Inflation, the Higgs field and the resolution of the Cosmological Constant Paradox

2017 J. Phys.: Conf. Ser. 880 012026

(http://iopscience.iop.org/1742-6596/880/1/012026)

IOP Conf. Series: Journal of Physics: Conf. Series **880** (2017) 012026 doi:10.1088/1742-6596/880/1/012026

Inflation, the Higgs field and the resolution of the Cosmological Constant Paradox

Francesco De Martini

Dipartimento di Fisica, Universita' La Sapienza, I- 00185 Roma, Italy

E-mail: francesco.demartini@uniroma1.it

Abstract. The nature of the scalar field responsible for the cosmological inflation, the "inflaton", is found to be rooted in the most fundamental concept of the Weyl's differential geometry: the parallel displacement of vectors in curved space-time. Within this novel dynamical scenario, the standard electroweak theory of leptons based on the $SU(2)_L \otimes U(1)_Y$ as well as on the conformal groups of spacetime Weyl's transformations is analyzed within the framework of a general-relativistic, co-covariant scalar-tensor theory that includes the electromagnetic and the Yang-Mills fields. A Higgs mechanism within a spontaneous symmetry breaking process is identified and this offers formal connections between some relevant properties of the elementary particles and the dark energy content of the Universe. An "Effective Cosmological Potential": V_{eff} is expressed in terms of the dark energy potential: $V_\Lambda \equiv M_\Lambda^2$ via the "mass reduction parameter": $\zeta \equiv \sqrt{\frac{|V_{eff}|}{|V_\Lambda|}}$, a general property of the Universe. The mass of the Higgs boson, which is considered a "free parameter" by the standard electroweak theory, by our theory is found to be proportional to the geometrical mean: $M_H \propto \sqrt{M_{eff} \times M_P}$ of the Planck mass, M_P and of the mass $M_{eff} \equiv \sqrt{|V_{eff}|}$ which accounts for the measured Cosmological Constant, i.e. the measured content of vacuum-energy in the Universe. The experimental result obtained by the ATLAS and CMS Collaborations at CERN in the year 2012: $M_H = 125.09(GeV/c^2)$ leads by our theory to a value: $M_{eff} \sim 3.19 \cdot 10^{-6}(eV/c^2)$. The peculiar mathematical structure of V_{eff} offers a clue towards the resolution of a most intriguing puzzle of modern quantum field theory, the "Cosmological Constant Paradox".

1. Introduction: Weyl geometry

A huge step forward in theoretical cosmology, and today a very important landmark of modern science, was the proposal by A. Starobinsky followed one year later by A.Guth, A. D. Linde, A. Albrecht and P.J. Steinhard [1, 2, 3, 4] of the *inflation*, an epoch of fast accelerating expansion of the early Universe that caused the Universe to expand through about 70 e-folds in a very small fraction of a second [5]. This expansion, driven by a scalar field called "inflaton", was originally argued to solve the problem of why the universe is so smooth at large scales. Moreover it later turned out to provide a consistent solution also to a host of different crucial problems among which the growth of structures in the Universe arising from magnified quantum fluctuations, the no observation of magnetic monopoles, the isotropy of the cosmic microwave background etc. [6, 7]. Over the years, the undeniable success of this idea was however somewhat questioned by the failure of finding the physical mechanism underlying the fundamental nature of the "inflaton" concept. In the present letter it is claimed that the fundamental nature of this scalar field is indeed *geometrical*, based on the conformal differential geometry introduced by Hermann Weyl

IOP Conf. Series: Journal of Physics: Conf. Series **880** (2017) 012026 doi:10.1088/1742-6596/880/1/012026

two years after the publication of the first Einstein paper on General Relativity (GR) [8, 9]. This geometry rests on the following general statement:" *All Laws of Physics are invariant under* (A) *any change of coordinates including time and* (B) *under any change of calibration (i.e. gauge)*" The first part, (A) expresses the well known *covariance* of the GR based on the Riemann geometry. The second part (B), a significant addition by Weyl to Riemann's, expresses the *conformal* property of the metric space-time theory. Accordingly, the above statement is usually cast in a simpler form: "*All Laws of Physics are conformally-covariant*" (or *Weyl covariant* or, in short, *co-covariant*).

The rationale of the Weyl geometry can be outlined as follows. Taking Planck's constant ($\hbar$) and the velocity of light (c) to be constant by definition, any physical quantity X, e.g. represented by a scalar, a tensor, a spinor etc, can be assigned a unit that is a power of a physical length $\tilde{L}$: $X \to \tilde{L}^{W(X)}$. Furthermore, to that quantity can be assigned a transformation law $X \to e^{W(X)\lambda(x)}X$ under a conformal mapping. $W(X)$ is a (positive or negative) real number dubbed "*Weyl weight of X*" (or "dimensional number of X") and $\lambda(x)$ is a regular, real function of the space-time coordinates. Thus, the conformal mapping is a "unit transformation" amounting to a local space-time redefinition of the unity of length, i.e. of the "*calibration*". This concept is rooted into the most basic operation of the differential geometry i.e. the "parallel displacement" of any vector in a non-Euclidean manifold. In fact, according to Weyl, the parallel displacement from two infinitely nearby points P and $P+dP$ in spacetime acts on the length ℓ of any vector by inducing a calibration change $\delta\ell = \ell\phi_\rho dx^\rho$, where ϕ_ρ is a universal "*Weyl vector*", defined in the whole space-time. In summary, the metric structure of the Weyl geometry implies two fundamental forms: the *quadratic* Riemannian one, $g_{\rho\sigma}dx^\rho dx^\sigma$, being $g_{\rho\sigma} = g_{\sigma\rho}$ the metric tensor, and the Weyl *linear* one $\phi_\rho dx^\rho$. In Riemann's geometry it is always $\phi_\rho = 0$. The parallel displacement is *integrable* iff a scalar *Weyl potential* ϕ exists such as $\phi_\rho = \partial_\rho\phi$. As we shall see, the "*Weyl vector*" $\phi_\rho(x)$ and the corresponding scalar "Weyl potential" $\phi(x)$, both defined in the whole space-time spanned by the x^ρ coordinates play a basic role in the present work: indeed the "inflaton" is identified with ϕ [5].

We first note that under a physics perspective the Weyl geometry indeed consists of an abelian local scale-invariance gauge theory implying the following group of transformations

$$\begin{aligned} \phi_\rho &\to \phi_\rho + \partial_\rho\lambda(x) \\ g_{\rho\sigma} &\to e^{2\lambda}g_{\rho\sigma}. \end{aligned} \tag{1}$$

The inflaton may be considered a "gauge field" [8, 10]. The insightful perspective offered by this theory was supported by P.A.M. Dirac in a 1973 seminal paper [11]:
There is a strong reason in support of the Weyl's theory. It appears as one of the fundamental principles of Nature that the equations expressing basic laws should be invariant under the widest possible group of transformations. The confidence that one feels in Einstein GR theory arises because its equations are invariant under the wide group of transformations of curvilinear coordinates in Riemann space. The passage to Weyl geometry is a further step in the direction of widening the group of transformations underlying the physical laws. One has to consider transformations [...] which impose stringent conditions on them.
The stringent conditions alluded by Dirac imply in the first place the correct choice by definition of several constant units in terms of which the physical quantities are measured: these units must be mutually independent in the sense that a dimensionless number cannot be constructed with them. As in the relativistic quantum theory, it is conventional to take $\hbar, c, m_e$ (the electron mass) to be *constant by definition* [9, 10]. Other gauges, e.g. by replacing m_e by the gravitational constant G, lead in general to different theories which are mutually connected by

conformal mapping. Furthermore, the Lagrangian density L from which the dynamical equations are derived as well any measurable quantity X attained as result of the theory must be co-covariant, and then expressed by the zero value of the Weyl weight $W(L) = 0$, $W(X) = 0$. At the end of the calculation a careful application of the initial and boundary conditions must be undertaken in order to select the physically admissible results among the special solutions of the theory. The special solutions have the property that particle masses are independent of positions in space-time and the gravitational constant G is a true constant. Thus the statement that all electrons have the same mass, all hydrogen atoms have the same size etc. may be taken to be true by definition. All this circumvents the original criticism to the Weyl theory [8, 13, 14, 15]. As it will be shown below, the requirement of full covariance under conformal mapping is necessary only to determine the formal structure of the theory.

The implied meaning of the present work may be further enlightened by the fact that after 1970 several axiomatic approaches for deducing the projective and conformal structure of space-time were carried out by using basic concepts such as light rays and freely falling particles [13, 14, 15]. The highly remarkable fact is that all these investigations ended by assigning a Weylan, not a Riemannian structure to space-time. This raised a long lasting, and as yet unresolved interesting dilemma that could be understood by the possibility of the onset of a cosmological "phase transition" by which the now perceived Riemannian structure is induced via a dynamical process that breaks the Weyl invariance of the vacuum at some critical temperature T_c.

2. Inflation and the Weyl-Dirac Theory

All these concepts will be applied to a very general Weyl-Dirac conformal scalar-tensor theory involving the mass m_E of an elementary particle, e.g. an electron. In order to keep the conformal structure of the theory, the mass of the particle is expressed in form of a "mass field" $m_E \to k_E \cdot \mu(x)$, being the dimensionless coupling constant: k_E an intrinsic particle's property, and $\mu(x)$ a real scalar field, function of space-time, with weight $W(\mu) = -1$ [12]. The gravitational constant G appearing in the Einstein gravitational equation has $W(G) = +2$ and so it cannot be regarded as constant in the present approach. We define a dimensionless constant: α and assume that the ratio l_C/l_P between the particle's Compton length $l_C = \hbar/mc$ and the Planck length is independent of position in space and time. By applying the Dirac's Lagrange-multiplier method, the simplest form of the co-covariant Lagrangian density in $D = 4$ can be expressed in the scalar-tensor form [9, 11]:

$$L = \sqrt{-g}\left\{\alpha\mu^2\left[\bar{R} + 2V(T,\phi,\mu)\right] - D_\rho\mu D^\rho\mu - \frac{1}{4}\phi_{\rho\sigma}\phi^{\rho\sigma}\right\} \tag{2}$$

where $\bar{R} = R + R_W$ is the overall Riemann-Weyl curvature scalar and D_ρ is the Weyl co-covariant derivative [8, ?]. In particular, the Weyl curvature scalar in D=4 is: $R_W = 6[\phi_\rho\phi^\rho - \phi^\rho_{|\rho}]$. The generic potential $V(T,\phi)$, related to the cosmological constant Λ, and function of the temperature T, accounts for the self-interaction of the scalar field ϕ [28]. As we shall see immediately, the stringent conditions implied by the Weyl symmetry on each co-covariant addendum X appearing in the expression of L in Eq. (2), i.e. $W(L) = W(X) = 0$, impose a well defined exponential expression on the function $V(T,\phi)$.

In addition to the above considerations, we must further impose in the present context the general condition that the action: $I_\phi = \int dx^4\sqrt{-g}L_\phi$ is stationary respect to variations in ϕ, where: $L_\phi \equiv \left[\frac{1}{2}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi - V(T,\phi)\right]$ expresses the effect of the inflation field in the $D = 4$

space-time (Weinberg, 2008). This condition is expressed by:

$$\nabla_B \phi = -\frac{\partial V(\phi, T)}{\partial \phi} \tag{3}$$

being: $\nabla_B \phi \equiv \phi^{\rho}_{|\rho}$ the Laplace-Beltrami differential operator of the field: ϕ [?]. In the theory above the Lagrangian L, Eq. (2), is written in terms of the *geometrical* Weyl potential ϕ_W within a Weyl-Dirac scalar-tensor theory, e.g. according to the Texts by P.A.M. Dirac and E.A. Lord [11, 9] (Cfr. pages 410 and 197, respectively). On the other hand, the Lagrangian L_ϕ is written in terms of the *physical* "inflaton" field ϕ_I, e.g. according to the Texts by S. Dodelson and S. Weinberg (2008) [6, 7] (Cfr. pages 152 and 526, respectively). According to the basic conjecture of our theory the two fields should be considered as complementary aspects of the same entity: $\phi_W \equiv \phi_I \equiv \phi$. Accordingly, there is no conflict between the corresponding lagrangian theories but rather Eq. (3), i.e. the Euler-Lagrange result of the variation respect ϕ of L_ϕ, provides the necessary mathematical relation leading to the completion of the general theory in closed form, i.e. with no approximations.

Let us now make the simplifying assumption that the inflationary system is "trapped" in local minimum of the potential, a significant dynamical condition largely considered in the literature [6] :$V' \equiv \frac{\partial V(\phi,T)}{\partial \phi} = 0$. The variation of L respect to the relevant fields leads to the dynamical equations of the Weyl conformal theory. This one is selected to be integrable, so that $\phi_{\rho\sigma} = 0$. In particular, the variation respect $g_{\rho\sigma}$ leads for $\alpha = \frac{1}{7} \simeq 0.1428$ to the following Einstein equation [9]:

$$R_{\rho\sigma} - \frac{1}{2} g_{\rho\sigma} R = \left[\partial_\rho \phi \partial_\sigma \phi - \frac{1}{2} g_{\rho\sigma} \partial_\eta \phi \partial^\eta \phi + g_{\rho\sigma} V(T, \phi) \right] \tag{4}$$

being $R_{\rho\sigma}$ and R the Ricci tensor and the curvature scalar of the Riemann geometry. The expression within square brackets at the r.h.s. of Eq. (4) may be cast in the standard form: $K^2 T^*_{\rho\sigma} \equiv K^2 [\partial_\rho \phi \partial_\sigma \phi - g_{\rho\sigma} L_\phi]$ with: $K^2 = \frac{8\pi G}{c^4}$, by the rescaling: $\phi \rightarrow K\phi$ and: $V(\phi) \rightarrow K^2 V(K\phi)$ [i.e.:$\phi_{new} = K^{-1} \phi_{old}$]. The Eq. (4) is a somewhat suggestive result since it reproduces exactly the basic Einstein equation of the inflation theory reported in the standard texts on cosmology [6, 7, 28]. Note that our expression of $T^*_{\rho\sigma}$ is obtained, within the quoted simplifying assumption, by the formal application of the standard Euler-Lagrange variational procedures to the expression in Eq. (2) of the $D = 4$ Weyl's scalar curvature which is absent in Riemann's geometry [?]. Indeed, in modern texts an equation similar to Eq. (4) is attained via the introduction of an exotic "quintessence" object wich is assumed to represent artificially a modified matter model [16]. As said, by our novel interpretation of the inflaton field, the present work suggests the geometrical nature of the quintessence. All this may lead to several consequences of dynamical relevance, as for instance a straightforward, dynamically driven interchange between the weylan to riemannian symmetry conditions [30]. Note that all the variational calculations, e.g. leading to Eq. (4) are carried out by keeping the explicit spacetime dependence of the field: $\mu(x)$. Only after the completion of the variational process we have taken advantage of the conformal gauge invariance of the theory by choosing μ to be a *constant* field. An extended discussion on the conformal Weyl gauge theory is found in: [?, 9]. As anticipated, the Weyl symmetry imposes a restriction to the explicit form of the massive inflaton potential. In virtue of Eq. (1), since for any physical quantity: $X \rightarrow e^{\lambda(x) W(X)} X$, and because here: $W(V_\Lambda) = -2, W(M_P^2) = -2, W(\sqrt{-g}) = +4$, a possible expression to be inserted in Eq. (2) may be cast in the co-covariant form as an exponential: $V_\Lambda(T, \phi) \propto e^{-2K\phi}$ or as a superposition of exponentials. Another co-covariant solution could be: $V_\Lambda(T, \phi) \propto (\phi_\rho \phi^\rho)$ possibly associated with a superposition of exponentials. Another co-covariant solution apt to a superposition could be: $V_\Lambda(T,\phi) \propto (\phi^{\rho}_{|\rho})$. Etc. All these formal generalizations, by implying the definition of a finite set of positive or negative scalar parameters of the superpositions $C_n(T)$, lead in generakl to a $V_\Lambda(T,\phi)$ function showing an unlimited number of local maxima or minima for $K\phi$ ranging from zero to infinity [16]. We notice nevertheless that in general T, and hence any $C_n(T)$, is not constant within the evolution of the inflaton $K\phi$, e.g. during the rolling down of the Universe system towards levels of lower free energy in an expansion phase or, on the contrary, in a phase of re-heating. In spite of these complications, we may recognize that the theory above is able to account reasonably for the so far inscrutable paths of the Universe evolution.

As suggested by the title of this work, in the next Section we shall adopt a broader perspective by inquiring about the connections existing between the two relevant scalar fields that, albeit supposed to belong to distant theoretical domains, today play a major role in the vast scenario of modern physics. In order to accomplish this task we need to consider a more complete Lagrangian than the one given by Eq. (2). We make a connection with the electroweak theory of leptons [26, 27] and we consider the following:

$$\hat{L} = \sqrt{-g}\left\{\alpha\mu^2\left[\bar{R} + 2V(T,\phi,\mu)\right] - D_\rho\mu D^\rho\mu + |\lambda|\mu^4 - \frac{1}{4}f^{\rho\sigma}f_{\rho\sigma} - \frac{1}{4}F^l_{\rho\sigma}F^{l\rho\sigma}\right\} \tag{5}$$

where the skew - symmetric tensors are defined as follows: $f_{\rho\sigma} \equiv (\partial_\sigma A_\rho - \partial_\rho A_\sigma)$ for the $U(1)_Y$ gauge field and: $F^l_{\rho\sigma} \equiv (\partial_\sigma b^l_\rho - \partial_\rho b^l_\sigma + g\epsilon_{jkl}b^j_\rho b^k_\sigma)$ for the $SU(2)_L$ non-abelian gauge fields of the Yang-Mills theory. Here we introduce the gauge vector bosons: $\vec{A_\rho}$, i.e. the electromagnetic vector potential, for the group $U(1)_Y$ and the three components in the isospin space: $\left(b^1_\rho, b^2_\rho, b^3_\rho\right)$ of the gauge vector boson: $\vec{b_\rho}$ for $SU(2)_L$. Y is the "weak hypercharge" operator. The term proportional to $|\lambda|$ is added for dynamical stability against unbounded field oscillations. In the present more advanced theory the parameter α, assumed to be a true constant, cannot be considered a free parameter. In facts the starting point of a rigorous standard contruction of any lagrangian theory leading to the field equations of GR consists of the adoption of the following density: $\frac{1}{2K^2}\sqrt{-g}R + \tilde{L}$ where $\tilde{L}$ is the lagrangian density of all fields other than gravitation [9]. Accordingly, we shall set: $\alpha \cdot \mu^2 = \frac{c^4}{16\pi G} = \frac{M_P^2}{16\pi}$ within $\hat{L}$, henceforth. $M_P = 1.22 \times 10^{19}\frac{GeV}{c^2}$ is the Planck mass.

3. The Higgs field and the Vacuum Energy in the Universe

The wide conceptual scenario opened by the preceeding chapter and the structure of the Lagrangian $\hat{L}$, Eq. (5) offer the possibility of inquiring about the implications of the dark energy content of the Universe within some relevant aspects of the submicroscopic world of the elementary particles. The supposed mass generation properties of the Higgs field, and his pervasiveness that parallels analogous aspects of the inflationary field has already stimulated in recent years a wealth of research in the field referred to as: "Higgs inflation" [17, 18, 19]. In what follows we shall find that the connection between the two fields can be demonstrated in the context of the present conformally-covariant theory in which the "classical" GR approach based on the lagrangian Eq. (5) is associated with a spontaneously broken $SU(2)_L \otimes SU(1)_L$ gauge theory. We shall briefly consider this theory in the framework of the standard electroweak "*theory of leptons*" due to Steven Weinberg and Abdus Salam [24, 25]. Let us introduce a *complex* iso-doublet of scalar fields [26, 27]:

$$\vec{\mu} \equiv \begin{pmatrix} \mu^+ \\ \mu^0 \end{pmatrix} \tag{6}$$

that transforms as a $SU(2)_L$ doublet with heavy hypercharge: $Y_\mu = +1$ according to the Gell Mann-Nishijima relation for the electric charge: $Q = I_3 + \frac{1}{2}Y$. The weak-isospin projection I_3 and the weak-hypercharge are commuting operators: $[I_3, Y] = 0$ [26, 27]. Next, we introduce in the expression of$\hat{L}$, Eq. (5) the following replacement: $D_\rho \mu D^\rho \mu \to (D_\rho \vec{\mu})^\dagger \cdot (D^\rho \vec{\mu})$ where D_ρ expresses the Weyl-covariant, i.e. co-covariant, derivative, and the dot represents scalar vector multiplication in the isospin space. For the sake of simplicity we still keep the symbols: $\mu^2 \equiv (\vec{\mu}^\dagger \cdot \vec{\mu})$, and: $\mu^4 \equiv (\vec{\mu}^\dagger \cdot \vec{\mu})^2$ since these scalar quantities do not unsettle the *real* structure of the GR theory. The gauge co-covariant derivatives are:

$$D_\rho \vec{\mu} = [\partial_\rho + \phi_\rho]\, \vec{\mu} + \frac{i}{2}\left[g' Y A_\rho + g \hat{\tau} \cdot \hat{b_\rho}\right] \vec{\mu} \tag{7}$$

where the components of the vector $\hat{\tau}$: (τ_1, τ_2, τ_3) are the Pauli spin operators, g represents the coupling constant of the weak-isospin group $SU(2)_L$ and $\frac{g'}{2}$ represents the coupling constant of the weak-hypercharge group $U(1)_Y$. The presence of the inflaton field ϕ_ρ multiplied by $\vec{\mu}$ in the first *real* square bracket at the r.h.s. of the Eq. (7) is due to the weight $W(\vec{\mu}) = -1$ within the co-covariant derivative [**?**]. The Eq. 7 reproduces the spontaneous symmetry breaking theory of leptons [26, 27] by further introducing within the standard theory the formal change: $\partial_\rho \to [\partial_\rho + \phi_\rho]$. It will be shown that this change provides a contribution to the interaction between the Higgs and the inflaton fields we are now dealing with. We shall show that a larger contribution to this effect, proportional to α, is due to the $\phi_\rho \phi^\rho$ and $\phi^\rho_{|\rho}$ terms in the curvature R_W [**?**]. In summary, and most interesting, all that shows that it is precisely the conformally-covariant structure of the Weyl's geometry that establishes the connection between the two fields protagonists of the present analysis.

Let us briefly outline the standard theory of leptons on the basis of the classic texts [26, 27]. Consider in general terms the kinetic term of the dynamical equation for the scalar field $\vec{\mu}$:

$$\{(D_\rho \vec{\mu})^\dagger \cdot (D^\rho \vec{\mu}) - |\lambda| \mu^4\} = \{(\partial_\rho \vec{\mu})^\dagger \cdot (\partial^\rho \vec{\mu}) - \langle \mu \rangle^2 \mu^2 - |\lambda| \mu^4\} \tag{8}$$

If the mass term $\langle \mu \rangle^2$ is negative the continuous symmetry of the system's hamiltonian does not coincide with the symmetry of the vacuum and the condition of dynamical "spontaneous symmetry breaking" takes place. In virtue of a theorem [29] a possible Goldstone boson is associated with a generator of the gauge group that does not leave the vacuum invariant. We investigate this case by choosing the following vacuum expectation value of the scalar field Eq. (6):

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{9}$$

where the vacuum field is: $v = \sqrt{\frac{-\langle \mu \rangle^2}{|\lambda|}}$. The vacuum is left invariant by any group generator G if: $G\langle \phi \rangle_0 = 0$. In our case we find all the $SU(2)_L$ and $U(1)_Y$ group generators: $\hat{\tau}_i, (i = 1, 2, 3)$ and Y operating on $\langle \phi \rangle_0$ break the symmetry of the vacuum. However the $U(1)_{EM}$ symmetry generated by the electric charge preserves the invariance since: $Q\langle \phi \rangle_0 \equiv \frac{1}{2}(\tau_3 + Y)\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. The photon, therefore, remains massless and the other three gauge bosons will acquire mass. These are the heavy bosons: $W^\pm_\rho \equiv \frac{[b^1_\rho \mp i b^2_\rho]}{\sqrt{2}}$. and: $Z_\rho \equiv \frac{[-g' A_\rho + g b^3_\rho]}{\sqrt{g^2 + g'^2}}$. Expressed in very popular terms, this is the process referred to by Abdus Salam in a sentence reported in Ref. [12]: *"The massless Yang-Mills particles "eat" the Higgs particles (or fields) in order to gain weight, and the swallowed Higgs particles become ghosts"*. Upon expansion of the Lagrangian (8) about the shifted minimum of the Higgs potential we can investigate the small oscillations around the vacuum v of of the "Higgs field", this one expressed by the field θ. This dynamics is expressed by the Lagrangian for small oscillations:

$$\bar{L} = \frac{1}{2}\Big[(\partial_\rho\theta)(\partial^\rho\theta) + 2\langle\mu\rangle^2\theta^2\Big] + \frac{v^2}{8}\left[g^2 \mid b_\rho{}^1 - ib_\rho^2 \mid^2 + (-g'A_\rho + gb_\rho^3)^2\right] + ... \qquad (10)$$

plus interaction terms. As shown by this equation, the Higgs field has acquired a $(mass)^2 \equiv (M_H)^2 = 2 \mid \langle\mu\rangle \mid^2$. The above sentences express in a very summary form the well known results of the standard electroweak theory.

We may now inquire about the effects on the theory of the change: $\partial_\rho \rightarrow [\partial_\rho + \phi_\rho]$ introduced in our present analysis by the co-covariant derivative (7) as well of the general relativistic structure of the overall co-covariant lagrangian $\hat{L}$ (5). Note first that nothing in the standard formulation of the electroweak theory based on the $SU(2)_L \otimes U(1)_Y$ group specifies the mass of the Higgs boson and none of the conceivable applications to the conventional processes depends in any way upon the value of M_H. It may therefore appear that M_H can indeed be considered a "free parameter" of the standard electroweak theory even if general constraints have been considered by some authors by imposing certain requirements of internal consistency [10].
This is not the case with our present theory. Since the Eq. (8) is formally included into the extended lagrangian *Eq.* (5), we can easily transfer to the last one all the physical conceptions and the theoretical considerations addressed so far to Eq. (8). In particular, the Riemann curvature R and the Weyl curvature R_W appearing in *Eq.* (5) are now involved in the spontaneous symmetry breaking scenario of the electroweak theory. On the other hand, they can also be expressed in terms of the relevant cosmological quantities: i.e. the inflation potential: $V_\Lambda(T,\phi)$, its ϕ-derivative: $V'_\Lambda(T,\phi)$ and the inflation vector field:ϕ_ρ. Consequently, and very important, the dynamical behaviour of all interactions affecting the properties of the elementary particles, including the Higgs field, is directly determined by these cosmological quantities, and then by the overall dynamics of the Universe. We believe that this is a striking result.
The present theory is very general and applies to all kinds of potential functions: $V_\Lambda(T,\phi)$. However, in view of what we believe to be the most interesting result of the present work, i.e. the resolution of the "Cosmological Constant Paradox" (referred to as: "Λ-Paradox", hereafter), in what follows we shall mostly deal with the exponential functions of ϕ: i.e. $V_\Lambda(T,\phi) \propto e^{-n\phi}$ and to the linear superpositions of these functions. There n is a "real number", i.e. either positive or negative, integer or fractional [16]. Within this restriction we claim the "universality" of the theory and then in what follows we shall consider only the case: n=2, as an example. For any other (n) the theoretical steps can be reproduced identically albeit the final numerical results of the calculations will be different.
Let us first define an "Effective Cosmological Potential":

$$V_{eff}(T,\phi) \equiv \left[V_\Lambda(T,\phi) + \frac{1}{2}\frac{\partial V_\Lambda(T,\phi)}{\partial\phi}\right] \qquad (11)$$

that can be either positive or negative depending on the sign and size of the proportionality parameters of $V_\Lambda(T,\phi)$ and/or of its ϕ-derivative. In case of a negative derivative a very small value of $|V_{eff}|$ may result from the sum of two very large contributions with opposite sign. As said, the contribution V_Λ is the vacuum energy, i.e. the dark-energy content of the Universe, conceptually connected with the Einstein's "Cosmological Constant" [28]. The quantity $|V_{eff}| \equiv M_{eff}^2$ represents the corresponding *measured* quantity. The above effect, expressed by the size of the "mass-reduction parameter": $\zeta \equiv \sqrt{\frac{|V_{eff}|}{|V_\Lambda|}} << 1$, indeed a general property of the Universe, can lead to a consequence of cosmological relevance since it represents a clue towards the resolution of the celebrated "Λ-Paradox" [20]. A conformally-covariant

solution of the Paradox based on Eq. (11) will be given later in this paper.
In agreement with our program and with the discussion above, we turn our attention to the evaluation of the mass of the Higgs boson, M_H. By considering Eq. (10), by collecting all terms proportional to μ^2 in $Eq.$ (5) and by expressing the Riemann curvature scalar in terms of $V_\Lambda(T,\phi)$, $V'_\Lambda(T,\phi)$ and: $\phi_\rho\phi^\rho$ [**?**], we find the expression:

$$M_H^2 = 24\alpha|V_{eff}| \tag{12}$$

We find that the spontaneous symmetry breaking condition implies: $V_{eff} < 0$. Remind the expression of α and make the simplifying assumption: $M_H^2 \cdot (2|\mu|^2) \rightarrow M_H^4$. This leads immediately to an expression of the Higgs mass:

$$M_H \simeq \left[\frac{3}{\pi}(M_{eff}^2 \cdot M_P^2)\right]^{\frac{1}{4}} \tag{13}$$

Interestingly enough, according to our theory the mass of the Higgs boson is proportional to the geometrical mean of the Planck mass and of the mass M_{eff} accounting for the *measured* cosmic vacuum-energy: $M_H = \left[\frac{3}{\pi}\right]^{\frac{1}{4}} \times \sqrt{M_P \cdot M_{eff}}$.
In conclusion, the present theory, which lies on the Weyl's geometrical foundations, prescribes that the value of the mass of any elementary particle belonging to the submicroscopic world depends on the average vacuum energy content of the Universe. This because it is precisely the Higgs field the source of the mass of all elementary quantum particles.
By taking into account the experimental datum measured by the joint ATLAS and CMS Collaborations at CERN in the year 2012:$M_H = 125.09(GeV/c^2)$ the corresponding size of the measurable dark-energy potential is found: $V_{eff} = 1.717 \cdot 10^{-12}(eV/c^2)^2$ [31, 32, 33].

The conceptual relevance of the "Effective Cosmological Potential" may be further enlightened by carring out the variational procedure respect $g_{\rho\sigma}$ on the Lagrangian $\hat{L}$ expressed by Eq. (5). For the sake of completeness we also include here the energy-momentum tensor $T_{\rho\sigma}$ due to external, unspecified matter and fields. The Euler-Lagrange equation consists of the Einstein's equation:

$$R_{\rho\sigma} - \frac{1}{2}g_{\rho\sigma}R - g_{\rho\sigma}\bar{\eta}\left[2V_{eff}(T,\phi) + \frac{1}{2}\partial_\rho\phi\partial^\rho\phi\right] = K^2T_{\rho\sigma} \tag{14}$$

where:$\bar{\eta} \equiv [3-(2\alpha)^{-1}]$. This equation is interesting because it shows that the generally very large "Inflation Potential" $|V_\Lambda(T,\phi)|$ appearing in the Lagrangian $\hat{L}$, Eq. (5) is replaced by the far smaller: $|V_{eff}(T,\phi)|$ within the corresponding Einstein equation. The expression within the square brackets in (14) is the *measured* "Cosmological Constant", Λ:

$$\Lambda \equiv \bar{\eta}\left[2V_{eff}(T,\phi) + \frac{1}{2}\partial_\rho\phi\partial^\rho\phi\right] \tag{15}$$

As we shall see: $|V_{eff}(T,\phi)| << |V_\Lambda(T,\phi)|$, and then: $\zeta << 1$. We stress here that the formal replacement $|V_\Lambda(T,\phi| \rightarrow |V_{eff}(T,\phi)|$ is precisely due to the application of the dynamical equation (3) to the expression of the Weyl-curvature R_W given in Ref. [**?**]. If the Weyl curvature is considered nonexistent in the source lagrangian $\hat{L}$, Eq. (5), as assumed in the context of Riemann's differential geometry (where: $R_W = 0$) the replacement $|V_\Lambda(T,\phi| \rightarrow |V_{eff}(T,\phi)|$ is not realized. This is precisely the origin and the real essence of the Λ-Paradox. Indeed, we shall see that the above replacement is the key argument for the resolution of the Paradox we shall propose later in this paper.

IOP Conf. Series: Journal of Physics: Conf. Series **880** (2017) 012026 doi:10.1088/1742-6596/880/1/012026

4. The Vector - Meson particle

If the "integrability condition" of the Weyl geometry, i.e. $\phi_\rho = \partial_\rho \phi$ and: $\phi_{\rho\sigma} = 0$, is relaxed, the quantities ϕ and ϕ_ρ must be considered independent variables of the theory and this one gets even richer. In particular, the inner structure of the Weyl geometry does not change, since the parallel displacement of vectors involves directly the vector-field ϕ^ρ, as said. Leaving aside all complications and deferring an exact analysis to future work, we may immediately apply the mathematical methods adopted in the previous Sections to the Lagrangian $\hat{L}$, Eq. (5). The variation respect to the *Weyl vector* $\phi_\rho(x)$ leads, via the gauge-fixing $\partial_\rho \phi^\rho = 0$, to the following Proca equation expressing the dynamics of a massive vector-meson, ϕ^ρ:

$$\left[\nabla_B + (\xi \frac{M_P c}{\hbar})^2\right] \phi^\rho = 0 \tag{16}$$

where M_P is the Planck mass and $\xi = \sqrt{2}(\beta)^{-1} \ll 1$ expresses the coupling of the particle to gravitons. It has been found that the massive meson ϕ^ρ, while strongly interacting with gravitons, does not interact with any $spin - \frac{1}{2}$ or $spin - 1$ elementary particle of the Standard Model. In other words, it is not coupled minimally to photons, hadrons and light or massive leptons [30, 34]. All these properties make this geometrical entity eligible for being considered an optimum candidate for Cold Dark Matter (CDM), the elusive object wich is now under (baffling) investigation in a large number of laboratories around the world. As it is well known, the (CDM) may consist of a Weakly Interacting Massive Particle (WIMP), a stable SUSY particle, a light neutralino with a mass of the order of $10^2 (GeV/c^2)$ or even an "axion" particle with a mass as low as $10^{-5} (GeV/c^2)$ [37, 16]. Large concentrations of (CDM) have been detected in zones of the Universe characterized by a large inhomogeneous gravitational energy density. Indeed the CDM predominately clusters on the scale of galaxies. In summary, today the size of the mass and any other physical property of (CDM) is inferred from some sophisticated quantum theory based on an attributed "physical model".

Waiting for any novel, convincing experimental evidence, we refrain from considering further the putative identification of (CDM) with the ϕ^ρ-particle. We tend nevertheless to believe, according to a perspective biased by our "geometrical model", that the mass of this particle, i.e. the value of the parameter ξ in Eq. (16), must depend, once again, on a property of the Universe via some dynamical mechanism similar to the one already considered in this work. If the exact size of the mass of the ϕ^ρ-particle is presently out of reach, we are nevertheless able to determine the average size of this quantum excitation in the Universe. The Riemann curvature R is obtained by a standard variational procedure applied to $\hat{L}$ in order to find the Einstein equation.Then, both sides of this equation are multiplied by the metric tensor, by keeping in mind that, in D=4: $g_{\rho\sigma} g^{\rho\sigma} = 4$. It is expressed by:

$$R = 2\bar{\eta} \left[4|V_{eff}| - \phi_\rho \phi^\rho\right] \tag{17}$$

This equation is interesting since it shows the relative sizes of the two dominant contributions to the Universe curvature, i.e. the vacuum-energy, V_{eff} and the energy V_{DM} associated with the vector-meson ϕ^ρ. The mass of the ϕ^ρ field is:

$$M_{DM} \simeq \sqrt{\langle \phi_\rho \phi^\rho \rangle} \simeq 2\sqrt{|V_{eff}|} = 2.62 \cdot 10^{-6} (eV/c^2) \tag{18}$$

since the energy density of the Universe is "critical" and the Universe is "flat", i.e.$R \simeq 0$. This was indeed the main result of the BOOMERANG experiment carried out on (CMB) by Paolo De Bernardis and his group in the year 2000 [35]. These results were confirmed by recent measurement carried out by the PLANCK mission [36].

In conclusion, we propose here the following conjecture of cosmological relevance:
"The result of all experiments carried out on the cosmic vacuum-energy, i.e. dark-energy or zero-point-energy or cosmological-constant, by our terrestrial or extraterrestrial measurement apparata (i.e. carried by orbiting satellites or spatial vehicles) always consists of the reduced value of the effective cosmological potential, V_{eff} and never of the full value of V_Λ. This last quantity, a quantum-mechanical property of the Universe, is inaccessible to the human perception". The conjecture is proved by the results of the BOOMERANG, PLANCK and other recent astrophysical experiments. It is also consistent with our equations: (12), (17), (18).
As an alternative solution, the above conjecture may raise a profound epistemological and gnoseological issue whether the very *calibration* process within the parallel displacement of vectors in curved spaces determines at a very fundamental level all measurement processes, i.e. is *an intrinsic feature* of all measurement processes, and reveals itself when the experiments are carried out over cosmological spaces and times.

5. The Cosmological Constant Paradox and its solution

The following quote by Richard Feynman is enlightening [38]: *"..Such a mass density would, at first sight at least, be expected to produce very large gravitational effects which are not observed. It is possible that we are calculating in a naive manner, and, if all of the consequences of the general theory of relativity (such as the gravitational effects produced by the large stresses implied here) were included, the effects might cancel out; but nobody has worked all this out. It is possible that some cutoff procedure that not only yelds a finite energy density for the vacuum-state but also provides relativistic invariance may be found. The implications of such a result are at present completely unknown."*.

Let us analyze the Λ-Paradox in *"a naive manner"*, as in Feynman's words. The vacuum energy in the Universe E_{vac}, i.e. the "zero-point energy" of the quantum fields associated with all existing quantum particles is evaluated by (QFT). For simplicity we consider here only the "photon", which is the subject of (QED) (Quantum Electro-Dynamics), the chapter of (QFT) accounting for the electromagnetic (e.m.) phenomena. To carry out the calculation of E_{vac} we should first evaluate the spatial density ρ_{em} of the available $\vec{k} - modes$, i.e. of the spatial vectors over which the photons propagate in the free space. Afterwards, each mode, which is modelled by QED as a quantum-mechanical oscillator, is multiplied by 2-times (because of the two orthogonal polarizations) the oscillator's "zero-point energy", which is $(h\omega/2)$, where (h) is the Planck constant. The frequency ω is taken to range from zero to a cutoff that may be assumed to be determined by the Planck length, $l_P = 1.616 \times 10^{-33}(cm)$: $\omega_c = (2\pi c/l_P)$. At last, the resulting spatial energy-density must be multiplied by the volume of the Universe, W_U in order to get the final result: E_{vac} . We can amuse ourselves by carrying out the sequence of these operations, and more than that, by putting numbers at the end of every step of the calculation. I limit myself to give here the final result.The size of final vacuum-energy content is found: $E_{vac} = \frac{2\pi\hbar c}{L_P^4} \times W_U = \frac{2\pi c^7}{\hbar G^2} \times W_U = 2.9 \times 10^{98}(Joules/cm^3) \times W_U(cm^3)$ Since the linear size of our Universe evaluated by a (CMB) analysis is: $L_U \sim 124$ billions of light-years, the volume is: $W_U \sim (1.61 \cdot 10^{87})(cm^3)$. Then, the value of the content of the vacuum-energy, due only to photons, is: $E_{vac} \sim 5 \times 10^{185}(Joules) \sim 3 \times 10^{204}(eV)$. To be more conservative, the frequency cutoff ω_c could be determined by the size of the Compton wavelength of the proton: $\lambda_c \simeq 2 \cdot 10^{-14}$ cm. In this case all the numerical figures for the energies and for the energy densities given above should be multiplied by the factor $\sim 10^{-76}$. Even in this case the size of all numbers keeps being impressive ! We should remind here that in the vast realm of modern Science, the (QED) theory and, in particular, the "vacuum-field" concept were the most tested paradigms, ever. Uncountable and impressively precise experiments involving the atomic

physics, the optical spontaneous emission, the Casimir effect, the atomic spectra, the Lamb's shift, the electron's magnetic moment etc. were the landmarks of the great success of the XX century Physics. The calculation carried out for photons should now be extended to the other existing particles and the corresponding energy contributions should be added, without any reasonable chance of mutual cancellations.
Interestingly enough, the Λ-Paradox was also analyzed in the framework of the electro-weak theory of leptons by Chris Quigg (2013) [10].
The Λ-Paradox consists of the mysterious, humongous discrepancy existing between the enormous size of the overall *calculated*: $(E_{vac}/c^2)^2 = |V_\Lambda(T,\phi)|$ and the very small size of the "Cosmological Constant" Λ, Eq. (15), *measured* today.

Our present theory offers a conformally-covariant solution of the Λ-Paradox. Consider the following expression for the inflationary potential which accounts for the overall vacuum-energy content of the Universe calculated by the standard methods of (QFT) as shown above:

$$V_\Lambda(T,\phi) = \tilde{C}(T) \times \exp\,[2(\epsilon - 1)]\phi \tag{19}$$

Where $\tilde{C}(T)$ is assumed independent of $\mu(x)$ and of $g_{\rho\sigma}(x)$. We have already noted that, since for any physical quantity: $X \rightarrow e^{\lambda(x)W(X)}X$, and $W(\sqrt{-g}) = +4$ the above solution corresponding to $n = 2$ is indeed a conformally covariant solution. The quantity ϵ is a dimensionless real number: $\epsilon \approx 0$. Plugging Eq. (19) into Eq. (11) gives the ratio: $\frac{|V_{eff}|}{|V_\Lambda|} = \epsilon$. This is indeed a most drastic "mass-reduction" effect with the parameter: $\zeta = \sqrt{|\epsilon|} \sim 10^{(-210)}$, where this number only accounts for the e.m. field considered by the above numerical evaluation of E_{vac}. By this argument, the enormous content of vacuum-energy in the Universe, calculated by the (QFT) methods as shown above, and expressed by $|V_\Lambda(T,\phi)|$ is made consistent by our theory, i.e. by our Eqs. (11) and (19), with the very small value of $|V_{eff}(T,\phi)|$ and then, of the "effective" Cosmological Constant, Λ, Eq. (15), *measured* today [6, 7]. Therefore the Λ-Paradox is resolved by our theory for n=2.
As already stressed, for any real number (n) the resolution of the Λ-Paradox is achieved quite generally by our theory for any exponential potential function: $V_\Lambda(T,\phi) \propto e^{-n\phi}$ by inserting in the Lagrangian expressed by Eg. (5) the corresponding value of the parameter $\eta(n)$. In the case of a linear superposition: $\sum_j V_\Lambda^j(T,\phi)$ of exponential functions: $V_\Lambda^j(T,\phi) \propto e^{-n_j\phi}$, where n_j are generally uncorrelated real numbers, it suffices to plug within the square brackets of the Lagrangian $\hat{L}$, Eq. (5) the sum: $\sum_j \eta(n_j)V_\Lambda^j(T,\phi)$. For the sake of clarity, it is convenient to collect here all these results valid for the general case: $V_{eff} = [V_\Lambda + (n)^{-1}V'_\Lambda]$, the Higgs boson mass: $M_H^2 = 2 \cdot \eta(n) \cdot \alpha|V_{eff}|$, the cosmological constant:$\Lambda = [\frac{\eta(n)}{2} + \frac{\bar{\eta}}{2}]$. $\eta(n) \equiv [(6 - \alpha^{-1}) \times n]$, $\bar{\eta} \equiv [3 - (2\alpha)^{-1}]$ and: n = any real number, positive or negative, integer or fractional. For $n \neq 2$ the co-covariance of the solution of the Λ-Paradox is attained by the multiplication of $\tilde{C}(T)$ with $(M_P)^\gamma$ with a suitable γ-exponent.
The Λ-Paradox cannot be resolved in the context of Riemann's differential geometry.

6. Conclusions

In conclusion, by our present work we have established several bridges between at least three cultural domains of the scientific endeavour that traditionally are rather disconnected: the foundations of the differential geometry, here focussed on the inspiring ideas by Hermann Weyl, the general relativistic cosmology, here focused on the inflation process, and the modern field-theory focused on some crucial interactive properties of the elementary quantum particles. Our aim was to search, by an unitarian perspective, for any possible conceptual and theoretical connection existing between the pervasive quantum fields that actively dwell the most remote corners of the Universe. We believe that precisely there can be found the key for undersanding the inner essence of Nature.

As a final comment to our somewhat unusual proposal let's identify here some key arguments of the underlying logical scenario. (A) The main result of our work is the expression of V_{eff}, Eq. (11) by the inclusion of the ϕ-derivative: V'. This is possible in virtue of Eq. (3), indeed the key-equation of our work. (B) Since Eq. (3) is the Euler-Lagrange equation of a variational procedure on the "physical" Lagrangian L_ϕ, the quantity ϕ must be a "physical" field, and not a mere "geometrical" entity. Otherwise, Eq. (3) would be meaningless. (C) The same "physical" Eq (3) establishes a direct and necessary link from physics to geometry via the expression $\phi^{\rho}_{|\rho}$ appearing in the Weyl curvature R_W. All this implies a lucky intersection of several nicely interweaved physical-geometrical concepts. The Riemann's geometry, indeed a beautiful theory, proves to be *too simple* within our scenario. The lack in its foundational premises of the "calibration", i.e. of the "gauge" concept, moves it out of the game.

In summary, the geometrical mechanism proposed by the present work represents a unifying scenario by which a unique quantum field appears to play, by different routes and under different forms, an essential role in determining the evolution of the Universe "at large" as well as, at the microscopic level and via the dynamics of the Weyl scalar curvature R_W, of the everyday quantum phenomenology [39, 40].This appears to be a glimpse into quantum gravity.

Acknowledgments

Thanks are due to Enrico Santamato, Valeria Ferrari and Claus Kiefer for valuable advices and collaboration. Most of the content of the present work was first presented at the Conference "Quantum and Beyond" held in Vaxjo, Sweden, in the month of June 2016.

References

[1] Starobinsky A 1980 *Physics Letters* **91** 99
[2] Guth A 1981 *Physical Review D***23** 347
[3] Linde A D 1982 *Physics Letters B* **108** 389
[4] Albrecht A and Steinhardt P 1982 *Phys. Rev. Lett.* **48** 1220
[5] De Martini F and Santamato E 2016 *ArXiv:1609.01149[gr-qc]3044* URL `http://arxiv.org/abs/1609.01149`
[6] Dodelson S 2003 *Modern Cosmology* (New York : Academic Press)
[7] Weinberg S 2008 *Gravitation and Cosmology* (New York: Wiley)
[8] Weyl H 1952 *Space, Time, Matter* 4th ed (New York: Dover Publications, Inc.)
[9] Lord E A 1979 *Tensors, Relativity and Cosmology* (New Delhi, New York: Tata McGraw-Hill Co.)
[10] Quigg C 1983 *Theories of the Strong, Weak and Electromagnetic interactions* (Menlo Park: Benjamin Publishing Co.)
[11] Dirac P 1973 *Proceedings Royal Society of London A* **333** 403
[12] Jammer M 2000 *Concept of Mass* (Princeton: Princeton University Press)
[13] Ehlers J and Schild A 1973 *Commununications Mathematical Physics* **32** 119
[14] Trautman A 2012 *Gen. Rel. and Gravitation* **44** 1581
[15] Scholz E 2012 *ArXiv:1206.1559[gr-qc]* URL `http://arxiv.org/abs/1206.1559`
[16] Amendola L and Tsujikawa S 2010 *Dark Energy* (New York: Cambridge University Press)
[17] Bezrukov F L and Shaposhnikov M E 2014 *Physics Letters B* **734** 249
[18] Salvio A and Mazumdar A 2015 *Physics Letters B* **750** 194
[19] Hamada Y and Kawai H and Oda K 2015 *Physical Review D* **92** 4509
[20] Zee A 2013 *Einstein gravity in a nutshell* (New York: Princeton University Press)
[21] Higgs P 1964 *Phys. Rev. Letters* **13** 508
[22] Englert F and Brout R 1964 *Phys. Rev. Letters* **13** 321
[23] Guralnik G and Hagen C and Kibble T 1964 *Phys. Rev. Letters* **13** 585
[24] Weinberg S 1964 *Phys. Rev. Letters* **19** 1264

IOP Conf. Series: Journal of Physics: Conf. Series **880** (2017) 012026 doi:10.1088/1742-6596/880/1/012026

[25] Salam A 1964 *Elementary Particle Physics* (Stockolm: Almqvist, Wicksell)
[26] Weinberg S 2013 *The Quantum Theory of Fields, I and II* (Cambridge, New York: Cambridge University Press)
[27] Georgi H 2009 *Weak interactions and Modern Particle Physics* (New York: Dover Publications, Inc.)
[28] Penrose R and Hawking S *The nature of space and time* (New York: Princeton University Press)
[29] Goldstone J and Salam A and Weinberg S 1962 *Physical Review* **127** 965
[30] Hochberg D and Plunien G 1991 *Physical Review D* **43** 3358
[31] Aad G and ATLAS collaboration 2012 *Physics Letters B* **718** 1
[32] Chatrchyan S and CMS collaboration 2012 *Physics Letters B* **718** 30
[33] ATLAS and CMS collaborations 2015 *Physical Review Letters* **114** 191803
[34] Chen H 1988 *Physical Review Letters* **61** 2182
[35] De Bernardis P 2000 *Nature* **404** 955
[36] PLANCK 2015 *ArXiv:1502.01589v3[gr-qc]* URL `http://arxiv.org/abs/1502.01589v3`
[37] Sanders R H 2010 *The Dark Matter problem* (New York: Cambridge University Press)
[38] Feynman R P 1965 *Quantum Mechanics and Path Integrals* (New York: McGraw-Hill Publishing Co.)
[39] De Martini F and Santamato E 2014 *Physica Scripta* **T163** 014015
[40] Santamato E and De Martini F 2015 *Foundations of Physics* **45** 858