Beyond Standard Model Physics

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Abstract

One era came to an end 4th of July last year when two experiments at the Large Hadron Collider announced the discovery of a resonance consistent with the properties of the Standard Model Higgs boson. The Higgs boson was the last missing piece of the Standard Model and after it mass is unambiguously measured, all the nineteen free parameters of the model are fixed. Although the Standard Model is in agreement with a great number of experimental measurements, it cannot explain all the observations.

In this thesis we investigate extensions of the Standard Model where the standard Higgs sector is replaced by a strongly interacting sector. We begin with a brief introduction to the Standard Model and review the reasons why there must be something beyond the Standard Model. After developing tools to study strongly interacting theories, we continue with the Minimal Walking Technicolor model and examine how it possibly manifests itself at the Large Hadron Collider.

The last chapter of the thesis in devoted to an extension which only adds more particles on top of the Standard Model. One of the new particle is stable and electrically neutral explaining the existence of a type of matter in the universe which does not shine light i.e. dark matter.

Resumé

En æra sluttede i den 4. juli sidste år, da to eksperimenter på Large Hadron Collider annoncerede opdagelsen af en resonans i overensstemmelse med egenskaber af Standardmodellens Higgs boson. Med målingen af Higgsbosonens masse er alle Standarmodellens nitten frie parametre fastlagt. Standardmodellen er med høj præcision i overensstemmelse med et stort antal af observabler ved forskellige energi-skalaer.

I denne afhandling undersøger vi udvidelser af standardmodellen, hvor standard Higgs sektor erstattes af en stærkt vekselvirkende sektor. Vi begynder med en kort introduktion til Standardmodellen og gennemgå årsagerne til, at der må være noget ud over Standardmodellen. Efter udviklingen af værktøjer til at studere stærkt vekselvirkende teorier, fortsætter vi med Minimal Walking Technicolor modellen og undersøger, hvordan det muligvis manifesterer sig ved Large Hadron Collider.

Det sidste kapitel i afhandlingen er dedikeret til en forlængelse, som kun tilføjer flere partikler i forlængelse af Standardmodellen. En af de nye partikler, der er stabil og elektrisk neutral, forklarer eksistensen af en slags stof i Universet, som ikke skinner lys.

Preface

The research for this thesis was done at the Centre for Cosmology and Particle Physics Phenomenology (CP^3-Origins), University of Southern Denmark. I would like to thank my advisor Francesco Sannino for the opportunity to do this PhD in Odense, and for all the guidance during these years. I would also like to thank all the people in CP^3 for the encouraging working atmosphere.

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Tuomas Hapola

List of Publications

The following publications are included in the thesis:

I **Pseudo Goldstone Bosons Phenomenology in Minimal Walking Technicolor**
T. Hapola, F. Mescia, M. Nardecchia and F. Sannino
Eur. Phys. J. **C72**, 2063 (2012) [arXiv:1202.3024 [hep-ph]]

II **W' and Z' limits for Minimal Walking Technicolor**
J. R. Andersen, T. Hapola and F. Sannino
Phys. Rev. **D85**, 055017 (2011) [arXiv:1105.1433 [hep-ph]]

III **Composite Higgs to two Photons and Gluons**
T. Hapola and F. Sannino
Mod. Phys. Lett. **A26**, 2313 (2011) [arXiv:1102.2920 [hep-ph]]

IV **Perturbative Extension of the Standard Model with 125 GeV Higgs and Magnetic Dark Matter**
K. Dissauer, M. T. Frandsen, T. Hapola and F. Sannino
Accepted for publication in Phys. Rev. D [arXiv:1211.5144 [hep-ph]]

Contents

Chapter 1.

Introduction

The Large Hadron Collider (LHC) is the biggest scientific instrument ever created. It accelerates and collides protons along the 27 kilometers long tunnel excavated beneath the Franco-Swiss border. In addition to the main accelerator, several other smaller machines are needed to inject the protons into the LHC. Protons, which result from hydrogen atoms by removing electrons, are first accelerated in the linear collider. From the linear collider, protons are injected into the proton-booster synchrotron, followed by the proton synchrotron and the super proton synchrotron. From the super proton synchrotron, protons with 450 GeV energy are finally injected into the LHC. Protons are then accelerated to obtain the wanted collision energy.

Proton beams consist of 2808 bunches with $1.15 \cdot 10^{11}$ protons in each bunch. At the collision point, the transverse size of both beams is squeezed to a size of tens of microns. Together with bunch crossing interval of the order of tens of nano seconds this should yield an instantaneous luminosity up to 10^{34} cm^{-2}s^{-1}. High instantaneous luminosity and frequent collision rate are desired to be able to study rare processes. The downside is that there will be tens of simultaneous events per bunch crossing i.e. events are piling up.

When two protons collide they break apart into quarks and gluons. High energy collision of these quarks and gluons carrying a small fraction of the total collision energy is what we are actually interested in, and what we can calculate using the Standard Model (SM). This hard process is accompanied by low momentum transfer processes and the experimental challenge is to pin down what happens in the hard interaction.

The main physics goal of the LHC is to complete the dynamical picture of the SM. A first step towards this goal was taken when the two biggest experiments, ATLAS and

CMS, announced the discovery of a resonance consistent with the properties of the SM Higgs boson [1, 2]. This particle was the final missing piece of the SM and after its mass is measured, all the nineteen free parameters of the SM are fixed. The next big step is to find out what lies beyond the SM. Despite of its success it cannot accommodate all the observations and there are a number of theoretical reasons why it cannot be the ultimate theory of nature.

This thesis consists of four original research papers [I-IV] and an introductory and summary parts presented below. In Chapter 2, the SM of the particle physics and its problems are discussed. The chapter begins with a discussion about the symmetries and why the symmetries are the primary reason for the predictive power of the SM. In Chapter 3, we introduce chiral symmetry breaking using the quantum chromodynamics (QCD) as an example. Chapter 4 gives an introduction to Technicolor and Chapter 5 summarizes the results of papers [I-III]. The last Chapter 6 introduces the Dark Matter problem and summarizes the results of paper [IV].

Chapter 2.

Introduction to Elementary Particle Physics

2.1. Symmetries

Symmetry means invariance under a set of transformations. For, example a square is a symmetric geometric object which looks exactly the same if we rotate it by an angle of 90 degrees, or by $n \in \mathbb{Z}$ times this angle. The set of all symmetry transformations form a symmetry group of the object. Of course, not all transformations are symmetric. A rotation is a continuous transformation, but we could have used a discrete transformation as an example as well.

Similarly, the laws of nature can be symmetric. This means the form of the equations describing the law is maintained under a change of variables and/or space-time coordinates. Hence, it is natural to categorize the symmetries accordingly to geometric symmetries and internal symmetries. The geometric symmetries act on space-time coordinates and the internal symmetries do not act on them.

We can relate the continuos symmetries of the equation of motions to conserved quantities. The precise mathematical formulation of this is given by the Noether's theorem [3].

Theorem. *(Noether) If the equations of motion are invariant under a continuous transformation with n parameters, there exist n conserved quantities.*

This applies to both geometric and internal symmetries.

In classical mechanics the basic space time dependent transformations are translation in time, translations in space and rotations in space. If the equations of motion are invariant under these transformations, the corresponding conserved quanties, or conservation laws, are

Time independence	$\rightarrow$	Energy conservation
Translation independence	$\rightarrow$	Momentum conservation
Rotational independence	$\rightarrow$	Angular momentum conservation

In order to discuss the relativistic particles, it is convenient to introduce Minkowski space $\mathcal{M}$ which is a real 4-dimensional vector space parametrized by coordinates $x^\mu = (t, \vec{r})$ and equipped with the metric

$$(ds)^2 = (dx)^2 = dt^2 - (d\vec{r})^2. \tag{2.1}$$

The Lorentz transformations $x \to x' = \Lambda x$ and the space-time translations $x \to x' = x + a$ ($a \in \mathcal{M}$) both form a group of transformations leaving the Minkowski metric invariant. A semi-direct product[1] of these groups form the group of isometries of Minkowski space-time, called the Poincaré group.

Describing an elementary particle should not depend on its position in space-time or if the observer is in uniform motion relative to it. Thus, all the elementary particles can be classified according to representations of the Poincaré group. Moreover, the mathematical description of a particle and its interactions should be invariant under the Poincaré group.

Let us write down a Lagrangian which is invariant under the Poincaré transformations

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu \psi) = \bar{\psi}(i\partial\!\!\!/ \psi). \tag{2.2}$$

It describes a Dirac fermion $\psi(x)$ and is in addition invariant under a global $U(1)$ phase transformation

$$\psi(x) \to e^{ie\alpha}\psi(x), \tag{2.3}$$

where e and α are space-time independent constants. If we take the α to be a space-time dependent function, equation (2.3) is no longer invariant under the $U(1)$ transformation.

[1] The group of translations is Abelian.

In order to restore the invariance, we replace the partial derivative with the covariant derivative

$$D_\mu = \partial_\mu + ieA_\mu, \tag{2.4}$$

where the gauge field A_μ transforms as

$$A_\mu \to A_\mu - \partial_\mu \alpha(x). \tag{2.5}$$

Local phase transformations are called gauge transformations and the invariance under these transformations is known as gauge invariance. The procedure is called gauging and it fixes the form of interactions. The local, and also global, $U(1)$ symmetry is a continuous symmetry and the corresponding conserved quantity, in this case, is the electric charge. This can be generalized to non-abelian compact Lie groups. It is important to note the role of internal and space time symmetries.

The internal and space-time symmetries are described in terms of Lie groups. There is no non trivial way to combine these symmetries according to Coleman-Mandula theorem [4]. If we consider a more general algebraic structure than a Lie group, namely a graded Lie group, this can be omitted. The Haag-Lopuszánski-Sohnius theorem [5] states that the most general graded Lie group of symmetries of a local field theory is the N-extended super Poincaré group. This allows non trivial mixing between internal and space-time symmetries leading to symmetry relating bosons and fermions.

Up to this point, we have considered only exact symmetries. Later, it will be important to differentiate what actually is symmetric, Lagrangian or the solutions, and at what scale the symmetry manifests itself and, if broken, how it is broken.

Explicit breaking can occur via non-invariant terms in the Lagrangian. This does not mean that the symmetry cannot be used to draw conclusions if the breaking is small. Some of the Lagrangian's classical symmetries can be spoiled by the quantum effects. This is called an anomalous breaking and the term driving the breaking is called an anomaly. It is important that in the end all the anomalies are cancelled. Otherwise the renormalizability of the theory is destroyed. If the Lagrangian is more symmetric than the quantum states, the symmetry is said to be spontaneously broken. Especially interesting is if the vacuum is not invariant under the same symmetries as the Lagrangian.

2.2. Goldstone's Theorem

Consider an action which is invariant under the infinitesimal transformation

$$\phi_i \to \phi_i' = \phi_i + \theta^a (\delta\phi)_i^\alpha, \tag{2.6}$$

where , for generality, ϕ_i can be either fermonic or bosonic. Following from the Noether's theorem, we have a conserved charge

$$Q^a(t) = \int d^3x J_0^a(x), \tag{2.7}$$

where the corresponding current is

$$J_\mu^a(x) = -\frac{\delta\mathcal{L}}{\delta(\partial^\mu\phi_i)}(\delta\phi)_i^a. \tag{2.8}$$

Let us take a small side step at this point for further convenience. The conjugate field is defined as

$$\pi_i(x) = \frac{\delta\mathcal{L}}{\delta\partial^0\phi_i}. \tag{2.9}$$

Using the canonical commutation relations, or the anti-commutation relations in the fermion case,

$$[\pi_i(\vec{x},t), \phi_j(\vec{y},t)] = -i\delta^3(\vec{x}-\vec{y})\delta_{ij}, \tag{2.10}$$

one can show that the conserved charges generate the algebra

$$[Q^a(t), Q^b(t)] = if^{abc}Q^c(t). \tag{2.11}$$

From this relation, it follows that also the unintegrated currents satisfy

$$[J_0^a(\vec{x},t), J_0^b(\vec{x},t)] = if^{abc}J_0^c(\vec{x},t)\delta^3(\vec{x}-\vec{y}). \tag{2.12}$$

This is the basis of current algebra which was extensively employed during the early days of modern particle physics.

Let Q be a symmetry charge, and the corresponding conserved current J_μ, so that

$$Q|0\rangle \neq 0 \tag{2.13}$$

The vacuum state $|0\rangle$ is not annihilated by Q and therefore does not represent the true vacuum of the theory. Current conservation implies that

$$\begin{aligned} 0 &= \int d^3x \, [\partial_\mu J^\mu(x), \phi(0)] \\ &= \partial_0 \int d^3x \left[J^0(x), \phi(0)\right] + \int_S d\vec{S} \left[\vec{J}(x), \phi(0)\right], \end{aligned} \tag{2.14}$$

for the field $\phi(0)$. We take the surface S to be large enough so that the last term vanishes. This gives

$$\frac{d}{dt} \, [Q(t), \phi(0)] = 0. \tag{2.15}$$

Then

$$\langle 0| \, [Q(t), \phi(0)] \, |0\rangle = C \neq 0, \tag{2.16}$$

which, after inserting a complete set of intermediate states and integrating over $\vec{x}$ gives

$$\sum_n (2\pi)^3 \delta^3(\vec{p}_n) \left[\langle 0|J^0(0)|n\rangle\langle n|\phi(0)|0\rangle e^{-iE_nt} - \langle 0|\phi(0)|n\rangle\langle n|J^0(0)|0\rangle e^{iE_nt}\right] = C \neq 0. \tag{2.17}$$

Now, if $E_n \neq 0$ the opposite frequency parts cannot cancel and the expression cannot be constant. In order to satisfy the previous equation the intermediate states have to be massless. The existence of these states is proven by the fact that $C \neq 0$. Thus for every broken generator there must be massless state satisfying

$$\langle n|\phi(0)|0\rangle \neq 0, \qquad \langle 0|J^0(0)|n\rangle \neq 0. \tag{2.18}$$

This result is called the (Nambu-)Goldstone's theorem [6].

2.3. Superconductivity

The original motivation to formulate the Higgs mechanism came from condensed matter physics and superconductivity. This section serves as an introduction to spontaneous symmetry breaking and will also be used later on to motivate the idea of dynamical electroweak symmetry breaking.

Superconductivity is the phenomenon of vanishing electrical resistivity at low temperatures and expulsion of magnetic fields from the interior of the sample (the Meissner effect). This phenomenon can be described near the critical temperature, T_c, with the Ginzburg-Landau model which is a generalization of the Landau model for phase transitions.

Let us begin with a Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + iqA_\mu)\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4, \tag{2.19}$$

which couples a complex scalar field ϕ to Maxwell's theory [7]. This Lagrangian is invariant under local $U(1)$ phase rotations. In the static limit, (2.19) reduces to the Ginzburg-Landau Lagrangian

$$\mathcal{L} = \frac{1}{2}(\nabla \times A)^2 + |(\nabla - iqA)\phi|^2 + m^2|\phi|^2 + \lambda|\phi|^4, \tag{2.20}$$

where $m^2 = a(T - T_c)$ near the critical temperature. Above the critical temperature $m^2 > 0$ and the minimum of the potential is at $|\phi| = 0$. When the temperature is below the critical one, the minimum is at $|\phi|^2 = -\frac{m^2}{2\lambda}$. The Vacuum is not anymore invariant under $U(1)$ rotations, which in the static limit are

$$\phi(x) \to e^{i\alpha(x)}\phi(x), \qquad \vec{A} \to \vec{A} + \frac{i}{q}\nabla\alpha(x). \tag{2.21}$$

The conserved current, associated with $U(1)$ rotations, is

$$\vec{j} = -i\left(\phi^*\nabla\phi - \phi\nabla\phi^*\right) - 2q|\phi|^2\vec{A}. \tag{2.22}$$

When $T < T_c$ and the field ϕ varies slowly over the medium:

$$\vec{j} = -\frac{qm^2}{2\lambda}\vec{A} \equiv -k^2\vec{A}. \tag{2.23}$$

This is called the London equation. The Ohm's law

$$\vec{E} = R\vec{j} \tag{2.24}$$

defines the resistance R. From the London equation and current conservation one can deduce that the resistance R must be zero. Thus, we have superconductivity. The Meissner effect can be derived taking the curl of Ampere's equation

$$\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \cdot \nabla) - \nabla^2 \vec{B} = \nabla \times \vec{j}, \tag{2.25}$$

$$\Rightarrow \nabla^2 \vec{B} = k^2 \vec{B}, \tag{2.26}$$

where we have used Gauss law. In one dimension $B_x \sim e^{-kx}$, indicating that the magnetic fields are expelled from a superconductor with penetration depth characterized by $1/k$. By writing equation (2.26) in a covariant form

$$\Box A_\mu = -k^2 A_\mu, \tag{2.27}$$

we can see that the photon has acquired a mass k.

This behavior can be explained in terms of a microscopic theory known as the BCS theory of superconductivity [8]. The main assumption of the BCS theory is that there exists an attractive force between the electrons inside the medium. Thus, the electrons will pair and form a bound state called the Cooper pair. At low temperatures these fall into the same quantum state forming a Bose-Einstein condensate. In this case, the complex ϕ above would be a many particle wave function. The coefficients m^2 and λ can be calculated from the BCS theory which effectively coincides with the Ginzburg-Landau model near the phase transition. The attractive interaction arises when the electron phonon interactions overcome the repulsive Coulomb interaction. The nuclei in a crystal oscillate about their equilibrium positions and the quanta of these vibrations are the phonons.

2.4. Standard Model

The Standard Model (SM) of particle physics is a gauge field theory based on the gauge symmetry group $SU(3) \times SU(2)_L \times U(1)_Y$ [9–11]. The field content of the SM is summarized in Table 2.1 The following notation is used:

Gauge Fields				
Symbol	Associate charge	Group	Coupling	Representation
B	Weak hypercharge	$U(1)_Y$	g'	$(\mathbf{1}, \mathbf{1}, 0)$
W	Weak isospin	$SU(2)_L$	g	$(\mathbf{1}, \mathbf{3}, 0)$
G	color	SU(3)	g_s	$(\mathbf{8}, \mathbf{1}, 0)$
Fermions				
Symbol	Name	Baryon number	Lepton number	Representation
Q_L	Left-handed quark	$\frac{1}{3}$	0	$(\mathbf{3}, \mathbf{2}, \frac{1}{3})$
$(u^c)^i_L$	Right-handed up quark	$\frac{1}{3}$	0	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3})$
$(d^c)^i_L$	Right-handed down quark	$\frac{1}{3}$	0	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3})$
L_L	Left-handed lepton	0	1	$(\mathbf{1}, \mathbf{2}, -1)$
$(e^c)^i_L$	Right-handed lepton	0	1	$(\mathbf{1}, \mathbf{1}, 2)$
Scalars				
Symbol	Name			Representation
H	Higgs boson			$(\mathbf{1}, \mathbf{2}, 1)$

Table 2.1.: The field content of the SM.

$$Q^i_L = \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L \qquad L^i_L = \begin{pmatrix} \nu^i_e \\ e^i \end{pmatrix}_L , \tag{2.28}$$

where $u(d)$ is called an up(down) type quark, e a lepton, ν_e a neutrino and i is the generation index. The SM contains three generations with a naming convention:

$$\begin{aligned} u &\in \{u,\ c,\ t\}, \\ d &\in \{d,\ s,\ b\}, \\ e &\in \{e,\ \mu,\ \tau\}, \\ \nu_e &\in \{\nu_e,\ \nu_\mu,\ \nu_\tau\}. \end{aligned}$$

For further convenience the Lagrangian of the SM is useful to divide into four parts:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}. \tag{2.29}$$

The first term contains kinetic terms for the gauge bosons

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}, \tag{2.30}$$

where

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{2.31}$$
$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk}W^j_\mu W^k_\nu, \tag{2.32}$$
$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc}G^b_\mu G^c_\nu. \tag{2.33}$$

The structure constants are defined as $[\tau^i, \tau^j] = i\epsilon^{ijk}\tau^k$ and $[\lambda^a, \lambda^b] = if^{abc}\lambda^c$, where τ^i and λ^a are the generators of $SU(2)$ and $SU(3)$ groups respectively. The second term contains kinetic terms for the fermions

$$\mathcal{L} = \bar{Q}_L i\not{D} Q_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + \bar{L}_L i\not{D} L_L + \bar{e}_R i\not{D} e_R, \tag{2.34}$$

where the covariant derivative is

$$D_\mu = \partial_\mu - ig'YB_\mu - ig\frac{\tau^i}{2}W^i_\mu - ig_s\frac{\lambda^a}{2}G^a_\mu. \tag{2.35}$$

Here Y denotes the hypercharge of a respective particle. Of course the last two terms in the covariant derivative can be absent if the particle is not charged under the corresponding force.

At this stage all the fermions are massless. A Majorana mass term is not possible because all the fermions carry hypercharge. A Dirac mass term is not allowed because the left-handed and right-handed fermions are not in complex conjugated representations. Thus the fermion sector possesses five accidental global $U(3)$ symmetries for the right and left-handed quarks and leptons.

The Yukawa Lagrangian violates these symmetries

$$\mathcal{L}_Y = -Y_u^{ij}\bar{Q}^i_L\epsilon H u^j_R - Y_d^{ij}\bar{Q}^i_L H d^j_R - Y_e^{ij}\bar{L}^i_L H e^j_R + \text{h.c.}, \tag{2.36}$$

where Y_u, Y_d and Y_e are 3×3 matrices and

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}. \tag{2.37}$$

is the Higgs doublet. A subset of $[U(3)]^5$ symmetries is left intact corresponding to the baryon and lepton numbers. In the end, only a linear combination of these two symmetries is an accidental global symmetry of the SM, separately these symmetries are anomalously broken. When the Higgs field acquires a vacuum expectation value (vev),

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{2.38}$$

the Yukawa interactions form mass terms for the fermions. The neutrinos are still massless because the minimal SM does not contain right-handed neutrinos. Formation of a Majorana mass term after the symmetry breaking is forbidden by the accidental baryon minus lepton number symmetry.

The last part of the Lagrangian is the part containing only the Higgs and the electroweak gauge bosons

$$\begin{aligned} \mathcal{L}_H &= (D_\mu H)^\dagger D^\mu H - V(H), \\ V(H) &= -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2, \end{aligned} \tag{2.39}$$

where the covariant derivative is

$$D_\mu = \partial_\mu + i\frac{g}{2}\sigma \cdot W_\mu + i\frac{g'}{2}B_\mu. \tag{2.40}$$

The mass terms for the gauge bosons follow from the kinetic term after the Higgs field acquires the vev.

2.4.1. Custodial symmetry

The Higgs Lagrangian is $SU(2)_L \times U(1)_Y$ invariant by construction, but has also an accidental symmetry. To illustrate better this accidental symmetry we can write the Lagrangian in another form. Instead of a doublet, let us form a bi-doublet:

$$\Phi = \begin{pmatrix} h^{0*} & h^+ \\ -h^- & h^0 \end{pmatrix}. \tag{2.41}$$

Now we can write the Higgs Lagrangian as

$$\mathcal{L}_H = \mathrm{Tr}\left(\mathrm{D}_\mu \Phi\right)^\dagger \mathrm{D}^\mu \Phi + \mu^2 \mathrm{Tr}\Phi^\dagger\Phi - \lambda\left(\mathrm{Tr}\Phi^\dagger\Phi\right)^2, \tag{2.42}$$

where the covariant derivative is

$$D_\mu \phi = \partial_\mu \Phi + i\frac{g}{2}\sigma \cdot W_\mu \Phi - i\frac{g'}{2} B_\mu \Phi \sigma^3. \tag{2.43}$$

The electroweak symmetry acts as follows

$$\begin{aligned} SU(2)_L: &\qquad \Phi \to L\Phi, \\ U(1)_Y: &\qquad \Phi \to \Phi e^{-\frac{i}{2}\sigma^3\theta}. \end{aligned} \tag{2.44}$$

We can make the global symmetry manifest by taking the hypercharge interactions to vanish, $g' \to 0$. In this limit the Higgs Lagrangian has a global $SU(2)_R$ symmetry

$$SU(2)_R: \qquad \Phi \to \Phi R^\dagger. \tag{2.45}$$

Therefore the Higgs Lagrangian has $SU(2)_L \times SU(2)_R$ symmetry which breaks down to $SU(2)_{L+R}$ when the Higgs field aquires a vev

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}. \tag{2.46}$$

This breaking pattern yields three Goldstone bosons which are eaten by the Higgs mechanism providing masses to the weak gauge bosons.

$$\begin{aligned} M_W^2 &= \frac{1}{4} g^2 v^2 \\ M_Z^2 &= \frac{1}{4}(g^2 + g'^2)v^2. \end{aligned} \tag{2.47}$$

Thus, at tree level

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1, \tag{2.48}$$

where the θ_W is the Weinberg angle. In the limit $g' \to 0$ the W^+, W^- and Z bosons form a triplet under the $SU(2)_{L+R}$ explaining why the masses are degenerate in this limit. Notably the radiative corrections must be proportional to g'^2 protecting the tree level value $\rho = 1$. For this reason this symmetry is called the custodial symmetry.

2.4.2. Comments

There are a few notes which should be kept in mind for later reference. The theory is not invariant under axial gauge transformations and one has to check that the gauge anomalies are cancelled. The gauge anomaly cancellation is shown in many quantum field theory text books (see, for example, [7]). Because of the $SU(2)_L$ gauge group, the SM suffers also from the Witten topological anomaly, reviewed in Appendix A. This anomaly cancels because there is an even number of left-handed fermion doublets in the theory.

There is no reason for the Yukawa couplings in equation (2.36) to be diagonal. These matrices can be brought to diagonal form using bi-unitary transformations. The electromagnetic and neutral weak currents will remain intact under the diagonalization and there is no flavor changing neutral currents. This is easily seen as all generations are replicas of each other and unitary diagonalization matrices always give rise to the unit matrix. Also, the charged current for the leptons is flavor diagonal provided that the neutrinos are mass degenerate. This is true in the SM. Only the system of charged weak currents involving quarks is effected by the mixing. The non-existence of the flavor changing neutral currents in the SM is called the GIM mechanism [12]. Nowadays it looks trivial, but back in the day it was proposed only three quarks were thought to exist and it predicted the charm quark.

The gauge structure $SU(2)_L \times U(1)_Y$ and the electroweak sector was tested thoroughly at the Large Electron-Positron collider (LEP). Colliding electron- positron pairs at the Z-pole enabled the precise measurements of several observables [13]. These precision observables agree well with the theoretical predictions. By doing a global fit one can examine how consistent the theory is or determine the validity of the model by over constraining the system. This is now possible because we know the Higgs mass [14].

2.5. Going Beyond

Before discussing different strategies and ways to go beyond the standard model, it is necessary to discuss why there should be something more than the SM. I will first list some examples of observations which are not explained within the SM and then give a few theoretical arguments.

2.5.1. Empirical proof

The first solid result which requires something beyond the SM is that neutrinos must be massive because they are observed to oscillate. The most minimal extension of the SM would be to add right-handed neutrinos when one would be able to write down Majorana mass terms for neutrinos or/and Yukawa interactions. It is interesting to point out that the most stringent limits on the neutrino mass scale comes from the cosmological measurements [15] and not from the dedicated neutrino experiments. The neutrino experiments, which study neutrino oscillations, can access only to the mass differences [16].

The next observational evidence does not come from the particle physics experiments but from the cosmology (see for example [17]). Only roughly five percents of the energy content of the universe is made up of ordinary matter. Dark energy accounts approximately 72 percents and dark matter (DM) circa 23 percents. We do not know much about dark energy. The same applies for DM but at least we know that it behaves gravitationally like ordinary matter but does not shine light, which is where the name comes from.

2.5.2. Theoretical Arguments

At the Planck scale around 10^{19} GeV, gravity becomes a strong force. Due to the non-renormalizabilty of gravity, the SM can be at most an effective theory up to the Planck scale. Once we have accepted the existence of a cut-off, we run into problems. The radiative corrections to the Higgs mass diverge quadratically with this high energy cut-off driving theory to be strongly coupled because $\lambda_H \sim m_H^2/v^2$. The quadratic corrections and the huge **hierarchy** between the electroweak scale $v \approx 246$ GeV and the Planck scale implies that there must be new physics at lower scales to meet the observed Higgs mass. Otherwise, one must **unnaturally fine-tune** the Higgs mass order by order so that the physical mass is the observed one. To this respect the experimental success of the SM is unreasonable.

One could also ask that, why there is only one scalar field and three families of fermions. The hierarchy in the fermion masses is also an interesting question. Without prior knowledge one would expect the masses to be in the order of the electroweak scale. Only the top quark mass fits this expectation.

2.5.3. Something potentially dangerous

Let us examine two potential problems, namely triviality and the vacuum stability. The Higgs self-coupling runs as a function of the renormalization scale μ as [18]

$$\frac{d\lambda}{dt} = \beta_\lambda, \tag{2.49}$$

where $t = \ln \mu$. At one-loop the beta function reads

$$\beta_\lambda = \frac{3}{4\pi^2}\left[\lambda^2 + \frac{1}{2}\lambda y_t^2 - \frac{1}{4}y_t^4 - \frac{1}{8}\lambda(3g^2 + g'^2) + \frac{1}{64}(3g^4 + 2g^2g'^2 + g'^4)\right], \tag{2.50}$$

where y_t is the top quark Yukawa coupling. Studying two different regimes $\lambda >> g,\ g',\ y_t$ and $\lambda << g,\ g',\ y_t$ we can set both a lower and upper bound on the Higgs mass.

Let us consider first the regime $\lambda >> g,\ g',\ y_t$. At this limit it is straightforward to solve equation (2.49)

$$\lambda(\mu) = \frac{\lambda(v)}{1 - \frac{3\lambda(v)}{4\pi^2}\ln\left(\frac{\mu^2}{v^2}\right)}. \tag{2.51}$$

The self coupling will hit a Landau pole at the scale $\mu = ve^{2\pi/3\lambda(v)}$. This allows us to determine $\lambda^{\max}(v)$ by taking $\lambda(\mu) = \infty$. This yields

$$\lambda^{\max}(v) = \frac{4\pi^2}{3\ln\left(\frac{\mu^2}{v^2}\right)} \tag{2.52}$$

The Higgs mass is related to the self-coupling through the relation $m_h^2 = 2\lambda v^2$. Plugging in the maximum value for the self coupling gives:

$$m_h < \sqrt{\frac{8\pi^2 v^2}{3 \ln\left(\frac{\mu^2}{v^2}\right)}}. \tag{2.53}$$

Using the Planck mass as a cut-off for the theory, the numerical value of the upper limit is $m_h < 160$ GeV.

The lower limit is set by examine the region $\lambda << g,\ g',\ y_t$. The electroweak vacuum must be lower than the symmetric vacuum and it must be bounded below. The beta function in this region is given as

$$\beta_\lambda = \frac{1}{16\pi^2}\left[-3y_t^4 + \frac{3}{16}\left(2g^4 + (g^2 + g'^2)^2\right)\right] = \frac{3}{16\pi^2 v^4}\left[2m_W^4 + m_Z^4 - 4m_t^4\right]. \tag{2.54}$$

This is clearly negative and thus the vacuum is stable until the scale when $\lambda(\mu) < 0$. Requiring $\lambda(\mu) > 0$ we obtain a condition

$$\lambda(v) + \beta_\lambda \ln\left(\frac{\mu^2}{v^2}\right) > 0. \tag{2.55}$$

Again, using the relation between the self-coupling and the Higgs mass we get

$$m_h^2 > -2v^2 \beta_\lambda \ln\left(\frac{\mu^2}{v^2}\right). \tag{2.56}$$

If we now plug in the numbers, we find that the lower bound is actually larger than the upper bound. The one loop result thus does not make any sense. A more careful analysis using a 2-loop renormalization group improved effective potential suggests that we are currently just below the lower bound for stability [19].

Notice that the uncertainties are still sizable; mainly coming from the Higgs mass and from the top quark mass uncertainties. Even if the Higgs potential is not absolutely stable, it is enough if the decay time to another vacuum is longer than the age of the universe.

2.5.4. How to go Beyond

As we saw in the last section we have some solid observational reasons to extend the SM. The theoretical problems are more difficult to use as a guiding principle. There is no theorem which states that nature should respect conceptual nicety. I would like to get rid of these problems. However, one must consider that after the discovery of the Higgs, and if it is confirmed to be SM like, one could argue that maybe these are not so severe after all. Even if there is a more fundamental model solving these theoretical problems, we do not know the scale of the new physics where this new model manifests itself. If the scale is high enough, particle physics experiments cannot directly explore it. This is, of course, a problem because, in the end, physics should be discipline explaining observed physical phenomena.

If we want to extend the SM, which problem we should tackle first. Or should we try to kill all the problem at once. Traditionally, the hierarchy problem has been the driving force for BSM physics. The cure to other unsatisfactory features are then adapted to one's favorite model to solve the hierarchy problem, if possible. The point here is that these, usually tremendously complicated, models give raise to a bunch of new problems which the SM does not suffer.

From the experimental point of view, it might be too restrictive to only explore "complete" models which try to explain everything. A more meaningful approach may be signature based studies [20]. The LHC collaboration have already published results based on the simplified models. From the theorist point of view this can look like going back in time, but without any hint of new physics the situation is difficult.

I should stress that there is still much to do within the SM. Our understanding, for example, of QCD is still mostly limited to the perturbative regime.

Chapter 3.

QCD and Chiral Symmetry Breaking

The QCD Lagrangian for N_f quark flavors reads

$$\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i(i\not{D} - m_i)\psi_i, \tag{3.1}$$

where the covariant derivative is

$$D_\mu = \partial_\mu - ig_s G_\mu. \tag{3.2}$$

The basic parameters of QCD are the dimensionless bare coupling g_s and the bare quark masses m_i. In order to keep physics invariant under the renormalization, changing the renormalization point must be offset by changes in the renormalized physical parameters as a function of the energy. The amount by which the bare coupling must be shifted is given by the beta function

$$\mu\frac{dg_s}{d\mu} = \beta(g_s). \tag{3.3}$$

The leading contribution to the beta function reads

$$\beta(g_s) = -\beta_0\frac{g_s^3}{(4\pi^2)} + \mathcal{O}(g_s^5) = -\frac{g_s^3}{(4\pi^2)}\left(11 - \frac{2}{3}N_f\right) + \mathcal{O}(g_s^5) \tag{3.4}$$

The number of flavors in QCD is six. This means that the beta function is negative and the theory is asymptotically free.

Integrating equation (3.3) and defining a scale Λ_{QCD} at which g_s diverges, we have

$$g_s(\mu)^2 = \frac{(4\pi)^2}{\beta_0 \ln\left(\mu^2/\Lambda_{QCD}^2\right)}. \tag{3.5}$$

The scale Λ_{QCD} is renormalization group invariant and the coupling g_s depends on it. Hence, the theory is actually characterized by this scale and not by the dimensionless coupling constant in the Lagrangian. This phenomenon is called dimensional transmutation.

The renormalization point dependence of the quark mass is given by the γ function

$$\mu\frac{dm_i}{d\mu} = -\gamma(g_s)m_i. \tag{3.6}$$

At one loop level the γ function is given as

$$\gamma(g_s) = \frac{g_s^2}{2\pi^2} + \mathcal{O}(g_s^4). \tag{3.7}$$

Let us forget the masses for a moment. The Lagrangian without mass term is invariant under independent left-handed and right-handed (chiral) $U(N_f)$ rotations

$$\psi_L \to U_L\psi_L, \qquad \psi_R \to U_R\psi_R, \qquad U_{L(R)} \in U(N_f), \tag{3.8}$$

where $\psi_{L(R)} = \frac{1}{2}(1\pm\gamma^5)\psi$. The axial current is not conserved at quantum level and the true global symmetry of massless QCD is

$$G = SU(N_f)_L \times SU(N_f)_R \times U(1)_V. \tag{3.9}$$

A generally accepted picture is that the quarks condensate and pick up a non-zero vacuum expectation value

$$\left\langle \bar{\psi}_i\psi^i \right\rangle \neq 0. \tag{3.10}$$

This breaks the global chiral symmetry down to maximal diagonal subgroup

$$SU(N_f)_V \times U(1)_V. \tag{3.11}$$

According to the Goldstone theorem $N_f - 1$ massless Goldstone bosons should appear. Next, we will discuss how to write down an effective theory describing the Goldstone bosons.

3.1. Effective Lagrangian

If a theory with a global symmetry G posseses a vacuum state which respects only a subgroup H of G, the action of G on this state generates a manifold of degenerate vacua. Any given state in this manifold is unchanged by a group of transformations isomorphic to H. Hence the set of transformations of degenerate vacua is isomorphic to the coset space G/H. Transformations in G/H correspond to directions of variations of the Lagrangian in which the effective action is level at its minimum. Quantizing the excitations along these direction produces zero mass particles one for each orthogonal direction in G/H.

Next we will figure out how the Goldstone bosons fields transform following reference[21]. The Goldstone theorem states that after the breaking there are $\dim(G) - \dim(H)$ Goldstone bosons. The group G acts on the Goldstone bosons through some representation

$$\pi \xrightarrow{g} \phi(g, \pi), \qquad g \in G. \tag{3.12}$$

Because ϕ is a representation, we have the composition law

$$\phi(g_1, \phi(g_2, \pi)) = \phi(g_1 g_2, \pi). \tag{3.13}$$

Let us consider the image of the origin, $\phi(g, 0)$. The set of elements h which map the origin onto itself forms a subgroup $H \in G$. Furthermore $\phi(gh, 0) = \phi(g, 0)$ according to the composition law for any $g \in G$, $h \in H$. The function $\phi(g, 0)$ thus maps G/H on the space of Goldstone bosons fields. As the dimension of the coset space is equal to the number of the Goldstone bosons, these can be identified with the coordinates of G/H. Thus the Goldstone bosons are said to live on the coset space.

Every element of G can be uniquely decomposed as $g = \xi h$ where ξ is an element of one of the equivalent classes $\{gh, h \in H\}$. Using the composition property

$$\phi(h, \xi') = \phi(h, \phi(g', \xi)) = \phi(hg', \xi) \tag{3.14}$$

we can find out the transformation law for ξ

$$g\xi = \xi' h. \tag{3.15}$$

i.e. the standard acton of G on the G/H space. Thus the only freedom left is the choice of representatives in the coset space.

In the case of $G = SU(2)_L \times SU(2)_R$ and $H = SU(2)_V$ quotient space is $SU(2)$. The pion fields are the three coordinates needed to parametrize the manifold, or group in this case. The usual choice of coordinates is [22]

$$U(x) = e^{i\frac{\pi}{F}}, \qquad \pi(x) = \pi^a(x)\tau_a. \tag{3.16}$$

The pion decay constant, F, is defined via relation

$$\left\langle 0|j^{\mu 5a}|\pi^b(p)\right\rangle = -ip^\mu F\delta^{ab}e^{ipx}, \tag{3.17}$$

where $j^{\mu 5a}$ is the relevant current to considerer the pion as a Goldstone boson. The transformation law of $U(x)$ follows from the relation $g\xi(x) = \xi(x)'h$

$$U'(x) = V_R U(x) V_L^\dagger, \tag{3.18}$$

where $V_{R(L)}$ is the $SU(2)_{R(L)}$ transformation. The leading term in the derivative expansion is

$$\mathcal{L} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right). \tag{3.19}$$

If we include masses for the quarks, one can write down, in leading order, a term that breaks symmetry

$$\mathcal{L}_{sb} = \frac{1}{2}F^2\left(B\mathrm{Tr}\left[MU^\dagger\right] + B^*\mathrm{Tr}\left[M^\dagger U\right]\right), \tag{3.20}$$

where B is a normalization constant. Expanding the symmetry breaking term we can read of the pion mass

$$m_\pi = (m_u + m_d)B + ... \tag{3.21}$$

The first term in the expansion is a constant which is related to the vacuum expectation value of the fermion condensate:

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = -F^2B + ... \tag{3.22}$$

The two flavors are denoted with u and d. Combining the two equations above we end up with the Gell-Mann-Oakes-Renner relation [23]

$$F^2m_\pi^2 = (m_u + m_d)|\langle 0|\bar{u}u|0\rangle| + ... \tag{3.23}$$

This allows one to study chiral symmetry breaking on the lattice by examining how the mass of the pion scales as a function of quark masses. If the condensate has a non zero value, the pion mass squared should approach zero linearly when the quark masses approach to zero.

3.2. Banks-Casher Relation

There is a way to compute the value of the condensate directly [24]. First step is to evaluate the fermion propagator as an eigenmode expansion [25]. Let us start by expanding ψ in terms of eigenfunctions of the Dirac operator $i\not{D}$

$$\psi = \sum_n b_n u_n, \tag{3.24}$$

where the b_n are Grassman variables and

$$i\not{D}u_n(x) = \lambda_n u_n(x), \qquad \int d^4x\bar{u}_m u_n = \delta_{mn}. \tag{3.25}$$

We can write the QCD partition function in a specific background gauge field configuration as

$$\begin{aligned}\mathcal{Z} = \int D[\bar{\psi}]D[\psi]D[A^\mu]e^{i\int d^4x\mathcal{L}_{QCD}} &= \int \Pi_n db_n^* db_n D[A^\mu]e^{(i\lambda_n - m)b_n^* b_n} \\ &= -\int D[A^\mu]\Pi_n(i\lambda_n - m).\end{aligned} \tag{3.26}$$

The quark propagator is then

$$\begin{aligned}\left\langle 0\left|\psi(x)\bar{\psi}(y)\right|0\right\rangle &= \frac{1}{\mathcal{Z}}\int \Pi_n db_n^* db_n \sum_i b_i u_i(x) \sum_j b_j^* \bar{u}_j(y) e^{(i\lambda_n - m)b_n^* b_n} \\ &= \frac{1}{\mathcal{Z}}\sum_n u_n(x)u_n^*(y)\Pi_{n\neq m}(m - i\lambda_n) \\ &= \sum_n \frac{u_n(x)u_n^*(y)}{m - i\lambda_n}\end{aligned} \tag{3.27}$$

The non-zero eigenvalues come in complex-conjugate pairs. The quark condensate is evaluated by taking $x = y$, integrating over volume and averaging over all gauge configurations

$$\langle 0|\bar{q}q|0\rangle = \frac{1}{V}\int d^4x \left\langle \bar{\psi}(x)\psi(x)\right\rangle = -\frac{2m}{V}\sum_{\lambda>0}\frac{1}{\lambda_n^2 + m^2}. \tag{3.28}$$

Call the mean number of eigenvalues in an interval $d\lambda$ per unit volume, $\rho(\lambda)d\lambda$. The spectral density $\rho(\lambda)$ can be introduced into above equation by taking the infinite volume limit, at which $1/V\sum_n \to \int d\lambda\rho(\lambda)$, yielding

$$\langle 0|\bar{q}q|0\rangle = -2m\int_0^\infty d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2}. \tag{3.29}$$

By taking the zero mass limit we arrive to the Banks-Casher relation

$$\langle 0|\bar{q}q|0\rangle = -\pi\rho(0). \tag{3.30}$$

Although this is, in principle, a straightforward way to study the chiral symmetry breaking using lattice techniques, the practice is not so straightforward. See for example [26] and references therein.

3.3. Vafa-Witten Theorem

A natural question to ask at this point: Is there something special with the chiral symmetries compared to vector symmetries? Vafa and Witten [27] have proved that this is indeed the case; vector symmetries cannot be spontaneously broken. Following reference [25], let us go through the argument.

Considering the two flavor QCD with equal quark masses $m_u = m_d = m \neq 0$ the vector symmetry in question is the $SU(2)_V$. If this symmetry were broken there would be Goldstone bosons associated with the scalar currents.

Let us consider Euclidean correlator

$$C_\Gamma = \left\langle 0 \left| J^{\bar{u}d}(x) J^{\bar{d}u}(y) \right| 0 \right\rangle, \tag{3.31}$$

where $J^{\bar{u}d} = \bar{u}\Gamma d$ are quark currents with

$$\Gamma = 1,\ \gamma^5,\ i\gamma_\mu,\ \gamma_\mu\gamma^5,\ i\sigma_{\mu\nu}. \tag{3.32}$$

Using the results derived in the section 3.2, we can present the correlators as

$$C_\Gamma(x,y) = \frac{1}{\mathcal{Z}} \left[\Pi_n (m - i\lambda)\right]^2 \mathrm{Tr}\left\{\Gamma \mathcal{G}(x,y) \Gamma \mathcal{G}(y,x)\right\}, \tag{3.33}$$

where $\mathcal{G}(x,y)$ is the Euclidean Green function of the u- and d-quarks in a given gauge field background

$$\mathcal{G}(x,y) = \sum_k \frac{u_k(x) u_k^\dagger(y)}{m - i\lambda_k}. \tag{3.34}$$

Note that we have explicitly employed the fact that the masses are degenerate. In addition, we have assumed that the common mass is real i.e. the θ angle of the QCD vacuum is zero.

Using the symmetry $u_k \to \gamma^5 u_k$, $\lambda_k \to -\lambda_k$ we can show that

$$\begin{aligned} \gamma_5 \mathcal{G}(x,y) \gamma_5 &= \sum_k \frac{[\gamma^5 u_k(x)][\gamma^5 u_k(y)]^\dagger}{m - i\lambda_k} = \sum_k \frac{u_k(x) u_k^\dagger(y)}{m + i\lambda_k} = \left[\sum_k \frac{u_k(y) u_k^\dagger(x)}{m - i\lambda_k}\right]^\dagger \\ &= \mathcal{G}^\dagger(y,x). \end{aligned} \tag{3.35}$$

The Green function can be expanded over the full basis

$$\mathcal{G}(x,y) = s(x,y) + \gamma^5 p(x,y) + i\gamma_\mu v_\mu(x,y) + \gamma_\mu \gamma^5 a_\mu a_\mu(x,y) + \frac{1}{2} i\sigma_{\mu\nu} t_{\mu\nu}, \tag{3.36}$$

yielding

$$\begin{aligned}\mathrm{Tr}\left\{\gamma^5\mathcal{G}(x,y)\gamma^5\mathcal{G}(y,x)\right\} &= \mathrm{Tr}\left\{|\mathcal{G}(x,y)|^2\right\} \\ &= 4\left(|s|^2+|p|^2+|v_\mu|^2+|a_\mu|^2+|t_{\mu\nu}|^2\right).\end{aligned} \tag{3.37}$$

On the other hand, using equation (3.35), we have

$$\begin{aligned}\mathrm{Tr}\left\{\mathcal{G}(x,y)\mathcal{G}(y,x)\right\} &= \mathrm{Tr}\left\{\mathcal{G}(x,y)\gamma^5\mathcal{G}^\dagger(x,y)\gamma^5\right\} \\ &= 4\left(|s|^2+|p|^2-|v_\mu|^2-|a_\mu|^2+|t_{\mu\nu}|^2\right).\end{aligned} \tag{3.38}$$

This then implies

$$|C_{\gamma^5}(x,y)| \geq |C_\Gamma(x,y)| \tag{3.39}$$

in any given background gauge field configuration.

By inserting a complete set of states and explicitly displaying the time evolution operator

$$\left\langle 0 \left| J^{\bar{u}d}(x)J^{\bar{d}u}(y,t)\right| 0\right\rangle = \sum_n \left\langle 0 \left| J^{\bar{u}d}(x,0)\right| n\right\rangle \left\langle n \left| e^{-E_n t}J^{\bar{d}u}(y,0)\right| 0\right\rangle \tag{3.40}$$

we see that the asymptotic behavior is dominated by the lightest state

$$C_\Gamma \propto e^{-m_\Gamma t} \tag{3.41}$$

The equation (3.39) implies that a pseudo scalar state is lighter than any other state, and, in particular, lighter than any scalar state

$$m_{PS} \leq m_S. \tag{3.42}$$

If the vector symmetry were broken there would be massless scalar Goldstone bosons in the spectrum. The mass inequality above shows that there must also be massless pseudo scalar states in the spectrum. We have assumed that $m_u = m_d \neq 0$ meaning that the theory has no exact axial symmetry and thus no reason for massless pseudo scalar states to exist. This completes the argument that vector symmetries cannot be spontaneously broken.

3.4. Gauging

If we gauge a $SU(2)_L \times U(1)_Y$ subgroup of $SU(2)_L \times SU(2)_R$, the formation of the quark condensate spontaneously breaks the $SU(2)_L \times U(1)_Y$ down to $U(1)_Q$. Thus we can achieve spontaneous symmetry breaking and massive gauge bosons within the Standard Model without the standard Higgs sector. This is called dynamical electroweak symmetry breaking.

Gauging the subgroup means that we have to replace the ordinary derivative in equation (3.19) with the following covariant derivative:

$$D_\mu U = \partial_\mu \Phi + i\frac{g}{2}\sigma \cdot W_\mu U - i\frac{g'}{2}B_\mu U \sigma^3. \tag{3.43}$$

Expanding the matrix U, gives the following mass terms for the gauge bosons

$$m_W = \frac{gF_\pi}{2}, \quad m_Z = \sqrt{g'^2 + g^2}\frac{F_\pi}{2}, \quad m_A = 0. \tag{3.44}$$

The value of the decay constant in our normalization is $F_\pi \approx 93MeV$. Hence QCD cannot provide the observed masses for the SM gauge bosons.

We have to remember here that the symmetry is already broken explicitly by gauging. In the next section, we will study how the spontaneous and explicit breaking work together.

3.5. Vacuum Alignment

Here we will follow original references [28, 29]. Let us consider a theory with fermions in the representation r of $SU(N)$, with global symmetry G breaking to its subgroup H. The generators of G are denoted as G^a, the generators of H as T^i, and the generators of G/H as X_z (orthogonal generators of G). The generators are normalized as $\mathrm{Tr} G_a G_b = \delta_{ab}$. To each generator of G corresponds a symmetry current $J^\mu_a(x) = \bar{\psi}_L(x)\gamma^\mu G_a \psi_L(x)$. Because G/H is a symmetric space, we can define a parity operator P so that

$$P^2 = 1, \qquad PT_iP = +T_i, \qquad PX_zP = -X_z. \tag{3.45}$$

The global symmetry G of the theory is spontaneously broken by the vacuum state $|0\rangle$ to a subgroup H. The generators T_i satisfy $T_i|0\rangle = 0$. The set of degenerate vacuum may be written as $\{\exp(i\alpha_z X_z)|0\rangle\} = \{|\alpha\rangle\}$. According to the Goldstone theorem, each current involving an X_z can crete a single massless boson from the vacuum.

$$\langle \pi_y(p)|J_z^\mu(0)|0\rangle = -ip^\mu f_\pi \delta_{yz} \tag{3.46}$$

The generators X_z correspond to a single irreducible representation of H, thus the decay constant is the same for all the Goldstone bosons.

Depending on if the representation r is real or complex we can have different global symmetries and symmetry breaking patterns.

$$\begin{array}{ccc} SU(2N) & \supset SU(N)\times SU(N)\times U(1) \\ \downarrow & \downarrow \\ O(2N) \text{ or } \mathrm{Sp(2N)} & \supset \quad SU(N)\times U(1) \end{array} \tag{3.47}$$

A small perturbation, ΔH, breaking the symmetry may lift the degeneracy of the vacuum

$$\Delta E(\alpha) = \langle \alpha|\Delta H|\alpha\rangle = \langle 0|e^{-i\alpha_z X_z}(\Delta H)e^{i\alpha_z X_z}|0\rangle. \tag{3.48}$$

To identify $|0\rangle$ with the true vacuum $\Delta E(\alpha)$ should have a minimum at $\alpha_y = 0$. Hence, we have two requirements for the vacuum energy

$$\frac{\partial}{\partial\alpha_y}\Delta E(\alpha)|_{\alpha=0} = i\langle 0|[X_y,\Delta H]|0\rangle = 0, \tag{3.49}$$

$$\frac{\partial^2}{\partial\alpha_y\partial\alpha_z}\Delta E(\alpha)|_{\alpha=0} = -\langle 0|[X_y,[X_z,\Delta H]]|0\rangle \geq 0. \tag{3.50}$$

The second derivative is proportional to the Goldstone boson mass matrix. The normalization is derived in [30, 31]

$$m_{yz}^2 = -\frac{1}{f_\pi^2}\langle 0|[X_y,[X_z,\Delta H]]|0\rangle\,. \tag{3.51}$$

If we consider two flavor QCD, in which the perturbation is given by the quark masses, we find:

$$m_\pi^2 \sim \frac{m_u + m_d}{f_\pi^2} \tag{3.52}$$

We are interested here in a situation where the perturbation originates from the electroweak gauge boson exchange. The weak interactions determine their own symmetry breaking pattern by their own choice of vacuum orientation. Let us gauge a subgroup G_w of G. The spontaneous breaking determines another subgroup H of G. The relative alignment is physically important. There can be also another subgroup S which is the maximal set of elements of G which commutes with G_w.

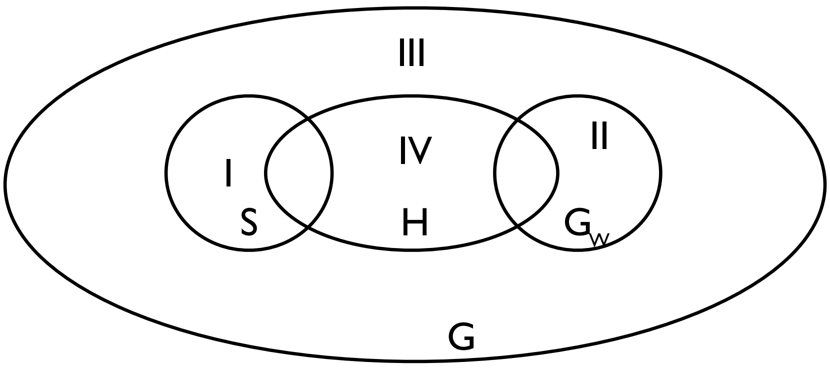

Figure 3.1.: Alignment of subgroups.

The $G_w \times S$ symmetry remains the exact symmetry of the Lagrangian and therefore the Goldstone bosons remain massless in regions I and II of Fig. Figure 3.1. The Goldstone bosons in region III acquire mass, because they do not correspond to exact symmetries. The gauge bosons in region I are coupled to broken symmetries and will receive mass through the Higgs mechanism. The Goldstone bosons in II will remain in the spectrum as physical states. The gauge bosons in IV will remain massless. The overlap of G_w and H is dynamically determined. It depends on which of the initially degenerate vacua is preferred by the perturbations.

The G_w couplings to fermions can be written as:

$$\mathcal{L} = A_\mu^A \bar{\psi} \gamma^\mu G_A \psi = A_\mu^A J_A^\mu, \tag{3.53}$$

where the G_A with capital indices are defined to absorb all the numerical factors. The G_A can be decomposed as:

$$G_A = T_{I(A)} + X_{Z(A)} \tag{3.54}$$

and the corresponding currents as:

$$J_A^\mu = J_{I(A)}^\mu + J_{Z(A)}^\mu . \tag{3.55}$$

Here we have used the parity we defined earlier, it forbids the mixed terms.

The leading order perturbation is due to one-gauge bosons exchange,

$$\Delta H = -\frac{1}{2} \int d^4x \Delta^{\mu\nu}(x) T \left[J_{\mu A}(x) J_{\nu A} \right], \tag{3.56}$$

where $\Delta^{\mu\nu}$ is the gauge boson propagator.

From the parity and Schur's lemma it follows that the only H invariant term in the products $X_x X_y$ and $T_i T_j$ are δ_{xy} and δ_{ij} respectively. This allows us to write

$$\langle 0|T J_{\mu i} J_{\nu j}|0\rangle = \langle 0|J_T J_T|0\rangle \, \delta_{ij} = \langle 0|J_T J_T|0\rangle \, \mathrm{Tr}\left(T_i T_j\right), \tag{3.57}$$

$$\langle 0|T J_{\mu z} J_{\nu y}|0\rangle = \langle 0|J_X J_X|0\rangle \, \delta_{zy} = \langle 0|J_X J_X|0\rangle \, \mathrm{Tr}\left(X_z X_y\right). \tag{3.58}$$

Thus

$$\begin{aligned} \langle 0|T J_{\mu A} J_{\nu A}|0\rangle &= \left\langle 0|T J_{\mu I(A)} J_{\nu I(A)}|0\right\rangle + \left\langle 0|T J_{\mu Z(A)} J_{\nu Z(A)}|0\right\rangle \\ &= \langle 0|J_T J_T|0\rangle \, \mathrm{Tr}\left(T_{I(A)} T_{I(A)}\right) + \langle 0|J_X J_X|0\rangle \, \mathrm{Tr}\left(X_{Z(A)} X_{Z(A)}\right) \\ &= \langle 0|J_T J_T|0\rangle \, \mathrm{Tr}\left(T_{I(A)} G_{(A)}\right) + \langle 0|J_X J_X|0\rangle \, \mathrm{Tr}\left(X_{Z(A)} G_{(A)}\right) \\ &= \langle 0|J_T J_T|0\rangle \, \mathrm{Tr}\left(G_{(A)} G_{(A)}\right) + \langle 0|J_X J_X - J_T J_T|0\rangle \, \mathrm{Tr}\left(X_{Z(A)} G_{(A)}\right). \end{aligned} \tag{3.59}$$

The first term in the last line is α independent. The α dependence is factored out in the second term and the vacuum energy density reads:

$$\langle 0|\Delta H|0\rangle = \Delta E(0) = E_0 + \left\{ \frac{1}{2} \int d^4x \Delta^{\mu\nu} \left\langle 0\left|J_{\mu T} J_{\nu T} - J_{\mu X} J_{\nu X}\right|0\right\rangle \right\} \mathrm{Tr}\left(X_{Z(A)}\right)^2 . \tag{3.60}$$

The preferred vacuum is the one which minimizes $\mathrm{Tr}\left(X_{Z(A)}\right)^2$. The second condition in equation (3.49) can now be written as:

$$m_{xy}^2 = \frac{1}{2}\frac{\partial^2}{\partial\alpha_x\partial\alpha_y}\mathrm{Tr}\left(X_Z(A)\right)^2 \times M^2 \geq 0, \tag{3.61}$$

where

$$M^2 = \frac{1}{f_\pi^2}\int d^4x\Delta^{\mu\nu}\left\langle 0|J_{\mu T}J_{\nu T} - J_{\mu X}J_{\nu X}|0\right\rangle. \tag{3.62}$$

The α dependence is, as stated earlier, only in the trace. In next section we will see how one can estimate the factor M^2.

3.6. Weinberg Sum Rules

Properties of the underlying theory can be linked to the effective theory via Weinberg sum rules (WSRs) [32]. To derive the sum rules let us consider the time ordered product of two currents

$$i\Pi_{\mu\nu}^{ab}(q) \equiv \int d^4x e^{-iqx}[\left\langle 0|J_{\mu,V}^a(x)J_{\nu,V}^b(0)|0\right\rangle - \left\langle 0|J_{\mu,A}^a(x)J_{\nu,A}^b(0)|0\right\rangle] \tag{3.63}$$

where $a, b = 1, \ldots, \mathrm{N}^2 - 1$ and the current read as

$$J_{\mu,V}^a = \bar{Q}T^a\gamma^\mu Q, \qquad J_{\mu,A}^a = \bar{Q}T^a\gamma^\mu\gamma^5 Q. \tag{3.64}$$

In the chiral limit

$$\Pi_{\mu\nu}^{ab}(q) = (q_\mu q_\nu - g_{\mu\nu}q^2)\delta^{ab}\Pi(q^2), \tag{3.65}$$

where the function $\Pi(q^2)$ obeys the unsubtracted dispersion relation

$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty ds\frac{\mathrm{Im}\Pi(\mathrm{s})}{s+Q^2}. \tag{3.66}$$

We are looking now a situation where $SU(N_f)_L \times SU(N_f)_R$ symmetry breaks down to $SU(N_f)_V$. For the next step let us study the high energy behavior of

$$G^{ab}_{\mu\nu}(q) \equiv \int d^4 x e^{-iqx} [\langle 0| J^a_{\mu,L}(x) J^b_{\nu,R}(0)|0\rangle], \tag{3.67}$$

which is equal to 1/4 times the $VV - AA$ product and transforms as the adjoint representation $(N_f^2 - 1, N_f^2 - 1)$ under $SU(N_f)_L \times SU(N_f)_R$. We can employ the operator product expansion to this task because the expansion coefficient functions respect the global symmetry. The asymptotic behavior is dictated by the lowest-dimension operator in expansion of $J^a_{\mu,L}(x) J^b_{\nu,R}(0)$ which has non-zero vacuum expectation value. It must be a singlet under the stability group and transform as the adjoint under the global symmetry. Lowest dimensional operator to satisfy these constraints is a four-fermion operator of dimension [mass]^6. Thus $G^{ab}_{\mu\nu}(q) \sim q^{-4}$ and $\Pi(Q^2) \sim Q^{-6}$.

Therefore, by expanding the right hand side of equation (3.66), we find the first and the second WSR:

$$\frac{1}{\pi}\int_0^\infty ds \mathrm{Im}\Pi(s) = 0, \qquad \frac{1}{\pi}\int_0^\infty ds\ s\mathrm{Im}\Pi(s) = 0. \tag{3.68}$$

Assuming that it is reasonable saturate the integral with the lowest lying narrow resonances, the vector and axial vector mesons as well as the Goldstone boson, we can write

$$\mathrm{Im}\Pi(s) = \pi F_V^2 \delta(s - M_V^2) - \pi F_A^2 \delta(s - M_A^2) - \pi F_\pi^2 \delta(s). \tag{3.69}$$

Substituting this to WSRs, we arrive to following relations

$$F_V^2 - F_A^2 = F_\pi^2, \qquad F_V^2 M_V^2 - F_A^2 M_A^2. \tag{3.70}$$

The precision parameter S [33] is related to the $VV - AA$ vacuum polarization and can be expressed as

$$S = 4\int_0^\infty \frac{ds}{s} \mathrm{Im}\bar{\Pi}(s) = 4\pi \left[\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right], \tag{3.71}$$

where $\mathrm{Im}\bar{\Pi}$ is the same as $\mathrm{Im}\Pi$ without the Goldstone boson contribution. This is commonly referred as the zeroth WSR.

Chapter 4.

Technicolor

An attractive feature of dynamical electroweak symmetry breaking is that it does not suffer from the naturalness, hierarchy and triviality problems like the elementary scalar Higgs in the SM. In order to understand that large separation of scales arises naturally in asymptotically free, strongly coupled theories, let us solve equation (3.3) for a large scale Λ in terms of the bare coupling

$$\Lambda_{QCD} = \Lambda e^{-\frac{8\pi}{g_s^{(0)} b_0}}. \tag{4.1}$$

If we take the scale to be the Planck scale, a coupling of order $g_s^{(0)} \sim 0.4$ will generate a strong coupling scale $\Lambda_{QCD} \sim 300$ MeV. Therefore, the scale of symmetry breaking generated by the dimensional transmutation is naturally exponentially smaller than the cut-off of the theory.

In technicolor theories new massless fermions, so called techniquarks, are included into the SM without the Higgs sector [34, 35]. They feel a new QCD-like strong force, corresponding to a gauge group $\mathrm{SU(N_{TC})}$, which causes the formation of techniquark condensate breaking the global chiral symmetry G down to $H \in G$. In order to achieve correct breaking of the electroweak sector, $SU(2)_L \times U(1)_Y$ group has to be embedded properly into G, as discussed in Section 3.5.

There is an intriguing similarity between superconductivity and technicolor. The SM with the minimal Higgs sector corresponds to the Ginzburg-Landau theory and technicolor would correspond to the BCS theory.

4.1. Historical Setup

Let us consider a model with N_f techniquarks in the fundamental representation of $SU(N_{TC})$. The left-handed techniquarks are arranged into $SU(2)_L$ doublets while the right handed ones are singlets

$$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L, \quad U_R, \quad D_R. \tag{4.2}$$

The hypercharge assignment is taken to be anomaly free. In analogue with QCD, this theory has a scale Λ_{TC} at which the technicolor gauge coupling $\tilde{g}$ diverges. Also the techni-pion decay constant F_{TC} is defined like the pion decay constant. Choosing the techni-pion decay constant appropriately, we reproduce correct masses for the gauge bosons.

Approximating QCD with the Nambu-Jona-Lasinio (NJL) model and using the Dyson-Schwinger equations, one can show that QCD obey the following scaling rules [36]

$$f_\pi \sim \sqrt{N_c}\Lambda_{QCD}, \qquad \langle \bar{q}q \rangle \sim N_c \Lambda_{QCD}. \tag{4.3}$$

Additionally, the technicolor model we are considering should follow these scaling rules, allowing us to write

$$F_{TC} \sim \sqrt{\frac{N_{TC}}{3}} \left(\frac{\Lambda_{TC}}{\Lambda_{QCD}} \right) f_\pi, \qquad v = \sqrt{N_D} F_{TC} \sim \sqrt{\frac{N_D N_{TC}}{3}} \left(\frac{\Lambda_{TC}}{\Lambda_{QCD}} \right) f_\pi, \tag{4.4}$$

Rearranging the latter expression gives an estimate for the technicolor scale

$$\Lambda_{TC} \sim \Lambda_{QCD} \frac{v\sqrt{3}}{f_\pi \sqrt{N_D N_{TC}}}. \tag{4.5}$$

Let us summarize: Using a new strong sector to replace the SM Higgs sector, we can correctly break the electroweak symmetry and give correct masses for the electroweak gauge bosons. At the same time we get rid of the hierarchy, fine-tuning and naturalness problems. A downside is that this mechanism alone cannot generate masses for the SM fermions.

4.2. Extended technicolor and Walking

Effective Couplings and Mass Terms

Let us extend the symmetry of the theory so that above the scale $\Lambda_{ETC} > \Lambda_{TC}$ the symmetry group of the theory is G_{ETC} [37,38]. Here ETC refers to extended technicolor and TC to technicolor. Accommodating technifermions and fermions into a same irreducible representation of G_{ETC}, there are ETC gauge bosons connecting the SM fermions to technifermions. When the symmetry breaks,

$$G_{ETC} \to G_{TC} \times G_{SM}, \tag{4.6}$$

gauge boson of the G_{ETC} become massive. At low energies, we can write down effective four-fermion interactions:

$$\alpha_{ab}\frac{(\bar{Q}_L T^a Q_R \bar{\psi}_R T^b \psi_L)}{\Lambda_{ETC}^2} + \beta_{ab}\frac{(\bar{Q} T^a Q \bar{Q} T^b Q)}{\Lambda_{ETC}^2} + \gamma_{ab}\frac{(\bar{\psi}_L T^a \psi_R \bar{\psi}_R T^b \psi_L)}{\Lambda_{ETC}^2} + \dots, \tag{4.7}$$

The first term is responsible for giving masses for the SM fermions

$$m_f \sim \frac{g_{ETC}^2}{M_{ETC}^2}\langle \bar{Q}Q\rangle_{ETC}. \tag{4.8}$$

In the preceding equation g_{ETC} is the ETC gauge coupling constant, M_{ETC} ETC gauge boson mass and $\langle \bar{Q}Q\rangle_{ETC}$ the techniquark condensate evaluated at the ETC-scale. The mass hierarchy between families can be achieved breaking G_{ETC} in several steps

$$G_{ETC} \to G_n \to \dots \to G_1 \to G_{TC} \times G_{SM}. \tag{4.9}$$

During the every step some of the gauge bosons become massive and produce a mass term for desired fermions. This scenario is called tumbling.

The second term in (4.7) can induce mass for the pseudo-Goldstone bosons [36]. Using equation (3.51), mass term for the technician reads

$$m_{\pi_{TC}}^2 \sim \frac{g_{ETC}^2}{F_\pi^2 M_{ETC}^2}\langle (\bar{Q}Q)^2\rangle_{ETC}. \tag{4.10}$$

The two scales, Λ_{TC} and Λ_{TC}, can be connected using the renormalization group equations [39]:

$$\langle \bar{Q}Q \rangle_{ETC} = \exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} d(\ln\mu)\gamma(\alpha(\mu)) \right) \langle \bar{Q}Q \rangle_{TC}, \tag{4.11}$$

where γ is the anomalous dimension of the operator $\bar{Q}Q$. We are talking about QCD-like asymptotically free gauge theory with running coupling constant. Hence, $\gamma \ll 1$ at large energies (see equation (3.7)) and $\langle \bar{Q}Q \rangle_{ETC} \sim \langle \bar{Q}Q \rangle_{TC}$. Using the scaling relations, we can write the mass terms as:

$$m_f \sim \frac{g_{ETC}^2 F_\pi^3}{M_{ETC}^2}, \quad m_{\pi_{TC}} \sim \frac{g_{ETC} F_\pi^2}{M_{ETC}}. \tag{4.12}$$

Flavor Changing Neutral Current

Finally, consider the γ_{ab}-term in (4.7). This operator mediates flavor changing neutral currents which must be suppressed. For example, operator

$$\pounds_{|\Delta S|=2} = \frac{4 g_{ETC}^2 V_{ds}^2}{M_{ETC}^2} \bar{d}\gamma^\mu P_L s \bar{d}\gamma_\mu P_L s + \text{h.c.}, \tag{4.13}$$

effects on the mass difference, ΔM_K, between the mixed eigenstates of the neutral kaons [40, 41]. In the above equation V_{ds} is the mixing factor of the quarks and P_L is the projection operator. The kaon decay constant is defined as $\langle 0|\bar{d}\gamma_\mu\gamma^5 s|\bar{K}(q)\rangle = i\sqrt{2} f_K q_\mu$ taking numerical value $f_K \approx 110\text{MeV}$. The mass difference is given as

$$\begin{aligned} \Delta M_K &= 2\text{Re} \langle K|\pounds_{|\Delta S|=2}|\bar{K}\rangle \\ &= \frac{4 g_{ETC}^2 \text{Re}(V_{ds}^2)}{M_{ETC}^2 M_K} \langle K|\bar{d}\gamma^\mu P_L s \bar{d}\gamma_\mu P_L s|\bar{K}\rangle \\ &\approx \frac{g_{ETC}^2 \text{Re}(V_{ds}^2)}{M_{ETC}^2} f_K^2 M_K^2 \end{aligned} \tag{4.14}$$

The experimental value for this is $\Delta M_K \approx 3.5 \times 10^{-15}\text{GeV}$ [42]. Assuming the mixing factor is the same oder of magnitude as the corresponding Cabibbo angle $V_{ds} \approx 0.1$, we can obtain numerical estimate for the technipion mass using (4.12):

$$m_{\pi_{TC}} \lesssim 10\text{GeV}. \tag{4.15}$$

A particle with this small mass should be seen in the experiments, thus the mass has to be bigger. We can increase the mass by making Λ_{ETC} to be smaller. On the other hand large suppression of the flavor changing neutral currents requires just the opposite. Again, we need something more.

Walking Dynamics

Intuitively the problem can be solved if there is a great difference between condensate at scales Λ_{TC} and Λ_{ETC}, since $m_f \propto \langle \bar{Q}Q \rangle_{ETC}$. Because of this, we can achieve large enough masses and suppressed flavor changing neutral currents at the same time. This idea was first introduced in reference [43] without providing an explicit model.

The coupling constant of a given Yang-Mills theory can behave also differently than in QCD, see Figure 4.1. If the β-function has a non-trivial fixed point α^*, the scale evolution of the theory stops when it flows to this point $\beta(\alpha^*) = 0$. If we formulate the theory such that it almost reaches the fixed point, its scale evolution slows down near fixed point but does not stop. Technicolor model with $\beta(\mu) \ll 1$ between $\Lambda_{TC} \leq \mu \leq \Lambda_{ETC}$ is called walking technicolor model.

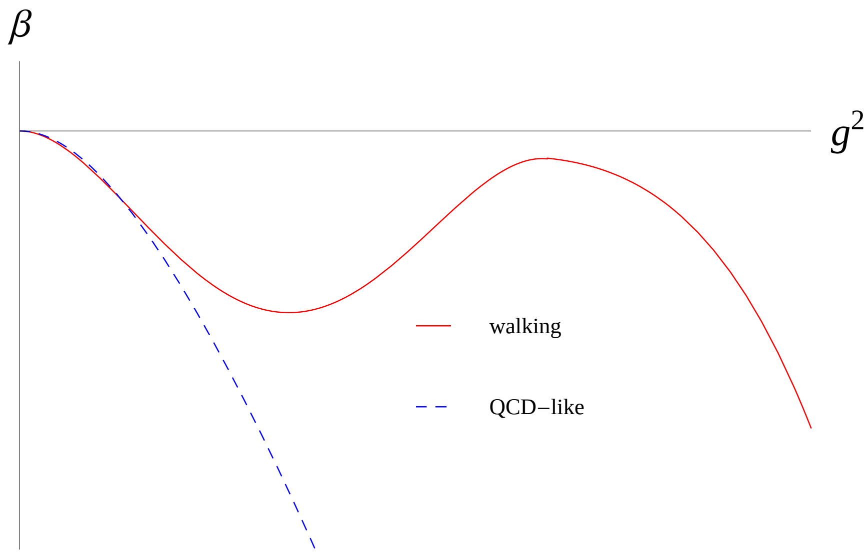

Figure 4.1.: Different possible *beta* functions for $SU(N)$ gauge theory.

It is also important that $\alpha^* \gtrsim \alpha_c$ where the subscript refers to the critical value at which the condensate forms. If the fixed point is achieved first, the scale evolution stops and the condensate can not form. On the other hand, formation of the condensate can affect the behavior of the β-function, and drive the evolution away from the vicinity of the fixed point too soon.

Dyson-Schwinger Analysis

Dyson-Schwinger equation for the fermion self energy is derived in Appendix B

$$\begin{aligned}\not{p} - m_0 + C_2(r) \int \frac{d^4k}{(2\pi)^4} \alpha(p,k) \gamma^\mu D_{\mu\nu}(p-k) S(K) \Gamma^\nu(p-k;k,p) \\ = iS^{-1}(p),\end{aligned} \tag{4.16}$$

where $S(p)$ is the fermion propagator, $D_{\mu\nu}(p-k)$ the gluon propagator and $\Gamma^\nu(p-k;k,p)$ the three point function. $C_2(r)$ denotes the Casimir invariant of the fermion representation. Note also that the coupling constant and the generator of the fermion representation are taken out from the three point function. Writing the fermion propagator in the form:

$$iS^{-1}(p) = Z(p^2)\not{p} - \Sigma(p^2), \tag{4.17}$$

where $Z(p^2)$ is the wave function renormalization factor and $\Sigma(p^2)$ is the self-energy, yields:

$$\begin{aligned}&\Sigma(p^2) + [1 - Z(p^2)]\not{p} = \\ &m_0 + C_2 \int \frac{d^4k}{(2\pi)^4} \alpha(p,k) \gamma^\mu G_{\mu\nu}(p-k) \frac{Z(k^2)\not{k} + \Sigma(k^2)}{Z(k^2)^2 k^2 + \Sigma(k^2)^2} \Lambda^\nu(p-k;k,p).\end{aligned} \tag{4.18}$$

Let us approximate the gluon propagator with the free propagator in the Landau gauge and the three point function with the tree level vertex factor. Writing the substitutions explicitly, we can identify equations for the self-energy and the renormalization factor

$$\Sigma(p^2) = m_0 + 3C_2 \int \frac{d^4k}{(2\pi)^4} \alpha(p,k)^2 \frac{1}{(p-k)^2} \frac{\Sigma(k^2)}{Z(k^2)^2 k^2 + \Sigma(k^2)^2} \tag{4.19}$$

$$\begin{aligned}Z(p^2) =& 1 + C_2 \int \frac{d^4k}{(2\pi)^4} \alpha(p,k)^2 \frac{1}{p^2(p-k)^2} \frac{Z(k^2)}{Z(k^2)^2 k^2 + \Sigma(k^2)^2} \\ &\left[\frac{k \cdot p(p-k)^2 + 2p \cdot (p-k) k \cdot (p-k)}{(p-k)^2}\right].\end{aligned} \tag{4.20}$$

Performing angular integrals we find $Z(p^2) = 1$ and

$$\Sigma(p^2) = m_0 + \frac{3C_2}{4\pi}\int_0^{\Lambda^2} dk^2 \alpha(p,k)\frac{\Sigma(k^2)}{k^2+\Sigma(k^2)^2}\left[\frac{k^2}{p^2}\theta(p^2-k^2)+\theta(k^2-p^2)\right]. \tag{4.21}$$

We can convert this to a differential equation by differentiating with respect to p^2, multiplying by p^4 and differentiating again. Using a notation $\lambda = \frac{3C_2\alpha}{4\pi} = \frac{\alpha}{\alpha_c 4}$ and $x = p^2$ we get

$$2\frac{d\Sigma(x)}{dx} + x\frac{d^2\Sigma(x)}{dx^2} + \frac{\lambda\Sigma(x)}{m^2+x} = 0, \tag{4.22}$$

with the following boundary condition

$$\frac{d}{dx}\left(x\Sigma(x)\right)|_{x=\Lambda^2} = m_0. \tag{4.23}$$

The solution of this differential equation is a hypergeometric function [44]

$$\Sigma(x) = MF\left(\frac{1}{2}+\frac{\omega}{2},\frac{1}{2}-\frac{\omega}{2},2;-\frac{x}{M^2}\right), \tag{4.24}$$

where $\omega = \sqrt{1-\frac{\alpha}{\alpha_c}}$. In the asymptotic region this solution can be expressed as

$$\frac{\Sigma(x)}{M} = c\left(\frac{x}{M^2}\right)^{-(1-\omega)/2} + d\left(\frac{x}{M^2}\right)^{-(1+\omega)/2}. \tag{4.25}$$

For strong coupling region, $\alpha > \alpha_c$, the solution becomes oscillating:

$$\frac{\Sigma(x)}{M} = 2\sqrt{cd}\sqrt{\frac{M^2}{x}}\sin\left(\frac{\omega'}{2}\ln\frac{x}{M^2}+\frac{1}{2i}\ln\left(\frac{-c}{d}\right)\right), \tag{4.26}$$

where $\omega' = \sqrt{\frac{\alpha}{\alpha_c}-1}$. In the weak coupling region, the solution reads

$$\frac{\Sigma(x)}{M} = 2\sqrt{-cd}\sqrt{\frac{M^2}{x}}\sinh\left(\frac{\omega}{2}\ln\frac{x}{M^2}+\frac{1}{2}\ln\left(\frac{-c}{d}\right)\right), \tag{4.27}$$

Only the oscillating solution can non-trivially satisfy the boundary condition (4.23) in the chiral limit $m_0 \to 0$ [45]. Thus we can identify the coupling α_c with the critical coupling for the chiral symmetry breaking. Using these solutions one can calculate the anomalous dimension at the critical coupling [46]:

$$\gamma(\alpha_c) = 1. \tag{4.28}$$

Let us return to consider the renormalization group analysis (4.11), when $\beta(\mu) \ll 1$ during the interval $\Lambda_{TC} \leq \mu \leq \Lambda_{ETC}$. Now we know that anomalous dimension of the operator $\bar{Q}Q$ is in this case $\gamma = 1$, leading to the following relation between the techniquark condensates

$$\langle \bar{Q}Q \rangle_{ETC} \approx \frac{\Lambda_{ETC}}{\Lambda_{TC}} \langle \bar{Q}Q \rangle_{TC}. \tag{4.29}$$

Note the enhancement in comparison with the QCD like case, $\gamma \ll 1$, yielding a correction to the fermion and technipion mass terms (4.12). Walking over two orders of magnitude produces an upper limit for the technipion mass

$$\frac{\Lambda_{ETC}}{\Lambda_{TC}} \sim 10^2 \rightsquigarrow m_{\pi_{TC}} \lesssim 1\text{TeV}, \tag{4.30}$$

which is large enough to be out of range of the todays accelerators.

The Conformal Window

The lack of full understanding of dynamics of strongly coupled theories using the perturbation theory, makes model building a challenging problem. We would like to construct near conformal theories, but we cannot just simply using pen and paper tell exactly which models fit into this category. As we have seen, lattice calculations can be used for this task. However, before doing some time consuming calculations, we would like to know what is worth calculating. Let us use the results from Dyson-Schwinger analysis to estimate which theories could be interesting.

The two loop β function for a generic $SU(N)$ gauge theory with fermions in the representation R is

$$\begin{aligned}
\beta(g) &= -\beta_0 \frac{g^3}{(4\pi)^2} - \beta_1 \frac{g^5}{(4\pi)^4}, \\
2N\beta_0 &= \frac{11}{3} C_2(G) - \frac{4}{3} T(R) \\
(2N)\beta_1 &= \frac{34}{3} C_2^2(G) - \frac{20}{3} C_2(G) T(R) - 4 C_2(R) T(R),
\end{aligned} \tag{4.31}$$

Table 4.1.: Group theoretical quantities. Representations are written in terms of Young tableaux.

r	$T(r)$	$C_2(r)$	$d(r)$
$\square$	$\frac{1}{2}$	$\frac{N^2-1}{2N}$	N
G	N	N	N^2-1
$\square\square$	$\frac{N+2}{2}$	$\frac{(N-1)(N+2)}{N}$	$\frac{N(N+1)}{2}$

where $C_2(R)$ is the quadratic Casimir and $T(R)$ the trace normalization factor. These two are related via

$$N_f C_2(R) d(R) = T(R) d(G), \tag{4.32}$$

where $d(R)$ is the dimension of the representation. The group theoretical quantities for representations we need are given in Table 4.1.

Theory loses asymptotic freedom when the first coefficient changes sign. This defines the upper limit for the conformal window:

$$N_f^I = \frac{11}{4} \frac{C_2(G)}{T(r)} \frac{d(G)}{d(R)}. \tag{4.33}$$

When the second coefficient changes sign the theory develops a Banks-Zaks fixed point. Therefore, if the number of flavors is smaller than

$$N_f^{III} = \frac{17 C_2(G)}{10 C_2(G) + 6 C_2(r)} \frac{d(G) C_2(G)}{d(R) C_2(r)} \tag{4.34}$$

. the spontaneous symmetry breaking cannot be triggered. Thus the lower boundary of the conformal window must be somewhere between these two values. Using the critical coupling, given by the DS analysis, and identifying it with the coupling at the fixed point, we acquire the DS estimate for the lower boundary

$$N_{f\,SD}^{II} = \frac{17 C_2(G) + 66 C_2(r)}{10 C_2(G) + 30 C_2(r)} \frac{C_2(G)}{T(r)}. \tag{4.35}$$

. The conformal windows for the fundamental, adjoint, two-index symmetric and two-index anti-symmetric representation are showed in figure 4.2.

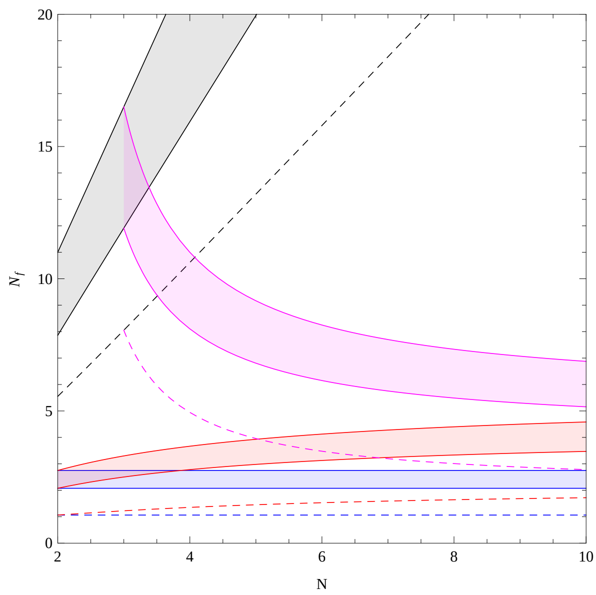

Figure 4.2.: Phase diagram for SU(N) theory with fermions in the fundamental (upper lines) and two-index symmetric representation (lower lines). Dashed line is the lower bound achieved via SD-analysis.

Minimal models of Walking Technicolor

Using the conformal windows in Figure 4.2 as a guide for model building, several walking models with minimal flavor content have been constructed.

- The *Minimal Walking Technicolor (MWT)* is $SU(2)_{TC}$ gauge theory with two Dirac flavors of technifermions transforming according to the two-index symmetric representation of the gauge group.
- The *Next-to Minimal Walking Technicolor (MWT)* is $SU(3)_{TC}$ gauge theory with three Dirac flavors of technifermions transforming according to the two-index symmetric representation of the gauge group.
- The *Ultra Minimal Walking Technicolor (MWT)* is $SU(2)_{TC}$ gauge theory with matter in two different representations of the gauge group. Two Dirac fermions in the fundamental representation and two Weyl fermions in the adjoint of $SU(2)_{TC}$.

The global symmetry breaking patterns for these models are:

$$\begin{aligned} SU(2)_L \times SU(2)_R \times U(1)_V \to SU(2) \times U(1)_V & \qquad \text{NMWT} \\ SU(4) \to SO(4) & \qquad \text{MWT} \\ SU(4) \times SU(2) \times U(1) \to Sp(4) \times SO(2) \times Z_2 & \qquad \text{UMT} \end{aligned} \tag{4.36}$$

We have omitted here the fundamental representation which is disfavored by electroweak precision measurements (for detailed discussion see, for example, [47, 48]).

Ideal Walking

Let us take a step backwards and what has been stated. The MWT model seems to already be inside the conformal window. This is further confirmed by the lattice simulations [49]. Should we discard the model based on this observation?

The conformal windows in Fig Figure 4.2 are drawn for technicolor theories in isolation without taking into account effects from the SM interactions and from the ETC interactions. The framework of ideal walking [50] consistently takes into account the effects from the four-fermion interactions for different representations as a function of number of favors and number of colors. As a result of this analysis the conformal window is shown to shrink for all the representations. In other words, the four-fermion interactions can drive a conformal theory to non-conformal phase.

Another desired results is that the anomalous dimension of the mass increases beyond unity, which was the value from the DS analysis. This helps to yield the correct mass for the top quark.

Chapter 5.

Minimal Walking Technicolor

In the MWT, the extended gauge group is $SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ and the field content of the technicolor sector is constituted by the techni-fermions, Q, U^c and D^c, and one techni-gluon all transforming according to the adjoint representation of $SU(2)_{TC}$.

The model suffers from the Witten topological anomaly [51] which is cured by adding a new fermionic weak doublet L singlet under technicolor gauge group [48]. Furthermore, the gauge anomalies cancel when introducing the $SU(2)_L$ singlets E^c and N^c with the hypercharge assignment below[1]:

	Field	$SU(2)_{TC}$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Techniquarks	$Q = \begin{pmatrix} U \\ D \end{pmatrix}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{2}$	$\frac{y}{2}$
	U^c	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{y+1}{2}$
	D^c	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$-\frac{y-1}{2}$
New Leptons	L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{3y}{2}$
	N^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{3y-1}{2}$
	E^c	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\frac{3y+1}{2}$

(5.1)

The parameter y can take any real value [48]. We refer to the states L, E^c and N^c as the New Leptons. The condensate which correctly breaks the electroweak symmetry

[1] We use the two component Weyl notation throughout this section.

is $\langle UU^c + DD^c \rangle$. To discuss the symmetry properties of the theory it is convenient to arrange the technifermions as a column vector, transforming according to the fundamental representation of $SU(4)$

$$\hat{Q} = \begin{pmatrix} U \\ D \\ U^c \\ D^c \end{pmatrix} , \tag{5.2}$$

The breaking of $SU(4)$ to $SO(4)$ is driven by the following condensate

$$\langle \hat{Q}^T E \hat{Q} \rangle \tag{5.3}$$

The matrix E is a 4×4 matrix defined in terms of the 2-dimensional unit matrix as

$$E = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} . \tag{5.4}$$

The above condensate is invariant under an $SO(4)$ symmetry.

5.1. Goldstone Bosons

5.1.1. Effective Theory

The symmetry breaking pattern of the MWT model is $SU(4) \to SO(4)$. This leaves us with nine broken generators with associated Goldstone bosons. As in Section 3.1 ,the resulting low energy effective theory can be organized in a derivative expansion with cut-off scale $4\pi F$, where F is the Goldstone boson decay constant. We first introduce the matrix:

$$U = \exp\left(i\frac{\sqrt{2}}{F}\Pi^a X^a \right) E \, , \tag{5.5}$$

where Π^a are the 9 Goldstone bosons and X^a are the 9 broken generators (see Appendix of [I]). The matrix U transforms under $SU(4)$ in the following way:

$$U \rightarrow gUg^T \ , \qquad g \in SU(4). \tag{5.6}$$

The leading term appearing in the Lagrangian is:

$$L_U = \frac{F^2}{2} \mathrm{Tr} \left[D_\mu U D^\mu U^\dagger \right] \ . \tag{5.7}$$

The electroweak covariant derivative reads:

$$D_\mu U = \partial_\mu U - i \left[G_\mu U + U G_\mu^T \right] \tag{5.8}$$

where

$$G_\mu \ = \ g_2 \ W_\mu^a \ L^a + g_1 \ B_\mu \left(-R^{3^T} + \sqrt{2} \ y \ S^4 \right) \ . \tag{5.9}$$

Here g_1 and g_2 are the hypercharge and the weak couplings, respectively. The value of F is fixed in order to reproduce correctly the electroweak symmetry breaking $F = \frac{2m_W}{g_2}$.

The gauging of the electroweak interactions breaks explicitly the $SU(4)$ symmetry group down to $SU(2)_L \times U(1)_Y \times U(1)_V$, while the spontaneous symmetry breaking leaves invariant an $SO(4)$ subgroup. The remaining unbroken group is $U(1)_Q \times U(1)_V$. The $U(1)_Q$ factor is the symmetry group associated to the electromagnetism while the $U(1)_V$ leads to the conservation of the technibarion number. A simple illustration of the spontaneous and explicit breaking of the $SU(4)$ symmetry is presented in Figure 5.1. Among the 9 physical degrees of freedom, 3 are eaten up by the longitudinal components

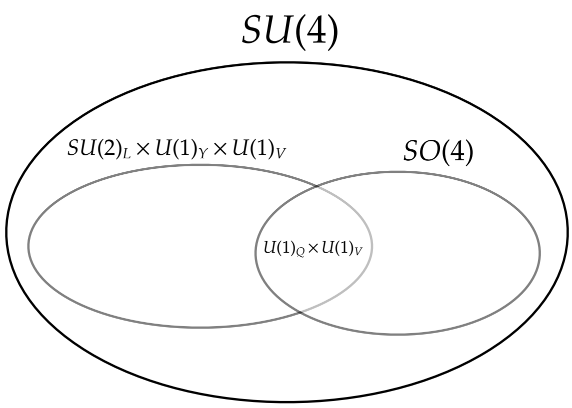

Figure 5.1.: Spontaneous and explicit breaking of the $SU(4)$ symmetry.

of the SM gauge bosons while the remaining 6 Goldstone bosons carry technibaryon number and will be denoted by Π_{UU}, Π_{UD} and Π_{DD}. Because Goldstone bosons carry technibaryon number, we refer to these states also as technibaryons.

The bosons can be classified according to the unbroken group $U(1)_V \times U(1)_Q$ in the following way:

Boson	U$(1)_V$ charge	U$(1)_Q$ charge	Linear Combination
W_L^+	0	$+1$	$\frac{\Pi^1 - i\Pi^2}{\sqrt{2}}$
W_L^-	0	-1	$\frac{\Pi^1 + i\Pi^2}{\sqrt{2}}$
Z_L	0	0	Π^3
Π_{UU}	$+1$	$y-1$	$\frac{\Pi^4 + i\Pi^4 + \Pi^6 + i\Pi^7}{2}$
Π_{DD}	$+1$	$y+1$	$\frac{\Pi^4 + i\Pi^4 + \Pi^6 + i\Pi^7}{2}$
Π_{UD}	$+1$	y	$\frac{\Pi^8 + i\Pi^9}{\sqrt{2}}$
$\Pi_{UU}^\dagger$	-1	$-y+1$	$\frac{\Pi^4 - i\Pi^4 + \Pi^6 - i\Pi^7}{2}$
$\Pi_{DD}^\dagger$	-1	$-y-1$	$\frac{\Pi^4 - i\Pi^4 + \Pi^6 - i\Pi^7}{2}$
$\Pi_{UD}^\dagger$	-1	$-y$	$\frac{\Pi^8 - i\Pi^9}{\sqrt{2}}$

(5.10)

Electroweak interactions split the technipion masses according to the following pattern [52]:

$$\Delta m^2_{\Pi_{UU}} = \frac{m^2_{\rm walk}}{g_1^2 + g_2^2}\left[g_1^2(1+2y)^2 + g_2^2\right] \tag{5.11}$$

$$\Delta m^2_{\Pi_{DD}} = \frac{m^2_{\rm walk}}{g_1^2 + g_2^2}\left[g_1^2(4y^2-1) + g_2^2\right] \tag{5.12}$$

$$\Delta m^2_{\Pi_{UD}} = \frac{m^2_{\rm walk}}{g_1^2 + g_2^2}\left[g_1^2(1-2y)^2 + g_2^2\right] . \tag{5.13}$$

In models with walking dynamics the value of $m_{\rm walk}$ can be even of few hundreds GeV [52]. Furthermore it is also possible to introduce a common mass term for the pseudo Goldstone bosons by adding the following term to the Lagrangian (5.7):

$$-\frac{m^2_{\rm etc} F^2}{4} {\rm Tr}\left[U^\dagger B_V U B_V\right] \tag{5.14}$$

with

$$B_V = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} . \tag{5.15}$$

This term was already added in [53] and it is expected to emerge from a more complete theory of SM fermion mass generation. It is expected to emerge from a four-techniquark interaction term, and preserves $SU(2)_L \times SU(2)_R \times U(1)_V$ of $SU(4)$, which contains the $SU(2)_V$ custodial symmetry group.

It is useful to know the transformation properties of the matrix U with respect to the electroweak gauge group. Under $SU(4)$ the matrix U transforms as a two index symmetric tensor, or in other words U transforms like the irreducible representation **10** of $SU(4)$:

$$U \to gUg^T \ , \qquad g \in SU(4). \tag{5.16}$$

Knowing the embedding of the electroweak generators in the $SU(4)$ algebra it is possible to decompose the representation **10** according to the $SU(2)_L \times U(1)_Y$ group:

$$\mathbf{10} \quad \to \quad \mathbf{3}_y + \mathbf{2}_{1/2} + \mathbf{2}_{-1/2} + \mathbf{1}_{-y+1} + \mathbf{1}_{-y} + \mathbf{1}_{-y-1} \ . \tag{5.17}$$

The identification of these representations inside the U matrix is given by:

$$\begin{array}{ccc} \mathbf{3}_y & \to & T_L \\ \mathbf{2}_{1/2} & \to & H_2 \\ \mathbf{2}_{-1/2} & \to & H_1 \\ \mathbf{1}_{-y-1} & \to & S_1 \\ \mathbf{1}_{-y} & \to & S_2 \\ \mathbf{1}_{-y+1} & \to & S_3 \end{array} \qquad U = \frac{1}{F}\begin{pmatrix} T_L & \frac{H_1}{\sqrt{2}} & \frac{H_2}{\sqrt{2}} \\ \frac{H_1^T}{\sqrt{2}} & S_1 & \frac{S_2}{\sqrt{2}} \\ \frac{H_2^T}{\sqrt{2}} & \frac{S_2}{\sqrt{2}} & S_3 \end{pmatrix} \tag{5.18}$$

By expanding the exponential in equation (5.5) to the second order in the number pseudo Goldstone boson fields, we identify the following phenomenologically relevant interaction Lagrangian for the first LHC searches:

$$T_L = i \begin{pmatrix} \Pi_{UU} & \frac{\Pi_{UD}}{\sqrt{2}} \\ \frac{\Pi_{UD}}{\sqrt{2}} & \Pi_{DD} \end{pmatrix} \qquad S_1 = i\Pi^\dagger_{UU} \qquad S_2 = i\Pi^\dagger_{UD} \qquad S_3 = i\Pi^\dagger_{DD} \tag{5.19}$$

$$\frac{H_1}{\sqrt{2}\,F} = \begin{pmatrix} 1 - \frac{\Pi^\dagger_{UD}\Pi_{UD}+2\,\Pi^\dagger_{UU}\Pi_{UU}}{4\,F^2} \\ -\frac{\Pi^\dagger_{UD}\Pi_{DD}+\Pi^\dagger_{UU}\Pi_{UD}}{2\sqrt{2}\,F^2} \end{pmatrix} \qquad \frac{H_2}{\sqrt{2}\,F} = \begin{pmatrix} -\frac{\Pi^\dagger_{DD}\Pi_{UD}+\Pi^\dagger_{UD}\Pi_{UU}}{2\sqrt{2}\,F^2} \\ 1 - \frac{\Pi^\dagger_{UD}\Pi_{UD}+2\,\Pi^\dagger_{DD}\Pi_{DD}}{4\,F^2} \end{pmatrix} . \tag{5.20}$$

Depending on the specific choice of the hypercharge assignment y, one can construct different Yukawa-type interactions involving the matrix U. Odd integer values are required for y to avoid stable composite states with fractional electric charge[2]. In general, electrically charge stable states are excluded by cosmology, see for example discussion in [54].

Within this setup the allowed Yukawa terms are

$$\begin{aligned} \mathcal{L}_{2HDM} &= H_2 q u^c + H_1 q d^c + H_1 l e^c + H_2 L N^c + H_1 L E^c \\ &+ H_1^c q u^c + H_2^c q d^c + H_2^c l e^c + H_1^c L N^c + H_2^c L E^c + \text{ h.c.} \end{aligned} \tag{5.21}$$

$$\mathcal{L}_{\Pi\psi\psi} = \begin{cases} y=-3 & S_3^\dagger e^c e^c + \text{ h.c.} \\ y=-1 & S_1^\dagger e^c e^c + S_2 ll + T_L^\dagger ll + \text{ h.c.} \\ y=+1 & S_3 e^c e^c + S_2^\dagger ll + T_L ll + \text{ h.c.} \\ y=+3 & S_1 e^c e^c + \text{ h.c.} \end{cases} \tag{5.22}$$

$$\mathcal{L}_{\Pi LL} = \begin{cases} y=-1 & S_1 E^c E^c + \text{ h.c.} \\ y=+1 & S_3 N^c N^c + \text{ h.c.} \end{cases} \tag{5.23}$$

$$\mathcal{L}_{\Pi L\psi} = \begin{cases} y=-5 & S_1 E^c e^c + \text{ h.c.} \\ y=-3 & S_1^\dagger L l + S_1 N^c e^c + S_2 E^c e^c + \text{ h.c.} \\ y=-1 & S_2^\dagger L l + S_2 N^c e^c + T_L^\dagger L l + \text{ h.c.} \\ y=+1 & H_1^c L e^c + H_2 L e^c + H_1 N^c l + H_2^c N^c l + S_3 L l + S_3 N^c e^c + \text{ h.c.} \end{cases} \tag{5.24}$$

H_i^c is defined as $H_i^c \equiv -i\sigma_2 H_i^*$, and the e^c and l symbols stand respectively for $SU(2)_L$ singlet and doublets states of the 3 lepton families (e, μ, τ). In equations (5.21)-(5.24),

[2] Bound states made by one technifermion and one technigluon have electric charge $-\frac{y \pm 1}{2}$

for simplicity, we omitted the appropriate $SU(2)_L$ contractions, the flavor indices and the coupling constants in front of each term. The interactions in (5.21)-(5.24) are valid for generic ETC models. Specific models can provide further symmetries which can be exploited to reduce the number of operators in (5.21)-(5.24). Moreover, it is worth noticing that quarks can couple, to this order, only to the Higgs sector (via $\mathcal{L}_{2HDM}$), i.e. quadratic in the number of technipions.

5.1.2. Results

The allowed values for y can be further restricted by requiring that the lightest state, among the pseudo Goldstone bosons and the new leptons, is not electrically charged and stable. According to this, we discard the cases $y = \{-5, 3\}$ and the surviving values of y are $\{-3, -1, 1\}$.

For every y, the neatest process to be investigated at the LHC involving the PGBs for which we will derive relevant constraints is

$$pp \to \Pi_2^\dagger \Pi_2 \to (\ell_i^- \ell_j^-)(\ell_h^+ \ell_k^+) \qquad \text{where } i, j, h, k = e, \mu, \tau. \tag{5.25}$$

The flavor structure of the coupling $\lambda^{ij}\Pi_2 l_i^+ l_j^+$ depends on the ETC sector, but given that this sector is unknown it is possible to have different lepton pairs in the final state. In order to simplify our analysis, we assume decays only into a pair of same sign muons.

The ATLAS collaboration has studied the production of a doubly charged Higgs boson at the LHC in [55] with 1.6 fb^{-1} of integrated luminosity. This study was performed by examining the invariant mass distribution of the same sign muon pairs. The doubly charged particle is assumed to decay into two muons with 100% branching fraction.

We have used the number of observed events and the number of expected background events reported in table 2 of [55] to calculate the 95% exclusion limit for the pseudo Goldstone boson production using the modified frequentists CL$_s$ method [56, 57]. The value for the coupling between the pseudo Goldstone boson and the muons is chosen according to [55], to ensure that the pseudo Goldstone bosons are decaying before the detector. This yields a lower limit of 286 GeV for the pseudo Goldstone boson mass. The exclusion plot is presented in Figure 5.2 for 1.6 fb^{-1}.

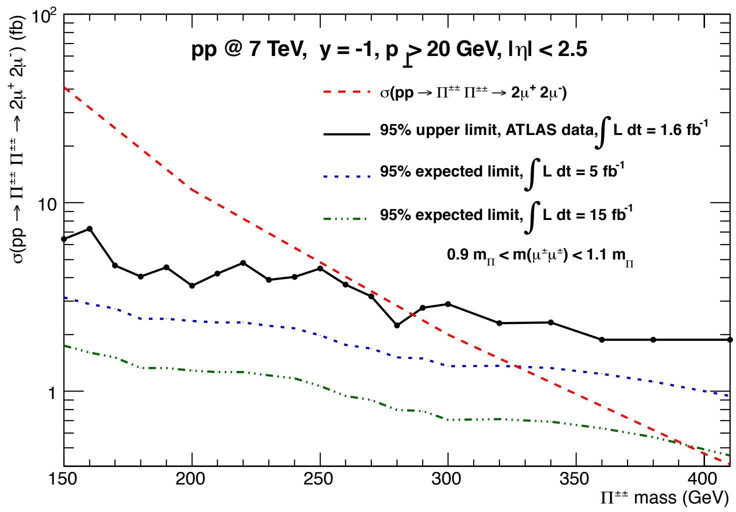

Figure 5.2.: Exclusion for a doubly charged PGB with $y = -1$ based on the ATLAS data.

5.1.3. Summary

This sector of the Minimal Walking Technicolor model was for the first time investigated in [I]. Strongly coupled theories are in many occasions studied in terms of vector resonances. Nevertheless, it is possible that pseudo Goldstone bosons are the the first sign of strong dynamics at the LHC.

The signature which is identified to be the most important can also arise from many theories with doubly charged Higgs bosons. Usually in these models, producing a single doubly charged Higgs is more economical. If doubly charged scalar states are observed to be produced only in pairs, this would favor our model. Of course the reverse is true as well.

5.2. Vector Resonances

Different walking technicolor models posses different global symmetries and symmetry breaking patterns. The precision measurements dictates that all these different extension of the SM have to contain at least the following chiral symmetry breaking pattern:

$$SU(2)_L \times SU(2)_R \to SU(2)_V. \tag{5.26}$$

Thus the NMWT model serves as a template to study vector resonance phenomenology at the LHC. Based on the symmetry breaking pattern, the low energy spectrum is described in terms of lightest spin one vector and axial-vector iso-triplets $V^{\pm,0}$ and $A^{\pm,0}$ as well as lightest iso-singlet scalar resonance H. The corresponding states in the QCD are the $\rho^{\pm,0}$, $a_1^{\pm,0}$ and the $f_0(600)$.

All the results in paper [II] are obtained using a linearly realized effective Lagrangian. How to write down this effective Lagrangian is described in details in [53, 58]. Out of author's own interest, let us write down a non-linear effective Lagrangian, supplemented with a scalar singlet state (i.e. the Higgs), and compare it with the Linear realization.

5.2.1. Non-linear Lagrangian

In order to consistently introduce mass terms for the vector mesons the Generalized Hidden Local Symmetry (GHLS) method is employed [59]. Formalism is based on the observation that a nonlinear sigma model based the manifold G/H is gauge equivalent to a linear model with symmetry group $G_{\text{Global}} \times G_{\text{local}}$. The vector mesons are taken to be gauge bosons of the G_{local} symmetry and a subgroup of G_{Global} is gauged under the electroweak interactions. The gauge equivalence is straightforward to verify by using the equations of motion of the gauge bosons and choosing a special gauge for the local group.

In our case $G_{\text{Global}} = SU(2)_L \times SU(2)_R$ and $G_{\text{local}} = SU(2)_L \times SU(2)_R$. A basic dynamical variable of the $G_{\text{Global}} \times G_{\text{local}}$ model is a unitary matrix field $U(x)$ [59–61] transforming as

$$U(x) \to U(x') = \tilde{h}(x)U(x)\tilde{g}^{\dagger}, \tag{5.27}$$

where $\tilde{h}(x) \in G_{\text{local}}$ and $\tilde{g} \in G_{\text{Global}}$. It is convenient to decompose the $U(x)$ as

$$U(x) = \xi_L^{\dagger}(x)\xi_M(x)\xi_R(x) \tag{5.28}$$

The variables $\xi(x)$ transform as

$$\xi_{L(R)} \to h_{L(R)}(x)\xi_{L(R)}(x)g_{L(R)}^{\dagger}, \qquad \xi_M \to h_L(x)\xi_M(x)h_R^{\dagger}(x), \tag{5.29}$$

where $g_{L(R)} \in \left[SU(2)_{L(R)}\right]_{\text{Global}}$ and $h_{L(R)} \in \left[SU(2)_{L(R)}\right]_{\text{Local}}$. We can construct the Lagrangian by introducing the covariant Maurer-Cartan 1-forms defined as:

$$\hat{\alpha}^{\mu}_{L,R,M}(x) = D^{\mu}\xi_{L,R,M}(x) \cdot \xi^{\dagger}_{L,R,M}(x). \tag{5.30}$$

The covariant derivatives for the different ξ fields are given as

$$D_{\mu}\xi_L(x) = \partial_{\mu}\xi_L(x) - iL_{\mu}(x)\xi_L(x) + i\xi_L(x)\mathcal{L}_{\mu}(x) \tag{5.31}$$

$$D_{\mu}\xi_R(x) = \partial_{\mu}\xi_R(x) - iL_{\mu}(x)\xi_R(x) + i\xi_R(x)\mathcal{R}_{\mu}(x) \tag{5.32}$$

$$D_{\mu}\xi_M(x) = \partial_{\mu}\xi_M(x) - iL_{\mu}(x)\xi_M(x) + i\xi_M(x)R_{\mu}(x), \tag{5.33}$$

where $\mathcal{L}_{\mu} = gW^a_{\mu}T^a$ and $\mathcal{R}_{\mu} = g'B_{\mu}T^3$ are the ordinary electroweak gauge bosons introduced by gauging the $SU(2)_L \times U(1)_Y$ subgroup of G_{Global} symmetry. The G_{Local} gauge bosons are defined to include the gauge coupling $\tilde{g}$

$$L_{\mu} = \tilde{g}\frac{V_{\mu} - A_{\mu}}{\sqrt{2}}, \qquad R_{\mu} = \tilde{g}\frac{V_{\mu} + A_{\mu}}{\sqrt{2}}. \tag{5.34}$$

Finally the Lagrangian is

$$\mathcal{L} = \mathcal{L}_{kin} + a\mathcal{L}_V + b\mathcal{L}_A + c\mathcal{L}_M + d\mathcal{L}_{\pi}, \tag{5.35}$$

where

$$\mathcal{L}_{kin} = -\frac{1}{2}\mathrm{Tr}\left[\mathrm{W}^{\mu\nu}\mathrm{W}_{\mu\nu}\right] - \frac{1}{4}\mathrm{B}^{\mu\nu}\mathrm{B}_{\mu\nu} - \frac{1}{2}\mathrm{Tr}\left[\mathrm{V}^{\mu\nu}\mathrm{V}_{\mu\nu} + \mathrm{A}^{\mu\nu}\mathrm{A}_{\mu\nu}\right], \tag{5.36}$$

$$\mathcal{L}_V = F^2\mathrm{Tr}\left[\hat{\alpha}_{\parallel\mu}\hat{\alpha}_{\parallel}\mu\right], \quad \mathcal{L}_{\mathrm{A}} = \mathrm{F}^2\mathrm{Tr}\left[\hat{\alpha}_{\perp\mu}\hat{\alpha}^{\mu}_{\perp}\right], \quad \mathcal{L}_{\mathrm{M}} = \mathrm{F}^2\mathrm{Tr}\left[\hat{\alpha}_{\mathrm{M}\mu}\hat{\alpha}^{\mu}_{\mathrm{M}}\right], \tag{5.37}$$

$$\mathcal{L}_{\pi} = F^2\mathrm{Tr}\left[(\hat{\alpha}_{\perp\mu} + \hat{\alpha}_{\mathrm{M}\mu})(\hat{\alpha}^{\mu}_{\perp} + \hat{\alpha}^{\mu}_{\mathrm{M}})\right]. \tag{5.38}$$

The perpendicular and parallel projections are defined as

$$\hat{\alpha}^{\mu}_{\parallel,\perp} = (\xi_M\hat{\alpha}^{\mu}_R\xi^{\dagger}_M \pm \hat{\alpha}^{\mu}_L)/2. \tag{5.39}$$

And the field strengths are

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig\left[W_\mu, W_\nu\right], \tag{5.40}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{5.41}$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i\tilde{g}\left[V_\mu, V_\nu\right], \tag{5.42}$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\tilde{g}\left[A_\mu, A_\nu\right]. \tag{5.43}$$

The Lagrangian (5.35) can be extended to include states which are singlets under the stability group H. After adding a scalar state the Lagrangian reads

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{kin} + k_a(\chi)a\mathcal{L}_V + k_b(\chi)b\mathcal{L}_A + k_c(\chi)c\mathcal{L}_M + k_d(\chi)d\mathcal{L}_\pi \\ &+ k_h(\chi)\frac{1}{2}\partial_\mu h \partial^\mu h - \mathcal{V}(\chi),\end{aligned} \tag{5.44}$$

where $\chi = h/v$ and v is an arbitrary scale. The arbitrary functions $k_x(\chi)$ must satisfy

$$k_a(0) = k_b(0) = k_c(0) = k_a(0) = k_h(0) = 1 \tag{5.45}$$

for a proper normalization of the Goldstone boson kinetic terms. The function $k_d(\chi)$ stands in front of a mixed term and the former requirement does not apply. Nevertheless, we will choose it to satisfy the same condition. One can derive the interactions with h expanding $k_x(\chi)$ around $\chi = 0$.

5.2.2. Linear Lagrangian

Let us write down the linearly realized effective Lagrangian for comparison:

$$\begin{aligned}\mathcal{L}_{\rm boson} &= -\frac{1}{2}{\rm Tr}\left[\widetilde{W}_{\mu\nu}\widetilde{W}^{\mu\nu}\right] - \frac{1}{4}\widetilde{B}_{\mu\nu}\widetilde{B}^{\mu\nu} - \frac{1}{2}{\rm Tr}\left[F_{{\rm L}\mu\nu}F_{\rm L}^{\mu\nu} + F_{{\rm R}\mu\nu}F_{\rm R}^{\mu\nu}\right] \\ &+ m^2\,{\rm Tr}\left[C_{{\rm L}\mu}^2 + C_{{\rm R}\mu}^2\right] + \frac{1}{2}{\rm Tr}\left[D_\mu M D^\mu M^\dagger\right] - \tilde{g}^2\, r_2\,{\rm Tr}\left[C_{{\rm L}\mu} M C_{\rm R}^\mu M^\dagger\right] \\ &- \frac{i\,\tilde{g}\,r_3}{4}{\rm Tr}\left[C_{{\rm L}\mu}\left(MD^\mu M^\dagger - D^\mu M M^\dagger\right) + C_{{\rm R}\mu}\left(M^\dagger D^\mu M - D^\mu M^\dagger M\right)\right] \\ &+ \frac{\tilde{g}^2 s}{4}{\rm Tr}\left[C_{{\rm L}\mu}^2 + C_{{\rm R}\mu}^2\right]{\rm Tr}\left[MM^\dagger\right] + \frac{\mu^2}{2}{\rm Tr}\left[MM^\dagger\right] - \frac{\lambda}{4}{\rm Tr}\left[MM^\dagger\right]^2\end{aligned} \tag{5.46}$$

where $\widetilde{W}_{\mu\nu}$ and $\widetilde{B}_{\mu\nu}$ are the ordinary electroweak field strength tensors, $F_{{\rm L/R}\mu\nu}$ are the field strength tensors associated to the vector meson fields $A_{{\rm L/R}\mu}$, and the $C_{{\rm L}\mu}$ and $C_{{\rm R}\mu}$

fields are

$$C_{\mathrm{L}\mu} \equiv A_{\mathrm{L}\mu} - \frac{g}{\tilde{g}}\widetilde{W}_\mu \ , \quad C_{\mathrm{R}\mu} \equiv A_{\mathrm{R}\mu} - \frac{g'}{\tilde{g}}\widetilde{B}_\mu \ . \tag{5.47}$$

The 2×2 matrix M is

$$M = \frac{1}{\sqrt{2}}\left[v + H + 2\ i\ \pi^a\ T^a\right] \ , \qquad a = 1, 2, 3 \tag{5.48}$$

where π^a are the Goldstone bosons produced in the chiral symmetry breaking, $v = \mu/\sqrt{\lambda}$ is the corresponding VEV, H is the composite Higgs, and $T^a = \sigma^a/2$, where σ^a are the Pauli matrices. The covariant derivative is

$$D_\mu M \ = \ \partial_\mu M - i\ g\ \widetilde{W}^a_\mu\ T^a M + i\ g'\ M\ \widetilde{B}_\mu\ T^3 \ . \tag{5.49}$$

When M acquires its VEV, the Lagrangian of equation (5.46) contains mass mixing terms for the spin-1 fields.

5.2.3. Comparison

The new vector resonances mass mixes with the SM gauge bosons. This induces the couplings between the new vectors and the SM fields. The masses depend on the parameters $a,\ b,\ c,\ d$ in the nonlinear model and on the parameters $r_1,\ r_2,\ r_3,\ k$ in the linear model. This in addition to a parameter fixing the scale and the gauge couplings. If we are interested in signatures at the LHC not involving the Higgs, it seems that at leading order we can define a mapping between the parameters of these two realizations of the model.

In the linear Lagrangian the symmetry $SU(2)_L \times SU(2)_R$ fixes the Higgs couplings to be given in terms of parameters $a,\ b,\ c,\ d$. In the non-linear realization all the couplings involve independent free parameters. The couplings between the new vector states and the Higgs are important when we calculate decay widths for the vector resonances. This can, without a doubt, have an effect on the results, even if we study processes without the Higgs. In Figure 5.3 we have plotted the decay width of the R_1 vector resonance in the linear model and in the non-linear model taking $k_x(\chi) = 0$. Notice here that the discussion applies to any theory with the same global symmetry. Connection to the underlying gauge theory can be achieved via the WSRs, which impose relations between the parameters. In rest of the chapter we will use the Linear Lagrangian.

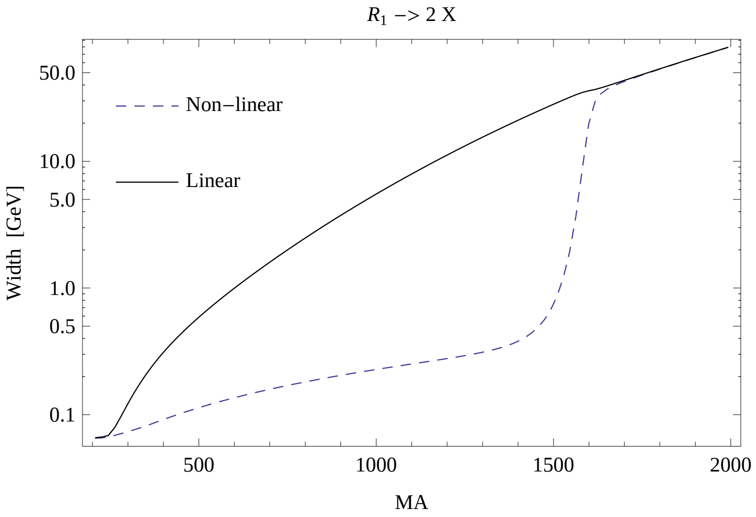

Figure 5.3.: The decay width of the R_1 vector resonance.

5.2.4. Results

As already mentioned, the new vector and axial-vector states mix with the SM gauge eigenstates yielding the ordinary SM bosons and two triplets of heavy mesons, $R_1^{\pm,0}$ and $R_2^{\pm,0}$, as mass eigenstates. The couplings of the heavy mesons to the SM particles are induced by the mixing. Important here is how the heavy vectors couple to the fermions. In the region of parameter space where R_1 is mainly an axial-vector and R_2 mainly a vector sate, the dependence of the couplings to the SM fermions as a function of $\tilde{g}$ is very roughly

$$g_{R_{1,2}f\bar{f}} \sim \frac{g^2}{\tilde{g}} \tag{5.50}$$

where g is the electroweak gauge coupling. The full coupling constant is also a function of M_A, but this dependence is very weak.

To constrain the parameter space of the model we use the CMS results [62], which report limits for a W' boson decaying to a muon and a neutrino at $\sqrt{s} = 7$ TeV in the mass range 600 - 2000 GeV for the resonance.

The relevant calculations are performed using MadGraph [63], using the CTEQ6L parton distribution functions [64], and the implementation for the NMWT model [65]. reference [62] reports experimental limits for the muon channel together with a combined analysis with the electron channel [66]. Because of the missing energy in the final state, the invariant mass of the resonance cannot be reconstructed, and the following transverse

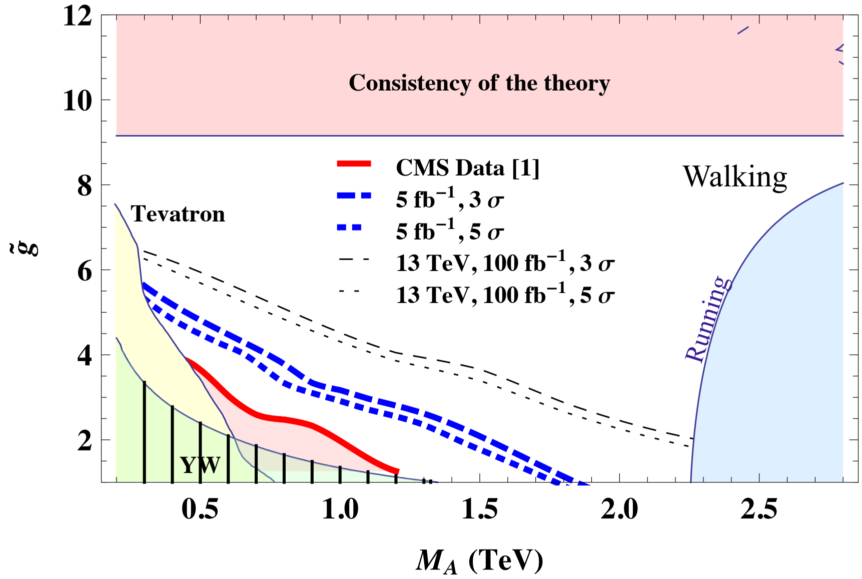

Figure 5.4.: Bounds in the (M_A, $\tilde{g}$) plane of the NMWT parameter space: (i) CDF direct searches of the neutral spin one resonance excludes the uniformly shaded area in the left, with $M_H = 200$ GeV and $s = 0$. (ii) The 95 % confidence level measurement of the electroweak precision parameters W and Y excludes the striped area in the left corner. (iii) Imposing the modified WRS's excludes the uniformly shaded area in the right corner. (iv) The horizontal stripe is excluded imposing reality of the axial and axial-vector decay constants. (v) The area below the thick uniform line is excluded by the CMS data [62]. (vi) Dashed and dotted lines are expected exclusions using different values of the integrated luminosity and center of mass energy.

mass variable is utilized

$$M_T = \sqrt{2 \cdot p_T E_T^{\text{miss}} \cdot (1 - \cos \Delta\phi_{\mu,\nu})}. \tag{5.51}$$

In the experimental analysis, the cut on the transverse mass is adjusted in bins of the mass of the sought-after resonance. In addition to the transverse mass cut, the lepton acceptance $|\eta| < 2.1$ is used. The resulting cross section is then compared with the limits reported in the experimental analyses.

Exploring the signal from the process $pp \to R_{1,2} \to l\nu$ we are able to limit the possible values for the parameters M_A and $\tilde{g}$. The theoretical limits as well as the limits from the Tevatron are described in [67]. The M_A, $\tilde{g}$ plane of the parameter space is presented in Figure 5.4 for $M_H = 200GeV$ and $s = 0$.

The uniformly shaded region on the left is excluded by the CDF searches of the resonance in the the $p\bar{p} \to e^+e^-$ process. The striped region in the lower left corner is excluded by the measurements of the electroweak W and Y parameters [68] adapted for models of MWT in [69]. Avoiding imaginary decay constants for the vector and axial-vector sets an upper bound for the $\tilde{g}$, i.e. excludes the uniformly shaded in the upper part of the figure. The near conformal (walking) dynamics modifies the WRS's, compared to a running case like QCD, as explained above [70]. Imposing these modified sum rules excludes the lower right corner of the parameter space. The CDF exclusion limit is sensitive, indirectly, to the mass of the composite Higgs and the coupling s via properties of the new heavy spin one states. However, the edge of the excluded area varies only very weakly as a function of s and M_H. The CMS search imposes a 95 % CL exclusion bound described with the thick solid (red) line. The thick dashed and dotted lines (blue) are three and five sigma exclusion limits for 7 TeV and 5 fb^{-1}. The thin dotted and dashed lines describe the reach of the LHC with 100 fb^{-1} at 13 TeV. The three and five sigma exclusion limits are calculated using poisson distribution, following [71]. Due to the effective description, we have not employed the K-factors when calculating the exclusion limits.

Comparing the three sets of lines for the LHC, the increase in the horizontal direction follows roughly the increase in luminosity. The small role of the center of mass energy can be understood by exploring the behavior of the cross section as a function of the center of mass energy and comparing it with the scaling with $\tilde{g}$, obtained from equation (5.50).

5.2.5. Summary

The limits on the masses are significantly lower that for the W' with SM like couplings. The reason for this can be traced back to fermion couplings which arise via mixing. The ATLAS Collaboration has performed a similar analysis with this model and reported results in [72]. The exclusion plot derived by ATLAS is shown in Figure 5.5

As evidenced in Figure 5.5, the leptonic final states are not enough to explore the whole parameter space. Interesting possibility is to use associate Higgs production to find vector resonances. For example, in the process $pp \to R_1 \to ZH$ the new vector state has only one coupling with the fermions which will help with large values of $\tilde{g}$. An example invariant mass distribution is given in Figure 5.6. Here the Higgs is taken to decay to

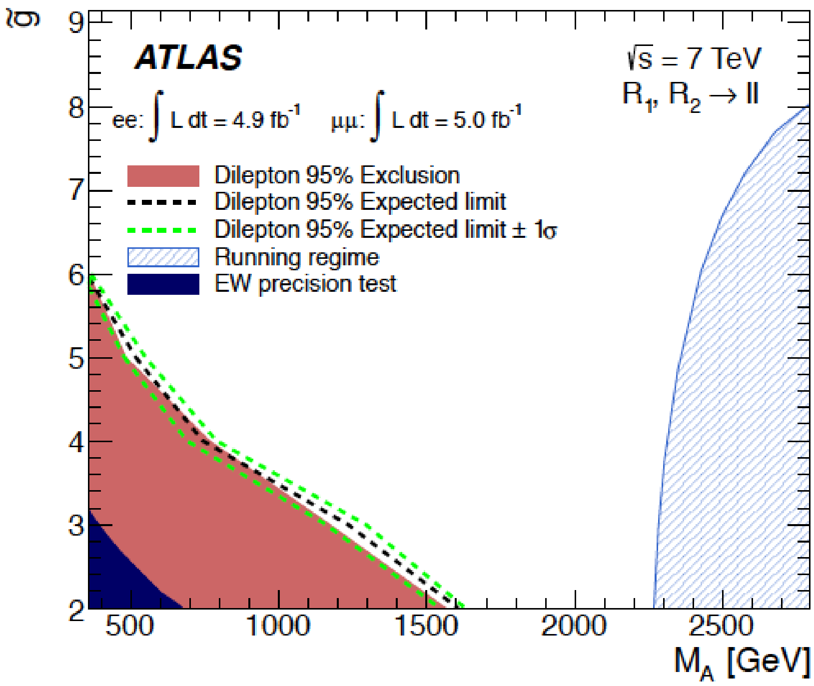

Figure 5.5.: Excluded parameter space based on the ATLAS measurements. The plot is taken from [72]

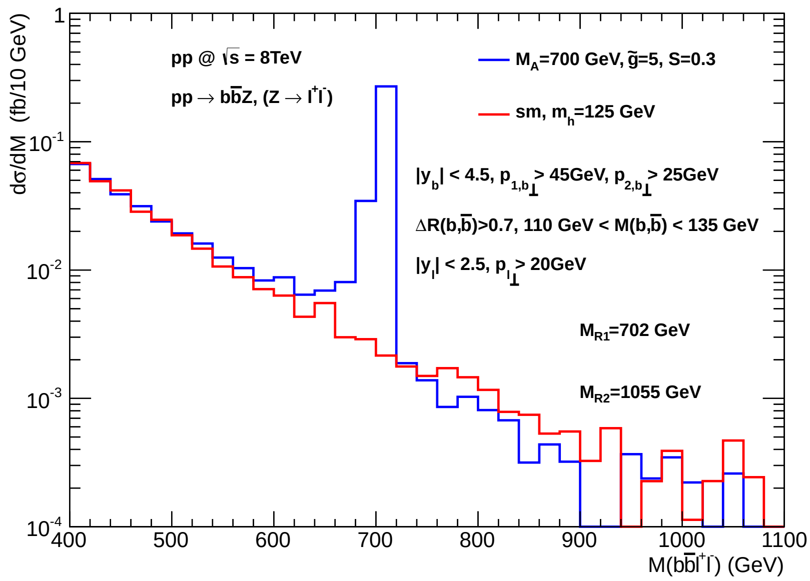

Figure 5.6.: Production cross section as a function of invariant mass M_{HZ}.

b-quark pair and the Z boson to leptons. To effectively use this channel requires a great deal of integrated luminosity and efficient b-tagging.

5.3. Technicolor Higgs

In technicolor the Higgs sector is replaced with a strongly interacting sector. The lightest scalar state plays a role of the Higgs bosons, provided that it is lighter compared to other sates. In QCD like models, mass of the composite Higgs is estimated to be roughly of the order of $M_H \sim 1$ TeV, which is of course way too heavy when compared to the observed Higgs mass. In walking models the mass of the Higgs can be as low as few hundreds of GeV [48, 73]. In order to explain why one scalar state if much lighter than other states the model, it is convenient to identify it with the Goldstone boson of the approximate scale symmetry. This scenario goes under the name techni-dilation [74].

What we have stated is not quite the complete picture. The Higgs here is composed of massless techniquarks and all the mass is given by the dynamics. The dynamical mass is not the physical mass; one has to consider also the effect from the radiative corrections. The SM top-induced radiative corrections can reduce the composite Higgs mass down to observed value [73]. This is like an inverse of the little hierarchy problem causing problems in several BSM models.

5.3.1. Hybrid Model

Inspired by the hybrid models used to calculate the two photon decay of the sigma meson in QCD, we calculated in [I] the effect of techniqurks on the Higgs two photon decay. The contribution to the process is modeled by re-coupling, in a minimal way, the composite Higgs to the techniquarks Q via the following operator:

$$L_{TC-2\gamma} = \sqrt{2}\,\frac{M_Q}{v_{weak}}\left[\overline{Q}_L^{\,t}\cdot H D_{Rt} + \overline{Q}_L^{\,t}\cdot (i\,\tau_2\,H^*)U_{Rt}\right] + \text{h.c.}\ , \tag{5.52}$$

where M_Q is the dynamical mass of the techniquark and $t = 1, \ldots, d[r]$ is the Technicolor index and $d[r]$ the dimension of the representation under which the techniquarks transform. If the model suffers from the Witten anomaly, a lepton family is added

$$L_{L-2\gamma} = \sqrt{2}\,\frac{M_E}{v_{weak}}\,\overline{L}_L\cdot H E_R + \sqrt{2}\,\frac{M_N}{v_{weak}}\overline{L}_L\cdot (i\,\tau_2\,H^*)N_R + \text{h.c.}\ , \tag{5.53}$$

where M_E and M_N are the fermion masses. The techniquark dynamical mass is intrinsically linked to the technipion decay constant F_π via the Pagels-Stokar formula [75, 76] $M_Q \approx \frac{2\pi F_\pi}{\sqrt{d(r)}}$.

The one loop Higgs decay width to two photons for any weakly interacting elementary particle contributing to this process can be neatly summarized as [77]:

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2 G_F M_h^2}{128\sqrt{2}\pi^3} \left| \sum_i n_i \, Q_i^2 \, F_i \right|^2 , \tag{5.54}$$

where i runs over the spins, n_i is the multiplicity of each species with electric charge Q_i in units of e. The F_i functions are given by

$$F_1 = 2 + 3\tau + 3\tau(2-\tau) f(\tau), \tag{5.55}$$

$$F_{1/2} = -2\tau[1 + (1-\tau) f(\tau)], \tag{5.56}$$

$$F_0 = \tau \left(1 - \tau f(\tau)\right), \tag{5.57}$$

where $\tau = \frac{4m_i^2}{M_h^2}$ and

$$f(\tau) = \begin{cases} \left(\arcsin\sqrt{\frac{1}{\tau}}\right)^2, & \text{if } \tau \geq 1 \\ -\frac{1}{4}\left(\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right) - i\pi\right)^2, & \text{if } \tau < 1 \end{cases} . \tag{5.58}$$

The lower index of the function F represent the spin of each particle contributing in the process.

5.3.2. Results

We plot in Figure 5.7 the intrinsic dependence on the dimension of the Technicolor matter representation $d[r]$ according to the ratio:

$$R = \frac{\Gamma_{SM}(h \to \gamma\gamma) - \Gamma_{TC}(h \to \gamma\gamma)}{\Gamma_{SM}(h \to \gamma\gamma)}. \tag{5.59}$$

For any odd representation we included the Lepton contribution and therefore we could not simply join the points. Interestingly when $d[r]$ is around 7 the contribution from the techniquarks and from the SM fermions (mainly the top) cancels the W contribution leading to $\Gamma_{TC} \approx 0$.

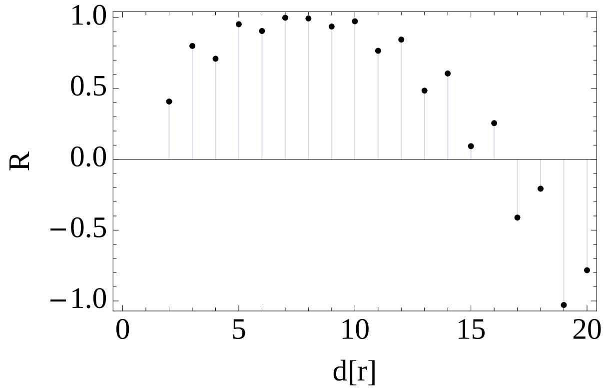

Figure 5.7.: Difference of the Walking TC and SM decay widths divided by the SM width plotted with respect to number of colors. $m_h = 120$.

5.3.3. Summary

Above calculation is a very crude estimate of a result which cannot be calculated exactly. We have also a problem with the double counting; the techni-pions eaten by the electroweak gauge bosons are techniquark bound sates. Moreover, for reasonable matter representations, the effect is too small to be observed at the LHC.

Chapter 6.

Dark Matter Model Building

Let us forget the problems with the SM Higgs sector for the rest of the thesis. A large number of observational evidences suggest that there is non-luminous matter in the universe [78]. This dark matter cannot be explained by the particles we know, thus requires physics beyond the SM. There are a number of experiments around the world which are trying to detect dark matter directly. The situation with these experiments is quite interesting because they do not mutually agree; for example DAMA/LIBRA [79], GoGeNT [80] and CRESST [81] claims to have seen dark matter while CDMS [82], XENON [83] and PICASSO [84] experiments exclude the parameter regions where observations have been made. However, there are number of caveats that need to be discussed as one tries to interpret these results as discussed for example in [85].

6.1. The model

Effect of a magnetic dipole moment of the dark matter on the direct detection experiments has been discussed in [85, 86] using effective operator approach. In in [IV] we have provided a renormalizable model which can, at low energies, provide this effective operator. On top of the SM we add a vector-like heavy electron (E), a complex scalar (S) and a SM singlet Dirac fermion (χ). The associated renormalizable Lagrangian is

$$\begin{aligned} \mathcal{L}_{S\overline{E}\chi y} &= \mathcal{L}_{\rm SM} + \bar{\chi} i\partial\!\!\!/ \chi - m_\chi \bar{\chi}\chi + \overline{E} i D\!\!\!\!/ E - m_E \overline{E} E - (S\overline{E}\chi y + \text{h.c.}) \\ &+ D_\mu S^\dagger D^\mu S - m_S^2 S^\dagger S - \lambda_{HS} H^\dagger H S^\dagger S - \lambda_S (SS^\dagger)^2 \ , \end{aligned} \tag{6.1}$$

where $D^\mu = \partial^\mu - ie\frac{s_w}{c_w} Z^\mu + ieA^\mu$, s_w and c_w represent the sine and cosine of the Weinberg angle. We assume the new couplings y, λ_{HS} and λ_S to be real and the bare mass squared of the S field to be positive so that the electroweak symmetry breaks via the SM Higgs doublet (H). The interactions among χ, our potential dark matter candidate, and the SM fields occur via loop-induced processes involving the $S\bar{E}\chi$y operator in (6.1).

These extra interactions have an effect on the vacuum stability we investigated in the introduction. However, the effect is very small because the scalar S does not acquire a vacuum expectation value, as argued in [87]. Both of the new fermions can mix with the SM leptons. For the sake of simplicity, we do not consider the effect of the mixing, even though it can lead to interesting consequences. For example, sizable mixing can explain the discrepancy between the measured and calculated muon anomalous magnetic moment.

At the one loop level χ develops the following magnetic-type interactions

$$\mathcal{L}_5 = \frac{\lambda_\chi}{2}\bar{\chi}\sigma_{\mu\nu}\chi F^{\mu\nu} - \frac{s_w}{2\,c_w}\lambda_\chi\bar{\chi}\sigma_{\mu\nu}\chi Z^{\mu\nu}, \tag{6.2}$$

where $F^{\mu\nu}$ and $Z^{\mu\nu}$ are the photon and Z field strength tensors, and λ_χ is related to the electromagnetic form factor $F_2(q^2)$ as follows:

$$\lambda_\chi = \frac{F_2(q^2)}{2m_\chi}e\ . \tag{6.3}$$

The explicit derivation of the one-loop–induced form factor can be found in the first appendix of [IV]. It is enlightening to report the analytic form for a few interesting limits to learn about the dependence upon couplings and masses. We start by considering the static limit $F_2(0)$, useful for dark matter direct detection experiments. To reduce the parameter space we take the masses of E and S to be degenerate $m_E = m_S = M$ leading to the simplified expression

$$F_2(0) = \frac{y^2}{8\pi^2}\left(\frac{2}{z}\sqrt{\frac{2+z}{2-z}}\tan^{-1}\left(\frac{z}{\sqrt{4-z^2}}\right) - 1\right), \tag{6.4}$$

where $z = \frac{m_\chi}{M}$. For small z we have

$$F_2(0) = \frac{y^2}{16\pi^2}\left(z + \frac{z^2}{3}\right) + \mathcal{O}\left(z^3\right)\ , \tag{6.5}$$

which for $z = 0$ gives $\lambda_\chi = ey^2/(32\pi^2 M)$. It is also interesting to consider a dark matter candidate with mass of the order of the electroweak scale or slightly higher. For this purpose a simple estimate for the electromagnetic form factor can be deduced by setting

$z = 1$ (i.e. $m_\chi = M$), and M around the electroweak scale, yielding $F_2(0) = y^2 \dfrac{\sqrt{3}\pi - 3}{24\pi^2}$ and therefore $\lambda_\chi = ey^2 \dfrac{\sqrt{3}\pi - 3}{48\pi^2 M}$. From the two limits it follows that the scale of the magnetic moment is controlled by the common mass of the heavy states M.

6.2. Results

It was shown in [85] that there is a region of parameter space able to alleviate the tension between the experiments when using magnetic moment interactions. It was observed that one can, in fact, bring DAMA, CoGeNT and CRESST signals to overlap while being marginally consistent with CDMS, XENON and PICASSO experiments (see Fig. 3 of ref. [85]). A best fit result leads to a dark matter mass around 10 GeV and a constant magnetic moment of about 1.5×10^{-18} e cm, which corresponds to $32\pi^2 M/y^2 = \frac{e}{\lambda_\chi} \sim 10$ TeV. For example for $M \approx 500$ GeV we find $y \approx \pi$ meaning that the underlying dynamics of the model can be explored at the electroweak scale.

The contour plot of equal λ_χ in the (M,y) plane is shown in Figure 6.1 together with the exclusion regions obtained using the following information: The CMS constraints on charged long-lived particles, The XENON100 results [83] after having taken into consideration the threshold effects [85] and the Higgs to two gamma constraints.

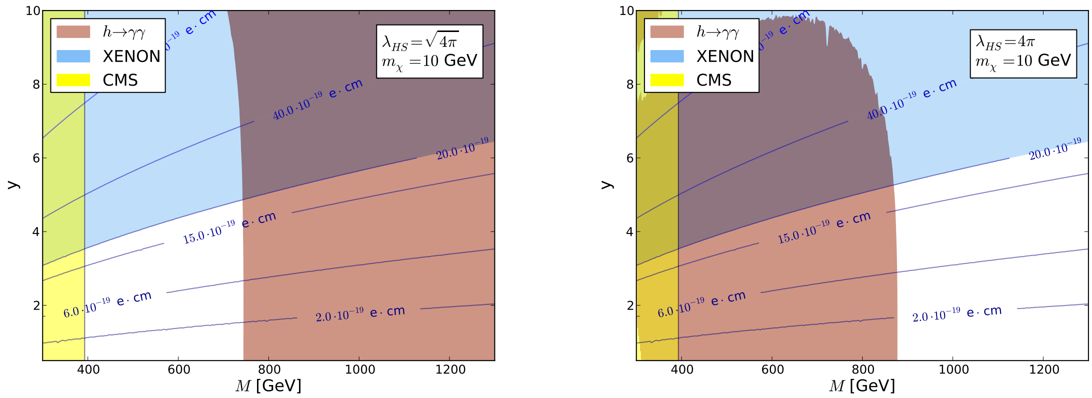

Figure 6.1.: Strength of the magnetic moment λ_χ in the (M, y) plane with $m_\chi = 10$ GeV and for $\lambda_{HS} = \sqrt{4\pi}$ (left panel) and $\lambda_{HS} = 4\pi$ (right panel). The unshaded region is the one allowed by experiments.

6.3. Summary

The extension of the SM presented here can be tested at the LHC while providing a dark matter candidate interacting with ordinary matter via magnetic operators which can be simultaneously investigated or observed in dark matter experiments. In most cases the link between the experiments can be established on within a renormalizable model. Number of papers have been studying limits on the dark matter production at the LHC using the same effective operators as for the direct detection. This can be, and is, dangerous due to vastly different energy scales.

Appendix A.

Witten Anomaly

Let us consider an Euclidean SU(2) Yang-Mills theory in the compact space $\mathbf{S}^4$. Following Witten, the global SU(2) anomaly is based on the observation that the fourth homotopy group of SU(2)

$$\Pi^4(\mathrm{SU}(2)) = \mathbf{Z}_2 \tag{A.1}$$

is nontrivial [51]. Homotopy groups are invariant under a homeomorphism, hence they are topological invariants [88]. The homeomorphism classify spaces according to whether can they be continuously deformed one to another. Thus homotopy groups offers a less restrictive way to classify spaces since spaces that have same homotopy groups are not necessarily homeomorphic. In physics homotopy groups are usually used to classify maps rather than spaces.

The group manifold of the SU(2) is $\mathbf{S}^3$. Thus the gauge transformations are maps $U(x) : \mathbf{S}^4 \to \mathbf{S}^3$. The equation (A.1) can be interpreted so that there exists different types of gauge transformations which approach unity at infinity[1]. The fact that fourth homotopy group is $\mathbf{Z}_2$ means that there is a topologically non-trivial gauge transformation that cannot be smoothly deformed to the identity but when the transformation is done twice it can be deformed to the identity.

Due to the topologically non-trivial mapping, for the every gauge field there is a conjugate field

$$A_\mu^U = U^{-1} A_\mu U - i U^{-1} \partial_\mu U. \tag{A.2}$$

[1] $\mathbf{S}^4$ is achieved from the $\mathbf{R}^4$ with the one-point compactification $\mathbf{R}^4 \bigcup \{\infty\}$. Thus the gauge transformations have to possess a constant value at the infty.

And they both give exactly the same contribution to the functional integral

$$Z = \int d\psi d\bar{\psi} dA_\mu \exp\left(-\int d^4x [\frac{1}{2g^2}\mathrm{tr}(F^2) + \bar{\psi} i\not{D}\psi]\right), \tag{A.3}$$

where ψ denotes a single left-handed fermion doublet. The Dirac operator $i\not{D}$ is hermitian, thus the eigenvalues of the operator are real. Since the gamma matrices γ_μ and γ^5 anticommute, for every eigenvalue λ there is an eigenvalue $-\lambda$

$$i\not{D}\psi = \lambda\psi, \quad i\not{D}(\gamma^5\psi) = -\lambda(\gamma^5\psi). \tag{A.4}$$

If the doublet ψ contains Weyl fermions, the integral over the fermion field is

$$\int d\psi d\bar{\psi} \exp\left(-\int d^4x \bar{\psi} i\not{D}\psi\right) = \sqrt{\det i\not{D}}, \tag{A.5}$$

where the determinant is the product of eigenvalues [89]. The square root stands for the product of the half of the eigenvalues.

We can choose that the square root of the eigenvalues is the product of positive eigenvalues. The key point is that there exists a possibility for the positive and negative eigenvalues to interchange their places under the non-trivial gauge transformation. One possible eigenvalue flow is represented in the figure A.1. If the number of flows interchanging positive and negative eigenvalue pairs is odd, it follows that

$$\det i\not{D}(A) = -\det i\not{D}(A^U). \tag{A.6}$$

The gauge fields A_μ and A_μ^U contribute equally to the functional integral (A.3), which means that it vanishes in this case.

In order to show that this kind of eigenvalue flow happens, Witten defines an instanton-like gauge field

$$A_\mu^{t(\tau)} = (1 - t(\tau))A_\mu + t(\tau)A_\mu^U, \quad 0 \geq t(\tau) \geq 1 \tag{A.7}$$

and considers a five dimensional Dirac equation

$$\not{D}^{(5)}\Psi = \sum_{i=1}^{5} \gamma^i \left(\partial_i + \sum_{a=1}^{3} A_i^a T^a\right)\Psi = 0. \tag{A.8}$$

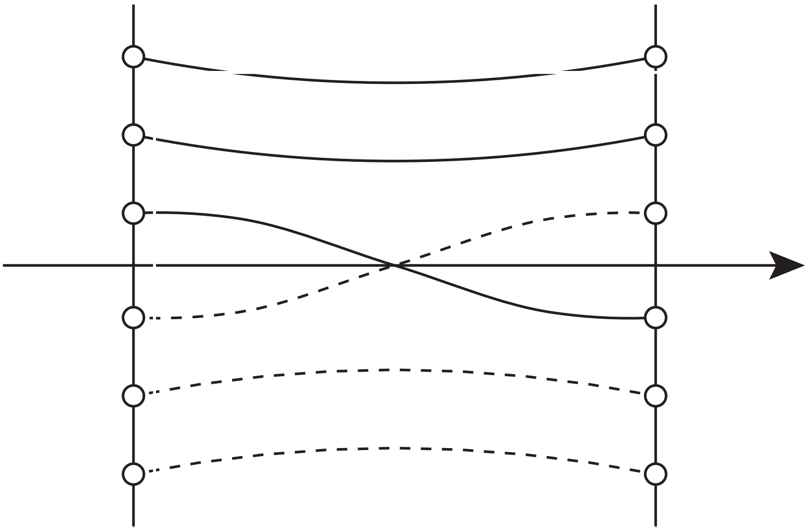

Figure A.1.: Eigenvalue flow under the non-trivial mpping.

The fifth component is the path parameter τ so that $t(\tau) = 1$ when $\tau \to +\infty$ and $t(\tau) = 0$ when $\tau \to -\infty$. Since $A_5^a = 0$ and the gamma matrices are real and symmetric we can write the equation (A.8) as

$$\frac{d\Psi}{d\tau} = -\gamma^\tau \not{D}^4 \Psi, \tag{A.9}$$

where $\not{D}^4$ is a four dimensional Dirac operator for each τ. To solve this we write

$$\Psi(x, \tau) = F(\tau)\phi^\tau(x), \tag{A.10}$$

where $\phi^\tau(x)$ is the eigenfunction of the operator $\gamma^\tau \not{D}^4$. In the adiabatic limit $\frac{d\phi^\tau(x)}{d\tau} = 0$ and the equation (A.9) simpliffies to a form

$$\frac{dF(\tau)}{d\tau} = -\lambda(\tau)F(\tau). \tag{A.11}$$

The solution to this equation is

$$F(\tau) = F(0) \exp\left(-\int_0^\tau dx \lambda(x)\right). \tag{A.12}$$

Now the logic goes as follows: According to mod 2 index theorem [90], the five-dimensional Dirac operator has an odd number of zero eigenvalues. This impose that equation (A.9) has an odd number of solutions. On the other hand equation (A.12) is normalizable only if $\lambda(\tau)$ is positive for $\tau \to +\infty$ and negative for $\tau \to -\infty$. This implies that there have to be odd number of eigenvalue pairs interchanging their places confirming the equation (A.6) to hold in this case.

Vanishing path integral (A.3) causes a problem, because for each gauge invariant operator the vacuum expectation value is

$$\langle 0|W|0\rangle = \frac{\int \mathcal{D}[A,\bar{\psi},\psi]\ W\ e^{-S}}{\int \mathcal{D}[A,\bar{\psi},\psi]\ e^{-S}} = \frac{"0"}{0}, \tag{A.13}$$

which is not well defined. Thus the SU(2) gauge theory with odd number of Weyl fermion doublets is inconsistent.

The hypothesis of adiabaticy is crucial in this derivation and criticism against this point is represented in the reference [91]. However, there are also two other ways to state the SU(2) anomaly. One is based on the U(1) anomaly and the rotation in the center of the SU(2), which is free from perturbative anomalies [92]. In the third approach SU(2) group is embedded into SU(3) group and the global anomaly result from the non-abelian anomaly of the group SU(3) [93, 94]. These alternative derivations confirm the existence of the SU(2) anomaly.

Appendix B.

Dyson-Schwinger Equation

The Schwinger-Dyson equation is based on the simple observation the (functional) integral of a (functional) derivative is zero [39]

$$\int D[\psi] \frac{\delta}{\delta \psi} \equiv 0. \tag{B.1}$$

This of course requires that the field ψ vanishes at spatial infinity. If we consider the generating functional of a QCD like theory

$$\begin{aligned} \mathcal{Z}\left[J, \eta, \bar{\eta}\right] &= \int D\left[G, \psi, \bar{\psi}\right] e^{iS\left[G,\psi,\bar{\psi},J,\eta,\bar{\eta}\right]}, \\ S\left[G, \psi, \bar{\psi}, J, \eta, \bar{\eta}\right] &= \int d^4x \left[\mathcal{L} + J_\mu G^\mu + e\bar{t}a\psi + \bar{\psi}\eta\right], \end{aligned} \tag{B.2}$$

where J_μ, η and $\bar{\eta}$ are the source fields, we have

$$0 = \int D\left[G, \psi, \bar{\psi}\right] \frac{\delta S\left[G, \psi, \bar{\psi}, J, \eta, \bar{\eta}\right]}{\delta \bar{\psi}^{i\alpha}(x)} e^{iS\left[G,\psi,\bar{\psi},J,\eta,\bar{\eta}\right]}. \tag{B.3}$$

This can be written as

$$\left[\eta^{i\alpha} + \left((i\partial\!\!\!/ - m_i)\delta^\alpha_\gamma + g\gamma^\mu (T^a)^\alpha_\mu \frac{\delta}{\delta i J^{a\mu}}\right) \frac{\delta}{\delta i \bar{\eta}^{i\gamma}}\right] \mathcal{Z}\left[J, \eta, \bar{\eta}\right] = 0. \tag{B.4}$$

Differentiating with respect to $\eta^{j\beta}$ and dividing with $\mathcal{Z}$ give

$$\begin{aligned} &\delta^4(x-y)\delta^i_j\delta^\alpha_\gamma + (i\partial\!\!\!/_x - m_i)\delta^\alpha_\beta \frac{i}{\mathcal{Z}} \frac{\delta^2 \mathcal{Z}}{\delta \bar{\psi}^{i\gamma}(x)\delta\eta_{j\beta}(y)} \\ &+ g\gamma^\mu (T^a)^\alpha_\gamma \frac{1}{\mathcal{Z}} \frac{\delta}{\delta J^{a\mu}(x)} \frac{\delta^2 \mathcal{Z}}{\delta \eta^{i\gamma}(x)\delta\eta_{j\beta}(y)} = 0, \end{aligned} \tag{B.5}$$

which can be modified to a form

$$\begin{aligned}
&\delta^4(x-y)\delta^i_j\delta^\alpha_\gamma + (i\not{\partial}_x - m_i)\delta^\alpha_\beta i\left(\frac{\delta^2 \ln \mathcal{Z}}{\delta\bar{\psi}^{i\gamma}(x)\delta\eta_{j\beta}(y)} + \frac{\delta \ln \mathcal{Z}}{\delta\bar{\eta}^{i\gamma}(x)}\frac{\delta \ln \mathcal{Z}}{\delta\eta_{j\beta}(y)}\right)\\
&+g\gamma^\mu(T^a)^\alpha_\gamma\left(\frac{\delta^3\mathcal{Z}}{\delta J^{a\mu}(x)\delta\eta^{i\gamma}(x)\delta\eta_{j\beta}(y)} + \frac{\delta}{\delta J^{a\mu}(x)}\left(\frac{\delta \ln \mathcal{Z}}{\delta\bar{\eta}^{i\gamma}(x)}\frac{\delta \ln \mathcal{Z}}{\delta\eta_{j\beta}(y)}\right)\right.\\
&\left.+\frac{\delta \ln \mathcal{Z}}{\delta J^{a\mu}(x)}\left(\frac{\delta^2 \ln \mathcal{Z}}{\delta\bar{\psi}^{i\gamma}(x)\delta\eta_{j\beta}(y)} + \frac{\delta \ln \mathcal{Z}}{\delta\bar{\eta}^{i\gamma}(x)}\frac{\delta \ln \mathcal{Z}}{\delta\eta_{j\beta}(y)}\right)\right) = 0,
\end{aligned} \tag{B.6}$$

By setting sources to zero this simplifies to

$$\begin{aligned}
&\delta^4(x-y)\delta^i_j\delta^\alpha_\gamma + (i\not{\partial}_x - m_i)\delta^\alpha_\beta i\frac{\delta^2 \ln \mathcal{Z}}{\delta\bar{\psi}^{i\gamma}(x)\delta\eta_{j\beta}(y)}\Big|_{J,\bar{\eta},\eta=0}\\
&+g\gamma^\mu(T^a)^\alpha_\gamma\frac{\delta^3\mathcal{Z}}{\delta J^{a\mu}(x)\delta\eta^{i\gamma}(x)\delta\eta_{j\beta}(y)}\Big|_{J,\bar{\eta},\eta=0} = 0,
\end{aligned} \tag{B.7}$$

Next we need the full quark and gluon propagators

$$S(x-y)^i_j\delta^\alpha_\beta = -\frac{\delta^2 \ln \mathcal{Z}}{i\delta\bar{\eta}^{i\alpha}(x)i\delta\eta_{j\beta}(y)}\Big|_{J,\bar{\eta},\eta=0}, \tag{B.8}$$

$$D_{\mu\nu}(x-y)\delta^{ab} = \frac{\delta^2 \ln \mathcal{Z}}{i\delta J^{\mu a}(x)i\delta J^{\nu b}(y)}\Big|_{J,\bar{\eta},\eta=0}, \tag{B.9}$$

and also the full gluon-quark-quark vertex

$$\begin{aligned}
&\frac{\delta^3 \ln \mathcal{Z}}{i\delta J^{\mu a}(z)i\delta\bar{\eta}^{i\alpha}i\delta\eta_{j\beta}}\Big|_{J,\bar{\eta},\eta=0}\\
&= -\int d^4x'd^4y'd^4z'D_{\mu\nu}(z-z')S(x-x')^i_k g\Gamma^\nu(z',x',y')^k_l(T^a)^\beta_\alpha S(y'-y)^l_j.
\end{aligned} \tag{B.10}$$

With the aid of identity

$$\int d^4z S(x-z)^\alpha_\gamma\delta^i_k S^{-1}(z-y)^\gamma_\beta\delta^k_j = \delta^\alpha_\beta\delta^i_j\delta^4(x-y), \tag{B.11}$$

and we get the Dyson-Schwinger equation for the quark propagator

$$\begin{aligned}
&iS^{-1}(x-y)^i_j\delta^\alpha_\beta = (i\not{\partial} - m_i)\delta^i_j\delta^\alpha_\gamma\delta^4(x-y)\\
&+g^2\int d^4x_1d^4x_2\gamma^\mu(T^a)^\alpha_\gamma S(x-x_1)^i_k\Gamma^\nu(x_2,x_1,y)^k_j(T^a)^\gamma_\beta D_{\mu\nu}(x-x_2).
\end{aligned} \tag{B.12}$$

Similarly for the gluon propagator

$$\begin{aligned} D^{-1}_{\mu\nu}(x-y)\delta^{ab} &= -i\left\{\partial^2 g_{\mu\nu} - \left(1-\frac{1}{\xi}\right)\partial_\mu\partial_\nu\right\}\delta^{ab}\delta^4(x-y) \\ +ig^2 &\int d^4x_1 d^4x_2 \mathrm{Tr}\left[\gamma_\mu T^a S(x-x_1)\Gamma_\nu(y,x_1,x_2)T^b S(x_2-x)\right] \end{aligned} \tag{B.13}$$

Colophon

This thesis was made in $\LaTeX\,2_\varepsilon$ using the "hepthesis" class [95].

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