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To cite this article: Alfredo Iorio 2015 *J. Phys.: Conf. Ser.* **626** 012035

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What after CERN?

Opportunities from co-responding systems

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Abstract. We offer some arguments in favor of the construction of an experimental facility, where to test fundamental theories of Nature by using co-responding systems. Co-responding systems are physical systems such that certain behaviors of one system are clearly related to certain behaviors of the other system. Physical systems available at our energy scales, co-responding to the unreachable high-energy systems, are what we need, to attack from an experimental perspective the open questions beyond the reach of CERN. The focus here is on two scenarios with which we have some familiarity: hadron production in high energy scattering processes as a Unruh phenomenon, and graphene as a quantum field theory in a curved spacetime, the latter being our prime bet.

1. Introduction

There are many indications that this might be the right time to gather into one place theoretical, numerical and experimental efforts to study fundamental aspects of Nature reproduced in systems within our reach.

Sometimes this research is referred to as “analogue” or “artificial” physics [1]. Instead, we take the view that when, for instance, Hawking radiation is produced in a Bose-Einstein condensate [2], that is... Hawking radiation, with no further attribute.

We do not claim that testing, for instance, black hole physics at the black hole location it is not worthwhile. Surely, many astrophysical issues are better understood by studying the astrophysical black hole than studying some realization in condensed matter systems [3]. Nonetheless: first, this is hard; second, there are too many fundamental questions awaiting an answer since decades [4]; third, certain fundamental issues (like the mentioned Hawking radiation, the related Bekenstein-Hawking entropy, and all the way till the quantum gravity scenarios), can well be addressed by identifying their essential features, and by reproducing these features here and now.

We shall not venture in a proper scientific defense of this approach as a whole. We take, instead, a practical view-point, by presenting two cases where certain important fundamental questions can be addressed in co-responding systems. The two examples are singled-out purely on the basis of our own familiarity with them.



2. Entanglement vs. Bekenstein-Hawking entropies.

Co-responding system: high energy collisions

Let us here illustrate how, by considering hadron production in high energy collisions as the co-responding system of the Unruh radiation [5], can serve the scope of investigating fundamental questions, such as the nature of the Bekenstein-Hawking entropy.

What is customarily done, in the attempt to explain the mysteries of the gravitational entropy, that appears to give us information on the inaccessible interior region of a black hole (see, e.g., [6] and [7]), is to consider the black hole and the surrounding quantum relativistic matter as two separate systems, so that the entropy of the composite system is viewed as the sum of the two

$$S_{tot} = S_{BH} + S_{matt} \quad (1)$$

S_{matt} is customarily seen as an entanglement entropy, between modes of the quantum field that are near the horizon, on both sides, in and out, see, e.g., [8]. Less clear, if not fully obscure, is the origin of the S_{BH} part. There are proposals on the latter being also an entanglement entropy, [8], [9], but there are several issues that need to be solved before considering this proposal a tenable solution [7]. Although the holographic description holds for the entanglement entropy as well as for S_{BH} (see [9] and [10], for two different but related approaches), the most important obstacle to this interpretation is that standard quantum field computations lead to infinite entanglement entropy, unless one introduces an ad hoc cutoff to regulate the ultraviolet (UV) divergencies.

Let us, then, remove altogether the black hole from the picture, but let us retain an event horizon and quantum relativistic matter surrounding it. A way to do so is to accelerate the quantum matter relativistically, so that the spacetime becomes a Rindler spacetime [11]. According to the generally accepted wisdom of (1), and using a well known application to the Rindler spacetime of the Gibbons-Hawking Euclidean action formula, see [12], that gives $S_{Rindler} = \frac{1}{4}A/r_P^2$, this situation should lead to

$$S_{tot} = S_{Rindler} + S_{matt} = \frac{1}{4} \frac{A}{r_P^2} + \alpha \frac{A}{r^2}, \quad (2)$$

where r_P is the Planck length, r is the length scale characterizing the quantum dynamics, α is a numerical constant, and $A = \int dydz$ is a surface of constant acceleration, i.e., of constant Rindler coordinate ξ , and of constant Rindler time η , [11] (we suppose that, the topology of the specific Rindler spacetime under consideration, is spherically symmetric, with radius R , hence, $A = 4\pi R^2$). This topology is precisely the one of interest for the hadronization mechanism [5]).

Formula (2) is mysterious in many respects. In the first term, the appearance of r_P , hence of G , makes no sense in a context where there is no gravitational field behind the acceleration of the Rindler spacetime. In other words, while, of course, the formula makes perfect sense for the near-horizon approximation of a black hole (on which black-hole to use for the hadron production case, see [13]), the converse is no longer true. I.e., if the dynamics (force) behind the Rindler acceleration is not of gravitational origin, it makes no sense to have r_P , on the contrary, one expects r instead. In fact, r enters only in the second contribution to (2), i.e. to the entanglement entropy of the matter fields around the horizon, see [8]. The second term a) has a numerical constant α that cannot be fixed from the theory, and, most importantly, b) if one cannot fix the length scale r to a nonzero value, it presents the UV divergence we have recalled earlier. The other mystery of (2) is that, even if one can find a natural scale r , it would be natural to have $r \gg r_P$, and this means that the “gravitational” entropy of the Rindler spacetime would dominate over any form of matter entropy.

A solution to these puzzles comes from the hypothesis that entanglement entropy coincides with the “gravitational” entropy (that would be, then, better called “geometric” entropy), and

that there is one entropy all the time, at least in the Rindler case

$$S = \frac{1}{4} \frac{A}{r^2} \quad (3)$$

The factor $1/4$ is the signature of the geometric entropy. The scale fixed by r , and not by r_P , is the signature that this is the matter/entanglement entropy. Furthermore, since no gravity is involved, the Bekenstein upper bound [14] can be invoked all the time. This serves the important scope of fixing a nonzero r , all the time. Finally, at least in this spherically symmetric Rindler spacetime, one can also invoke the saturation of the Bekenstein bound, and this would fix all the parameters involved.

These theoretical arguments we now offer to the co-responding system of hadron production in high energy collisions, at zero chemical potential $\mu = 0$. The sequential quark-antiquark pair-productions, which is the basic ingredient for hadron production, generate a series of decelerations of constant magnitude that are a realization of the Unruh phenomenon in a Rindler spacetime [5]. By using $r = 1/k_T$, with k_T the transverse momentum (the characteristic small length scale of the quantum dynamic, here the quark-antiquark dynamics), in [15] it is shown that, the experimental result [16]

$$s/T \simeq 7, \quad (4)$$

with s entropy density and T freeze-out temperature, is well matched by the formula (3), that gives

$$s/T^3 = 3\pi^2/4 \simeq 7.4. \quad (5)$$

3. Curved spacetimes singularities and nonunitarity.

Co-responding system: graphene

In this second example, we point to the relation among curvature of the spacetime, singularities, and inequivalent quantizations/Hilbert spaces. The co-responding system is graphene physics [17]. The fundamental physics arena is the long-ongoing debate on unitarity, equivalence principle, quantum field theory (QFT) in curved spacetimes, and black-hole physics, see, e.g., [18].

Consider a metric that can encode intrinsic curvature, and that is the next-to-trivial with respect to the (1+1)-dimensional ((1+1)d) case,

$$g_{\mu\nu}(q) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & & \\ 0 & g_{ij} \end{pmatrix}, \quad (6)$$

with $\mu, \nu, \lambda, \dots = 0, 1, 2$ (time and space), $i, j, k, l, \dots = 1, 2$ (space only), $\partial_t g_{ij} = 0$, and we are in a coordinate frame $q^\mu = (t, x, y)$.

There are various reasons to choose the set-up (6). First, this way we can take advantage of the well-known differential geometry of surfaces. Then, since we focus on how quantization is affected in the process of going from a flat to a curved spacetime, and since we want to be in a simple but not trivial case, we choose the conformally flat (CF) sub-set of (6). This way, on the one hand, we shall be able to use the geometry of surfaces and the richness of the structure there, see [19]. On the other hand, being in (2+1)d, we know that very interesting spacetimes exist, e.g., the black-hole of [20], or the gravitational kink of [21].

The conformal structure is used in an “anomalous” sense, i.e., it is a classical symmetry, but quantum mechanically we are able to distinguish among Weyl-related configurations [19]. To single-out this phenomenon, we want to avoid the well-known scale/trace anomaly [22]. Being in (2+1)d, we can ignore the latter altogether. The issue becomes more involved when some sort of horizon is present, like the conformal Killing horizon of [3], for more details see [17].

In that case, an effective (1+1)d theory emerges, and anomalies of the standard kind (general relativistic or trace), will indeed play an important role. If one requires that no horizon is formed, the “quantum anomalies” we shall isolate persist in all cases where intrinsic curvature is present (hence, *also* when an horizon is present).

In two dimensions all metrics are CF [23], i.e. there exists $\tilde{q}^\mu \equiv (t, \tilde{x}, \tilde{y})$, such that

$$g_{\mu\nu}(\tilde{q}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -e^{2\sigma(\tilde{x}, \tilde{y})} & 0 \\ 0 & 0 & -e^{2\sigma(\tilde{x}, \tilde{y})} \end{pmatrix}. \quad (7)$$

Notice that we acted only on the 2d spatial part of the coordinates $q^\mu: (x, y) \rightarrow (\tilde{x}, \tilde{y})$. The latter coordinates are usually called “isothermal”. What we need is a CF metric in (2+1)d, which is obtained iff the surface has constant Gaussian (intrinsic) curvature \mathcal{K} , i.e.

$$\sigma = \sigma_H \quad \text{or} \quad \sigma = \sigma_L, \quad (8)$$

where $(\partial_x^2 + \partial_y^2)\sigma_H = 0$, i.e. σ_H is an harmonic function, corresponding to an intrinsically flat (7), and with $(\partial_x^2 + \partial_y^2)\sigma_L = -\mathcal{K} e^{2\sigma_L}$, i.e. σ_L is a Liouville function, corresponding to an intrinsically constant curved (7). On this see [19]. Further attention is necessary when dealing with the global properties of the surface, like the ones we are interested in here.

When σ is given by (8), then exists a third coordinate frame $Q^\mu = (T, X, Y)$, such that the metric (6) is

$$g_{\mu\nu}(Q) = e^{2\Sigma(Q)} \eta_{\mu\nu}, \quad (9)$$

where, of course, $\Sigma(\sigma)$. This is as far the classical background for the quantum field is concerned.

For the quantum field, the best setting to single-out the effect we are looking for, is to have a field that classically has the same dynamics in the flat and in the curved case, whereas, quantum mechanically the difference will show-up only for the quantum inequivalence we are investigating. As explained earlier, this is not trace anomaly. All of this points clearly to the use of the Dirac field that, for $m = 0$, enjoys the full local Weyl invariance, at the classical level [24] (here $\hbar = c = 1$)

$$\mathcal{A}_\Sigma = i \int d^3Q \sqrt{g} \bar{\psi}_\Sigma \nabla \psi_\Sigma = i \int d^3Q \bar{\psi} \not{\partial} \psi = A, \quad (10)$$

when

$$g_{\mu\nu}(Q) = e^{2\Sigma(Q)} \eta_{\mu\nu} \quad \text{and} \quad \psi_\Sigma(Q) = e^{-\Sigma(Q)} \psi(Q). \quad (11)$$

Clearly, with the use of the quantum implementation of Weyl symmetry, through the operator

$$U[\Sigma]: \psi \rightarrow \psi_\Sigma, \quad (12)$$

and by focusing on the conformally flat cases, we can have a clear understanding of the relation between the Hilbert/Fock spaces in the flat and in the curved cases.

What one needs to investigate is how intrinsic (Gaussian) curvature \mathcal{K} affects the operator $U[\Sigma]$, making it ill-defined/essentially singular, etc.. This would mean that quantum vacua, in the curved and in the flat case, are not mapped one to the other with a unitary operator, just like predicted by the failure of the Stone-von Neumann theorem in the presence of a non trivial topology [25]. The line of thoughts here is to link intrinsic curvature with a topological defect, hence with a singularity. Since, on the one hand, we are in a setting mimicking three-dimensional gravity [26], and since, on the other hand, the hexagonal lattice of graphene requires disclination defects for an intrinsically curved membrane, such a link should be possible along the lines of [27], where three dimensional gravity and defects in elastic media are merged (notice that inequivalent quantizations, in the presence of a singularity, were already found in [28]).

Work is in progress in this direction [29].

4. Conclusions

It might be truly worthwhile that the scientific community makes an effort towards the construction of an experimental facility, with the same motivations of CERN, but entirely dedicated to test fundamental phenomena of Nature by reproducing their key aspects on corresponding systems.

To name things is a crucial part of making them happens [30], thus, let us do it here. We name this facility HELIOS, for High Energy Laboratory for Indirect ObservationS. Of course, we would see no harm if the facility is called in any other way...

Seeds of HELIOS are probably in many places, right now. For what matters our own work in that direction, we are committed to set under control all possible aspects of graphene as a quantum field theory in curved spacetimes, and as a quantum gravity playground [17]. The enterprise is a tall-order, and we are proceeding by incremental steps. The first is to set under control the geometries of the graphene membranes that are of interest for the studies of fundamental physics [31] (see also [32], and [33] for an introductory overview).

Acknowledgments

The financial support of the Czech Science Foundation (GAČR), under contract no. 14-07983S, is acknowledged.

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