

Generalized Galileons: instabilities of bouncing and Genesis cosmologies and modified Genesis

M. Libanov,^{a,b} S. Mironov^a and V. Rubakov^{a,c}

^aInstitute for Nuclear Research of the Russian Academy of Sciences,
60th October Anniversary Prospect 7a, 117312 Moscow, Russia

^bMoscow Institute of Physics and Technology,
Institutskii per. 9, 141700 Dolgoprudny, Moscow Region, Russia

^cDepartment of Particle Physics and Cosmology, Physics Faculty,
M.V. Lomonosov Moscow State University,
Vorobjevy Gory, 119991, Moscow, Russia

E-mail: ml@ms2.inr.ac.ru, sa.mironov.1@physics.msu.ru, rubakov@ms2.inr.ac.ru

Received June 26, 2016

Accepted August 12, 2016

Published August 18, 2016

Abstract. We study spatially flat bouncing cosmologies and models with the early-time Genesis epoch in a popular class of generalized Galileon theories. We ask whether there exist solutions of these types which are free of gradient and ghost instabilities. We find that irrespectively of the forms of the Lagrangian functions, the bouncing models either are plagued with these instabilities or have singularities. The same result holds for the original Genesis model and its variants in which the scale factor tends to a constant as $t \rightarrow -\infty$. The result remains valid in theories with additional matter that obeys the Null Energy Condition and interacts with the Galileon only gravitationally. We propose a modified Genesis model which evades our no-go argument and give an explicit example of healthy cosmology that connects the modified Genesis epoch with kination (the epoch still driven by the Galileon field, which is a conventional massless scalar field at that stage).

Keywords: alternatives to inflation, initial conditions and eternal universe, cosmic singularity

ArXiv ePrint: [1605.05992](https://arxiv.org/abs/1605.05992)



Contents

1	Introduction and summary	1
2	Generalities	3
3	Bouncing Universe and original Genesis: no-go	5
4	Modified Genesis	6
4.1	Early-time evolution	6
4.2	From Genesis to kination: an example	7
4.3	Curvaton	12
5	Conclusion	12
A	Galileon and its perturbations	13

1 Introduction and summary

Bouncing and Genesis cosmologies are interesting scenarios alternative or complementary to inflation. Both require the violation of the Null Energy Condition¹ (NEC), and hence fairly unconventional matter. Candidates for the latter are generalized Galileons [2–12], scalar fields whose Lagrangians involve second derivatives, and whose field equations are nevertheless second order (for a review see, e.g., ref. [13]). Indeed, in the original Genesis model [14] as well as in its variants [15–22], the initial super-accelerating stage can occur without ghosts and gradient instabilities (although there is still an issue of superluminality [23, 24]). Likewise, bouncing Universe models with generalized Galileons can be arranged in such a way that no ghost or gradient instabilities occur at and near the bounce [25–28].

The situation is not so bright in more complete cosmological models. Known models of the bouncing Universe, employing generalized Galileons, are in fact plagued by the gradient instabilities, provided one follows the evolution for long enough time [29–33]. Gradient instabilities occur also in the known Genesis models, once one requires that the early Genesis regime turns into more conventional expansion (inflationary or not) at later times [20, 35, 36]. An intriguing exception is the model [20] in which Genesis-like super-accelerated expansion starts from the de Sitter, rather than Minkowski, epoch. We comment on this model in section 3.

One way to get around the gradient instability problem is to arrange the model in such a way that the quadratic in spatial gradients, wrong sign term in the action is small, and higher derivative terms restore stability at sufficiently high spatial momenta [20, 34]. There is also a possibility that the strong coupling momentum scale is low enough [33]. In both cases the exponential growth of trustworthy perturbations does not have catastrophic consequences, provided that the time interval at which the instability operates is short enough. Another option is to introduce extra terms in the action which are not invariant under general coordinate transformations [32, 35].

¹An exception is bounce of a closed Universe [1].

Clearly, it is of interest to understand whether gradient or ghost instabilities are inherent in all “complete” bouncing models and Genesis models with initial Minkowski space, which are based on classical generalized Galileons and General Relativity, or these instabilities are merely drawbacks of concrete models constructed so far. In the latter case, it is worth designing examples in which the gradient and ghost instabilities are absent.

It is this set of issues we address in this paper. We consider the simplest and best studied generalized Galileon theory interacting with gravity. The Lagrangian is (mostly negative signature; $\kappa = 8\pi G$)

$$L = -\frac{1}{2\kappa}R + F(\pi, X) + K(\pi, X)\square\pi, \quad (1.1)$$

where π is the Galileon field, F and K are smooth Lagrangian functions, and

$$X = \nabla_\mu\pi\nabla^\mu\pi, \quad \square\pi = \nabla_\mu\nabla^\mu\pi.$$

We also allow for other types of matter, assuming that they interact with the Galileon only gravitationally and obey the NEC:

$$\rho_M + p_M \geq 0. \quad (1.2)$$

To see that our observations are valid in any dimensions, we study this theory in $(d+1)$ space-time dimensions with $d \geq 3$; the case of interest is of course $d = 3$. We consider spatially flat FLRW Universe with the scale factor $a(t)$ where t is the cosmic time, and study spatially homogeneous backgrounds $\pi(t)$.

Our framework is quite general. In the Genesis case we require that neither $a(t)$ nor $\pi(t)$ has future singularity (i.e., $a(t)$, $\pi(t)$ and their derivatives are finite for all $-\infty < t < +\infty$). Our definition of the bouncing Universe is that the scale factor $a(t)$ either is constant in the past and future, $a(t) \rightarrow a_\mp$ as $t \rightarrow \mp\infty$, or diverges in one or both of the asymptotics (i.e., $a_- = \infty$ or/and $a_+ = \infty$), and that there is no singularity in between.

Somewhat surprisingly, our results for the bouncing and Genesis scenarios are quite different. In the bouncing Universe case, we show that the gradient (or ghost) instability is inevitable. This result is a cosmological counterpart of the observation that a static, spherically symmetric Lorentzian wormhole supported by the generalized Galileon always has the ghost or gradient instability [37] (see also ref. [38]); the technicalities involved are also similar.

Analogous no-go theorem does not hold in the Genesis case. Yet the requirement of the absence of the gradient and ghost instabilities strongly constrains the Galileon theories (i.e., Lagrangian functions F and K). In particular, the gradient or ghost instability (or future singularity) does exist, if the initial stage is the original Genesis [14] or its versions in which $a(t) \rightarrow \text{const}$ as $t \rightarrow -\infty$, which is the case, e.g., in the subluminal Genesis [15] as well as in the DBI [16, 17] and generalized Genesis [21, 22] models in which the Lagrangians have the general form² (1.1) (in the language of ref. [16, 17], the Lagrangians from this subclass do not contain terms \mathcal{L}_4 and \mathcal{L}_5).

Equipped with better understanding of the instabilities in the Genesis models with generalized Galileons, we propose a modified Genesis behavior in which the space-time curvature, energy and pressure vanish as $t \rightarrow -\infty$ and which is not inconsistent with the absence of the

²The reservation here has to do with the fact that we merely leave more complicated models aside. It is worth seeing whether our result can be generalized to all Horndeski-like Lagrangians.

gradient and ghost instabilities and the absence of future singularity. The pertinent Galileon Lagrangian is similar to ones considered in refs. [19, 21, 22]; in particular, the action is not scale-invariant. Starting from this Lagrangian, we give an example of a “complete” model, with Genesis at the initial stage and kination (the epoch still driven by the Galileon field which, however, is a conventional massless scalar field at that stage) at later times. This model is free of the gradient instabilities, ghosts and superluminal propagation about the homogeneous solution, while the kination stage may possibly be connected to the radiation domination epoch via, e.g., gravitational particle creation, cf. ref. [39].

This paper is organized as follows. In section 2 we discuss, in general terms, the conditions for the absence of the gradient and ghost instabilities in the cosmological setting. We show in section 3 that irrespectively of the forms of the Lagrangian functions $F(\pi, X)$ and $K(\pi, X)$, these conditions cannot be satisfied in the bouncing Universe scenario as well as in the Genesis models with time-independent past asymptotics of the scale factor. We propose a modified Genesis model in section 4, where we first study general properties and concrete example of early Genesis-like epoch which evades the no-go argument of section 3, then give an explicit example of healthy model connecting Genesis and kination and, finally, briefly discuss a spectator field whose perturbations may serve as seeds of the adiabatic perturbations. We conclude in section 5. For completeness, the general expressions for the Galileon energy-momentum tensor and quadratic Lagrangian of the Galileon perturbations are given in appendix.

2 Generalities

The general expression for the Galileon energy-momentum tensor is given in appendix, eq. (A.1). In the cosmological context the energy density and pressure are

$$\rho = 2F_X X - F - K_\pi X + 2dHK_X \dot{\pi}^3, \quad (2.1a)$$

$$p = F - 2K_X X \ddot{\pi} - K_\pi X, \quad (2.1b)$$

where H is the Hubble parameter and

$$X = \dot{\pi}^2.$$

Hereafter $F_\pi = \partial F / \partial \pi$, $F_X = \partial F / \partial X$, etc.

Our main concern is the Galileon perturbations χ of high momentum and frequency. The general expression for the effective quadratic Lagrangian for perturbations is again given in appendix, eq. (A.3). For homogeneous background we obtain

$$L^{(2)} = A\dot{\chi}^2 - \frac{1}{a^2}B(\partial_i \chi)^2 + \dots \quad (2.2)$$

where

$$A = F_X + 2F_{XX}X - K_\pi - K_{X\pi}X + 2dH\dot{\pi}(K_X + K_{XX}X) + \frac{2d}{d-1}\kappa K_X^2 X^2, \quad (2.3a)$$

$$B = F_X - K_\pi + 2K_X \ddot{\pi} + K_{X\pi}X + 2K_{XX}X \ddot{\pi} + 2(d-1)HK_X \dot{\pi} - \frac{2(d-2)}{d-1}\kappa K_X^2 X^2, \quad (2.3b)$$

and terms omitted in (2.2) do not contain second derivatives of χ . These terms are irrelevant for high momentum modes. The absence of ghosts and gradient instabilities requires $A > 0$, $B \geq 0$. In particular, if $B < 0$, there are ghosts (for $A < 0$) or gradient instability (for $A > 0$).

Our focus is on the coefficient B . Despite appearance, it can be cast in a simple form. To this end we make use of the Friedmann and covariant conservation equations

$$H^2 = \frac{2}{d(d-1)}\kappa(\rho + \rho_M) \quad (2.4a)$$

$$\dot{\rho} = -d \cdot H(\rho + p) \quad (2.4b)$$

$$\dot{\rho}_M = -d \cdot H(\rho_M + p_M) \quad (2.4c)$$

and hence

$$\dot{H} = -\frac{1}{d-1}\kappa[(\rho + p) + (\rho_M + p_M)]. \quad (2.5)$$

Here ρ and p are Galileon energy density and pressure, while ρ_M and p_M are energy and pressure of conventional matter, if any. The latter obey the NEC, eq. (1.2). We recall that we assume that conventional matter does not interact with the Galileon directly, so the covariant conservation equations (2.4b) and (2.4c) have to be satisfied separately.

Equations (2.1), (2.3b) and (2.5) lead to a remarkable relation

$$2BX = \frac{d}{dt} \left(2K_X \dot{\pi}^3 - \frac{d-1}{\kappa} H \right) - \frac{2(d-2)}{d-1} \kappa K_X \dot{\pi}^3 \left(2K_X \dot{\pi}^3 - \frac{d-1}{\kappa} H \right) - (\rho_M + p_M).$$

It is natural to introduce a combination

$$Q = 2K_X \dot{\pi}^3 - \frac{d-1}{\kappa} H \quad (2.6)$$

and write

$$2BX = \dot{Q} - \frac{2(d-2)}{d-1} \kappa K_X \dot{\pi}^3 Q - (\rho_M + p_M). \quad (2.7)$$

Another representation is in terms of the function

$$R = \frac{Q}{a^{d-2}},$$

namely

$$\frac{2BX}{a^{d-2}} = \dot{R} - \frac{d-2}{d-1} \kappa a^{d-2} R^2 - \frac{\rho_M + p_M}{a^{d-2}}.$$

Since we assume that the conventional matter, if any, obeys the NEC, the positivity of B requires

$$\dot{Q} - \frac{2(d-2)}{d-1} \kappa K_X \dot{\pi}^3 Q \geq 0 \quad (2.8)$$

and

$$\dot{R} - \frac{d-2}{d-1} \kappa a^{d-2} R^2 \geq 0. \quad (2.9)$$

As we now see, these requirements are prohibitively restrictive in the bouncing Universe case and place strong constraints on the Genesis models.

3 Bouncing Universe and original Genesis: no-go

We now show that the inequality (2.9) cannot be satisfied in the bouncing Universe scenario. We write it as follows,

$$\frac{\dot{R}}{R^2} \geq \frac{d-2}{d-1} \kappa a^{d-2}$$

and integrate from t_i to $t_f > t_i$:

$$\frac{1}{R(t_i)} - \frac{1}{R(t_f)} \geq \frac{d-2}{d-1} \kappa \int_{t_i}^{t_f} dt a^{d-2}. \quad (3.1)$$

Suppose now that $R(t_i) > 0$. Since $\dot{R} > 0$ in view of (2.9), R increases in time and remains positive. We have

$$\frac{1}{R(t_f)} \leq \frac{1}{R(t_i)} - \frac{d-2}{d-1} \kappa \int_{t_i}^{t_f} dt a^{d-2}. \quad (3.2)$$

Since $a(t)$ is either a constant or growing function of t at large t , the right hand side of the latter inequality eventually becomes negative at large t_f . Thus $R^{-1}(t_f)$ as function of t_f starts positive (at $t_f = t_i$) and necessarily crosses zero. At that time $R^{-1} = 0$, and $R = \infty$, which means a singularity.

A remaining possibility is that $R(t)$ is negative at all times. In particular, $R(t_f) < 0$. In that case a useful form of the inequality (3.1) is

$$\frac{1}{R(t_i)} \geq \frac{1}{R(t_f)} + \frac{d-2}{d-1} \kappa \int_{t_i}^{t_f} dt a^{d-2}. \quad (3.3)$$

Now, $a(t)$ is either a constant or tends to infinity as $t \rightarrow -\infty$, so the right hand side is positive at large negative t_i . Hence, there is again a singularity $R = \infty$ at $t_i < t < t_f$. This completes the argument.

The same argument applies to the original Genesis model [14] and many of its versions, like subluminal Genesis [15] and the DBI Genesis [16, 17], provided the Lagrangian has the general form (1.1). In these versions, the scale factor tends to a constant as $t \rightarrow -\infty$ and, assuming that the Universe ends up in the conventional expansion regime, the scale factor grows at large times. The integral in eq. (3.1) blows up at large t_f or large negative t_i , so the inequalities (3.2), (3.3) are impossible to satisfy without hitting the singularity³ $R = \infty$. In fact, in the models of refs. [14–17], one has $Q > 0$, which is consistent with healthy behavior at early times but implies either gradient (or ghost) instability or singularity in future.

At this point let us make contact with the model of ref. [20] in which the Genesis-like super-accelerated expansion starts from the de Sitter rather than Minkowski epoch, $d = 3$, $a \propto e^{\lambda t}$. In that case the integral in (3.3) is convergent as $t_i \rightarrow -\infty$. Hence, our argument does not work: one can have $R < 0$ at all times, leaving a room for the stable evolution. In fact, in the model of ref. [20], our parameter Q defined in (2.6) is constant in time and negative, while $B > 0$ in full accordance with (2.7). We generalize this construction in section 4.

³Note that if there is an initial singularity, there is no argument that would forbid Q to be negative and increasing towards zero at early times, cross zero at some intermediate time and continue to increase later on. This is what happens in the setup [18] where the NEC is satisfied at early times and is violated later on in the Genesis phase. The inequality (3.2) shows, however, that in that case there is either gradient (or ghost) instability or future singularity after Q crosses zero.

4 Modified Genesis

4.1 Early-time evolution

In this section we construct a model which interpolates between a stage similar to Genesis (in the sense that space-time curvature, energy and pressure vanish as $t \rightarrow -\infty$) and kination epoch at which the Galileon behaves as a conventional massless scalar field. The model is purely classical and does not have gradient or ghost instability at any time. We begin with the early Genesis-like stage, having in mind the observations made in section 3.

Since we would like the scale factor to increase at late times, and in view of the inequality (3.2), we require that at the Genesis-like stage $R < 0$ and hence

$$Q < 0.$$

This means that

$$H > \frac{2\kappa}{d-1} K_X \dot{\pi}^3. \quad (4.1)$$

Furthermore, the second term in the right hand side of eq. (2.8) must be larger than $|\dot{Q}|$ at the Genesis-like epoch: since H increases at that epoch from originally zero value, so does $|Q|$ (barring cancelations), and we have $\dot{Q} < 0$. Thus, besides the inequality (4.1) we require

$$\frac{2(d-2)}{d-1} \kappa K_X \dot{\pi}^3 |Q| > |\dot{Q}|. \quad (4.2)$$

Assuming power law behavior of Q , we see that the simplest option is that at large negative times

$$K_X \dot{\pi}^3 \propto (-t)^{-1}, \quad t \rightarrow -\infty. \quad (4.3)$$

From eq. (4.1) we deduce that H cannot rapidly tend to zero as $t \rightarrow -\infty$; we can only have

$$H = -\frac{h}{t}, \quad a(t) \propto \frac{1}{(-t)^h}, \quad h = \text{const}, \quad t \rightarrow -\infty, \quad (4.4)$$

in contrast to the conventional Genesis, in which $H \propto (-t)^{-3}$. Thus *both* energy density and pressure should behave like t^{-2} as $t \rightarrow -\infty$ (while in the original Genesis one has $p \propto t^{-4}$, $\rho \propto t^{-6}$).

Now, the no-go argument based on (3.3) is not valid provided that the integral in the right hand side is convergent at the lower limit of integration,⁴

$$\int_{-\infty}^t dt a^{d-2} < \infty. \quad (4.5)$$

Thus, we require that

$$h > \frac{1}{d-2}. \quad (4.6)$$

These are the general properties of the Genesis-like stage which is potentially consistent with the overall healthy dynamics. Note that unlike in the original Genesis scenario, the scale

⁴For the case of interest $d = 3$ (four space-time dimensions), eq. (4.5) implies that space-time is past-incomplete in the sense that past-directed null geodesics reach spatial infinity $a(t)|_{\mathbf{x}} = \infty$ at finite value of the affine parameter (and past-directed time-like geodesics reach spatial infinity in finite proper time), cf. ref. [41]. We leave this issue open in this paper.

factor does not tend to a constant as $t \rightarrow -\infty$. Yet the geometry tends to Minkowski at large negative times in the sense that the space-time curvature tends to zero, and so do the energy density and pressure.

One way to realize this scenario is to choose the Lagrangian functions in the following form

$$F = -f^2(\partial\pi)^2 + \alpha_0 e^{-2\pi}(\partial\pi)^4 \quad (4.7a)$$

$$K = \beta_0 e^{-2\pi}(\partial\pi)^2. \quad (4.7b)$$

The resulting Lagrangian is similar to those introduced in ref. [21, 22], although the particular exponential dependence that we have in (4.7) was not considered there. There is a solution

$$e^\pi = -\frac{1}{H_* t} \quad (4.8a)$$

$$H = -\frac{h}{t}, \quad t < 0, \quad (4.8b)$$

with time-independent H_* and h ; we relate them to the parameters α_0 , β_0 and f shortly. For this solution

$$Q = -\frac{1}{t} \left(2\beta_0 H_*^2 - \frac{d-1}{\kappa} h \right)$$

and

$$2BX = \frac{1}{t^2} \left(2\beta_0 H_*^2 - \frac{d-1}{\kappa} h \right) \left(1 - 2\frac{d-2}{d-1} \kappa \beta_0 H_*^2 \right).$$

Thus, one has $Q < 0$ and no gradient instability at early times ($B > 0$), provided that

$$\frac{d-1}{2\kappa(d-2)} < \beta_0 H_*^2 < \frac{d-1}{2\kappa} h. \quad (4.9)$$

Note that with this choice of parameters, the inequality (4.6) is satisfied, as it should.

The parameters H_* and h are related to the parameters of the Lagrangian via the field equations (2.4). Two independent combinations of these equations are

$$2\alpha_0 H_*^2 = \frac{d(d-1)}{\kappa} h^2 + \frac{(d-1)}{\kappa} h - 2\beta_0 H_*^2 - 2dh\beta_0 H_*^2 \quad (4.10a)$$

$$f^2 = \frac{d(d-1)}{\kappa} h^2 + \frac{3(d-1)}{2\kappa} h - \beta_0 H_*^2 - dh\beta_0 H_*^2 \quad (4.10b)$$

Using these relations and inequalities (4.9) one can check that $f^2 > 0$ and, importantly, the coefficient of the kinetic term in the Lagrangian for perturbations is positive, $A > 0$. The modified Genesis regime is stable.

4.2 From Genesis to kination: an example

A scenario for further evolution is as follows. The function $\pi(t)$ is monotonous, $\dot{\pi} > 0$. However, the Lagrangian functions F and K depend on π and hence on time in a non-trivial way. The variable Q remains negative at all times, but eventually (at $t = t_c$) \dot{Q} changes sign and Q starts to increase towards zero. Choosing $K_X > 0$, we find that at $t > t_c$ the stability condition $B > 0$ is satisfied trivially,

$$2BX = \dot{Q} - \frac{2(d-2)}{d-1} \kappa K_X \dot{\pi}^3 Q > 0, \quad t > t_c$$

One should make sure, however, that at $t < t_c$ the inequality (4.2) is always satisfied. Another point to check is that the coefficient of the kinetic term in the Lagrangian for perturbations is positive, $A > 0$, at all times.

To cook up a concrete example of a model in which the Genesis regime is smoothly connected to kination (the regime at which Galileon is a conventional massless scalar field dominating the cosmological expansion), the simplest way is to introduce a field

$$\phi = \phi(\pi)$$

in such a way that the solution is

$$\phi = t. \tag{4.11}$$

The general formulas of sections 2, 3 remain valid, with understanding that the Lagrangian functions are now functions of ϕ and $X = (\partial\phi)^2$; in particular, the combination Q is the same as in (2.6) with ϕ substituted for π . We choose the Lagrangian functions in the following form:

$$\begin{aligned} F &= -v(\phi)X + \alpha(\phi)X^2, \\ K &= \beta(\phi)X. \end{aligned}$$

On the solution (4.11) one has

$$F = -v(t) + \alpha(t), \quad F_X = -v(t) + 2\alpha(t), \quad K = K_X = \beta(t).$$

One way to proceed is to postulate suitable forms of $Q(t) < 0$ and $\beta(t)$, such that the inequality (4.2) is satisfied, evaluate $H = \frac{\kappa}{d-1}(2\beta - Q)$, reconstruct $v(t)$ and $\alpha(t)$ from the field equations and then check that $A > 0$ at all times. With the convention (4.11), once $v(t)$, $\alpha(t)$ and $\beta(t)$ are known, the Lagrangian functions are also known, $v(\phi) = v(t = \phi)$, etc.

As we anticipated in eqs. (4.3), (4.4), the initial behavior is

$$\begin{aligned} Q &= -\frac{\hat{q}}{t}, \quad \hat{q} < 0 \\ \beta &= -\frac{\hat{\beta}}{t}, \quad t \rightarrow -\infty \end{aligned}$$

with time-independent \hat{q} and $\hat{\beta}$. To satisfy the inequality (2.8), we impose the condition

$$\hat{\beta} > \frac{d-1}{2\kappa(d-2)}.$$

We would like the Galileon to become a conventional massless scalar field at large positive t , whose equation of state is $p = \rho$, and require that at large t the function $\beta(t)$ rapidly vanishes, while $H = (d \cdot t)^{-1}$, and hence

$$Q = -\frac{d-1}{d\kappa t}, \quad t \rightarrow +\infty.$$

It is convenient to introduce rescaled variables

$$\begin{aligned} Q &= -\frac{d-1}{d\kappa}P, \\ \beta &= \frac{d-1}{d\kappa}b, \end{aligned}$$

where P is positive. In terms of these variables the Hubble parameter is

$$H = \frac{\kappa}{d-1}(2K_X\dot{\phi}^3 - Q) = \frac{1}{d}(2b + P).$$

The combinations of the field equations, analogous to eqs. (4.10), give

$$\begin{aligned} \alpha &= -dH\beta + \frac{d(d-1)}{2\kappa}H^2 + \frac{d-1}{2\kappa}\dot{H} \\ &= \frac{d-1}{d\kappa} \left(bP + \frac{1}{2}P^2 + \dot{b} + \frac{1}{2}\dot{P} \right), \end{aligned} \quad (4.12a)$$

$$\begin{aligned} v &= -\dot{\beta} - dH\beta + \frac{d(d-1)}{\kappa}H^2 + \frac{3(d-1)}{2\kappa}\dot{H} \\ &= \frac{d-1}{d\kappa} \left(2b^2 + 3bP + P^2 + 2\dot{b} + \frac{3}{2}\dot{P} \right). \end{aligned} \quad (4.12b)$$

Finally, the coefficients in the Lagrangian for perturbations are

$$\begin{aligned} A &= \frac{d-1}{d\kappa} \left(4b^2 + 5bP + 2P^2 + 2\dot{b} + \frac{3}{2}\dot{P} \right), \\ B &= \frac{d-1}{d\kappa} \left(\frac{d-2}{d}bP - \frac{1}{2}\dot{P} \right). \end{aligned}$$

The asymptotics of the solution should be

$$\begin{aligned} t \rightarrow -\infty : \quad P &= -\frac{p_0}{t}, \quad p_0 > 0, \\ b &= -\frac{b_0}{t}, \quad b_0 > \frac{d}{2(d-2)}, \end{aligned} \quad (4.13a)$$

$$\begin{aligned} t \rightarrow +\infty : \quad P &= \frac{1}{t}, \\ b &= 0. \end{aligned} \quad (4.13b)$$

As a cross check, the late-time asymptotics (4.13b) imply $\alpha = 0$ and

$$v = -\frac{d-1}{2d\kappa} \cdot \frac{1}{t^2} < 0,$$

which corresponds to the Lagrangian of free massless scalar field, albeit written in somewhat unconventional form,

$$L_\phi^{t \rightarrow +\infty} = \frac{d-1}{2d\kappa} \frac{(\partial\phi)^2}{\phi^2}. \quad (4.14)$$

On the other hand, the early-time asymptotics (4.13a), according to eq. (4.12), give

$$\begin{aligned} \alpha &= \frac{c_1}{t^2}, \quad c_1 > 0, \\ v &= \frac{c_2}{t^2}, \quad c_2 > 0, \end{aligned}$$

so that the Lagrangian at early times ($\phi \rightarrow -\infty$) reads

$$L_\phi^{t \rightarrow -\infty} = -\frac{c_1}{\phi^2}(\partial\phi)^2 + \frac{c_2}{\phi^2}(\partial\phi)^4 - \frac{\hat{\beta}}{\phi}(\partial\phi)^2\Box\phi.$$

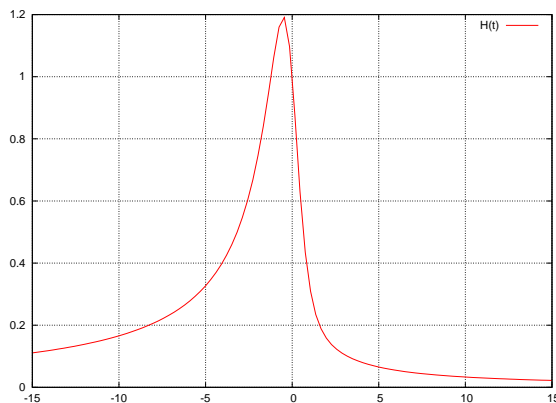


Figure 1. Hubble parameter in units of τ^{-1} as function of t/τ for the model of eqs. (4.15), (4.16).

Upon introducing new field at early times

$$\pi = -\ln(-\phi) ,$$

one writes the Lagrangian in the following form

$$L_{\pi}^{t \rightarrow -\infty} = -c_1(\partial\pi)^2 + (c_2 - \hat{\beta})e^{-2\pi}(\partial\pi)^4 + \hat{\beta}e^{-2\pi}(\partial\pi)^2\Box\pi .$$

This is precisely the form (4.7).

One more point to note is that if the parametric form of $b(t)$ and $P(t)$ is

$$b = \tau^{-1}\tilde{b}(t/\tau) , \quad P = \tau^{-1}\tilde{P}(t/\tau) ,$$

with \tilde{b} and \tilde{P} of order 1 (which is consistent with the asymptotics (4.13)), then for large τ the dynamics is sub-Planckian during entire evolution: in that case one has sub-Planckian $H \sim \tau^{-1}$, $\alpha \sim \kappa^{-1}\tau^{-2}$, etc.

A random example is

$$P = \frac{1}{\sqrt{t^2 + \tau^2}} , \tag{4.15a}$$

$$b = \frac{b_0}{\sqrt{t^2 + \tau^2}} \cdot \frac{1}{2} \left[1 - \text{th} \left(\mu \frac{t}{\tau} \right) \right] . \tag{4.15b}$$

With

$$d = 3 , \quad b_0 = 2 , \quad \mu = 1.1 \tag{4.16}$$

((3 + 1)-dimensional space-time) the system smoothly evolves from the modified Genesis regime to kination (free massless scalar field) regime with $A(t) > 0$ and $B(t) > 0$ for all t , and also with subluminal propagation of perturbations about this background (the latter property explains the choice $\mu = 1.1$: for $\mu = 1$, say, there is a brief time interval in which the perturbations propagate superluminally). The properties of this model are illustrated in figures 1–4. It is worth noting that although $A(t)$ and $B(t)$ at late times appear different in figure 2, they are actually the same,

$$A(t) = B(t) = \frac{1}{3\kappa t^2} , \quad t \rightarrow +\infty .$$

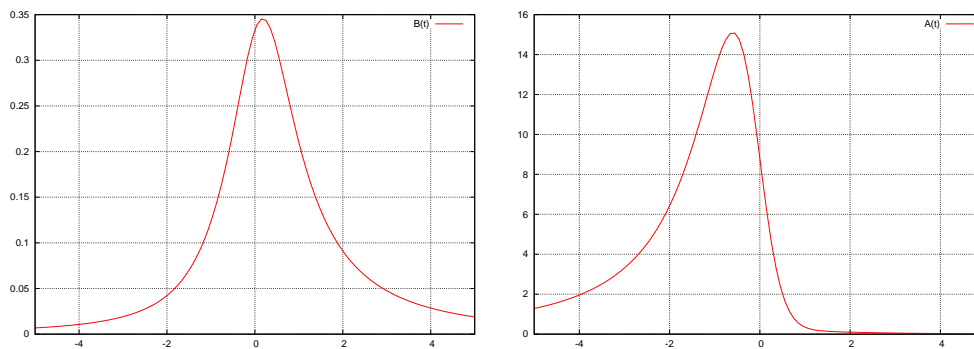


Figure 2. Left: the function B in units $\frac{2}{3\kappa}\tau^{-2}$ as function of t/τ for the model of eqs. (4.15), (4.16). Right: same for the function A . Note the larger scale as compared to the left panel, which implies small sound speed at early times.

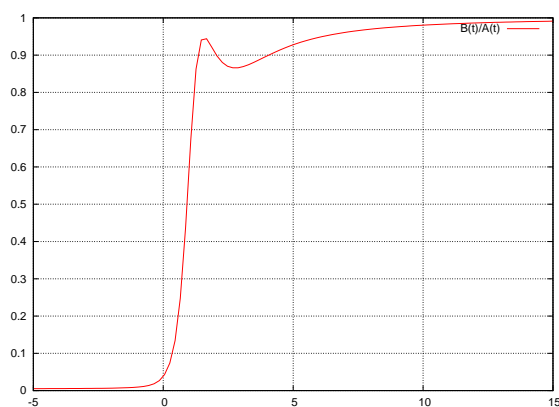


Figure 3. Sound speed squared, B/A , as function of t/τ for the model of eqs. (4.15), (4.16).

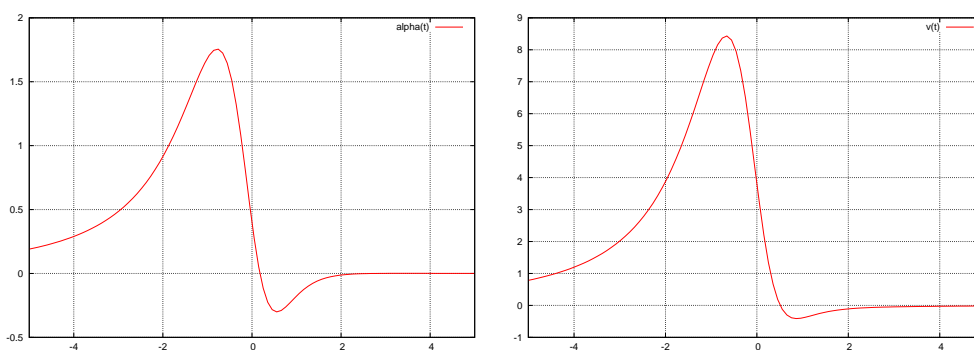


Figure 4. Left: the function α in units $\frac{2}{3\kappa}\tau^{-2}$ as function of t/τ for the model of eqs. (4.15), (4.16). Note that α rapidly vanishes at late times. Right: the same for the function v . Negative sign of v at late times corresponds to conventional sign of the scalar kinetic term.

The fact that these functions tend to zero as $t \rightarrow +\infty$ does not indicate the onset of strong coupling; this behavior rather has to do with the field redefinition from the canonical massless scalar field to the field ϕ described by the Lagrangian (4.14). The late-time theory is the theory of free massless scalar field, with no strong coupling or instabilities.

4.3 Curvaton

It is unlikely that the Galileon perturbations would produce adiabatic perturbations with nearly flat power spectrum. Like in ref. [14], the adiabatic perturbations may originate from perturbations of an additional scalar field, “curvaton”. Let us consider this point, specifying to 4-dimensional space-time, $d = 3$.

We are interested in the early modified Genesis stage when the Galileon Lagrangian functions have the form (4.7). The modified Galileon action is not invariant under the scale transformations

$$\pi(x) \rightarrow \hat{\pi}(x) = \pi(\lambda x) + \ln \lambda, \quad g_{\mu\nu}(x) \rightarrow \hat{g}_{\mu\nu} = g_{\mu\nu}(\lambda x).$$

One has instead $S(\hat{\pi}, \hat{g}_{\mu\nu}) = \lambda^{-2} S(\pi, g_{\mu\nu})$, just like for the Einstein-Hilbert action. Let us introduce a spectator curvaton field θ which is invariant under the scale transformations, $\theta(x) \rightarrow \hat{\theta}(x) = \theta(\lambda x)$, and require that its action be scale-invariant. This requirement gives

$$S_\theta = \int d^4x \sqrt{-g} e^{2\pi} (\partial\theta)^2.$$

In the background (4.8) this action reads

$$S_\theta = \int dt d^3x a^3 \left(\frac{1}{(H_* t)^2} \left(\frac{\partial\theta}{\partial t} \right)^2 - \frac{1}{(H_* t)^2} \frac{1}{a^2} \left(\frac{\partial\theta}{\partial x_i} \right)^2 \right),$$

where the scale factor is given by (4.4). Let us introduce conformal time

$$\eta = \int \frac{dt}{a(t)} = -\frac{1}{a_0(h+1)} (-t)^{h+1}.$$

Then the action for θ has the form

$$S_\theta = \int d\eta d^3x a_{\text{eff}}^2(\eta) \left[\left(\frac{\partial\theta}{\partial\eta} \right)^2 - \left(\frac{\partial\theta}{\partial x_i} \right)^2 \right],$$

where

$$a_{\text{eff}}(\eta) = -\frac{1}{H_*(h+1)\eta},$$

which is precisely the action of the massless scalar field in de Sitter space-time. We immediately deduce that the power spectrum of perturbations $\delta\theta$ generated at the modified Genesis epoch is flat. This is a pre-requisite for the nearly flat power spectrum of adiabatic perturbations, which may be generated from the curvaton perturbations at later stage.

5 Conclusion

We have seen in this paper that with generalized Galileons, it is possible to construct healthy Genesis-like cosmologies, albeit with somewhat different properties as compared to the original Genesis scenario. On the other hand, bouncing cosmologies with generalized Galileons are plagued by the gradient (or ghost) instability, at least at the level of second derivative Lagrangians of the form (1.1). This is not particularly surprising. The theory appears to protect itself [37] from having stable wormhole solutions which can be converted into time machines [40]. Technically the same protection mechanism, with radial coordinate and time interchanged, forbids the existence of spatially flat bouncing cosmologies. It would be interesting to understand how general are these features.

Acknowledgments

The authors are indebted to R. Kolevato, A. Sosnovikov, A. Starobinsky and A. Vikman for useful discussions and J.-L. Lehners and D. Pirtskhalava for helpful correspondence. This work has been supported by Russian Science Foundation grant 14-22-00161.

A Galileon and its perturbations

The Galileon energy-momentum tensor in the theory with the Lagrangian (1.1) reads

$$T_{\mu\nu} = 2F_X \partial_\mu \pi \partial_\nu \pi + 2K_X \square \pi \cdot \partial_\mu \pi \partial_\nu \pi - \partial_\mu K \partial_\nu \pi - \partial_\nu K \partial_\mu \pi - g_{\mu\nu} F + g_{\mu\nu} g^{\lambda\rho} \partial_\lambda K \partial_\rho \pi, \quad (\text{A.1})$$

We now consider Galileon perturbations, write $\pi = \pi_c + \chi$, and omit subscript c in what follows. We are interested in high momentum and frequency modes, so we concentrate on terms involving $\nabla_\mu \chi \nabla_\nu \chi$ in the quadratic effective Lagrangian or, equivalently, second order terms in the linearized field equation. A subtlety here is that the Galileon field equation involves the second derivatives of metric, and the Einstein equations involve the second derivatives of the Galileon [10], and so do the linearized equations for perturbations. The trick is to integrate the metric perturbations out of the Galileon field equation by making use of the Einstein equations [10]. In the cosmological setting this is equivalent to the approach adopted in ref. [11].

To derive the quadratic Lagrangian for the Galileon perturbations, one writes the full Galileon field equation

$$\begin{aligned} &(-2F_X + 2K_\pi - 2K_{X\pi} \nabla_\mu \pi \nabla^\mu \pi - 2K_X \square \pi) \square \pi + (-4F_{XX} + 4K_{X\pi}) \nabla^\mu \pi \nabla^\nu \pi \nabla_\mu \nabla_\nu \pi \\ &- 4K_{XX} \nabla^\mu \pi \nabla^\nu \pi \nabla_\mu \nabla_\nu \pi \square \pi + 4K_{XX} \nabla^\nu \pi \nabla^\lambda \pi \nabla_\mu \nabla_\nu \pi \nabla^\mu \nabla_\lambda \pi + 2K_X \nabla^\mu \nabla^\nu \pi \nabla_\mu \nabla_\nu \pi \\ &+ 2K_X R_{\mu\nu} \nabla^\mu \pi \nabla^\nu \pi + \dots = 0; \end{aligned}$$

hereafter dots denote terms without second derivatives. The subtle term is the last one here.

The linearized equation can be written in the following form

$$\begin{aligned} &-2[F_X + K_X \square \pi - K_\pi + \nabla_\nu (K_X \nabla^\nu \pi)] \nabla_\mu \nabla^\mu \chi \\ &-2[2(F_{XX} + K_{XX} \square \pi) \nabla^\mu \pi \nabla^\nu \pi - 2(\nabla^\mu K_X) \nabla^\nu \pi - 2K_X \nabla^\mu \nabla^\nu \pi] \nabla_\mu \nabla_\nu \chi \\ &+ 2K_X R_{\mu\nu}^{(1)} \nabla^\mu \pi \nabla^\nu \pi + \dots = 0, \quad (\text{A.2}) \end{aligned}$$

where the terms without the second derivatives of χ are omitted, and $R_{\mu\nu}^{(1)}$ is linear in metric perturbations. We now make use of the Einstein equations $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu}$, or

$$R_{\mu\nu} = \kappa \left(T_{\mu\nu} - \frac{1}{d-1} g_{\mu\nu} T^\lambda{}_\lambda \right),$$

linearize the energy-momentum tensor and obtain for the last term in eq. (A.2)

$$2K_X R_{\mu\nu}^{(1)} \nabla^\mu \pi \nabla^\nu \pi = -2\kappa K_X^2 \left[-\frac{2(d-2)}{d-1} X^2 \square \chi + 4X \nabla^\mu \pi \nabla^\nu \pi \nabla_\mu \nabla_\nu \chi \right] + \dots$$

The resulting linearized Galileon field equation is obtained from the following quadratic Lagrangian:

$$\begin{aligned} L^{(2)} &= [F_X + K_X \square \pi - K_\pi + \nabla_\nu (K_X \nabla^\nu \pi)] \nabla_\mu \chi \nabla^\mu \chi \\ &+ [2(F_{XX} + K_{XX} \square \pi) \nabla^\mu \pi \nabla^\nu \pi - 2(\nabla^\mu K_X) \nabla^\nu \pi - 2K_X \nabla^\mu \nabla^\nu \pi] \nabla_\mu \chi \nabla_\nu \chi \\ &- \frac{2(d-2)}{d-1} \kappa K_X^2 X^2 \nabla_\mu \chi \nabla^\mu \chi + 4\kappa K_X^2 X \nabla^\mu \pi \nabla^\nu \pi \nabla_\mu \chi \nabla_\nu \chi. \quad (\text{A.3}) \end{aligned}$$

We specify to spatially homogeneous Galileon background in section 2.

References

- [1] A.A. Starobinsky, *On a nonsingular isotropic cosmological model*, *Sov. Astron. Lett.* **4** (1978) 82 [*Pis'ma Astron. Zh.* **4** (1978) 155].
- [2] G.W. Horndeski, *Second-order scalar-tensor field equations in a four-dimensional space*, *Int. J. Theor. Phys.* **10** (1974) 363 [INSPIRE].
- [3] D.B. Fairlie, J. Govaerts and A. Morozov, *Universal field equations with covariant solutions*, *Nucl. Phys.* **B 373** (1992) 214 [hep-th/9110022] [INSPIRE].
- [4] D.B. Fairlie and J. Govaerts, *Universal field equations with reparametrization invariance*, *Phys. Lett.* **B 281** (1992) 49 [hep-th/9202056] [INSPIRE].
- [5] D.B. Fairlie and J. Govaerts, *Euler hierarchies and universal equations*, *J. Math. Phys.* **33** (1992) 3543 [hep-th/9204074] [INSPIRE].
- [6] M.A. Luty, M. Porrati and R. Rattazzi, *Strong interactions and stability in the DGP model*, *JHEP* **09** (2003) 029 [hep-th/0303116] [INSPIRE].
- [7] A. Nicolis and R. Rattazzi, *Classical and quantum consistency of the DGP model*, *JHEP* **06** (2004) 059 [hep-th/0404159] [INSPIRE].
- [8] A. Nicolis, R. Rattazzi and E. Trincherini, *The Galileon as a local modification of gravity*, *Phys. Rev.* **D 79** (2009) 064036 [arXiv:0811.2197] [INSPIRE].
- [9] C. Deffayet, S. Deser and G. Esposito-Farese, *Arbitrary p-form Galileons*, *Phys. Rev.* **D 82** (2010) 061501 [arXiv:1007.5278] [INSPIRE].
- [10] C. Deffayet, O. Pujolàs, I. Sawicki and A. Vikman, *Imperfect Dark Energy from Kinetic Gravity Braiding*, *JCAP* **10** (2010) 026 [arXiv:1008.0048] [INSPIRE].
- [11] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *G-inflation: Inflation driven by the Galileon field*, *Phys. Rev. Lett.* **105** (2010) 231302 [arXiv:1008.0603] [INSPIRE].
- [12] A. Padilla and V. Sivanesan, *Covariant multi-galileons and their generalisation*, *JHEP* **04** (2013) 032 [arXiv:1210.4026] [INSPIRE].
- [13] V.A. Rubakov, *The Null Energy Condition and its violation*, *Phys. Usp.* **57** (2014) 128 [arXiv:1401.4024] [INSPIRE].
- [14] P. Creminelli, A. Nicolis and E. Trincherini, *Galilean Genesis: An Alternative to inflation*, *JCAP* **11** (2010) 021 [arXiv:1007.0027] [INSPIRE].
- [15] P. Creminelli, K. Hinterbichler, J. Khoury, A. Nicolis and E. Trincherini, *Subluminal Galilean Genesis*, *JHEP* **02** (2013) 006 [arXiv:1209.3768] [INSPIRE].
- [16] K. Hinterbichler, A. Joyce, J. Khoury and G.E.J. Miller, *DBI Realizations of the Pseudo-Conformal Universe and Galilean Genesis Scenarios*, *JCAP* **12** (2012) 030 [arXiv:1209.5742] [INSPIRE].
- [17] K. Hinterbichler, A. Joyce, J. Khoury and G.E.J. Miller, *Dirac-Born-Infeld Genesis: An Improved Violation of the Null Energy Condition*, *Phys. Rev. Lett.* **110** (2013) 241303 [arXiv:1212.3607] [INSPIRE].
- [18] B. Elder, A. Joyce and J. Khoury, *From Satisfying to Violating the Null Energy Condition*, *Phys. Rev.* **D 89** (2014) 044027 [arXiv:1311.5889] [INSPIRE].
- [19] Z.-G. Liu, J. Zhang and Y.-S. Piao, *A Galileon Design of Slow Expansion*, *Phys. Rev.* **D 84** (2011) 063508 [arXiv:1105.5713] [INSPIRE].
- [20] D. Pirtskhalava, L. Santoni, E. Trincherini and P. Uttayarat, *Inflation from Minkowski Space*, *JHEP* **12** (2014) 151 [arXiv:1410.0882] [INSPIRE].

- [21] S. Nishi and T. Kobayashi, *Generalized Galilean Genesis*, *JCAP* **03** (2015) 057 [[arXiv:1501.02553](#)] [[INSPIRE](#)].
- [22] S. Nishi and T. Kobayashi, *Reheating and Primordial Gravitational Waves in Generalized Galilean Genesis*, *JCAP* **04** (2016) 018 [[arXiv:1601.06561](#)] [[INSPIRE](#)].
- [23] D.A. Easson, I. Sawicki and A. Vikman, *When Matter Matters*, *JCAP* **07** (2013) 014 [[arXiv:1304.3903](#)] [[INSPIRE](#)].
- [24] R. Kolevatov, *Superluminality in dilatationally invariant generalized Galileon theories*, *Phys. Rev. D* **92** (2015) 123532 [[arXiv:1508.00046](#)] [[INSPIRE](#)].
- [25] T. Qiu, J. Evslin, Y.-F. Cai, M. Li and X. Zhang, *Bouncing Galileon Cosmologies*, *JCAP* **10** (2011) 036 [[arXiv:1108.0593](#)] [[INSPIRE](#)].
- [26] D.A. Easson, I. Sawicki and A. Vikman, *G-Bounce*, *JCAP* **11** (2011) 021 [[arXiv:1109.1047](#)] [[INSPIRE](#)].
- [27] M. Osipov and V. Rubakov, *Galileon bounce after ekpyrotic contraction*, *JCAP* **11** (2013) 031 [[arXiv:1303.1221](#)] [[INSPIRE](#)].
- [28] T. Qiu, X. Gao and E.N. Saridakis, *Towards anisotropy-free and nonsingular bounce cosmology with scale-invariant perturbations*, *Phys. Rev. D* **88** (2013) 043525 [[arXiv:1303.2372](#)] [[INSPIRE](#)].
- [29] Y.-F. Cai, D.A. Easson and R. Brandenberger, *Towards a Nonsingular Bouncing Cosmology*, *JCAP* **08** (2012) 020 [[arXiv:1206.2382](#)] [[INSPIRE](#)].
- [30] M. Koehn, J.-L. Lehners and B.A. Ovrut, *Cosmological super-bounce*, *Phys. Rev. D* **90** (2014) 025005 [[arXiv:1310.7577](#)] [[INSPIRE](#)].
- [31] L. Battarra, M. Koehn, J.-L. Lehners and B.A. Ovrut, *Cosmological Perturbations Through a Non-Singular Ghost-Condensate/Galileon Bounce*, *JCAP* **07** (2014) 007 [[arXiv:1404.5067](#)] [[INSPIRE](#)].
- [32] T. Qiu and Y.-T. Wang, *G-Bounce Inflation: Towards Nonsingular Inflation Cosmology with Galileon Field*, *JHEP* **04** (2015) 130 [[arXiv:1501.03568](#)] [[INSPIRE](#)].
- [33] M. Koehn, J.-L. Lehners and B. Ovrut, *Nonsingular bouncing cosmology: Consistency of the effective description*, *Phys. Rev. D* **93** (2016) 103501 [[arXiv:1512.03807](#)] [[INSPIRE](#)].
- [34] P. Creminelli, M.A. Luty, A. Nicolis and L. Senatore, *Starting the Universe: Stable Violation of the Null Energy Condition and Non-standard Cosmologies*, *JHEP* **12** (2006) 080 [[hep-th/0606090](#)] [[INSPIRE](#)].
- [35] T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Galilean Creation of the Inflationary Universe*, *JCAP* **07** (2015) 017 [[arXiv:1504.05710](#)] [[INSPIRE](#)].
- [36] A. Sosnovikov, *Transition to the hot cosmological epoch in models with Galileon*, MSc Thesis, Moscow State University, Moscow Russia (2015).
- [37] V.A. Rubakov, *More about wormholes in generalized Galileon theories*, [arXiv:1601.06566](#) [[INSPIRE](#)].
- [38] V.A. Rubakov, *Can Galileons support Lorentzian wormholes?*, *Theor. Math. Phys.* **187** (2016) 743 [[arXiv:1509.08808](#)] [[INSPIRE](#)].
- [39] S. Nishi and T. Kobayashi, *Reheating and Primordial Gravitational Waves in Generalized Galilean Genesis*, *JCAP* **04** (2016) 018 [[arXiv:1601.06561](#)] [[INSPIRE](#)].
- [40] M.S. Morris, K.S. Thorne and U. Yurtsever, *Wormholes, Time Machines and the Weak Energy Condition*, *Phys. Rev. Lett.* **61** (1988) 1446 [[INSPIRE](#)].
- [41] A. Borde, A.H. Guth and A. Vilenkin, *Inflationary space-times are incomplete in past directions*, *Phys. Rev. Lett.* **90** (2003) 151301 [[gr-qc/0110012](#)] [[INSPIRE](#)].