26 A SUSY $SU(5) \times T'$ Unified Model of Flavour with large θ_{13}

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Abstract We present a SUSY $SU(5) \times T'$ unified flavour model with type I see-saw mechanism of neutrino mass generation with $\theta_{13} \approx 0.14$ close to the recent results from the Daya Bay and RENO experiments. The model predicts also values of the solar and atmospheric neutrino mixing angles, which are compatible with the existing data. The T' breaking leads to tri-bimaximal mixing in the neutrino sector, which is perturbed by sizeable corrections from the charged lepton sector. The model exhibits geometrical CP violation: all complex phases arise from the complex Clebsch–Gordan coefficients (CGs) of T'. Both normal and inverted ordering are possible for the light neutrino mass spectra. We give also predictions for the $2\beta 0\nu$-decay effective Majorana mass.

26.1 Introduction

The recent results of the short-baseline reactor experiments on θ_{13}, Daya Bay [1] and RENO [2], clearly indicate that the precise measurements era for neutrino physics has started. A non zero value of θ_{13} has been reported with an accuracy around 5σ by both experiments. More specifically, Daya Bay and RENO measured $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$ and $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$, respectively. Motivated by the fact that at present we know all three angles in the PMNS mixing matrix with a good precision, we tried to construct a unified model of flavour, which describes correctly the quark and charged lepton masses, the mixing and CP violation in the quark sector, the mixing in the lepton sector and predicts a value of the angle θ_{13} compatible with the recent data (all the details in [3]). The model is supersymmetric and is based on two main ingredients: i) a GUT embedding using $SU(5)$ as gauge group; this may eventually lead to a sizable θ_{13} [4] ii) a discrete family symmetry T', double-valued group of the tetrahedral symmetry T which is isomorphic to A_4; the latter has three inequivalent spinorial unitary irreducible representations which are relevant in the description of the quarks and lepton mixing. Moreover the complex CGs of the T' group can be source of CP violation, so-called "geometrical" CP violation.

We must notice that an interesting model based on $SU(5) \times T'$ as symmetry group was proposed in the literature [5] [6], but it is now ruled out by the current data on θ_{13}. In contrast, due to a non-standard Higgs sector content [4, 7], the model we are going to describe predicts a value for this angle compatible with the recent data.

The model presented in this talk includes three right-handed (RH) neutrino fields N_{lR}, $l = e, \mu, \tau$, which possess a Majorana mass term. The light active neutrino masses are generated by the type I see-saw mechanism and are naturally small. The corresponding Majorana mass term of the left-handed flavour neutrino fields $\nu_{lL}(x)$, $l = e, \mu, \tau$, is diagonalized by the so-called tri-bimaximal unitary matrix:

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix} . \tag{26.1}$$

Of course this mixing pattern has to be "corrected" in order to get a non zero value for θ_{13} in the standard PMNS mixing matrix, U_{PMNS}. Indeed from the simultaneous diagonalization of the neutrino and the charged lepton mass matrices the PMNS mixing matrix reads (RL convention):

$$U_{PMNS} = U^{\dagger}_{eL} U_{\nu} \tag{26.2}$$

Moreover, a relation between the charged leptons and the down-type quarks mass matrices is established through the SU(5) gauge symmetry in such a way that the antireactor mixing angle θ_{13} results connected to the Cabibbo angle θ^c: $\sin^2\theta_{13} \cong C^2(\sin^2\theta^c)/2$ where $C \cong 0.9$ is a constant determined from the fit.

26.2 Matter and Scalar Fields

The model we proposed in [3] is based on $SU(5)$ as gauge group and T' as discrete family symmetry plus an extra shaping symmetry, $Z_{12} \times Z_8^3 \times Z_6^2 \times Z_4$, which is required to select the correct vacuum alignments and to forbid unwanted terms and couplings in the superpotential. We impose as well the $U(1)_R$ symmetry, the continuous generalization of the usual R-parity. The three generations of matter fields are defined in the usual $\bar{\mathbf{5}}$ and $\mathbf{10}$, representations of $SU(5)$, $\bar{F} = (d^c, L)_L$ and $T = (q, u^c, e^c)_L$ and we introduce three heavy right-handed Majorana neutrino fields N as singlets under $SU(5)$. The Higgs sector is composed by a number of copies of Higgs fields in the $\mathbf{5}$ and $\bar{\mathbf{5}}$ representation of $SU(5)$ which contain as linear combinations the two Higgs doublets of the MSSM. To get realistic mass ratios between down-type quarks and charged leptons and to get a large reactor mixing angle we have introduced Higgs fields in the adjoint representation of $SU(5)$, $\bar{\mathbf{24}}$, which are as well responsible for breaking the GUT group. In Tab. 26.1 we summarize the charge assignments of the matter

	T_3	T_a	$\bar{F}$	N	$H_5^{(1)}$	$H_5^{(2)}$	$H_5^{(3)}$	$\bar{H}_5^{(1)}$	$\bar{H}_5^{(2)}$	$\bar{H}_5^{(3)}$	$\bar{H}''_5$	H''_{24}	$\tilde{H}''_{24}$
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{5}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\mathbf{24}$	$\mathbf{24}$
T'	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}''$	$\mathbf{1}''$

Table 26.1: Matter and Higgs field content of the model including quantum numbers.

and the Higgs fields under $SU(5) \times T'$ (the charge assignments under the extra symmetries are given in full detail in [3]). Note that the right-handed neutrinos N are accommodated in T' triplets in such a way that the tri-bimaximal mixing pattern arises in the neutrino sector before considering corrections from the charged lepton sector. Notice that the complex CGs, involved whenever the spinorial representation is used, is a source of CP violation in the quark and in the lepton sector.

The scalar sector of fields related to the breaking of T' is composed by 13 flavons. We introduce triplets with two possible alignment in flavour space:

$$\langle\phi\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}\phi_0\,, \quad \langle\tilde{\phi}\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}\tilde{\phi}_0\,, \quad \langle\xi\rangle = \begin{pmatrix}1\\1\\1\end{pmatrix}\xi_0\,. \tag{26.3}$$

The alignment $(0,0,1)$ is relevant for the quark and the charged lepton sector while the $(1,1,1)$ alignment couples only to the neutrino sector. For the doublets we considered two possible orthogonal alignments:

$$\begin{aligned}\langle\psi'\rangle &= \begin{pmatrix}1\\0\end{pmatrix}\psi'_0 \sim 2'\,, \quad \langle\psi''\rangle = \begin{pmatrix}0\\1\end{pmatrix}\psi''_0 \sim 2''\,,\\ \langle\tilde{\psi}'\rangle &= \begin{pmatrix}1\\0\end{pmatrix}\tilde{\psi}'_0 \sim 2'\,, \quad \langle\tilde{\psi}''\rangle = \begin{pmatrix}0\\1\end{pmatrix}\tilde{\psi}''_0 \sim 2''.\end{aligned} \tag{26.4}$$

Furthermore we have introduced six flavons in one-dimensional representations of T' which receive all non-vanishing (and real) vevs

$$\langle\zeta'\rangle = \zeta'_0\,, \quad \langle\zeta''\rangle = \zeta''_0\,, \quad \langle\tilde{\zeta}'\rangle = \tilde{\zeta}'_0\,, \quad \langle\tilde{\zeta}''\rangle = \tilde{\zeta}''_0\,, \quad \langle\rho\rangle = \rho_0\,, \quad \langle\tilde{\rho}\rangle = \tilde{\rho}_0\,. \tag{26.5}$$

The primes indicates the type of singlets 1, $1'$, $1''$. We assume here that all flavon vevs are real i.e. we considered as only source of CP violation the complex CGs introduced geometrically by the group T'. In the Appendix of [3] we show a superpotential that provides the desired flavon vev structure. The latter is obtained adding the so called "driving fields", fields that are gauge singlets but transform non trivially under T' and the extra shaping symmetry.

26.3 Yukawa couplings

When T' breaks and the flavons take their real vevs, one can write down at GUT scale the Yukawa coupling matrices (RL convention). In our model the elements of the Yukawa coupling matrices are generated dynamically through a number of effective operators up to dimension seven which structure is tightly related to the matter fields assignment under the T' discrete symmetry. CP violation in the quark and charged lepton sector is entirely due to the CGs of the T' discrete group. For the up-type quarks we find:

$$Y_u = \begin{pmatrix}\bar{\omega}a_u & ib_u & 0\\ ib_u & c_u & \omega d_u\\ 0 & \omega d_u & e_u\end{pmatrix}\,, \tag{26.6}$$

while in the down-type sector and the charged lepton sector the Yukawas read:

$$Y_d = \begin{pmatrix}\omega\, a_d & ib'_d & 0\\ \bar{\omega}\, b_d & c_d & 0\\ 0 & 0 & d_d\end{pmatrix} \quad \text{and} \quad Y_e = \begin{pmatrix}-\frac{3}{2}\,\omega\, a_d & \bar{\omega}\, b_d & 0\\ 6\, i\, b'_d & 6\, c_d & 0\\ 0 & 0 & -\frac{3}{2}\, d_d\end{pmatrix}\,, \tag{26.7}$$

where $\omega = (1+i)/\sqrt{2}$ and $\bar{\omega} = (1-i)/\sqrt{2}$. The ten parameters appearing in the matrices are (real) functions of the underlying parameters. Notice that in this model $b-\tau$ unification is not realized. Indeed in order to get fermion mass ratios compatible with experimental data we used new relations that have been recently proposed in the literature [7], for instance $y_\tau/y_b = -3/2$ and $y_\mu/y_s \approx 6$. Furthermore it was shown in [4] (see also [8]) that those new $SU(5)$ CGs might also give a large reactor neutrino mixing angle θ_{13}. More importantly, due to the $SU(5)$ symmetry of the model, Y_d and Y_e are related (and therefore the corresponding down quark and charged lepton mass matrices) and are expressed in terms of the same parameters. As a consequence, since Y_e (and as well Y_d) is a block diagonal matrix in the 1-2 sector it is diagonalizable by a rotation of angle θ^e_{12} (i.e. $U_{eL} \sim R_{12}(\theta^e_{12})$). In this way θ^e_{12} is related to the Cabibbo angle $\theta^c \cong 0.226$. Specifically using the results of a fit performed on the 10 parameters that appear in the Yukawas from GUT scale down to the electroweak scale (more details in [3]) we get:

$$|V_{us}| = \left|\frac{b_d}{c_d}\right| + \mathcal{O}(a_d)$$

$$\theta^e_{12} = \left|\frac{6ib'_d}{6c_d}\right| + \mathcal{O}(a_d) = \left|\frac{b'_d}{b_d}\right|\theta^c, \quad (b'_d = 0.9\,b_d)$$

One can also get an expression for the angle $\theta^{\mathrm{PMNS}}_{13}$ using the equation (26.2):

$$\theta^{\mathrm{PMNS}}_{13} = \frac{1}{\sqrt{2}}\theta^e_{12} = \frac{0.9}{\sqrt{2}}\theta^c.$$

This value is compatible with the recent Daya Bay and RENO results.

26.4 Neutrino Sector

The model includes three heavy right-handed Majorana neutrino fields N which are singlets under $SU(5)$ and form a triplet under T'. Through the type I seesaw mechanism we generate light neutrino masses. The neutrino sector is described by the following terms in the superpotential

$$\mathcal{W}_\nu = \lambda_1 NN\xi + NN(\lambda_2\rho + \lambda_3\tilde{\rho}) + \frac{y_\nu}{\Lambda}(N\bar{F})_1(H_5^{(2)}\rho)_1 + \frac{\tilde{y}_\nu}{\Lambda}(N\bar{F})_1(H_5^{(2)}\tilde{\rho})_1\,, \tag{26.8}$$

where we have given the T' contractions as indices at the brackets for non-renormalizable terms. From the superpotential we get the mass matrix for the Majorana right-handed neutrinos and the Dirac neutrino mass matrix

$$M_R = \begin{pmatrix} 2Z+X & -Z & -Z \\ -Z & 2Z & -Z+X \\ -Z & -Z+X & 2Z \end{pmatrix}, \quad M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\frac{\rho'}{\Lambda}, \tag{26.9}$$

where X, Z and ρ' are real parameters. The right-handed neutrino mass matrix M_R is diagonalized by the tri-bimaximal mixing (TBM) matrix such that the heavy RH neutrino masses read:

$$U^T_{TBM} M_R\, U_{TBM} = D_N = \mathrm{diag}(3Z+X, X, 3Z-X)$$

It is more convenient to change parametrization and to use $\alpha \equiv |3Z/X| > 0$ and $\phi \equiv \arg(Z) - \arg(X)$ so the diagonal Majorana mass matrix becomes :

$$\begin{pmatrix} 3Z+X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & 3Z-X \end{pmatrix} \longrightarrow |X| \begin{pmatrix} |1+\alpha e^{i\phi}|e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & |1-\alpha e^{i\phi}|e^{i\phi_3} \end{pmatrix}$$

where $\phi_1 = \phi_2 = \phi_3 = 0, \pi$. The light neutrino Majorana mass term is obtained via type I see-saw mechanism:

$$M_\nu \;=\; -M_D^T M_R^{-1} M_D = U_\nu^* \mathrm{diag}\,(m_1, m_2, m_3)\, U_\nu^\dagger\,,$$

where the unitary matrix U_ν that diagonalize the Majorana light mass matrix is proportional to U_{TBM}, precisely:

$$U_\nu = i\, U_{TBM}\, \mathrm{diag}\left(e^{i\phi_1/2}, e^{i\phi_2/2}, e^{i\phi_3/2}\right).$$

The masses of the light neutrinos result:

$$m_i = \left(\frac{\rho'}{\Lambda}\right)^2 \frac{1}{M_i}\,,\;\; i = 1, 2, 3 \quad m_i > 0$$

The value of the phase ϕ defines the type of the neutrino mass spectrum in the model since one can show that:

$$\Delta m^2_{31} \equiv \Delta m^2_A = \frac{1}{|X|^2}\left(\frac{\rho'}{\Lambda}\right)^4 \frac{4\alpha\cos\phi}{\left|1+\alpha\, e^{i\phi}\right|^2 \left|1-\alpha\, e^{i\phi}\right|^2}\,. \tag{26.10}$$

Thus, for $\cos\phi = +1$, we get $\Delta m^2_{31} > 0$, i.e., a neutrino mass spectrum with normal ordering (NO), while for $\cos\phi = -1$ one has $\Delta m^2_{31} < 0$, i.e., neutrino mass spectrum with inverted ordering (IO). In order to find the numerical values of the light masses one can use as input parameters the experimental values of Δm^2_{21} and $r = \frac{\Delta m^2_\odot}{|\Delta m^2_A|} = 0.032 \pm 0.006$. For a given type of neutrino mass spectrum, i.e., for a fixed $\phi = 0$ or π, one can find a value of the parameter α. It is easy in this way to get the value of the lightest neutrino mass, which together with the data on Δm^2_{21} and $\Delta m^2_{31(32)}$ allows to obtain the values of the other two light neutrino masses. Knowing the latter one can find also the two ratios of the heavy Majorana neutrino masses. In the case of NO neutrino mass spectrum ($\phi = 0$), there are two possible values of α and so there are two possible spectra (solution A and B):

$$m_1 \cong 4.44\times 10^{-3}\ \mathrm{eV}, m_2 \cong 9.77\times 10^{-3}\ \mathrm{eV}, m_3 \cong 4.89\times 10^{-2}\ \mathrm{eV},\ \text{solution A (NO)}. \tag{26.11}$$

$$m_1 \cong 5.89\times 10^{-3}\ \mathrm{eV}, m_2 \cong 1.05\times 10^{-2}\ \mathrm{eV}, m_3 \cong 4.90\times 10^{-2}\ \mathrm{eV},\ \text{solution B (NO)}. \tag{26.12}$$

The ratios of the heavy Majorana neutrino masses read for the solution (A) $M_1/M_3 \cong 11.0$ and $M_2/M_3 \cong 5.0$. For solution B we find: $M_1/M_3 \cong 8.33$ and $M_2/M_3 \cong 4.67$. In both cases we have $M_3 < M_2 < M_1$. For the IO spectrum ($\phi = \pi$), we find only one possible value for α and in this case the light neutrino masses read:

$$m_1 \cong 5.17\times 10^{-2}\ \mathrm{eV}, m_2 \cong 5.24\times 10^{-2}\ \mathrm{eV}, m_3 \cong 1.74\times 10^{-2}\ \mathrm{eV},\ \ (\mathrm{IO}), \tag{26.13}$$

i.e., the light neutrino mass spectrum is not hierarchical exhibiting only partial hierarchy. For the heavy Majorana neutrino mass ratios we obtain: $M_1/M_2 \cong 1.014$ and $M_3/M_2 \cong 3.01$. Thus, in this case N_1 and N_2 are quasi-degenerate in mass: $M_1 \cong M_2 < M_3$.

In this model is possible to predict also the values of observables such as the fundamental parameter of $2\beta 0\nu$-decay, the Majorana effective mass $\langle m \rangle$. At this purpose one has to find the values of the angles and phases of the PMNS mixing matrix and this can be done with standard formulae (see [4] for instance) recasting the PMNS mixing matrix in the standard parametrization. We list in table 26.2 the numerical values of the angles and phases found in our analysis. We found that the Dirac phase is $\delta \cong 84.3°$ and the values of the Majorana phases in the standard parametrization are not CP conserving. As one can see the value of δ predicted by the model is close to $\pi/2$: this implies that the magnitude of the CP violation effects in neutrino oscillations, is predicted to be relatively large. The rephasing invariant associated with the Dirac phase reads $J_{\mathrm{CP}} = \mathrm{Im}(U^*_{e1} U_{\mu 1} U_{e3} U^*_{\mu 3}) = 0.0324$. Finally we are

Quantity	Experiment (2σ ranges)	Model
$\sin^2\theta_{12}$	0.275 – 0.342	0.340
$\sin^2\theta_{23}$	0.36 – 0.60	0.490
$\sin^2\theta_{13}$	0.015 – 0.032	0.020
δ	-	84.3°
β_1	-	337.1° + ϕ_3
β_2	-	11.5° + ϕ_3 - ϕ_2

Table 26.2: Numerical results for the neutrino sector. The experimental results are taken from [9] apart from the value for θ_{13} which is the DayaBay result [1].

able to compute the value of the Majorana effective mass $\langle m \rangle$ for NO:

$$\langle m \rangle = 4.90\,(7.95) \times 10^{-3}\ \mathrm{eV}\,, \quad \text{solution A (B)},$$

and for IO:

$$\langle m \rangle = 2.17 \times 10^{-2}\ \mathrm{eV}.$$

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