FINITE MASS SUM RULES

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I INTRODUCTION

Duality has been, and continues to be, a very useful tool in two-body reactions. In this talk I shall review the recent progress that has been made in applying duality to many-body (≥ 3) final states. These applications are closely analogous to the two-body ones, and can conveniently be divided into three main areas:

(A) The use of Finite Mass Sum Rules (FMSR) to constrain high energy (triple-Regge) parameters.

(B) Two-component duality, in particular for the case of Pomeron-particle scattering.

(C) Semilocal duality in Reggeon-particle scattering.

As we shall see below, there is now substantial evidence for the expected dual features in (A) and (C). Both the theoretical and phenomenological evidence is rather controversial in (B), however. I shall mainly discuss the most recent contributions to the above areas.

II THE FMSR

The inclusive cross section for $a + b \to c + X$ measures, in the kinematic region where $s/M^2 \gg 1$ (fig 1), a Reggeon-particle total cross-section $i + b \to X$. Through unitarity, this means that the imaginary part of the forward elastic Reggeon-

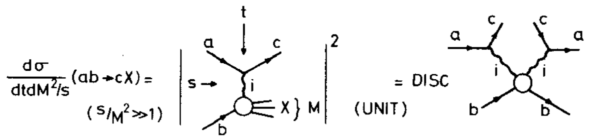

Fig. 1 The relation between the inclusive cross-section for $a+b \to c+X$ and the $b+i \to b+i$ amplitude. Only a single Reggeon i is assumed to be exchanged.

particle amplitude is directly measurable. Just as in ordinary $2 \to 2$ amplitudes, the FMSR[1] relate the low-M inclusive cross section to the high - M triple Regge limit (fig 2). Explicitly, they have the form

$$\int_0^N d\nu\,\nu^n \left[\frac{d\sigma}{dt\,d\nu}(ab\to cX) + (-1)^{n+1} \frac{d\sigma}{dt\,d\nu}(cb\to aX)\right] =$$

$$= \sum_{ijk} \frac{G_{ijk}(t)}{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)+n+1} (s/s_0)^{\alpha_i(t)+\alpha_j(t)-2}$$

$$(N/\nu_0)^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)+n+1} \qquad (1)$$

where G_{ijk} is the product of the Regge couplings in figure 2 and

$$\nu = M^2-t-m_b^2 \qquad (2)$$

The analytic properties of the Reggeon amplitude that allow the derivation of the FMSR (1) have been verified[1] in Regge pole models (ϕ^3 ladder diagrams, the dual resonance model). The validity of equ.(1) when Regge cuts are exchanged is unclear. When supported by the data, the FMSR therefore indicate either that the cuts are weak or that they have the same dual properties as poles.

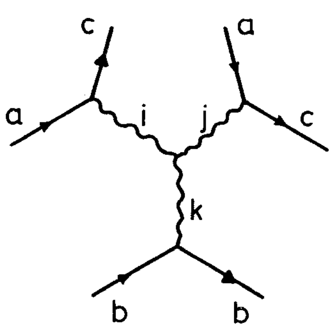

Fig. 2 The triple-Regge limit (s/M^2 and M^2 simultaneously large), with exchange of trajectories $\alpha_i(t)$, $\alpha_j(t)$ and $\alpha_k(o)$. This diagram will be referred to as an ijk diagram.

III THE FMSR AND TRIPLE-REGGE FITS

In order to conclusively test the FMSR we have to see whether the low M inclusive cross-section agrees with the data in the triple-Regge region according to equ.(1). The l.h.s. of equ.(1) can be reliably evaluated using the abundant data in the 10...30 GeV/c range. To determine the triple-Regge couplings on the r.h.s. however, we need data at much higher energies, where the constraints $s/M^2 >> 1$ and $M^2 >> 1$ can be satisfied over an extended region. The ISR-NAL data meet this criterion. Most of the data in this energy range are on the processes $pp \to pX$ and $\pi p \to pX$. At present we can therefore only test the FMSR for reactions like $ab \to aX$, with vacuum exchange allowed in all channels i,j and k of figure 2.

The FMSR were proposed before the ISR-NAL data were available. The first applications(2,3) thus, in effect, predicted the triple-Regge couplings G_{ijk}. The most comprehensive study(3) was done on the reaction $\pi^- p \to pX$ in the energy range 5...40 GeV/c. The main result was that the approximate scaling seen at small M^2/s implied a sizeable PPP coupling. This was contrary to the general belief at the time of the dominance of the PPR term (low-mass diffraction). The result was soon confirmed, however, by the scaling seen in the diffractive peak at high energies.

There are two recent fits(4,5) to all the existing data on $pp \to pX$ in the ISR-NAL range, complemented by the FMSR constraint provided by data at 10...30 GeV/c. Although Field and Fox(5) (FF) use essentially the same set of data as Roy and Roberts(4) (RR), the fitted triple-Regge parameters are quantitatively different in some respects. This is probably due to the large number of contributing triple-Regge terms, even assuming the absence of interference terms. In the following I shall effectively regard the two solutions as providing the limits of uncertainty using the present data.

Both RR and FF agree that the constraint provided by the FMSR is fully consistent with the NAL-ISR data. Figure 3 shows the FMSR integral as a function of P_{lab}, compared with the triple-Regge fits. Both the n=1 and n=3 moments agree remarkably well with the fits. The same is true at larger $|t|$ values. FF also show that fitting the triple-Regge parameters using only the high energy data changes the solution only a little. The FMSR integral is still in agreement with the fit. In fact, the agreement between the low-M data and the triple-Regge extrapolation is semi-local. This is shown in figure 4 where the solution of RR is compared(4) with the M-distribution at 21 GeV/c.

In figures 3 and 4, most of the cross section comes from diffractive production (i=P in figure 1). We thus have evidence that the $Pp \to Pp$ amplitude satisfies the FMSR. This is a remarkable result considering the qualitatively different nature of the pomeron singularity compared to ordinary Regge exchanges and physical particles. This conclusion should be

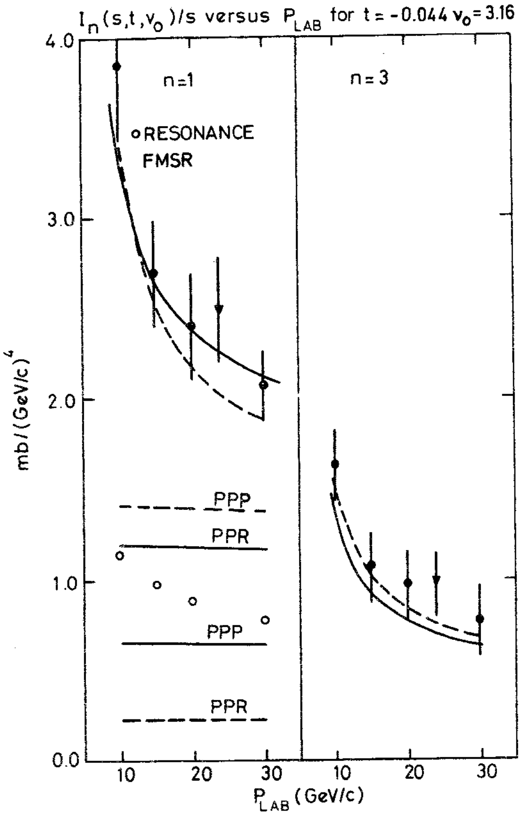

Fig. 3 The FMSR integral evaluated[(5)] using data in the 10 to 30 GeV/c region (solid points). The solid and dashed lines refer to the triple-Regge fits of FF (solution 1)[(5)] and RR[(4)], respectively. The PPP and PPR contributions are explicitly shown. The open circles give the resonance contribution to the FMSR[(5)].

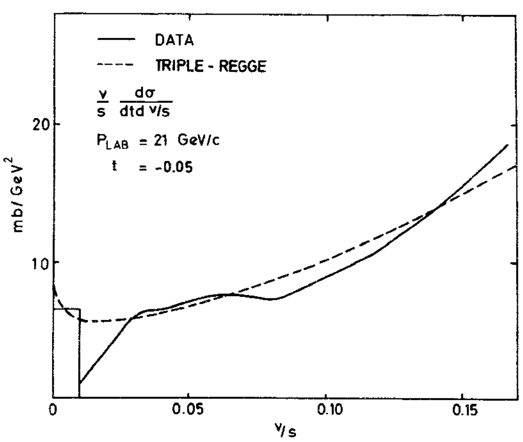

Fig. 4 The triple-Regge fit of RR compared[(4)] to the pp → pX data at 21 GeV/c.

strengthened as data on quantum number exchange reactions becomes available at high energies. The validity of the FMSR for non-pomeron exchanges can then be independently verified.

The validity of the FMSR can also be indirectly tested using only data in the 10...30 GeV/c range plus factorisation. Consider the reactions pp→pX and π-p→pX. The FMSR integrals can be expressed in terms of the triple-Regge diagrams of figure 2, where k is either P or f. From data on total cross-sections we know that the ratio of the pomeron trajectory couplings to $N\bar{N}$ and ππ is about equal to 2. Furthermore, the f trajectory couples in very nearly the same ratio. Hence the factorisation test is independent of the relative amounts of P and f exchange. The result of a recent FMSR analysis[(6)] is shown in figure 5. The ratio of the pp and π-p low-M integrals is indeed close to 2 in the whole energy range considered. The fact that the cut-off N can be chosen as low as 1 GeV^2 is particularly interesting. The Born term contributions (pp and π-p elastic scattering) are in a ratio ≃ 3:1 (again because of factorisation!). This "wrong" ratio is compensated by the inelastic events in the nearby M region, so that factorisation is semilocally satisfied.

IV TWO-COMPONENT DUALITY

Having seen that the FMSR indeed are satisfied, we can proceed to ask whether duality, as in 2→2 reactions, has two components. It is generally agreed that for non-pomeron exchanges i in figure 1 the situation should be completely analogous to that in 2→2. This is supported by the evidence for semilocal duality in Reggeon-particle scattering (see below).

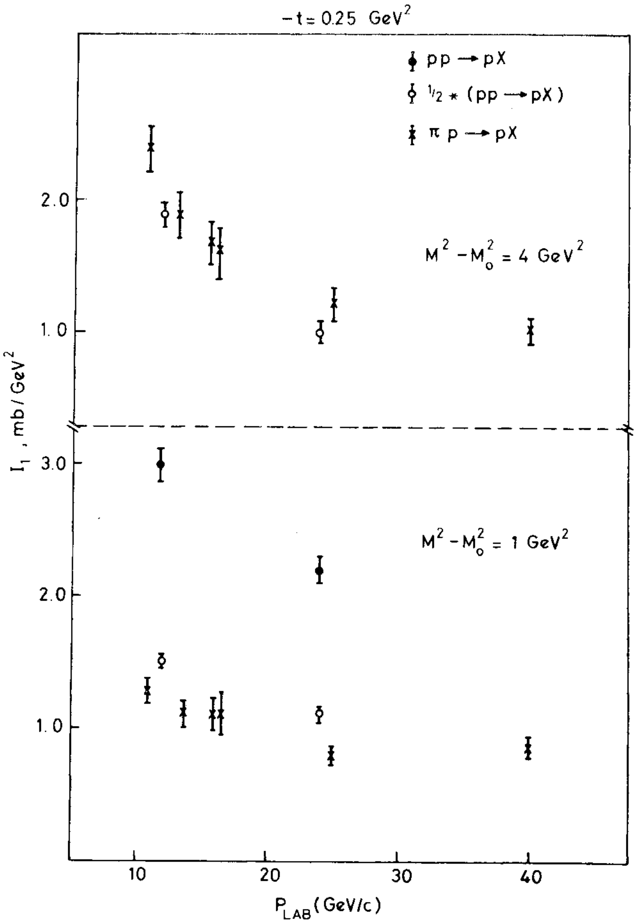

Fig. 5 Comparison[6] of the first moment FMSR integrals for π-p $\rightarrow$ pX and pp $\rightarrow$ pX for two different cut-offs M in the missing mass. $M_o = m_\pi$ (m_p) for the π^-p (pp) reaction.

The situation is much less clear for pomeron-particle amplitudes. On the one hand, Einhorn et al[7] have suggested that the resonances in the direct channel, at least partly, build up the pomeron in the crossed channel (ie, the PPP term). Their arguments are based on dual loop diagrams, and also imply that the PPR term should give a rather small contribution. On the other hand, a naive application of the f-dominance of pomeron couplings[8] would support an ordinary two-component duality scheme, with diffractive resonances building up the meson exchange (PPR term).

Experimentally, this question is hard to decide because of the ambiguity in separating diffractive resonances from the background. In fact, it is not even clear whether bumps like the A_1, Q and N^*(1410) should be called resonances at all. For the sake of this discussion, however, I shall treat them as ordinary resonances.

Since the PPP and PPR terms have a different M-dependence, they could in principle be distinguished by the moment dependence of the FMSR saturated with diffractive resonances. The resonance-background ambiguity makes this doubtful in practice, however, We are therefore limited to comparing the magnitude of the n=1 resonance FMSR with the fitted triple-Regge parameters.

The first indication that duality in pomeron-particle amplitudes may indeed be abnormal was seen by Chan et al[3] in their study of π-p$\rightarrow$pX. Their PPR term was too small to explain all of the resonance production in the missing mass. At least part of the resonance contribution had to be attributed to the PPP term. Capella[9] has shown that if the diffractive resonances are assumed to build up the entire PPP term (neglecting PPR), then estimating PPP from pp$\rightarrow$pN* gives rough agreement with the high energy (ISR) data. Abnormal duality seems thus consistent with the present data.

The triple-Regge fits of RR[4] and FF[5] make it possible to more quantitatively estimate the evidence for abnormal two-component duality. In figure 3 the resonance contribution (diffractive + non-diffractive) to the FMSR is shown[5] as open circles. The solid (dashed) lines represent the PPP and PPR terms in the solution of FF (RR). Although the sum PPP+PPR is consistent in the two solutions (as implied by the full FMSR), the relative magnitude of the two terms are very different. Taken at face value, both solutions would imply abnormal duality. However, while for RR this is because the PPR term is too small to account for the resonances, for FF the PPR term is too large. Evidently a compromise solution between the two could have roughly equal PPP and PPR contributions, making it impossible to decide for or against abnormal duality.

Roberts and Roy(10) have proposed another test of two-component duality in pomeron-particle amplitudes. They observe that the relative coupling of the trajectory **k** in figure 2 (with i=j=P) to different particles b in general depends on k. For example, if k=P the coupling is about equal for b=π and b=K, whereas the couplings differ by a factor 2 if k=f (neglecting(10) a possible f' exchange contribution). A comparison with data on $\pi^- p \to pX$ and $K^- p \to pX$ at 40 GeV/c is shown in figure 6. The ratio is close to 1 in the whole mass range, suggesting that both the diffractive resonances and their background build up the PPP term.

While suggestive, this result cannot be regarded as conclusive evidence for abnormal duality. On the one hand, there is the perennial problem of resonance-background separation which becomes rather serious in the A_3 - L region. At lower masses, on the other hand, f' exchange could make the ratio between the πp and Kp reactions smaller than 2.

In conclusion, the question of two-component duality remains unresolved. As we have seen above, if new data in the NAL-ISR range confirms either of the two triple-Regge solutions(4,5) (and not something in between!), the evidence for abnormal duality would become much stronger. Another possibility for settling the issue is to study the

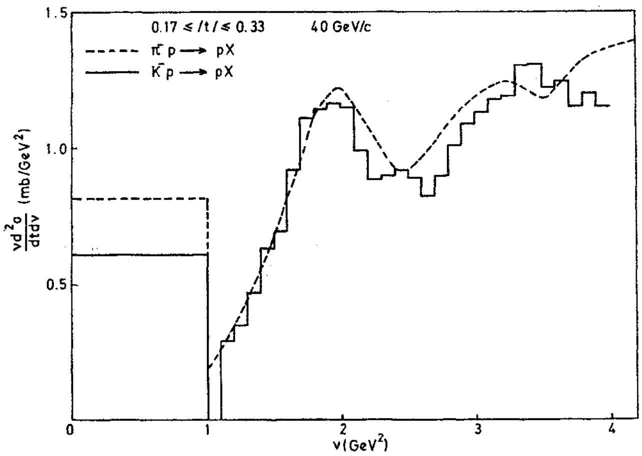

Fig. 6 Comparison(10) between the missing mass spectra in $\pi^- p \to pX$ and $K^- p \to pX$ at 40 GeV/c. The box represents the elastic (X = π, K) contribution.

diffractive production of the πN system in $aN \to a(\pi N)$ (a=π,K or N). The N* resonances in this case are familiar from $\pi N \to \pi N$ (with the possible exception of N*(1410)). A detailed comparison between the $PN \to \pi N$ and $\pi N \to \pi N$ reactions could reveal whether duality is similar in the two cases or not.

V SEMILOCAL DUALITY PREDICTIONS

In this section we shall discuss the duality predictions in cases where i in figure 1 is a non-pomeron trajectory(11). So far, there is very little data available on such reactions in the ISR-NAL energy range. However, in analogy to 2→2 amplitudes, we may use semilocal duality to predict the relative magnitudes of low-M contributions to $ib \to ib$. This makes it possible to take advantage of the abundant data on quasi two-particle processes in the 4...30 GeV/c range.

Requiring that the FMSR relation (1) should be valid for low cut-offs N we find that on the average at low values of M,

$$\frac{d\sigma_i}{dtdM^2}(ab\to cX) \propto s^{2\alpha_i(t)-2}(M^2)^{\alpha_k(0)-2\alpha_i(t)} \quad (3)$$

Here σ_i refers to the part of the cross-section due to the exchange of the Reggeon i. By separating the resonance and background contributions on the l.h.s. we have a single Regge term on the r.h.s, where k is ρ-f or P, correspondingly.

The most striking prediction of equ.(3) is that the cross-section increases faster with M the lower the trajectory $\alpha_i(t)$ is. There is already evidence for this in several reactions[11,12]. We shall here only discuss the recent evidence submitted to this conference.

The Rutherford-Saclay-Ecole Polytechnique collaboration[13] has studied $K^-p\to K^{*-}(890)X^+$ and $K^-p\to K^{*o}(890)X^o$ at 14.3 GeV/c. The exchanged trajectories i in figure 1 are π-B and f-ω-ρ-A_2. There are no interference terms between the unnatural and natural parity trajectories. From equ.(3) we expect the ratio of unnatural to natural parity exchange U/N to rise linearly with M^2. The data shown in figure 7 indeed show this rise in the small M^2/s (Regge) region. A similar behaviour was previously seen in the $K^+p\to K^{*+}X^+$ reaction at 8.2 GeV/c[14].

The relative M-dependence of B and ρ exchange can be studied in $\pi+p\to Res(M)\Delta^{++}$, where Res(M) denotes the (G=-1) resonance contributions to $\pi+p\to\Delta^{++}X$. It has been known for some time that the B exchange contribution to $\pi+p\to\omega^o\Delta^{++}$ is considerable[15]. Semilocal duality then predicts[12], according to equ.(3), that the B/ρ exchange ratio should be a factor $\approx M^2_{A2}/M^2_\omega$ = 2.8 larger in $\pi+p\to A_2^o\Delta^{++}$ than in $\pi+p\to\omega^o\Delta^{++}$. Thus A_2^o should be dominantly produced by unnatural parity exchange[16].

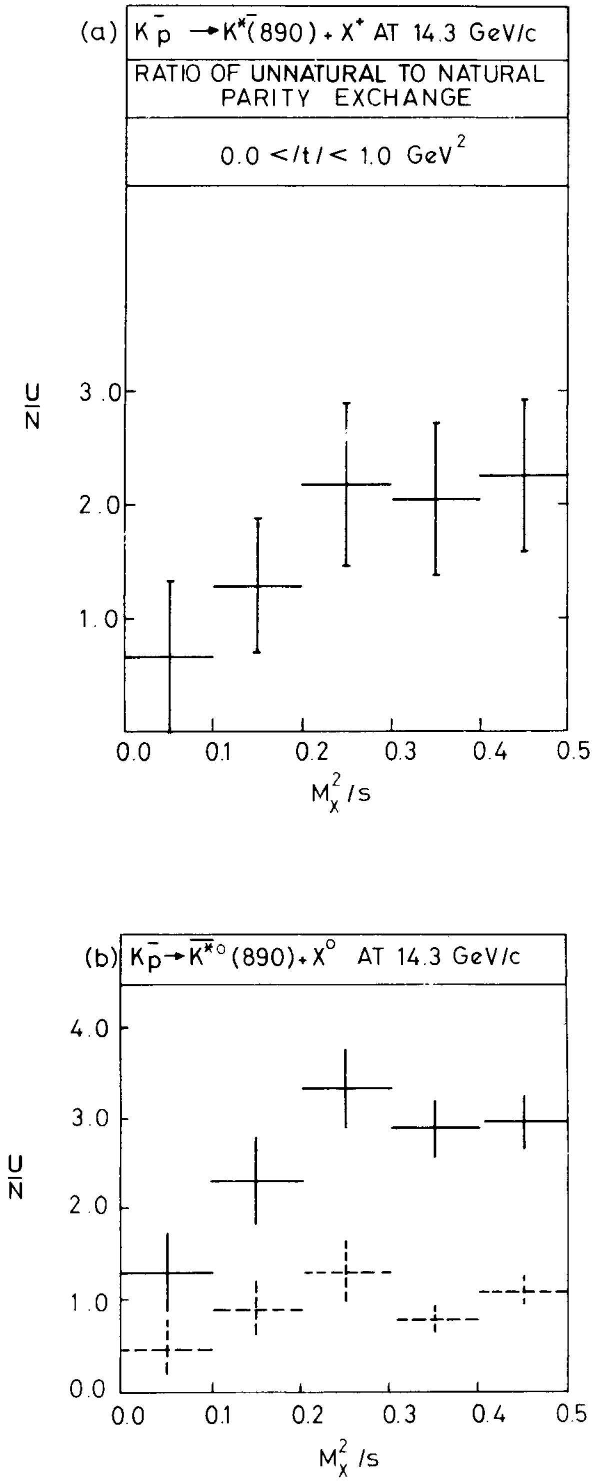

Fig. 7 The ratio between unnatural (U) and natural (N) parity exchange in (a) $K^-p \to K^{*-}(890)$ X and (b) $K^-p \to K^{*o}(890)$ X[13]. In (b) the t interval is 0.2 <|t|< 1.5 GeV² and the dashed data points give the ratio between the (s-channel) helicity one unnatural and natural parity exchanges.

Until recently there was no data on the naturality of the exchange in neutral A_2 production, so the prediction could not be tested. However, the Berkeley 7 GeV/c data have now been anlysed in this way[17]. Figure 8 shows the 3π mass distribution in $\pi + p \rightarrow (3\pi)^o \Delta^{++}$, separated into its exchange components. The unnatural/natural parity exchange ratio increases by a factor ≈ 1.5 from the ω to the A_2 region. It can also be clearly seen that the background is relatively more important in the natural parity exchange component. Making a resonance-background separation would therefore tend to further increase the unnatural parity exchange in A_2^o production.

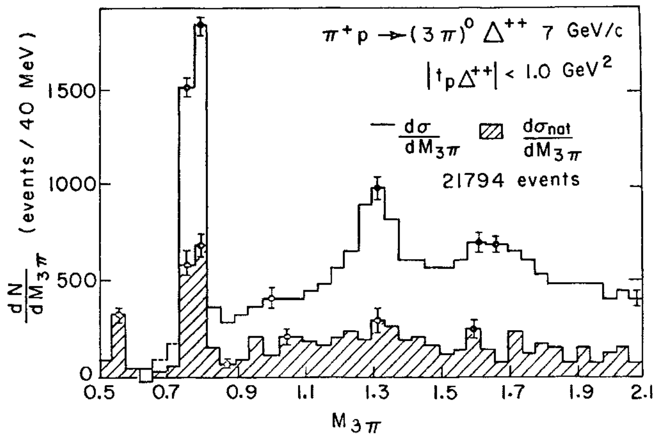

Fig. 8 The 3π mass distribution in the reaction $\pi^+ p \rightarrow (3\pi)^o \Delta^{++}$, separated into its natural and unnatural parity exchange components.[17]

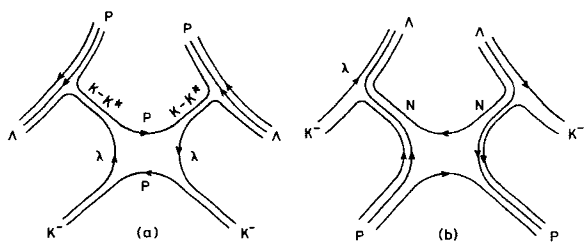

Fig. 9 (a) The duality diagram describing K^*-K exchange in $K^- p \rightarrow \Lambda X$. The resonances in the missing mass (with quark content $p\bar{p}$) are seen to be dual to ϕ-f' exchange. (b) The duality diagram describing nucleon exchange in $K^- p \rightarrow \Lambda X$. The resonances are dual to an exotic exchange.

Further evidence for the dominance of unnatural parity exchange has recently been found in $\pi^+ n \rightarrow A_2^o p$ at 6 GeV/c.[18] The unnatural parity exchange component in $\pi^- p \rightarrow A_2^- p$ is shown to be consistent, for all t, with the $I_t=1$ exchange cross section. This strongly suggests that B exchange dominates both the unnatural parity and $I_t=1$ amplitudes.

One of the advantages with Reggeon-particle amplitudes is the possibility to experimentally study duality features which are difficult to isolate in $2 \rightarrow 2$ amplitudes. An example of this is shown in figure 9a, where the resonances produced in $K^- p \rightarrow Res(M) \Lambda$ by K*-K exchange are expected[12,19] to be dual to the ϕ-f' trajectory. Evidence for this has been presented by the Amsterdam-CERN-Nijmegen Collaboration[20]. Their missing mass spectrum shown in figure 10a contains abundant resonance structure. Combining their data at 4.2 GeV/c with data at 3.93 and 14.3 GeV/c they are able to determine both $\alpha_k(o)$ and $\alpha_i(t)$ in eq.(3). For $\alpha_k(o)$ they obtain 0.06 ± 0.08, nicely consistent with the intercept of the ϕ-f' trajectory. The effective $\alpha_i(t)$ (figure 10b) lies between the K and K* trajectories, as expected.

The backward produced resonances in $K^- p \rightarrow \Lambda X$ should be dual to an exotic exchange[12], as shown in figure 9b. This constitutes another example of an unusual duality feature (duality in $N\bar{N} \rightarrow N\bar{N}$) that can be studied in Reggeon-particle amplitudes. The new data at 4.2 GeV/c[20] are shown in figure 11a. The background under the resonances is now more important. The data are analysed as in the forward production case. The effective intercept $\alpha_k(0)$ in equ.(3) is 0.13 ± 0.06. This is consistent with

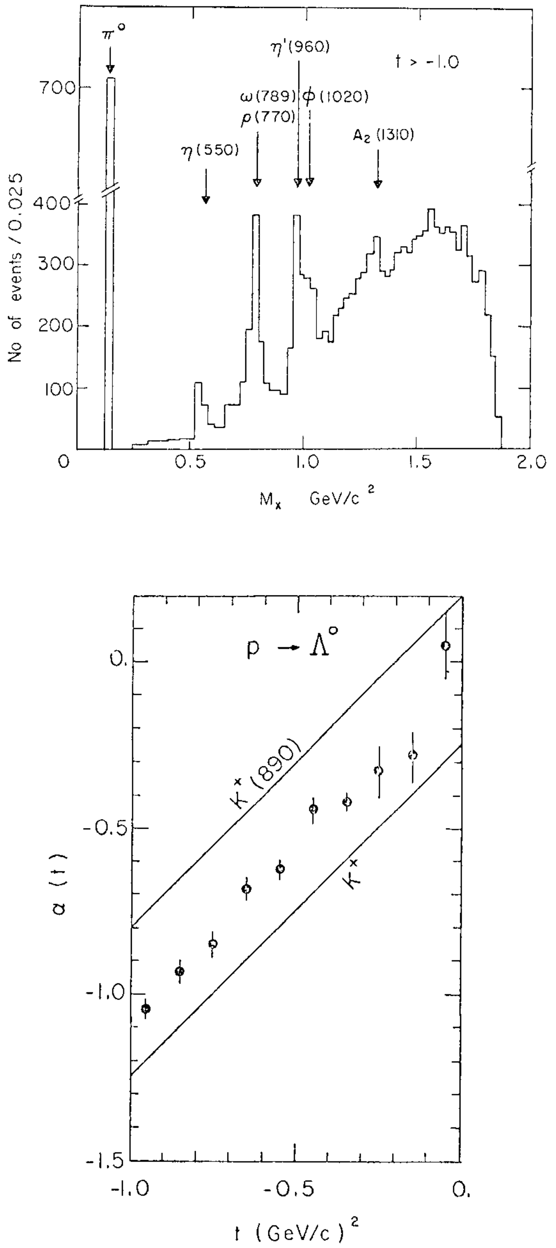

Fig. 10 (a) The missing mass distribution in $K^-p \to \Lambda X$ at 4.2 GeV/c in the forward direction[20]. (b) The effective trajectory exchanged in the $p\bar{\Lambda}$ channel. See text.

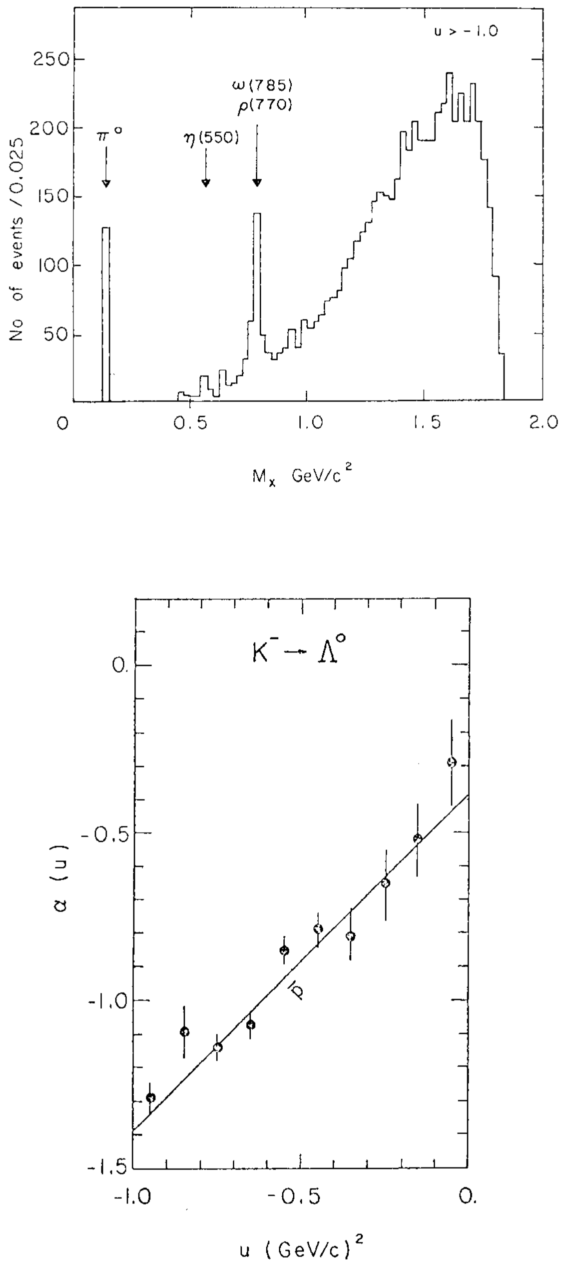

Fig. 11 (a) The missing mass distribution in $K^-p \to \Lambda X$ at 4.2 GeV/c in the backward direction[20]. (b) The effective trajectory exchanged in the $K^-\bar{\Lambda}$ channel, compared to the nucleon trajectory. See text.

theoretical prejudice and earlier analyses[12], which suggest $\alpha_k(0)=1$ for the background component and $\alpha_k(0)\simeq-0.5$ for the resonance component. The exchanged trajectory $\alpha_i(t)$ in equ.(3) is shown in figure 11b. It is very close to the expected nucleon trajectory (solid line).

VI PROSPECTS FOR THE FUTURE

As evidenced by the considerable number of recent studies of the FMSR and related questions, the field of Reggeon-particle physics has generated considerable interest. The fact that duality seems to work so well makes it clear that we can expect many new and exciting developments. Much work remains to be done along the lines that I have discussed above. However, I would like here to briefly mention two rather novel approaches that have recently been exploited.

In analogy to 2→2 FESR, we expect duality to provide many more constraints than those implied by the Reggeon-particle total cross-section FMSR. Non-forward Reggeon FESR's can in fact be realised in two different ways. The phenomenologically more obvious one is depicted in figure 12a. An FESR can be derived for the a+i→ b+c amplitude, provided the Toller angle associated with the Reggeon has a certain value[21]. This amplitude can be obtained through factorisation from a 2→3 amplitude at high energies.

Compared to the inclusive FMSR (figure 1), the novel and important aspects of the FESR for a+i→b+c are (in the case of saturation by resonances):

(i) Only resonances coupling to the two particles b and c can contribute

(ii) The dependence on momentum transfer ($s_{a\bar{b}}$) can be studied and restricts the polarization of the resonances.

A first application of these FESR has been made to study ρ and f production in $\pi^- p \to \pi^- \pi^+ n$ through π and A_2 exchange. Semilocal duality is well satisfied

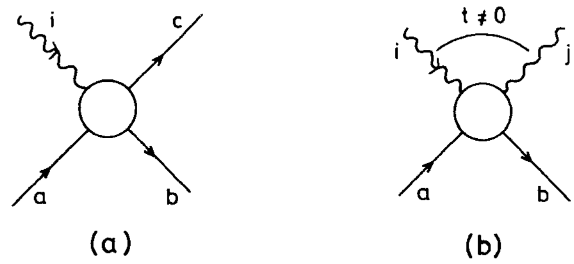

Fig. 12 (a) The a+i → b+c Reggeon amplitude. (b) The a+i → b+j (non-forward) amplitude.

by the data[22], and severely constrains the production amplitudes. Knowing the amplitude for ρ exchange one can, in fact, (correctly) predict f production.

Another (independent) set of duality constraints can be obtained from the a+i→b+j amplitude shown in figure 12b. Apart from the special case of figure 1 (forward elastic) unitarity cannot be used to relate this amplitude to experimentally observable quantities. However, an FESR can again be derived, and the ensuing constraints may be useful theoretically. Such an application has already been made[23] to approximate the overlap integral in the 2→2 unitarity equation. Also, in a resonance saturation scheme, the resonance contributions to the a+i→b+j amplitude can be explicitly calculated once their production mechanism is known. In this sense this FESR does restrict observable quantities.

Acknowledgements

This talk was made possible by the many informative discussions I have had with my colleagues. I am particularly indebted to Drs. Chan Hong-Mo, R Field, A Gula, C Quigg, R G Roberts, D P Roy and F Wagner.

REFERENCES

1 J Kwiecinski, Lett. Nuovo Cim. 3, 619 (1972)
M B Einhorn et al, Phys.Rev. D5, 2063 (1972)
A I Sanda, Phys. Rev. D6, 280 (1972).

2 R M Edelstein et al, Phys.Lett. 35B, 408 (1972)
S D Ellis and A I Sanda, Phys.Lett 41B, 87 (1972).

3 Chan Hong-Mo et al, Nucl.Phys. B54, 411 (1973).

4 D P Roy and R G Roberts, Rutherford Laboratory preprint RL-74-022, T79 (1974) (to be published in Nucl. Phys. B) and private communication from R G Roberts.

5 R D Field and G C Fox, Caltech preprint CALT-68-434 (1974).

6 A Gula, private communication.

7 M B Einhorn et al, Phys.Lett. 37B, 292 (1971); Phys. Rev. D6, 1675 (1972).

8 R Carlitz et al, Phys.Rev. D4, 3439 (1971)

9 A Capella, Phys.Rev. D8, 2047 (1973).

10 R G Roberts and D P Roy, Phys.Lett 47B, 247 (1973) and contribution to the Boston Meson Spectroscopy Conference, April 1974.

11 For reviews see, eg:
D P Roy, in Proceedings of the Eighth Rencontre de Moriond, Ed. J Tran Thanh Van, CNRS publication (1973), Vol. II p499.
R G Roberts, Acta Physica Polonica B5, 47 (1974)
P Hoyer, in Proceedings of the Symposium on Production of Heavy Excited Hadrons, Argonne National Laboratory, March 1974.

12 P Hoyer et al, Nucl. Phys. B56, 173 (1973); Phys.Lett. 44B, 258 (1973).

13 K Paler et al, Contribution to this Conference.

14 P Chliapnikov et al, Nucl.Phys. B37, 336 (1972).

15 See eg, I J Bloodworth et al, Nucl.Phys. B35, 79 (1971).

16 The same prediction has been made, based on SU_3 and factorisation, by G C Fox and A J G Hey, Nucl.Phys. B56, 386 (1973).

17 Group A, Lawrence Berkeley Laboratory, and private communication through F Wagner. The naturality of the exchange was analysed using the Δ^{++} decay, assuming a quark model type coupling for the Δ. For a discussion of this point, see F Wagner, Plenary talk at this conference.

18 H A Gordon et al, Caltech preprint CALT-68-447 (1974).

19 T Inami and H I Miettinen, Phys.Lett. 49B, 67 (1974).

20 Amsterdam-CERN-Nijmegen Collaboration, paper contributed to this conference.

21 P Hoyer and J Kwiecinski, Nucl. Phys. B60, 26 (1973).

22 P Hoyer et al., Phys. Rev. D10, 80 (1974).

23 Chan Hong-Mo and J E Paton, Phys. Lett. 46B, 228 (1973) and work in progress. For an account of this work, see
Chan Hong-Mo, Lectures given at the IXth Balaton Symposium on Particle Physics, June 1974.