is the pure one-photon exchange cross section. The functions ϕ_1 and ϕ_2 are constructed from the interference structure functions $\tau_\lambda\,(q^2,W^2)$ and the electroproduction ones $\sigma_\lambda^Q(q^2,W^2)$

$$\phi_1(q^2,W^2,E) = \frac{\tau_T(q^2,W^2) + \varepsilon\,\tau_L(q^2,W^2)}{\sigma_T^Q(q^2,W^2) + \varepsilon\,\sigma_L(q^2,W^2)}$$

$$\phi_2(q^2,W^2,E) = \frac{\sqrt{1-\varepsilon^2}}{2}\;\frac{\tau_{-1}(q^2,W^2) - \tau_{+1}(q^2,W^2)}{\sigma_T^Q(q^2,W^2) + \varepsilon\,\sigma_L^Q(q^2,W^2)}$$

where the kinematical quantity ε has been previously defined

$$\varepsilon^{-1} = 1 + 2\, tg^2 \frac{\theta}{2}\, \frac{\nu^2 + q^2}{q^2}$$

It is clear on these expressions that Φ_1 is parity conserving and Φ_2 parity violating.

2) Various asymmetries can be computed from equation (104). In order to eliminate higher order electromagnetic effects we only consider parity violating quantities and we define asymmetries for a given charge and opposite polarizations of the beam:

$$A_{\mp} = \frac{d\sigma_R^{\mp} - d\sigma_L^{\mp}}{d\sigma_R^{\mp} + d\sigma_L^{\mp}}$$

To lowest order in G we get from equation (104)

$$A_{\mp} = \frac{\sqrt{2}\, G}{e^2}\, q^2 \left[\pm \Phi_1 - a\, \Phi_2 \right] \tag{105}$$

In the Weinberg-Salam model the two parameters a and b are given by

$$a = 4 \sin^2 \theta_W - 1 \qquad\qquad b = 1$$

and the asymmetries are functions of the mixing angle θ_W. As previously we shall put $x = \sin^2 \theta_W$.

3) We now consider the simple model proposed by Weinberg for the hadronic current

$$J_Z^{\mu} = V_3^{\mu} + A_3^{\mu} - 2x\, J_Q^{\mu}$$

In the quark parton model based on U(3) symmetry the interference scaling functions $\tilde{F}_{\pm}(\xi)$ defined by

$$\mathrm{LIM}\ \frac{M\sqrt{\nu^2+q^2}}{\pi}\, \tau_{\pm 1}(q^2, W^2) = \tilde{F}_{\pm}(\xi)$$

can be expressed as linear combinations of the electroproduction neutrino and antineutrino strangeness conserving scaling functions.
The result is

$$\tilde{F}_{+}(\xi) = \frac{1}{6}\, G_{+}^{\nu}(\xi) + \frac{1}{12}\, G_{+}^{\bar{\nu}}(\xi) - 2x\, F_T^{Q}(\xi)$$

$$\tilde{F}_{-}(\xi) = \frac{1}{6}\, G_{-}^{\bar{\nu}}(\xi) + \frac{1}{12}\, G_{-}^{\nu}(\xi) - 2x\, F_T^{Q}(\xi)$$

Let us restrict now to the simple case where the proton and neutron scaling functions have been averaged

$$\tilde{F}_{\pm}^{N}(\xi) = \frac{1}{4} G_{\pm}^{N}(\xi) - 2x\, F_{T}^{eN}(\xi) \tag{106}$$

In the scaling limit the functions ϕ_1 and ϕ_2 are independent of the incident energy E and they can be computed in terms of electroproduction and weak scaling functions

$$\phi_1^N \longrightarrow \frac{1}{4} \frac{G_T^N(\xi)}{F_T^{eN}(\xi)} - 2x$$

$$\phi_2^N \longrightarrow \frac{1-\rho^2}{1+\rho^2} \frac{1}{8} \frac{G_{-1}(\xi) - G_{+1}(\xi)}{F_T^{eN}(\xi)}$$

By using the language of differential cross sections ($\Delta S = 0$) we get

$$\phi_1^N = \frac{1}{8} \frac{d\sigma_{cc}^{\nu N} + d\sigma_{cc}^{\bar{\nu} N}}{X\, d\sigma^{eN}} - 2x \tag{107}$$

$$\phi_2^N = \frac{1}{8} \frac{d\sigma_{cc}^{\nu N} - d\sigma_{cc}^{\bar{\nu} N}}{X\, d\sigma^{eN}} \tag{108}$$

where the notations are the same as in part B: $X = 2(G^2/e^4)q^4$.

Let us notice that the expression (108) for ϕ_2 is a trivial consequence of the simple isotopic spin structure assumed for the weak currents and it is independent of the quark parton model.

4) We first obtain a prediction for the sum of the asymmetries

$$A_-^N + A_+^N = \frac{1-4x}{2} \frac{G}{\sqrt{2}\, e^2} q^2 \frac{d\sigma_{cc}^{\nu N} - d\sigma_{cc}^{\bar{\nu} N}}{X\, d\sigma^{eN}} \tag{109}$$

Unfortunately the differential cross sections for neutrino and antineutrino are not well known and it is interesting to look at an averaged asymmetry defined as follows

$$\langle A_{\mp}^N \rangle = \int A_{\mp}^N X\, d\sigma^{eN} \Big/ \int X\, d\sigma^{eN} \tag{110}$$

It is clear, from equations (109) and (110) that averaged values of q^2 for neutrino and antineutrino reactions are involved and we obtain

$$\langle A_-^N + A_+^N \rangle = \frac{1-4x}{2} \frac{G}{\sqrt{2}e^2} \frac{\langle q^2 \rangle_\nu \sigma_{cc}^{\nu N} - \langle q^2 \rangle_\nu \sigma_{cc}^{\bar{\nu} N}}{\int X d\sigma^{eN}}$$

In the high energy limit we used scaling in the form

$$\frac{\sigma^{\nu,\bar{\nu}}}{\int X d\sigma^{eN}} \longrightarrow \frac{3}{2} \frac{B^{\nu,\bar{\nu}}}{I^{eN}}$$

all the parameters being defined in part B. The final result is

$$\langle A_-^N + A_+^N \rangle \longrightarrow \frac{1-4x}{2} \frac{3}{2I^{eN}} \frac{G}{\sqrt{2}e^2} \left[\langle q^2 \rangle_\nu B_{cc}^{\nu N} - \langle q^2 \rangle_{\bar{\nu}} B_{cc}^{\bar{\nu} N} \right]$$

Numerical estimates can be obtained using the CERN-Gargamelle data presented in part B

$$\langle q^2 \rangle_\nu \simeq (0.21 \pm 0.02)\, E \qquad \langle q^2 \rangle_{\bar{\nu}} \simeq (0.14 \pm 0.03) E$$

$$B_{CC}^{\nu N} \simeq 0.493 \pm 0.050 \qquad B_{CC}^{\bar{\nu} N} \simeq 0.180 \pm 0.020$$

and the electroproduction result

$$I^{eN} = 0.15 \pm 0.01$$

As expected the sum of averaged asymmetries increases linearly with the incident energy E

$$\langle A_-^N + A_+^N \rangle = (1-4x)(0.35 \pm 0.07) 10^{-4}\, E\ \mathrm{GeV}^{-1} \tag{111}$$

5) An analogous treatment can be done for the difference of asymmetries $A_-^N - A_+^N$ and numerical estimatescan be computed for the averaged value of that difference. However it is interesting to remark that from the CERN-Gargamelle results presented in Fig. 19 the ratio of scaling functions $G_T(\xi)/F_T^{eN}(\xi)$ involved in the function ϕ_1 is practically constant at least for $\xi > 0.3$ and the constant turns out to be consistent with the value of 0.9 predicted by a pure three valence quark model. Therefore for $\xi > 0.3$

$$\phi_1 \simeq 0.9 - 2x$$

and we obtain a prediction for the difference of asymmetries

$$A_-^N - A_+^N = (0.9 - 2x)\ 3.6\ 10^{-4}\ q^2\ \mathrm{GeV}^{-2} \tag{112}$$

III BREAKING OF SCALING

1) The scaling à la Bjorken of the structure functions has been observed in electroproduction at SLAC and DESY in a limited range of values for q^2 and W^2

$$1 \text{ GeV}^2 < q^2 < 12 \text{ GeV}^2 \qquad\qquad 2 \text{ GeV} < W < 7 \text{ GeV}$$

The neutrino and antineutrino experiments performed at CERN with the Gargamelle bubble chamber cover an analogous range for q^2 and W^2. Indirect evidence for scaling has been obtained.
A possible physical interpretation of this fact is the parton model for hadrons: the elementary constituents have very small dimensions and appear as point like in their interactions with the electromagnetic and weak currents.

2) Even if the Bjorken scaling is an asymptotic theoretical statement we must ask the question: what will happen at higher values of q^2 and W^2? In principle the experiments performed at N.A.L. covering a more extended range of the q^2, W^2 plane will answer that question and we shall come back on this point later. On phenomenological grounds we have two possibilities

a- we are in an asymptotic region and nothing new will appear; we have reached the ultimate constituents of hadrons and life is simple.

b- we are in a preasymptotic region and at larger values of q^2 and W^2 deviations of scaling will take place due for instance to the excitation of internal degrees of freedom of partons.

3) Let us first look at the possibility for partons to have a structure which can be represented by a form factor of the type proposed by Chanowitz and Drell:

$$F(q^2) = \frac{1}{1 + q^2/m_G^2}$$

The new mass scale m_G which may be associated in a more or less effective way to gluons is assumed to be very large as compared to the nulceon mass.

For instance the electroproduction structure function at large q^2 and fixed ξ will have the factorized form

$$F_2^{ep}(q^2,\xi) \rightarrow \frac{F_2^{ep}(\xi)}{[1+q^2/m_G^2]^2}$$

The parton structure has not yet been seen at SLAC. Although the analysis depends on the choice of the scaling variable the data put a lower bound on m_G of order 10 GeV. Crucial information will be provided by the NAL experiment with incident μ^- leptons and where values of q^2 as large as 40 GeV^2 can be reached.

For neutrino and antineutrino processes such a parton structure will compete with that due to the intermediate vector boson propagator. Unfortunately the breaking of scaling due to finite values of m_G and m_W produces analogous and indistinguishable effects.

An interesting consequence of the existence of a parton structure will also occur for time like photons in the annihilation process $e^+ + e^- \rightarrow$ HADRONS. The manifestation will now be an enhancement of the cross section of the resonance type. Something unexpected appears in the CEA and SPEAR experiments which may be associated to a parton structure or due to a totally different origin as for instance the production of new particles (heavy leptons or charmed and colored hadrons).

4) The results of the quark parton model for deep inelastic lepton scattering can equivalently be obtained in the framework of the light cone quark algebra supplemented by Wilson's operator product expansion. A better understanding of these simple results can be undertaken in a more systematic approach to asymptotic behaviour using the techniques of the renormalization group. Without giving any detail or proof we now briefly sketch some important steps of the method.

The hadronic tensor for inelastic lepton scattering is the Fourier transform of the one-particle matrix element of the product of two current operators. By taking advantage of the translational invariance equation (13) can be written as

$$M_{\mu\nu}^{\alpha\beta}(p,q) = \frac{M}{2\pi}\int e^{-iq\cdot x}\langle p|\, J_\nu^\beta(\tfrac{x}{2})\, J_\mu^\alpha(-\tfrac{x}{2})\,|p\rangle\, d_4x$$

We expand the hadronic tensor on a complete basis of covariants $I^j_{\mu\nu}$ the coefficients of that expansion being the structure functions

$$M_{\mu\nu}^{\alpha\beta}(p,q) = \sum_j I^j_{\mu\nu}\, F_j^{\alpha\beta}(q^2,\xi)$$

The $I^j_{\mu\nu}$'s are chosen so that to have simple properties for the structure functions the Bjorken conjecture about scaling holds. In this case we have

$$\lim_{\substack{q^2\to\infty \\ \xi \text{ fixed}}} F_j^{\alpha\beta}(q^2,\xi) = F_j^{\alpha\beta}(\xi)$$

We are interested in the behaviour of M(p,q) in the deep inelastic region for q spacelike and large with the target momentum p fixed. It is then convenient to expand the product of current operators near x = 0 by introducing an appropriate complete set of local operators O_n

$$J_\nu^\beta\left(\frac{x}{2}\right) J_\mu^\alpha\left(-\frac{x}{2}\right) = \sum_n C_{\mu\nu}^{\alpha\beta;n}(x)\, O_n(0) \tag{113}$$

In an analogous way the tensor $C_{\mu\nu}^{\alpha\beta;n}(x)$ is expanded on a Lorentz covariant basis and the Fourier transform $\tilde{C}(q)$ of the scalar coefficients C(x) can be studied by means of a generalization of the renormalization group equation of Gell-Mann and Low, the so-called Callan-Symanzik equation

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_n(g)\right] \tilde{C}_j^{\alpha\beta;n}\left(q^2/\mu^2, g\right) = 0 \tag{114}$$

where μ is the subtraction point introduced in the renormalization and g a dimensionless coupling constant. For conserved or partially conserved currents the anomalous dimension of the current operators J vanishes so that $\gamma_n(g)$ is simply the anomalous dimension of the operator O_n. The solution of this Callan-Symanzik equation can be expressed in terms of an auxiliary function $\bar{g}(t,g)$ defined by

$$\frac{\partial}{\partial t}\bar{g}(t,g) - \beta(g)\frac{\partial}{\partial g}\bar{g}(t,g) = 0 \tag{115}$$

where

$$t = \frac{1}{2}\log\frac{q^2}{\mu^2}$$

with the initial condition $\bar{g}(o, g) = g$. The result is

$$\tilde{C}_j^{\alpha\beta;n}\left(q^2/\mu^2, g\right) = \tilde{C}_j^{\alpha\beta;n}\left[1, \bar{g}(t,g)\right] \exp\left[-\int_0^t \gamma_n\left[\bar{g}(\tau,g)\right] d\tau\right] \tag{116}$$

The connection between the structure functions $F_j^{\alpha\beta}$ and the Wilson coefficients $\tilde{C}_j^{\alpha\beta;n}$ is obtained at the level of the various moments and the result is simply

$$\int_0^1 \xi^{n-1} F_j^{\alpha\beta}(q^2,\xi) = \tilde{C}_j^{\alpha\beta;n}(q^2)\, M_n \tag{117}$$

where M_n is the one-particle matrix element of the operator O_n. The asymptotic behaviour of these moments is controlled by that of the Wilson coefficients $\tilde{C}(q)$ which is determined, from equation (116) by the large t behaviour of the function $\bar{g}(t,g)$. The result is

$$\lim_{q^2\to\infty} \tilde{C}_j^{\alpha\beta;n}\left(q^2/\mu^2, g\right) = \tilde{C}_j^{\alpha\beta;n}(1, g_0)\left[q^2/\mu^2\right]^{-\frac{1}{2}\gamma_n(g_0)}$$

where g_0 is the renormalization group fixed point

$$\beta(g_0) = 0 \qquad\qquad \lim_{t\to\infty} \bar{g}(t,g) = g_0$$

The condition for Bjorken canonical scaling is then simply

$$\gamma_n(g_0) = 0 \quad \text{for all n's}$$

As shown in Part A in the case $\alpha=\beta$ the diagonal elements of the hadronic tensor in the helicity space are positive functions of their arguments. Therefore the corresponding momenta are positive functions of q^2 which, at fixed q^2, decrease when n increases. This positivity property enables to reduce the infinite number of constraints to two only.

It can be shown that the ultraviolet stable fixed point of the renormalization group must be at the origin $g_0 = 0$. This result is called asymptotic freedom because in this situation the strong interactions turn off for large space like momenta. Therefore if we insist to explain Bjorken scaling using the renormalization group approach the class of renormalizable theories for strong interactions one may consider is severely limited. Only gauge theories based on non Abelian gauge groups have the property of asymptotic freedom.

In an asymptotically free gauge theory the approach to asymptotic behaviour is not with a power law but with logarithmically vanishing correction terms. The functions $\beta(g)$ and $\gamma_n(g)$ are now expected to vanish around the origin according to

$$\beta(g) = -b_0 g^3 + O(g^5) \qquad\qquad \gamma_n(g) = \gamma_n g^2 + O(g^4)$$

Using equations (115) and (116) a straightforward computation gives the following result for the moments of the structure functions at large q^2

$$\int_0^1 \xi^{n-1} F_j^{\alpha\beta}(q^2,\xi)\,d\xi \longrightarrow \tag{118}$$

$$\longrightarrow \text{constant} \cdot M_n \left[\log \frac{q^2}{\mu^2}\right]^{-a_n} \left[\tilde{C}_j^{\alpha\beta;n}(1,0) + O\left(\frac{1}{\log q^2/\mu^2}\right)\right]$$

where $a_n = \gamma_n / 2b_0$ is a model dependent parameter and the rate of approach to this situation will obviously depend on the unknown scale μ.

Let us finally remark that the dependence on the indices α, β and j which is contained in the quantity $\tilde{C}(1,0)$ turns out to be the same as in free field theory. As a consequence the moments of the structure functions will satisfy all parton model relations and sum rules. The Adler sum rule is valid for all q^2 but the Gross-Llewellyn Smith sum rule is approached logarithmically

$$\int_0^1 \left[F_3^W(q^2,\xi) + F_3^{\bar{W}}(q^2,\xi)\right] d\xi = [GL]\ \mathcal{F}(q^2)$$

where the constant [GL] is its asymptotic value depending on the algebra and $\mathcal{F}(q^2)$ a function with the structure

$$\lim_{q^2\to\infty} \mathcal{F}(q^2) = 1 + O\left(\frac{1}{\log q^2/\mu^2}\right)$$

Analogously the dominance of spin $\frac{1}{2}$ partons is expressed by a Callan-Gross type relation

$$\frac{\int_0^1 \xi^n F_L(\xi,q^2)\,d\xi}{\int_0^1 \xi^n F_T(\xi,q^2)\,d\xi} \longrightarrow O\left(\frac{1}{\log q^2/\mu^2}\right)$$

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CONCLUSION

A complete study of the application of the quark parton model to electromagnetic and weak interactions would involve more topics than those considered here, namely the two sets of processes

(i) Semi-inclusive reactions where one or more hadrons in the final state is detected in coincidence with the final leptons

(ii) Electron-positron annihilation into hadrons.

The application of the parton model to semi-inclusive reactions implies new assumptions concerning the production mechanism and, to my point of view, the central point is to construct a model where the non observation of quarks and antiquarks as free particles in the final state - which is an experimental fact - appears as a natural consequence of the dynamics used to describe the production of hadrons. To my knowledge, the various proposals made are not totally satisfactory in this respect.

The description of annihilation processes with a quark parton model is generally made using a two-step mechanism; first a quark-antiquark pair is produced via one photon exchange or some less conventional way as that proposed by Pati and Salam, and then this quark-antiquark pair annihilates into hadrons in a way which again prevents the observation of a $q\bar{q}$ pair in the final state. The data produced by CEA and SPEAR lead a naive quark parton model into difficulties, the total cross section for $e^+e^- \rightarrow$ HADRONS being roughly constant between 9 and 25 GeV^2 for the squared total energy s. Therefore, the timelike region appears to behave differently from the spacelike one and satisfactory answers have not yet been given to this apparent contradiction, which is very important if experimentally confirmed.