Inflation driven by the Galileon field

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Abstract

We propose a new class of inflation model, *G-inflation*, which has a Galileon-like nonlinear derivative interaction of the form $G(\phi, (\nabla\phi)^2)\Box\phi$ in the Lagrangian with the resultant equations of motion being of second order. It is shown that (almost) scale-invariant curvature fluctuations can be generated even in the exactly de Sitter background and that the tensor-to-scalar ratio can take a significantly larger value than in the standard inflation models, violating the standard consistency relation. Furthermore, violation of the null energy condition can occur without any instabilities. As a result, the spectral index of tensor modes can be blue, which makes it easier to observe quantum gravitational waves from inflation by the planned gravitational-wave experiments such as LISA and DECIGO as well as by the upcoming CMB experiments such as Planck and CMBpol.

Inflation in the early universe is now a part of the standard cosmology to solve the horizon and flatness problem as well as to account for the origin of density/curvature fluctuations. It is most commonly driven by a scalar field dubbed as inflaton, and the research on inflationary cosmology has long been focused on the shape of the inflaton potential in the particle physics context. Its underlying physics is now being probed using precision observations of the cosmic microwave background and large scale structure which are sensitive only to the dynamical nature of the inflaton. Reflecting this situation, a number of novel inflation models have been proposed extending the structure of the kinetic function.

In this article, we propose a new class of inflation models, for which the scalar field Lagrangian is of the form

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi, \tag{1}$$

where K and G are general function of ϕ and $X := -\nabla_\mu\phi\nabla^\mu\phi/2$. The most striking property of this generic Lagrangian (1) is that it gives rise to derivatives no higher than two both in the gravitational- and scalar-field equations. In the simplest form the nonlinear term may be given by $G\Box\phi \propto X\Box\phi$, which has recently been discussed in the context of the so-called *Galileon* field [1, 2]. The general form $G(\phi, X)\Box\phi$ may be regarded as an extension of the Galileon-type interaction $X\Box\phi$ while maintaining the field equations to be of second-order [3]. So far the phenomenological aspects of the Galileon-type scalar field have been studied mainly in the context of dark energy and modified gravity. In this article, we discuss primordial inflation induced by this type of fields. See also [4, 5].

Now let us start investigating our model in detail. Assuming that ϕ is minimally coupled to gravity, the total action is given by

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_\phi\right]. \tag{2}$$

The energy-momentum tensor $T_{\mu\nu}$ derived from the action reads

$$T_{\mu\nu} = K_X\nabla_\mu\phi\nabla_\nu\phi + Kg_{\mu\nu} - 2\nabla_{(\mu}G\nabla_{\nu)}\phi + g_{\mu\nu}\nabla_\lambda G\nabla^\lambda\phi - G_X\Box\phi\nabla_\mu\phi\nabla_\nu\phi. \tag{3}$$

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The equation of motion of the scalar field is equivalent to $\nabla_\nu T_\mu^{\ \nu} = 0$. Here and hereafter we use the notation K_X for $\partial K/\partial X$ etc.

Taking the homogeneous and isotropic background, $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2$, $\phi = \phi(t)$, let us study inflation driven by the Galileon-like scalar field (1), which we call "*G-inflation*." The energy-momentum tensor (3) has the form $T_\mu^{\ \nu} = \mathrm{diag}(-\rho, p, p, p)$ with

$$\rho = 2K_X X - K + 3G_X H\dot{\phi}^3 - 2G_\phi X, \quad p = K - 2\left(G_\phi + G_X\ddot{\phi}\right)X. \tag{4}$$

Here, ρ has an explicit dependence on the Hubble rate H. The gravitational field equations are thus given by

$$3M_{\rm Pl}^2 H^2 = \rho, \quad -M_{\rm Pl}^2\left(3H^2 + 2\dot{H}\right) = p, \tag{5}$$

and the scalar field equation of motion reads

$$\begin{aligned} &K_X\left(\ddot{\phi} + 3H\dot{\phi}\right) + 2K_{XX}X\ddot{\phi} + 2K_{X\phi}X - K_\phi - 2\left(G_\phi - G_{X\phi}X\right)\left(\ddot{\phi} + 3H\dot{\phi}\right) \\ &+6G_X\left[(HX)^{\cdot} + 3H^2X\right] - 4G_{X\phi}X\ddot{\phi} - 2G_{\phi\phi}X + 6HG_{XX}X\dot{X} = 0. \end{aligned} \tag{6}$$

These three equations constitute two independent evolution equations for the background. Note that the appearance of the terms proportional to the Hubble parameter in Eqs. (4) and (6) reflects the fact that the Galileon symmetry is broken in the curved spacetime even if we constrain our functional form of the Lagrangian which possess its symmetry in the Minkowski spacetime.

We begin with constructing an exactly de Sitter background, taking K and G as

$$K(\phi, X) = K(X), \quad G(\phi, X) = g(\phi)X. \tag{7}$$

In this case, inflation is driven purely kinematically, although G-inflation does not preclude a potential-driven inflationary solution with an explicit ϕ-dependence in $K(\phi, X)$ in general; see Eq. (1). If $g(\phi) =$ const, *i.e.*, the Lagrangian has a shift symmetry $\phi \to \phi+$ const, we have an exactly de Sitter solution satisfying $\dot{\phi}$ =const,

$$3M_{\rm Pl}^2 H^2 = -K, \quad \mathcal{D} := K_X + 3gH\dot{\phi} = 0. \tag{8}$$

Let us now provide a simple example:

$$K = -X + \frac{X^2}{2M^3\mu}, \quad g = \frac{1}{M^3}, \tag{9}$$

where M and μ are parameters having dimension of mass. The de Sitter solution is given by

$$X = M^3\mu x, \quad H^2 = \frac{M^3}{18\mu}\frac{(1-x)^2}{x}, \tag{10}$$

where x $(0 < x < 1)$ is a constant satisfying $(1-x)/x\sqrt{1-x/2} = \sqrt{6}\mu/M_{\rm Pl}$. For $\mu \ll M_{\rm Pl}$, it can be seen that $x \simeq 1 - \sqrt{3}\mu/M_{\rm Pl}$ and hence the Hubble rate during inflation is given in terms of M and μ as $H^2 \simeq M^3\mu/(6M_{\rm Pl}^2)$. As the first term in $K(X)$ has the "wrong" sign, one may worry about ghost-like instabilities. However, as we will see shortly, this model is free from ghost and any other instabilities.

We now move on to study scalar perturbations in this model using the unitary gauge with $\delta\phi = 0$ and

$$\mathrm{d}s^2 = -(1+2\alpha)\mathrm{d}t^2 + 2a^2\partial_i\beta\mathrm{d}t\mathrm{d}x^i + a^2(1+2\mathcal{R}_\phi)\mathrm{d}\mathbf{x}^2. \tag{11}$$

In this gauge we have $\delta T_i^{\ 0} = -G_X\dot{\phi}^3\partial_i\alpha$, and hence this gauge does not coincide with the comoving gauge $\delta T_i^{\ 0} = 0$. Consequently, $\mathcal{R}_\phi$ in general differs from the comoving curvature perturbation $\mathcal{R}_c$. This point highlights the difference between the present model and the standard k-inflationary model described simply by $\mathcal{L}_\phi = K(\phi, X)$. It will turn out that the variable $\mathcal{R}_\phi$ is subject to an analogous wave equation to the familiar Sasaki-Mukhanov equation.

Expanding the action (2) to second order in the perturbation variables and then substituting the Hamiltonian and momentum constraint equations to eliminate α and β, we obtain the following quadratic action for $\mathcal{R}_\phi$:

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}\tau\mathrm{d}^3x\, z^2\left[\mathcal{G}(\mathcal{R}_\phi')^2 - \mathcal{F}(\vec{\nabla}\mathcal{R}_\phi)^2\right], \tag{12}$$

where

$$z := \frac{a\dot\phi}{H - G_X\dot\phi^3/2M_{\rm Pl}^2}, \tag{13}$$

$$\mathcal{F} := K_X + 2G_X\left(\ddot\phi + 2H\dot\phi\right) - 2\frac{G_X^2}{M_{\rm Pl}^2}X^2 + 2G_{XX}X\ddot\phi - 2\left(G_\phi - XG_{\phi X}\right), \tag{14}$$

$$\mathcal{G} := K_X + 2XK_{XX} + 6G_XH\dot\phi + 6\frac{G_X^2}{M_{\rm Pl}^2}X^2 - 2\left(G_\phi + XG_{\phi X}\right) + 6G_{XX}HX\dot\phi, \tag{15}$$

and the prime represents differentiation with respect to the conformal time τ. The squared sound speed is therefore $c_s^2 = \mathcal{F}/\mathcal{G}$. To avoid ghost and gradient instabilities we require the conditions $\mathcal{F} > 0$ and $\mathcal{G} > 0$. One should note that the above equations have been derived without assuming any specific form of $K(\phi, X)$ and $G(\phi, X)$.

It is now easy to check whether a given G-inflation model is stable or not. In the simplest class of models (7), we have

$$\mathcal{F} = -\frac{K_X}{3} + \frac{XK_X^2}{3K},\ \mathcal{G} = -K_X + 2XK_{XX} - \frac{XK_X^2}{K}, \tag{16}$$

where the "slow-roll" suppressed terms are ignored. For the previous toy model (9) one obtains $\mathcal{F} = x(1-x)/6(1-x/2)$ and $\mathcal{G} = 1 - x + (1-x/2)^{-1}$. Since $0 < x < 1$, both $\mathcal{F}$ and $\mathcal{G}$ are positive. In this model, the sound speed is smaller than the speed of light: $c_s^2 \leq (4\sqrt{2}-5)/21 \simeq 0.031 < 1$.

The power spectrum of $\mathcal{R}_\phi$ generated during G-inflation can be evaluated by writing the perturbation equation in the Fourier space as

$$\frac{\mathrm{d}^2u_k}{\mathrm{d}y^2} + \left(k^2 - \frac{\tilde z_{,yy}}{\tilde z}\right)u_k = 0, \tag{17}$$

where $\mathrm{d}y = c_s\,\mathrm{d}\tau$, $\tilde z := (\mathcal{F}\mathcal{G})^{1/4}\,z$, and $u_k := \tilde z\mathcal{R}_{\phi,k}$. Let us again focus on the class of models (7). Note that the sound speed c_s may vary rapidly in the present case, and hence one cannot neglect $\epsilon_s := \dot c_s/Hc_s$ even when working in leading order in "slow-roll." Indeed, one finds $\epsilon_s \simeq \eta X(\mathcal{G}_X/\mathcal{G} - \mathcal{F}_X/\mathcal{F})$. With some manipulation, one obtains $\tilde z_{,yy}/\tilde z \simeq (-y)^{-2}[2 + 3\epsilon\mathcal{C}(X)]$ with

$$\mathcal{C}(X) := \frac{K}{K_X}\frac{Q_X}{Q}, \quad Q(X) := \frac{(K - XK_X)^2}{18M_{\rm Pl}^4Xc_s^2\sqrt{\mathcal{F}\mathcal{G}}}. \tag{18}$$

It should be emphasized that scalar fluctuations are generated even from exactly de Sitter inflation. This is because, as mentioned before, the Galileon symmetry is broken in the de Sitter background, which is manifest from $\dot\phi = \text{const}$. This situation is in stark contrast with other inflation models: scalar fluctuations cannot be generated from the de Sitter background with $\dot\phi = 0$ in usual potential-driven inflation, while the exactly de Sitter background cannot be realized in k-inflation.

The normalized mode is given in terms of the Hankel function as

$$u_k = \frac{\sqrt{\pi}}{2}\sqrt{-y}H_\nu^{(1)}(-ky), \quad \nu := \frac{3}{2} + \epsilon\,\mathcal{C}, \tag{19}$$

from which it is straightforward to obtain the power spectrum and the spectral index:

$$\mathcal{P}_{\mathcal{R}_\phi} = \left.\frac{Q}{4\pi^2}\right|_{c_sk=1/(-\tau)}, \quad n_s - 1 = -2\epsilon\,\mathcal{C}. \tag{20}$$

The behavior of tensor perturbations in G-inflation is basically the same as in the usual inflation models and is completely determined geometrically. Therefore, the power spectrum and the spectral index of primordial gravitational waves are given by $\mathcal{P}_T = (8/M_{\rm Pl}^2)(H/2\pi)^2$ and $n_T = -2\epsilon$. However, it would be interesting to point out that the tensor spectrum can be blue in G-inflation with possible violation of the NEC. The positive tensor spectral index not only is compatible with current observational data, but also broadens the limits on cosmological parameters. Moreover, the amplitude of tensor fluctuation with such a blue spectral index is relatively enhanced for large frequencies, which makes its direct detection easier.

As a concrete example, let us come back again to the previous toy model (9), in which the tensor-to-scalar ratio is given by

$$r \simeq \frac{16\sqrt{6}}{3}\left(\frac{\sqrt{3}\mu}{M_{\rm Pl}}\right)^{3/2} \quad \text{for} \quad \mu \ll M_{\rm Pl}. \tag{21}$$

With the properly normalized scalar perturbation, $\mathcal{P}_{\mathcal{R}_\phi} = 2.4 \times 10^{-9}$, we can easily realize large r to saturate the current observational bound, exceeding the predictions of the chaotic inflation models. For example, for $M = 0.00425 \times M_{\rm Pl}$ and $\mu = 0.032 \times M_{\rm Pl}$ we find $r = 0.17$, which is large enough to be probed by the PLANCK satellite. Note that neither the standard consistency relation, $r = -8n_T$, nor the k-inflation-type consistency relation, $r = -8c_s n_T$, holds in our model.

In summary, we have proposed a novel inflationary mechanism driven by the Galileon-like scalar field. Our model —*G-inflation*— is a new class of inflation models with the term proportional to $\Box\phi$ in the Lagrangian, which opens a new branch of inflation model building. Contrary to the most naive expectation, the interaction of the form $G\left(\phi, (\nabla\phi)^2\right)\Box\phi$ gives rise to derivatives no higher than two in the field equations [3]. In this sense, G-inflation is distinct also from ghost condensation and B-inflation. After G-inflation, the universe is reheated through the gravitational particle production with successful thermal leptogenesis. We have also shown that G-inflation can generate (almost) scale-invariant density perturbations, possibly together with a large amplitude of primordial gravitational waves. These facts have great impacts on the planned and ongoing gravitational wave experiments and CMB observations. In a forthcoming paper we shall compute the non-Gaussianity of the curvature perturbation from G-inflation, which would be a powerful discriminant of the scenario in addition to the violation of the standard consistency relation.

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