The Time Inversion Symmetry In Case of Time Translation Existing

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In the case of time translation symmetry existing the quantum system is characterized by quasi-energy levels or quasi-energy bands. The time inversion symmetry does not impose additional constraints in itself on the quasi-energy states in comparison with the case of stationary states. However, since for systems with the quasi-energy spectrum the time inversion operator and the time translation operator commute each taken separately with the "Hamiltonian" $\mathcal{H} = H(t) - i\hbar\frac{\partial}{\partial t}$ but do not commute mutually in the general case, the simultaneous account of the time inversion symmetry and the time translation symmetry leads to additional constraints on the quasi-energy states.

The "Hamiltonian" $\mathcal{H}$ is simultaneously invariant both to the time inversion and to time translation transformations:

$$[\mathcal{H}, K_t] = [\mathcal{H}, T_{m\tau}] = 0, \tag{1}$$

where $K_t = UK_0I_t$ (UK_0 is the Wigner's operator of time inversion [1] and I_t is defined by $I_tt = -t, I_t^2 = 1$ [2]), $T_{m\tau}$ is the operator of time translations for a system in a strong oscillating field with the frequency $\omega = 2\pi/\tau$ when $m = 0, \pm1, \pm2, \cdots$ [3].

Let us consider a new group of transformations which is formed by the operators $K_tT_{m\tau}$. The new extended group of transformations by contrast to the time translation group is not an Abelian one because

$$T_{m\tau}I_t = I_tT_{-m\tau} \tag{2}$$

and hence, in the general case (besides the exeption considered below)

$$[K_t, T_{m\tau}] \neq 0. \tag{3}$$

If the Hamiltonian of system commutes with some two operators of transformations which do not commute with each other the spectrum of the eigenvalues of the Hamiltonian disintegrates into "multiplets of degenerate states" [4]. Since the operator K_t is antiunitary precautionary measures are needed while degeneralizing this results in case of the relationship (3).

Let us consider some eigenfunctions of the "Hamiltonian" $\mathcal{H}$ which is simultaneously an eigenfunction of the operator $T_{m\tau}$ with the eigenvalue $e^{\frac{i}{\hbar}m\xi\tau}$:

$$\mathcal{H} \mid \xi >= \xi \mid \xi >, \tag{4}$$

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$$T_{m\tau} \mid \xi >= e^{\frac{i}{\hbar}m\xi\tau} \mid \xi >, \tag{5}$$

where the following designation for the states of the quasi-energy is used ($\vec{x}$ include both the spatial and spin coordinates):

$$\mid \xi >= \varphi_{lm}(\vec{x},t)e^{-\frac{i}{\hbar}\xi t} = f_l(\vec{x}exp\{i(m\omega - \frac{\xi}{\hbar})t\}, l = 0,1,2,\cdots \tag{6}$$

Then

$$\mathcal{H}K_t \mid \xi >= \xi K_t \mid \xi >, \tag{7}$$

$$T_{m\tau}K_t \mid \xi >= T_{m\tau}I_tUK_0 \mid \xi >= I_tT_{-m\tau}UK_0 \mid \xi >= UK_0I_tT_{-m\tau} \mid \xi >= UK_0I_te^{-\frac{i}{\hbar}m\xi\tau} \mid \xi >$$

and hence

$$T_{m\tau}K_t \mid \xi >= e^{-\frac{i}{\hbar}m\xi\tau} \mid \xi > . \tag{8}$$

Thus, the spectrum of eigenvalues of the operator $\mathcal{H}$ consists of doublets of degenerated states which are eigenfunctions of the operator $T_{m\tau}$ with the eigenvalues of $e^{\pm\frac{i}{\hbar}m\xi\tau}$. Both these states transform into one in other at the time inversion operation K_t. In this case the commutator $[K_t, T_{m\tau}]$ is determined by the statement:

$$[K_t, T_{m\tau}] = 2i\sin(m\frac{\xi}{\hbar}\tau)K_t. \tag{9}$$

In particular, for $l = 1$ we have:

$$\mid \xi_1 >= \varphi_{1m}(\vec{x},t)e^{-\frac{i}{\hbar}\xi t} = f_1(\vec{x})e^{i(m\omega-\frac{\xi}{\hbar})t}. \tag{10}$$

Under the action of the commutator (9) upon the state $\mid \xi_1 >$ (in view of the reality of the function $f_1(\vec{x})$ and without taking into consideration the spin) we have

$$[K_t, T_{m\tau}] \mid \xi_1 >= 2i\sin(m\frac{\xi_1}{\hbar}\tau) \mid \xi_1 > . \tag{11}$$

According to (11) at

$$\frac{\xi_1}{\hbar\omega} = \frac{k}{m}, \quad k = 0, \pm1, \pm2, \cdots \tag{12}$$

the operators K_t and $T_{m\tau}$ are commuted as

$$[K_t, T_{m\tau}] = 0, \tag{13}$$

that is, in this case the state of the quasi-energy $\mid \xi_1 >$ is an eigenfunction of the operator $[K_t, T_{m\tau}]$ with the eigenvalue which is equal to zero. This corresponds to the presence of "singlets" (instead of "doublets") of the quasi-energy.

Note that for systems with an integer summary angular momentum the states $(1 + K_t)\varphi_{lm}(\vec{x},t)$ and $i(1 - K_t)\varphi_{lm}(\vec{x},t)$ are simultaneously eigenfunctions of the operators $\mathcal{H}$ and K_t. These states form a real basis of eigenfunctions whereas for systems with an half-integer summary angular momentum there is no such a real basis since the operator K_t in this case has not any eigenfunctions and this fact causes the Kramers degeneracy of the states of the quasy-energy.

References

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