EFFECTS OF FIELD PERTURBATIONS IN FFAG ACCELERATORS*

*G. Parzen***, *P. L. Morton*

Midwestern Universities Research Association, USA

INTRODUCTION

Because of the nonlinear magnetic guide fields in FFAG accelerators, the effects of perturbing magnetic fields have some aspects which are not found in accelerators with more linear guide fields. This paper will deal primarily with the effects due to the resonances excited by the perturbing fields and, in particular, the effects of the linear coupling resonances $\nu_r + \nu_z = n$, n being an integer, will be studied in detail.

In order to understand the effects of perturbing fields, it is helpful to first review the orbit properties of the unperturbed accelerator.

I. THE UNPERTURBED ACCELERATOR

The unperturbed accelerator has a median plane magnetic field which is given roughly by [1]

$$H_z = -B_0 (r/r_0)^k \{1 + \cos N\varphi\} \qquad (1.1a)$$

$$\varphi = \theta - (1/wN) \ln (r/r_0). \qquad (1.1b)$$

In the computer studies, which will be reported here, the parameters used were $N = 48$, $k = 84.4$, $1/w = 536.3$. This leads to a small amplitude tune $\nu_r = 9.778$, $\nu_z = 6.291$.

One important effect of the nonlinear magnetic guide field is that the tune ν_r, ν_z varies with the amplitude of the betatron oscillation. This variation of the tune with the oscillation amplitude is shown in Fig. 1, by plotting ν_z against ν_r for various initial values of r and z betatron oscillations. The diagram shown in Fig. 1 is called a tune space diagram and it indicates the range of tunes present in a beam that contains particles with all the possible stable betatron oscillation amplitudes. One may note that the range of tunes present is quite large, and that quite a number of resonances cross the region of tune space occupied by the beam. It seems likely that the strongest resonance which is shown in Fig. 1 is the linear coupling resonance $\nu_r + \nu_z = 16$ and it is the effects of this resonance which will be presented in detail in this paper.

The rapid variation of tune with oscillation amplitude shown in Fig. 1 has the disadvantage that many resonances will affect the betatron oscillations of the beam particles. On the other hand, it is believed that the rapid variation of tune with oscillation amplitude will limit the effects of the resonances. The picture that has been adopted is that the perturbation resonances will cause the betatron oscillations to grow for those particles whose tunes are near the resonance involved. However, the accompanying change in tune when the oscillations grow will move the particles away from the resonance, *limiting* the amount of growth in the betatron oscillations.

Perhaps the effect of the perturbation resonances that is of most concern is the coupling of vertical and horizontal betatron oscillations. In a multi-GeV FFAG accelerator, the particles will be injected having much larger horizontal oscillations than vertical oscillations and there is little room for coupling of the horizontal oscillations into vertical oscillations.

The coupling between vertical and horizontal oscillations is displayed in Fig. 2, where $y_{\max}$, the maximum value reached by the vertical oscillation at a particular azimuth is plotted against x_0 the initial value of the horizontal oscillation, and for each point, the initial vertical oscillations is very small. $x_{\max}$, the maximum value reached by the horizontal oscillation at a particular azimuth is also plotted against x_0 in Fig. 2. $x_{\max}$ may be considered as the amplitude of the horizontal oscillation which in these computer runs is different from x_0, the initial value of the horizontal oscillation.

All the betatron oscillation amplitudes given in this paper are given at the azimuthal position where the magnetic field reaches a maxi-

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* Supported by the U. S. Atomic Energy Commission.

The report was not read.

** On leave from the University of Notre Dame, Notre Dame, Indiana.

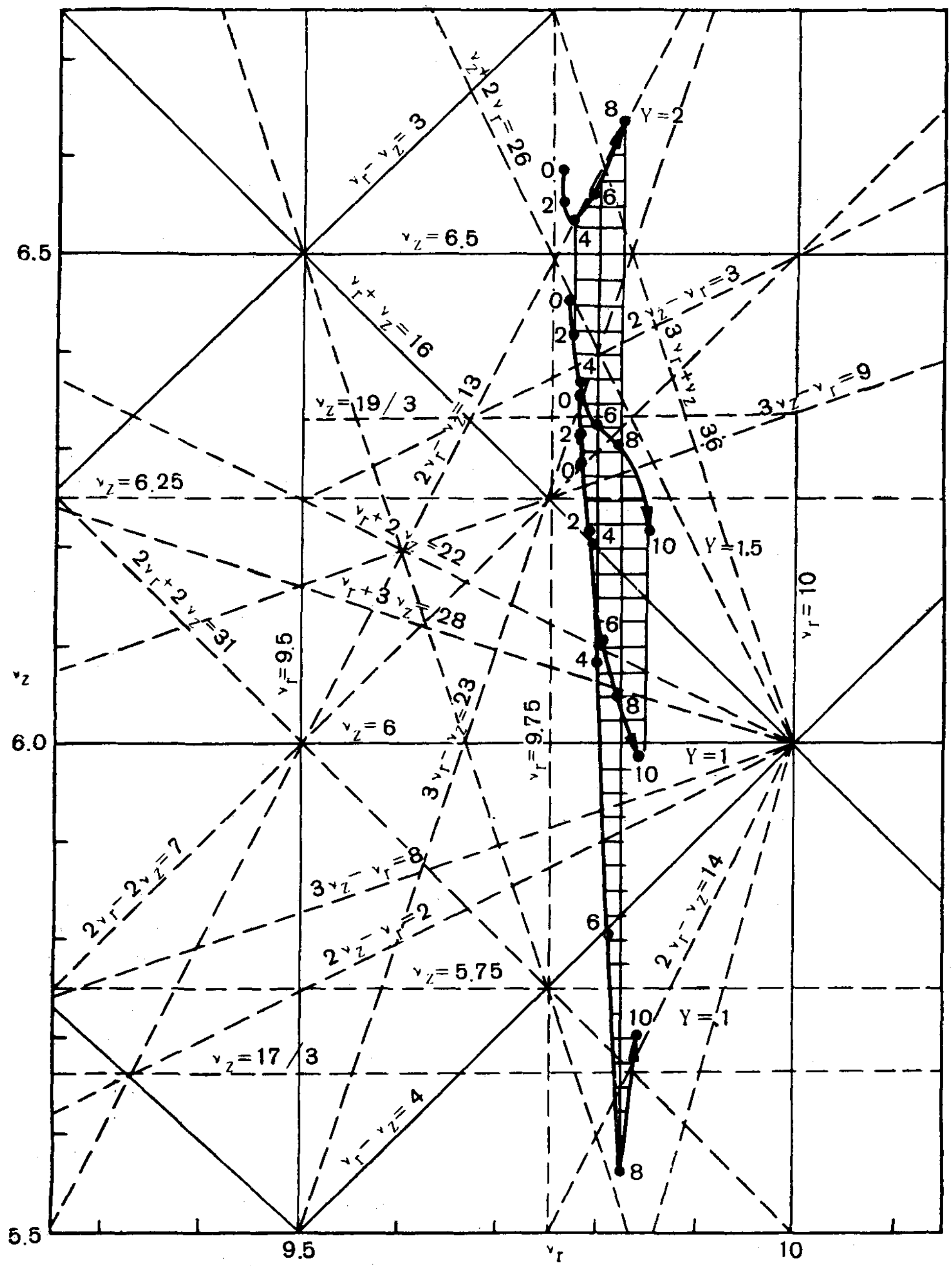

Fig. 1. A tune space diagram for the unperturbed spiral sector FFAG accelerator, showing the range of tunes, ν_r and ν_z, which is present in a beam containing particles with all the possible stable betatron oscillation amplitudes. Each curve of ν_z against ν_r is found by holding the initial vertical oscillation, y, constant at the value indicated, and doing a series of computer runs for various initial horizontal oscillations, x. x in varied by steps of 2 in units of 10^{-4} R where R is the average radius of the orbit. y is given in units of 10^{-4} R. x and y are the initial oscillations at the azimuthal position where the magnetic field reaches its maximum. The cross-hatched area indicates the entire range of tune space that may be occupied by the beam.

mum value. The peak betatron oscillation amplitudes are roughly a factor of 1.4 larger than the values given. One may note from Fig. 2 that the coupling present causes the vertical motion to reach 3 cm when the horizontal amplitude is 13.4 cm for an accelerator

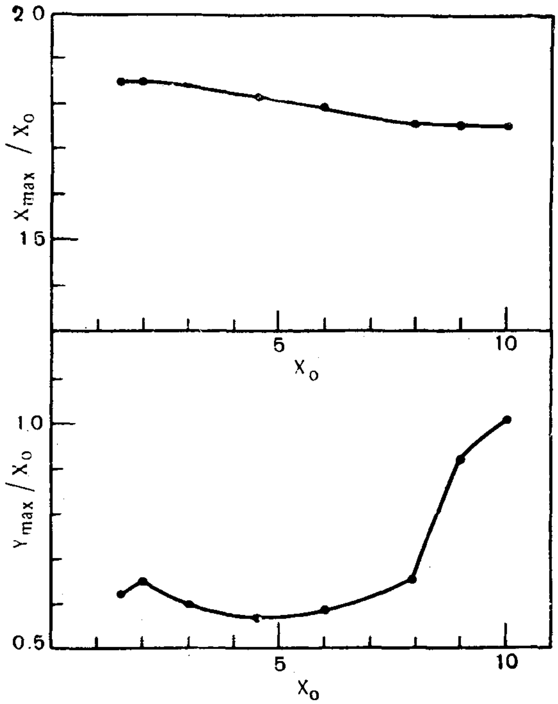

Fig. 2. A plot showing the degree of coupling between vertical and horizontal oscillations in the unperturbed accelerator. Each point is found by doing a computer run starting with a very small vertical oscillation, $y_0 = 10^{-5} R$, and with various values for the initial horizontal oscillation x_0. y_{max} is the largest value of the vertical oscillation reached at the particular azimuth where the magnetic field reaches a maximum. x_{max} is the corresponding value for the horizontal oscillation. x_0 is the initial horizontal oscaillation at this azimuth. x_0 is given in units of 10^{-4} R where R is the average radius of the orbit.

whose radius is 85 m. One may say that the coupling reaches a maximum of 22% for the range of oscillation amplitudes one is likely to use.

II. THE $\nu_r+\nu_z = 16$ RESONANCE

The perturbation resonance which seems likely to be the most troublesome is the nearest linear coupling resonance which for the accelerator being considered here is the $\nu_r + \nu_z = 16$ resonance. This resonance is primarily excited by the sixteenth harmonic of the radial field ΔH_r in the median plane. In order to study the effects of the $\nu_r + \nu_z = 16$ resonance, a perturbation in the magnetic guide field was applied which had only a sixteenth harmonic. The radial component of the perturbation is given by

$$\Delta H_r = -2 \times 10^{-4} B_0 (r/r_0)^k \cos 16\varphi \quad (2.1a)$$

$$\varphi = \theta - (1/wN) \ln (r/r_0), \quad (2.1b)$$

where k and $1/w N$ are the same as for the unperturbed accelerator. The magnitude of the perturbing harmonic is 2×10^{-4} compared with the main harmonics, the zeroth and forty-eighth, which have a magnitude of 1. The amount of perturbing harmonic, 2×10^{-4}, is the estimated tolerance on the perturbing harmonics.

In addition to the perturbation in the radial field, ΔH_r, there is a perturbation in the azimuthal field, ΔH_θ, also present. The ΔH_θ that is present is similar in form to ΔH_r and is such as to satisfy the Maxwell equation $(1/r)\ \partial H_r/\partial\theta = \partial H_\theta/\partial r$.

One observes in Fig. 1, that the linear tune, for the accelerator being considered, is above the $\nu_r + \nu_z = 16$ resonance line. However, as the radial oscillation is increased, while the vertical amplitude is held at some small value, the tune moves downward and crosses the $\nu_r + \nu_z = 16$ resonance. Fig. 3 shows the effect of the $\nu_r + \nu_z = 16$ resonance. A series of computer runs were done by varying the initial radial oscillation amplitude x_0 while holding the initial vertical oscillation amplitude constant at the small value $y_0 = 10^{-5}$ in units of the average radius of the orbit. The length of the runs were from 10 to 40 revolutions. In Fig. 3, y_{max}/x_0 is plotted against x_0 where y_{max} is the largest vertical oscillation achieved at a particular azimuth. x_{max}/x_0 is also plotted against x_0 where x_{max} is the largest radial oscillation achieved.

The effect of the $\nu_r + \nu_z = 16$ resonance is clearly shown in the large coupling which occurs when $x_0 = 3 \times 10^{-4}$ in units of the average radius. The coupling here reaches 50% as $y_{max}/x_{max} = 0.5$, and y_{max} reaches 2.55 cm for an accelerator with an 85-meter radius. In Fig. 3, $\nu_r + \nu_z$ is also plotted against x_0 and one may note that the greatest coupling occurs when $\nu_r + \nu_z = 15.9$ so that the tune is below

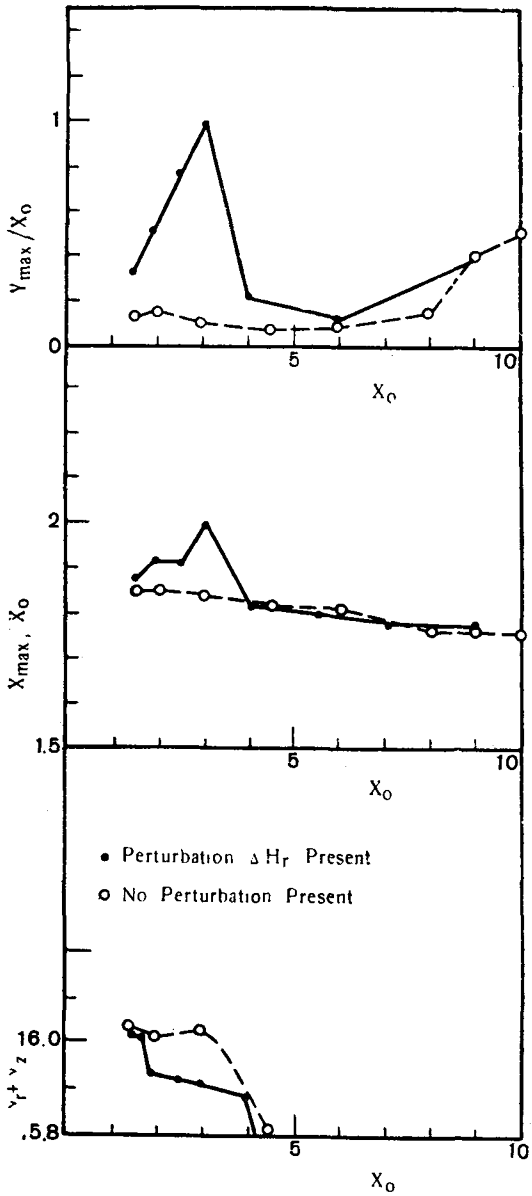

Fig. 3. This graph shows the effect of the $\nu_r + \nu_z = 16$ resonance when a field perturbation ΔH_r is present. Each point is found by doing a computer run starting with a small vertical oscillation, $y_0 = 10^{-5} R$, and with vartious values for the initial horizontal oscillation, x_0. y_{max} is the largest value of the vertical oscallation reached at the particular azimuth where the magnetic field reaches a maximum. x_{max} is the corresponding value for the horizontal oscillation. x_0 is given in units of 10^{-4} R where R is the average radius of the orbit. For the sake of comparison, corresponding results are also plotted for the unperturbed machine.

the resonance line $\nu_r + \nu_z = 16$ rather than when the tune is on the resonance line.

The picture which has been adopted is that when the vertical oscillation grows, the vertical tune ν_z tends to increase. Thus if one starts out with the tune on the resonance line, the tupe soon moves above the resonance line. If one starts with the tune just below the resonance line but close enough to cause growth in the vertical oscillation, then the tune moves up into the stop band and the growth continues until the tune moves above the resonance line.

Fig. 3 also plots x_{max}/x_0 which also has a peak showing the effect of the $\nu_r + \nu_z = 16$ resonance, although the peak here is not as marked as for the y_{max}/x_0 plot. One may note that the resonance causes a 10% growth in the horizontal oscillation. The tune which is plotted in Fig. 3 was obtained by Fourier analyzing the motion over ten revolutions of the particle. One finds that when the tune is measured according to this definition, that the tune will change as the particle continues to go around the machine. The tunes plotted in Fig. 3 were measured using the first ten revolutions of the particle. If one measures the tune again using the second ten revolutions, and the third, and so on, then one finds that the tune appears to move up until it is above the resonance line.

The above results show the effects of the $\nu_r + \nu_z = 16$ resonance. Computer studies have also been done on the $\nu_z = 6.5$ resonance with the same magnitude perturbing field. The effects found for the $\nu_z = 6.5$ resonances were weaker than for the $\nu_r + \nu_z = 16$ resonance.

Computer studies are planned for studying the effects of some of the nonlinear resonances. It is not entirely clear from the present study as to what choice of the tune ν_r and ν_z is the optimum one for the greatest stability against the effects of perturbing fields. Answering this question will require doing lengthy computer studies for several operating tunes. The operating tune employed in this study seems to be a satisfactory choice of tune.

REFERENCE

1. The MURA Staff. In: Proceedings of the International Conference on High Energy Accelerators (Brookhaven, 1961), p. 57.