Isospin dependence of nuclear level density of ${}^{28}Al$ considering symmetry energy and pairing corrections

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Abstract. In this study, the single-particle level densities are calculated by using the isospin dependent nuclear level density (NLD) formula. The calculations are performed using the experimental data for ²⁸Al achieved from ²⁷Al(n, γ)²⁸Al resonances. Considering the symmetry energy and parity corrections, the NLDs at the excitation energy, E=20MeV are calculated and the maximum possible isospin in the range from ground state up to this excitation energy is estimated. The first correction led to reduced level densities and the later resulted in decreased densities for some of the levels while increased densities for the others. It is observed that the maximum level density occurs for T=1 and at low energies for which the experimental data are available there is a good agreement between calculated and experimental data.

1. Introduction

Nuclear level density is one of the most interesting concepts of nuclear physics. Its application spreads from fundamental research of nuclear structure, activation methods, shielding, and reactor physics to astrophysics and cosmology. All fields, which intersect in some cases, need a deeper understanding of the behaviour of the NLD at high excitation energy. Among these fields, NLDs are important in theoretical estimate of various compound nuclear reactions. For instance, neutron-capture reaction rates are approximately proportional to the corresponding NLDs in the neutron resonance region [1].

It is often necessary to have an accurate estimate of the NLD of highly excited nuclei as a function of the number of neutrons and protons, the excitation energy, angular momentum, and other constants of the motion [2]. The initial work in the NLD was done by Bethe [3]. The so-called partition function method is by far the most widely used technique to calculate level densities, particularly in view of its ability to provide simple analytical formulae. In its simplest form, the NLD is evaluated for a gas of non-interacting fermions confined to the nuclear volume and having equally spaced energy levels [4]. Such a model corresponds to the zero order approximation of a Fermi gas model and leads to very simple analytical, though unreliable expressions for NLD. The angular momentum and isospin of the nuclear are good quantum numbers that can be studied. In nuclear reactions, the transition probabilities vary so rapidly with angular momentum that only the levels having the appropriate angular momentum are excited with reasonably large probabilities. Therefore, it is important to determine the density of levels with given angular momentum. More generally, one can compute the density of the levels for which any number of constants of motion takes given values. The isospin formalism is very convenient for representing a new constant of motion similar to an angular momentum. The well-known spin-dependence distribution of NLD by using the central limit theorem of the statistics is in the Gaussian form

$$\rho(E,M) = \frac{1}{\sqrt{2\pi\sigma^2}}\rho(E)\exp\left[-\frac{M^2}{2\sigma^2}\right]$$
(1)

where $\rho(E)$, M and σ^2 are the level density at excitation energy E, the z-component of the total angular momentum J, and the spin cut-off factor, respectively. The spin cut-off factor determines the level spin distribution. Since

$$\rho(E,J) = \rho(E,M=J) - \rho(E,M=J+1) \cong [-\frac{\partial}{\partial M}\rho(E,M)]_{M=J+1/2}$$
(2)

Then

$$\rho(E,J) \cong \frac{1}{\sqrt{2^3 \pi \sigma^6}} \rho(E)(2J+1) \exp\left[-\frac{(J+1/2)^2}{2\sigma^2}\right]$$
(3)

In this paper, the level density of the ${}^{28}Al$ is studied by using the spin and isospin dependence formulae obtained from the partition function method [5] and the limitation of the isospin value at excitation energy E=20MeV is presented.

2. Density states of a given angular momentum and isospin

The usual approach to the level density calculation takes the independent particle model as its starting point. The nucleus may simply be described as a set of independent fermions located in a potential well. The word "independent" implies that any particle placed in the well is in the state that is described with good quantum numbers, and the characteristics of this state are independent of other occupied states. The importance of independent-particle approximation is that one can write the partition function of the nucleus in a single form in terms of single particle energies. For a system of Z protons and N neutrons in quantum states i

$$N_{i} = \sum_{\nu} n_{i}(n,\nu) ; \quad \epsilon_{i} = \sum_{\nu} n_{i}(n,\nu)\epsilon(n,m,\nu) + \sum_{\nu} n_{i}(p,\nu)\epsilon(p,m,\nu)$$
$$Z_{i} = \sum_{\nu} n_{i}(p,\nu) ; \quad M_{i} = \sum_{\nu} n_{i}(n,\nu)m(\nu) + \sum_{\nu} n_{i}(p,\nu)m(\nu)$$
(4)

Where $n_i(n,\nu)$ and $n_i(p,\nu)$ are the neutron and proton occupation numbers for the one-particle state ν in the quantum state *i*, respectively. These numbers can take either the value 1 or 0 because of the exclusion principle. Also $m(\nu)$ is the projected angular momentum quantum number for symmetry axis, $\epsilon(n,m,\nu)$ and $\epsilon(p,m,\nu)$ are the energies of the single-particle states of neutrons and protons, respectively. The partition function is in the form [6]

$$Z(\alpha_n, \alpha_p, \beta, \gamma) = \sum_{i} \exp \left[\alpha_n N_i + \alpha_p Z_i - \beta \epsilon_i - \gamma M_i\right]$$
(5)
$$= \prod_{n,\nu} (1 + \exp \left[\alpha_n - \beta \epsilon(n, m, \nu) - \gamma m(\nu)\right])$$
$$\times \prod_{p,\nu} (1 + \exp \left[\alpha_p - \beta \epsilon(p, m, \nu) - \gamma m(\nu)\right])$$

The standard treatment of statistical mechanics includes other approximations which lose their validity in the systems containing small number of particles. Here, we assume that the numbers of the single-particle states are large and the states are sufficiently placed close together. Therefore,

the individual nucleon states in the central potential can be replaced by continuous distribution. Thus ϵ_i , the energies of single-particle states and also $m(\nu)$, single particle angular momentum are replaced by the continuous distribution. Ultimately, $\ln Z$ simplifies in the form of

$$\ln Z(\alpha_n, \alpha_p, \beta, \gamma) = \int_0^\infty d\epsilon \int_{-\infty}^\infty dm g_n(\epsilon, m) \ln \left(1 + \exp\left[\alpha_n - \beta\epsilon(n, m) - \gamma m\right]\right)$$
(6)
+
$$\int_0^\infty d\epsilon \int_{-\infty}^\infty dm g_p(\epsilon, m) \ln \left(1 + \exp\left[\alpha_p - \beta\epsilon(p, m) - \gamma m\right]\right)$$

Two factors $g_n(\epsilon, m)$ and $g_p(\epsilon, m)$ respectively define single - particle level densities for neutrons and protones. Considering that NLD is the inverse of Laplace transform of partition function, then

$$\rho = \left(\frac{1}{2\pi i}\right)^4 \int_{-i\infty}^{i\infty} e^{-\alpha_n N - \alpha_p Z + \beta \epsilon - \gamma M} Z(\alpha_n, \alpha_p, \beta, \gamma) d\alpha_n d\alpha_p d\beta d\gamma \tag{7}$$

In calculating this integral, the saddle point conditions are used, which give rise to

$$N = \left(\frac{\partial \ln Z}{\partial \alpha_n}\right)_0 \; ; \; \epsilon = \left(-\frac{\partial \ln Z}{\partial \beta}\right)_0 \; ; \; Z = \left(\frac{\partial \ln Z}{\partial \alpha_p}\right)_0 \; ; \; M = \left(-\frac{\partial \ln Z}{\partial \gamma}\right)_0 \tag{8}$$

Where ()₀ indices refer that these formulae were evaluated in the saddle points. Finally, the formula below for M-distribution is derived.

$$\rho(N, Z, E, M) = \frac{1}{12\sqrt{2}}g_0(\frac{g_0^2}{4g_ng_p})^{\frac{1}{2}} \langle m^2 \rangle^{-\frac{1}{2}} (g_0 E - \frac{M^2}{2\langle m^2 \rangle})^{-\frac{3}{2}} \exp\{2[\frac{1}{6}\pi^2(g_0 E - \frac{M^2}{2\langle m^2 \rangle})]^{\frac{1}{2}}\}$$
(9)

Where $g_0 = g_n + g_p$. In a similar manner, the density of states as a function of T_z , the Z-component of total isospin T, can be derived [5].

$$\rho(A, E, M, T_z) = \frac{1}{12\sqrt{2}}g\langle m^2 \rangle^{-\frac{1}{2}} (gE - 2T_z^2 - \frac{M^2}{2\langle m^2 \rangle})^{-\frac{3}{2}} \exp\{2[\frac{1}{6}\pi^2(gE - 2T_z^2 - \frac{M^2}{2\langle m^2 \rangle})^{\frac{1}{2}}\} (10)$$

where g is the total single particle level density. The density of states with given total isospin T, excitation energy E, and Z-component of angular momentum M is then given by

$$\rho(A, E, M, T) = \rho(A, E, M, T_z = T) - \rho(A, E, M, T_z = T + 1) \cong \left(-\frac{\partial}{\partial T_z}\rho(A, E, M, T_z)\right)_{T_z = T + \frac{1}{2}}$$
(11)

Similarly, the density of states with given total angular momentum J, excitation energy E and total isospin T may be given by

$$\rho(A, E, J, T) = \rho(A, E, M = J, T) - \rho(A, E, M = J + 1, T) \cong \left(-\frac{\partial}{\partial M}\rho(A, E, M, T)\right)_{M = J + \frac{1}{2}} (12)$$

In deriving eqn. (10) it is assumed that

$$gE > 2T_z^2 + \frac{M^2}{2\langle m^2 \rangle} \tag{13}$$

However, it should be noted that the eqn. (13) imposes some limitations on the values of M and T_z . By considering the eqns. (11, 12) these limitations would change the values of total isospin T and total angular momentum J. Thus, the excitation energy of the system is obtained in the form of

$$E = \frac{\pi^2}{6\beta^2}g + \frac{M^2}{2g\langle m^2 \rangle} + \frac{2T_z^2}{g}$$
(14)

J	0	1	2	3	4
Number of levels	13	21	18	10	3

Table 1. The number of levels and their distribution among various values of J.

Eqn. (14) represents the effect of isospin on excitation energy of the system. Since the states with different total isospin have different excitation energies, this formula is not a complete one. A perfect excitation energy formula, however, must be dependent on the total isospin. Jänecke has used such a formula to demonstrate the dependence of the excitation energy on the total isospin and also Z-component of isospin [7]. The results obtained by Jänecke were used in the current study for a more precise estimation of the excitation energy.

3. Calculation

Hibdon has presented a distribution of angular momentum of ²⁸Al states [8]. By examining total neutron cross section from 1keV to 450keV for neutron capture reaction ${}^{27}Al(n,\gamma){}^{28}Al$. The number of levels assigned to each value of the spin J is given in Table 1. The average value of the excitation energy is 7.95MeV and the experimentally determined density of all levels is $\rho({}^{28}Al, E = 7.95MeV) = 146MeV^{-1}$. The density of states, as a function of total angular momentum J and total isospin T, is given by eqn. (12). For light and medium nuclei it is reasonable to assume that $g_n \approx g_p$ and this equation would contain only three parameters g, $\langle m^2 \rangle$ and T. In this reaction, by considering the isospin conservation, the isospin of the states could be assumed T = 1. By fitting the calculated density states by experimental data the values of g = 2.71 and $\langle m^2 \rangle = 0.71$ were obtained with an error less than values suggested by Kanestrøm [5], who obtained g = 2.55 and $\langle m^2 \rangle = 0.65$. In Figure 1 and Figure 2, the calculated NLD fitted by experimental data are plotted. These Figures demonstrate the state density of ${}^{28}Al$ calculated in the present study and by Kanestrøm [5], respectively, both based on the experimental values reported by Hibdon [8].



Figure 1. The state density of ${}^{28}Al$ as a function of total angular momentum J. Experimental points shown by solid circles are taken from Hibdon [8]. The curve represents the plot of eq. (12) for g = 2.71 and $\langle m^2 \rangle = 0.71$.



Figure 2. The state density of ${}^{28}Al$ as a function of total angular momentum J. Experimental points shown by solid circles are taken from Hibdon [8]. The curve represents the plot of eq. (12) for the suggested values by Ref. [5], g = 2.55and $\langle m^2 \rangle = 0.65$.

In this study, the NLD for the isospins T = 1, 2, 3 at excitation energy E = 20 MeV are calculated by using eqns. (10, 11, 12). The results are demonstrated in Figure 3. At this excitation energy the NLD for isospin $T \ge 4$ is not a real number and the condition (13) cannot be satisfied which is supported by the energies of the ²⁸Al states. The energy levels of ²⁸Al show that its ground state has isospin T = 1 and the lowest state with isospin T = 2 occurs at excitation energy E = 5.989 MeV [9]. By using the pairing and symmetry energies, the total energies of excited nuclei for $A \le 80$ are presented [7]. The energy difference between the lowest excited states with isospins T and T', $\Delta_{T,T'}$ is obtained. For ²⁸Al, $\Delta_{3,2}$ and $\Delta_{4,1}$ are about 15 and 40 MeV, respectively. Therefore, it is expected that the lowest states with T = 3 and T = 4 occur at excitation energies 20 and 40 MeV, respectively. Therefore, the maximum Tat excitation energy E = 20 MeV was found to be 3 which is also in good agreement with our calculations.



Figure 3. The calculated level density of ${}^{28}Al$ as a function of the isospin T and the angular momentum J, at the excitation energy E = 20 MeV.

It is noteworthy that fitting the parameters g, $\langle m^2 \rangle$ and T, gave rise to the values of 2.71, 0.71 and 1 for these parameters, respectively. This indicates that excitation states of this resonance may be at T = 1 which is in agreement with isospin conservation.

4. Results and discussion

This study attempts to outline some features of calculated average level densities. The theory is applied to a light nucleus, ${}^{28}Al$. Here every attempt has been done to increase the accuracy of the calculations. So it would lead to the increase of single-particle level density parameter from 2.55 to 2.71 and $\langle m^2 \rangle$ from 0.65 to 0.71. The obtained excitation energy formula contains the new term that represents the dependence of excitation energy on the z-component of the isospin. Although the NLD formula obtained in this study suggests interesting properties, this method does not lead to the most perfect formula for excitation energy of the system. The presented method determines the maximum isospin that can be found in the desired excitation energy. Furthermore, this method can estimate the isospin of excited energies by putting the isospin as the third parameter to fit with the experimental results.

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