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Strongly Interacting Multi-component Fermions: From Ultracold Atomic Fermi Gas to Asymmetric Nuclear Matter in Neutron Stars

Hiroyuki Tajima¹, Tetsuo Hatsuda^{1,2}, and Yoji Ohashi³

¹ RIKEN, Nishina Center, 2-1, Hirosawa, Wako, Saitama, 351-0198, Japan

² RIKEN iTHES and iTHEMS, 2-1, Hirosawa, Wako, Saitama, 351-0198, Japan

³ Department of Physics, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama, Kanagawa 223-8522, Japan

E-mail: hiroyuki.tajima@riken.jp

Abstract. We investigate an asymmetric nuclear matter consisting of protons and neutrons with spin degrees of freedom ($\sigma = \uparrow, \downarrow$). By generalizing the Nozières and Schmitt-Rink theory for two-component Fermi gases to the four-component case, we analyze the critical temperature T_c of the superfluid phase transition. Although the pure neutron matter exhibits the dineutron condensation in the low-density region, the superfluid instability toward the deuteron condensation is found to take place as the proton fraction increases. We clarify the mechanism of the competition between the deuteron condensation and dineutron condensation. Our results would serve for understanding the properties of asymmetric nuclear matter realized in the interior of neutron stars.

1. Introduction

Two-component ultracold Fermi gases (such as ${}^6\text{Li}$ and ${}^{40}\text{K}$) are known to be the good testing grounds to study various strong coupling phenomena, because the strength of the pairing interaction between atoms can be controlled by a Feshbach resonance [1, 2, 3]. Indeed, the crossover from the weak-coupling BCS Fermi superfluid to the Bose-Einstein condensate (BEC) of tightly bound molecules, which was originally proposed theoretically [4, 5, 6, 7], has been realized by this novel technique [8, 9]. Such crossover behavior can be confirmed with high precision in various thermodynamic quantities such as the pressure, internal energy, chemical potential, as well as single-particle and collective excitations [10, 11, 12].

The BCS-BEC crossover of the ultracold Fermi gas is essentially dictated by the s -wave interaction with large scattering length in the atomic scale. This situation is analogous to the low-density pure neutron matter where the ${}^1\text{S}_0$ channel (spin-singlet and s -wave) has a scattering length $a_{\text{nn}}^{\text{T}} = -18.5$ fm in the nuclear scale [13]. Indeed, the current experimental results on the equation of state of the ${}^6\text{Li}$ superfluid Fermi gas near the s -wave Feshbach resonance and the numerical simulations of the pure neutron matter in the low density region have quantitative agreement after appropriate rescaling [12]. Although there are some differences between these two systems such as the effect of the effective range and the three-body force, the ultracold Fermi gas is expected to be a useful reference system for the study of pure neutron matter [14].



The above-mentioned agreement makes us expect that a two-component Fermi gas in the BCS-BEC crossover region may also be used as a good starting point for the study of an asymmetric nuclear matter, where protons (with spin $\sigma = \uparrow, \downarrow$) mix with neutrons (with spin $\sigma = \uparrow, \downarrow$). Recently, thermodynamic properties of an asymmetric nuclear matter have attracted much attention, such as astrophysical simulations for supernova explosion [15, 16, 17, 18, 19], the problem of massive neutron stars [20, 21], as well as the so-called symmetry energy [22, 23, 24] in nuclear physics.

In cold atomic Fermi gases, strong-coupling theories that explain various experimental results have been developed in the BCS-BEC crossover region [25, 26, 27]. Therefore, it is an interesting challenge to extend those theories to the four-component system, which may be applied to the physics of the proton-neutron mixture. Even in ultracold atomic Fermi gas, exploration toward the multi-component Fermi gases beyond the two-component would be one of the interesting future directions.

In this paper, we investigate strong-coupling properties of the low-density asymmetric nuclear matter, consisting of protons and neutrons. Extending the strong-coupling BCS-BEC crossover theory developed by Nozières and Schmitt-Rink (NSR) [5] for two-component Fermi gases, we examine the superfluid phase transition temperature T_c . We clarify how T_c varies with the density increased, as well as how the change of proton fraction affects the pairing-type (neutron-neutron pair vs. proton-neutron pair) responsible for the superfluid instability. Throughout this paper, we take the system volume to be unity for simplicity, and set $\hbar = k_B = 1$.

2. Formulation

We consider a mixture of interacting protons ($I = p$) and neutrons ($I = n$), described by the Hamiltonian

$$\begin{aligned}
H = & \sum_{\mathbf{k}, \sigma = \uparrow, \downarrow, I = n, p} \xi_{\mathbf{k}, I} c_{\mathbf{k}, \sigma, I}^\dagger c_{\mathbf{k}, \sigma, I} - \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}, I = n, p} U_{II}^T c_{\mathbf{k} + \mathbf{q}/2, \uparrow, I}^\dagger c_{-\mathbf{k} + \mathbf{q}/2, \downarrow, I}^\dagger c_{-\mathbf{k}' + \mathbf{q}/2, \downarrow, I} c_{\mathbf{k}' + \mathbf{q}/2, \uparrow, I} \\
& - \frac{U_{np}^T}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \left[c_{\mathbf{k} + \mathbf{q}/2, \uparrow, n}^\dagger c_{-\mathbf{k} + \mathbf{q}/2, \downarrow, p}^\dagger + c_{\mathbf{k} + \mathbf{q}/2, \uparrow, p}^\dagger c_{-\mathbf{k} + \mathbf{q}/2, \downarrow, n}^\dagger \right] \\
& \quad \times \left[c_{-\mathbf{k}' + \mathbf{q}/2, \downarrow, p} c_{\mathbf{k}' + \mathbf{q}/2, \uparrow, n} + c_{-\mathbf{k}' + \mathbf{q}/2, \downarrow, n} c_{\mathbf{k}' + \mathbf{q}/2, \uparrow, p} \right] \\
& - \frac{U_{np}^S}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \left[c_{\mathbf{k} + \mathbf{q}/2, \uparrow, n}^\dagger c_{-\mathbf{k} + \mathbf{q}/2, \downarrow, p}^\dagger - c_{\mathbf{k} + \mathbf{q}/2, \uparrow, p}^\dagger c_{-\mathbf{k} + \mathbf{q}/2, \downarrow, n}^\dagger \right] \\
& \quad \times \left[c_{-\mathbf{k}' + \mathbf{q}/2, \downarrow, p} c_{\mathbf{k}' + \mathbf{q}/2, \uparrow, n} - c_{-\mathbf{k}' + \mathbf{q}/2, \downarrow, n} c_{\mathbf{k}' + \mathbf{q}/2, \uparrow, p} \right] \\
& - U_{np}^S \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}, \sigma = \uparrow, \downarrow} c_{\mathbf{k} + \mathbf{q}/2, \sigma, n}^\dagger c_{-\mathbf{k} + \mathbf{q}/2, \sigma, p}^\dagger c_{-\mathbf{k}' + \mathbf{q}/2, \sigma, p} c_{\mathbf{k}' + \mathbf{q}/2, \sigma, n}. \quad (1)
\end{aligned}$$

where $\xi_{\mathbf{k}, I} = k^2/(2m) - \mu_I$ is the kinetic energy of a nucleon measured from the nucleon chemical potential μ_I . In this paper, we ignore difference between the proton mass m_p and neutron mass m_n , for simplicity ($m_p = m_n = m$). Also, we assume that the system is not spin-polarized. $c_{\mathbf{k}, \sigma, I}$ is the annihilation operator of a nucleon, where $\sigma = \uparrow, \downarrow$ represents the spin. $-U_{II}^T$ and $-U_{np}^S$ originate from the strong nuclear forces [28] and correspond to the attractive interactions in the isospin-triplet (spin-singlet) and the isospin-singlet (spin-triplet) channels, respectively. In this paper, we focus on the low-density and low-energy region, so that the nuclear forces are simply treated as the contact-type s -wave pairing interactions. We use the experimental values of the s -wave scattering lengths, $a_{nn}^T = -18.5$ fm, $a_{pp}^T = -7.8$ fm, $a_{np}^T = -23.75$ fm, and $a_{np}^S = 5.42$ fm [13], that are related to the “bare” interactions $-U_{II}^Z$ ($Z = S, T$) as

$$\frac{4\pi a_{II'}^Z}{m} = \frac{-U_{II'}^Z}{1 - U_{II'}^Z \sum_{\mathbf{k}} \frac{k_c}{k^2}}, \quad (2)$$

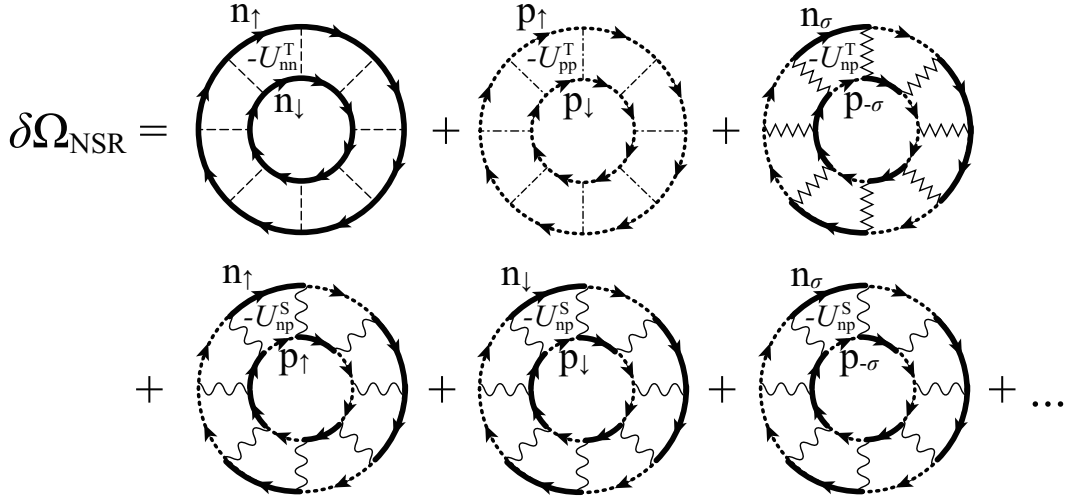


Figure 1. Feynman diagrams describing strong-coupling corrections ($\delta\Omega_{\text{NSR}}$) to the thermodynamic potential Ω of an interacting proton-neutron mixture. The solid (dotted) line shows the bare propagator of a neutron (proton). The neutron-neutron interaction $-U_{\text{nn}}^{\text{T}}$, proton-proton interaction $-U_{\text{pp}}^{\text{T}}$, isospin-triplet neutron-proton interaction $-U_{\text{np}}^{\text{T}}$, as well as isospin-singlet neutron-proton interaction $-U_{\text{np}}^{\text{S}}$ are all included in these diagrams on an equal footing, within the framework of NSR theory.

with k_c being an ultra-violet cutoff momentum. Note that the difference between a_{nn}^{T} and a_{pp}^{T} comes mainly from the Coulomb interaction. Our setup is justified in the low density region where the effective range or higher angular-momentum pairs are negligible. Our theory involves only nucleon masses and scattering lengths as input parameters, in contrast to the case with more realistic finite-range nuclear potential [22]. If we neglect the existence of protons, our model given by Eq. (1) is reduced to the ordinary BCS Hamiltonian in a two-component ultracold Fermi gas.

To examine many-body effects associated with the interactions $-U_{\text{II}'}^{\text{Z}}$ in Eq. (1), we extend the strong-coupling theory developed by Nozières and Schmitt-Rink (NSR) to the present proton-neutron mixture. The resulting strong-coupling corrections ($\delta\Omega_{\text{NSR}}$) to the thermodynamic potential ($\Omega = \Omega_0 + \delta\Omega_{\text{NSR}}$ with Ω_0 being the non-interacting part) are diagrammatically described in Fig. 1. The total Ω is then written as

$$\begin{aligned} \Omega = & -2T \sum_{\text{I}=\text{n,p}} \sum_{\mathbf{k}} \ln \left[1 + e^{-\frac{\xi_{\mathbf{k},\text{I}}}{T}} \right] \\ & + T \sum_{\mathbf{q}, i\nu_l} \left[\ln \left[1 - U_{\text{nn}}^{\text{T}} \Pi_{\text{nn}}(\mathbf{q}, i\nu_l) \right] + \ln \left[1 - U_{\text{pp}}^{\text{T}} \Pi_{\text{pp}}(\mathbf{q}, i\nu_l) \right] \right. \\ & \left. + \ln \left[1 - U_{\text{np}}^{\text{T}} \Pi_{\text{np}}(\mathbf{q}, i\nu_l) \right] + 3 \ln \left[1 - U_{\text{np}}^{\text{S}} \Pi_{\text{np}}(\mathbf{q}, i\nu_l) \right] \right], \end{aligned} \quad (3)$$

where ν_l is the boson Matsubara frequency. In Eq. (3),

$$\Pi_{\text{II}'}(\mathbf{q}, i\nu_l) = - \sum_{\mathbf{k}} \frac{1 - n_{\text{F}}(\xi_{\mathbf{k}+\mathbf{q}/2,\text{I}}) - n_{\text{F}}(\xi_{-\mathbf{k}+\mathbf{q}/2,\text{I}'})}{i\nu_l - \xi_{\mathbf{k}+\mathbf{q}/2,\text{I}} - \xi_{-\mathbf{k}+\mathbf{q}/2,\text{I}'}} \quad (4)$$

is the two-particle pair propagator. In the NSR scheme, the proton (μ_{p}) and neutron (μ_{n}) chemical potentials are determined from the equations for the number density $\rho_{\text{I}=\text{p,n}}$ of protons

and neutrons, that are derived from the NSR thermodynamic potential in Eq. (3) as,

$$\rho_I = -\frac{\partial\Omega}{\partial\mu_I}. \quad (5)$$

The superfluid phase transition temperature in the dineutron channel is determined by the Thouless criterion,

$$1 - U_{nn}^T \Pi_{nn}(\mathbf{q} = 0, i\nu_l = 0) = 0. \quad (6)$$

On the other hand, the superfluid instability being accompanied by deuteron (the isospin-singlet proton-neutron pair) condensation occurs when the following condition is satisfied,

$$1 - U_{np}^S \Pi_{np}(\mathbf{q} = 0, i\nu_l = 0) = 0. \quad (7)$$

For later convenience, we define the proton fraction Y_p by

$$Y_p = \frac{\rho_p}{\rho_n + \rho_p}. \quad (8)$$

In principle, other superfluid instabilities such as the proton superfluid are possible. However, we have numerically confirmed that only the dineutron condensation or the deuteron condensation occur.

3. Results

Figure 2(a) shows the superfluid phase transition temperature T_c in a proton-neutron asymmetric nuclear matter as a function of $(k_{F,n} a_{nn}^T)^{-1}$, where $k_{F,n}$ is the Fermi momentum of neutrons. In cold Fermi gas physics, similar interaction strength $(k_F a_s)^{-1}$ is frequently used, where k_F is the Fermi momentum of a Fermi atomic gas and the scattering length a_s describes a pairing interaction between Fermi atoms. In our model in Eq. (1), the case of pure neutron matter ($Y_p = 0$) is formally the same as the two-component Fermi gas, where a_s corresponds to a_{nn}^T . However, $a_{nn}^T = -18.5$ fm is fixed in a nuclear matter, unlike the tunable a_s in ultracold Fermi gases. Thus, the scaled interaction $(k_{F,n} a_{nn}^T)^{-1}$ changes via the change of the neutron density $\rho_n = k_{F,n}^3/(3\pi^2)$. In this sense, the x -axis in Fig. 2 (a) may be regarded as indicating the density dependence, that is, the neutron density increase as one moves from the left toward the unitary regime, $(k_{F,n} a_{nn}^T)^{-1} \sim 0$ in Fig. 2 (a).

Around the unitarity limit, or the region of high neutron density ($(k_{F,n} a_{nn}^T)^{-1} \simeq 0$), Fig. 2 (a) shows that T_c decreases with increasing proton fraction Y_p , indicating that the existence of protons disfavors the dineutron condensation and promotes the deuteron (proton-neutron pair) condensation, especially in the low-density regime $(k_{F,n} a_{nn}^T)^{-1} \lesssim -0.5$.

To understand the competition between the dineutron condensation and deuteron condensation in the asymmetric nuclear matter shown in Fig. 2 (a), it is important to recall that the isospin-singlet proton-neutron (isospin-triplet neutron-neutron) scattering length is positive (negative). Because of this difference, the “weak-coupling regime” $(k_{F,n} a_{nn}^T)^{-1} \lesssim -1$ in the neutron-neutron Cooper channel corresponds to the “strong-coupling regime” $(k_{F,n} a_{np}^S)^{-1} \gtrsim +1$ in the proton-neutron Cooper channel, giving high superfluid phase transition temperature of deuteron condensation. In “the strong-coupling limit” of the deuteron channel, $(k_{F,n} a_{np}^S)^{-1} \gg +1$, or $(k_{F,n} a_{nn}^T)^{-1} \ll -1$, T_c is conventionally given by the Bose-Einstein condensation temperature $T_{\text{BEC}}^{\text{deuteron}}$ in an ideal Bose gas with $\rho_n Y_p/(1 - Y_p)$ spin-1 deuterons

$$T_{\text{BEC}}^{\text{deuteron}} = \frac{\pi}{m} \left[\frac{\rho_n}{3\zeta(3/2)} \frac{Y_p}{1 - Y_p} \right]^{\frac{2}{3}}, \quad (9)$$

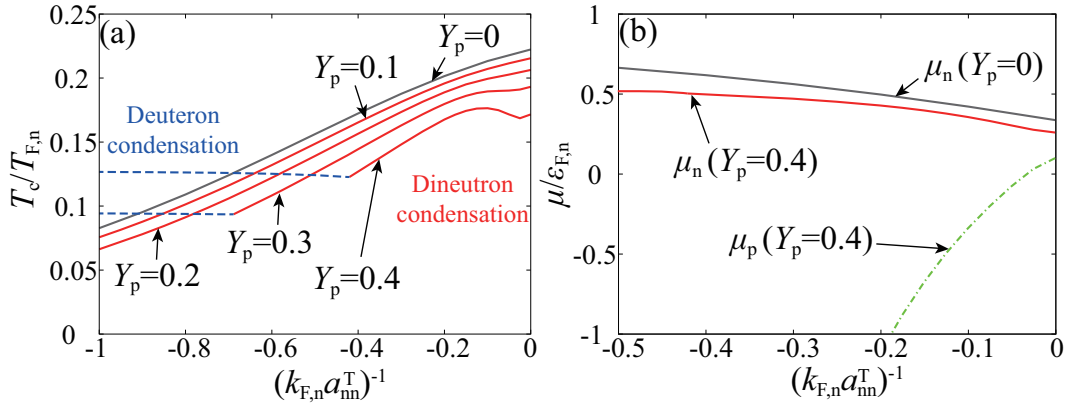


Figure 2. (a) Calculated superfluid phase transition temperature T_c , as a function of $(k_{F,n}a_{nn}^T)^{-1}$. The solid and dashed lines show T_c for the dineutron and the deuteron condensation, respectively. (b) Proton (μ_p) and neutron (μ_n) chemical potential at T_c . $k_{F,n}$ and $\varepsilon_{F,n} = k_{F,n}^2/(2m)$ are the Fermi momentum and the Fermi energy of the neutron component. In these figures, the neutron density ρ_n is fixed instead of the total density $\rho = \rho_n + \rho_p$.

where $\zeta(s)$ is the zeta function. Eq. (9) gives $T_{\text{BEC}}^{\text{deuteron}} = 0.126T_{F,n}$ when $Y_p = 0.4$.

Although the proton-neutron pairing interaction ($a_{np}^S = 5.42$ fm) is stronger than the neutron-neutron interaction ($a_{nn}^T = -18.5$ fm), one sees in Fig. 2 (a) that the superfluid instability is still dominated by dineutrons around the unitarity limit ($(k_{F,n}a_{nn}^T)^{-1} \simeq 0$), even when the system is close to the symmetric matter ($Y_p = 0.4$). This is because the neutron chemical potential μ_n is different from the proton chemical potential μ_p as shown in Fig. 2 (b), as a result of the nucleon density difference as well as the charge dependence, that is, the difference between a_{nn}^T and a_{pp}^T . We note that in the low density region, since the system is dominated by the deuteron formation, the proton chemical potential is given by $\mu_p \simeq -\mu_n + E_b$ where $E_b (< 0)$ is the binding energy of a deuteron. Then, the deuteron pairing is influenced by the “effective magnetic field” $h_{\text{eff}} = \mu_n - \mu_p$, which suppresses the superfluid instability as well known for metallic superconductor under an external magnetic field. Thus, to realize the deuteron condensation, the scaled interaction $(k_{F,n}a_{np}^S)^{-1}$ must be strong enough to overwhelm this depairing effect. On the other hand, h_{eff} does not suppress the neutron-neutron pairing, leading to the dineutron condensation near the unitarity limit even when $Y_p = 0.4$.

We note that T_c would be affected by the effective range which becomes non-negligible in the high density region ($(k_{F,n}a_{nn}^T)^{-1} \gtrsim -0.1$) [29]. Thus, to clarify the superfluid phase transition, as well as the competition between the dineutron condensation and deuteron condensation, we need to extend our present work to include such effect. In addition, the so-called Gorkov-Melik-Barkhadarov (GMB) correction [30] needs to be included to make quantitative prediction of T_c in the weak-coupling regime.

4. Summary

To summarize, we have theoretically investigated strong-coupling properties of a mixture of protons and neutrons with spin degrees of freedom ($\sigma = \uparrow, \downarrow$). Extending the theory of Nozières and Schmitt-Rink for a two-component Fermi gas to the four-component case, we have calculated the superfluid phase transition temperature. We showed that the deuteron condensation becomes more favorable than the dineutron condensation as the proton fraction is increased.

In this paper, we take only two-body pairing interactions and the associated two-particle

pairs in the Cooper-channels. Although this treatment is reasonable for small Y_p , we would need to consider triton and alpha particles when the system is close to the symmetric matter ($Y_p \simeq 0.5$) [31, 32]. Also, the three-body interaction is known to be crucial [22, 33] when the nucleon density approaches to the nuclear saturation density $\rho_0 \simeq 0.17 \text{ fm}^{-3}$. These are the important future problems to be examined. Besides the importance in nuclear physics, studying strong-coupling properties of multi-component Fermi gases beyond the two-component systems is an interesting future direction in the physics of cold Fermi gas.

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