SATURATION EFFECTS IN LOW X QCD EVOLUTION

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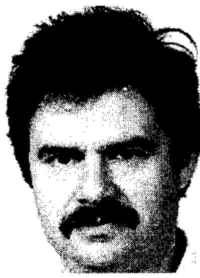

I review the approach to unitarization and saturation of partonic densities at high energy due to nonlinear effects in QCD evolution. The nonlinear generalization of QCD evolution equation is derived in the leading logarithmic approximation and its double logarithmic limit is discussed. It is shown that the nonlinearities at high density considerably slow down the evolution of the gluon distribution and the unitarity constraint is not violated. I argue that the intrinsic gluon distribution is strongly modified at low transverse momentum. Below $k_s \propto x^{-\alpha_s c}$ the intrinsic gluon distribution behaves like $\ln \frac{k_s}{k_\perp}$ rather than the standard low density behavior $\frac{1}{k_\perp^2}$.

1 Introduction

In this talk I am going to report on work published in [1]. The main direction of this work is to study a new semiperturbative regime, in which the QCD coupling constant is small but the fields involved are large. This regime arises when many particles are stacked in a small spatial region. If the density of particles is high (fields are large) the characteristic momentum transfer should be large as well, and the coupling should be small. However the most straightforward and naive perturbation theory assumes also low density and therefore is not directly applicable. It should be modified to take into account finite density effects.

In the context of hadronic physics one may expect this situation to arise in several interesting instances. The first candidate is deep inelastic scattering at low x. Here the standard BFKL picture is the rapid growth of partonic density at low x. When partons overlap in the transverse space nonlinear effects due to finite density must become important[a]. Another interesting

[a] The usual disclaimers are due of course when thinking about BFKL evolution in DIS. One should either think about DIS on a small object like "onium", whose wave function is dominated by small, perturbative configurations, or select carefully final states *a la* Mueller and Navelet. This is crucial for applicability of perturbation theory.

situation is scattering on a large nucleus. When the nucleus is moving fast the partons that in the rest frame are separated in the longitudinal direction get squeezed by Lorentz contraction and "sit" on top of each other. For very large nucleus this can lead to high surface density and again, nonlinear effects should arise. In the rest of this contribution I will discuss only the low x evolution.

Let me be more precise what do I mean by nonlinear effects in the evolution. Consider first the BFKL equation

$$\frac{\partial}{\partial \ln 1/x}\phi(k_\perp) = \alpha_s K(k_\perp, l_\perp)\phi(l_\perp) \tag{1}$$

It is written for the intrinsic gluon distribution, which is directly proportional to the intensity of the glue field

$$\phi(k_\perp) \propto k_\perp^2 < A_i^2(k_\perp) > \tag{2}$$

The BFKL kernel $K(k_\perp, l_\perp)$ is the probability of emission of an extra gluon into the hadronic wave function as x is lowered. In this linear evolution equation this probability does not depend on the density of gluons already present in the hadron. This obviously is a low density picture. As the density of the gluons rises (field intensity grows) one expects the emission probability itself to depend on the density. Therefore at very low x one expects schematically

$$K \to K[\phi] \tag{3}$$

This nontrivial dependence on the density should slow down the fast BFKL growth and eventually lead to unitarization of the cross section and possibly to saturation of the gluon density.

The aim of this work is to derive the nonlinear evolution equation which generalizes BFKL to this nonlinear high density regime.

2 Evolution

In the high density regime it is more convenient to use the language of classical fields rather than that of partons. We therefore describe the wave function of the hadron in terms of distribution of classical fields. The statistical weight of a given gluon field configuration (or probability to find a given field configuration in the hadronic wave function) is $Z[F_{\mu\nu}]$. We work in the frame where the hadron moves very fast. In this frame the only important configurations are the ones which are squeezed by the Lorentz contraction to a very thin disk and are practically static due to time dilation. They therefore have the form

$$F^{+i} = \delta(x^-)b^i(x_\perp) \tag{4}$$

with all other components of the field strength vanishing. Of course the staticity of the field is only approximate and only holds on the relevant resolution scale $\Delta t \sim \frac{1}{E} \sim \frac{x}{Q^2}$. In other words only the field modes with frequencies smaller than the resolution scale contribute in eq.(4). The same comment applies to the x^- dependence. Only modes with longitudinal wavelength smaller than $\Delta x^- \sim \frac{1}{x}$ are considered in eq.(4). Clearly when x is lowered the resolution is increased, and faster and longer wavelength quantum fluctuations become important. The picture is indeed very simple. Decreasing x amounts to further boosting our hadron. Under this further boost more field modes get squeezed into the $\delta(x^-)$ structure and more modes are frozen due to time dilation and therefore become static. It is clear then that the distribution of the background fields $Z[b^i]$ is x - dependent.

Technically we derive the evolution of Z with x by integrating out the fluctuations of the gluon field with higher frequencies and longer longitudinal wavelengths which due to the additional boost become part of the "background field" b_i at lower x. This integrating out is performed perturbatively to leading order in α_s.

The interaction of quantum fluctuations with the background field b^i is described by the action

$$S = -\int d^4x \frac{1}{4}\mathrm{tr}F^{\mu\nu}F_{\mu\nu} + \frac{i}{N_c}\int d^2x_\perp dx^- \delta(x^-)\partial^i b^i_b(x_\perp)\mathrm{tr}T_a W_{-\infty,\infty}[A^-](x^-, x_\perp) \quad (5)$$

where W is the Wilson line in the adjoint representation along the x^+ axis

$$W_{-\infty,+\infty}[A^-](x^-, x_\perp) = P\exp\left[+ig\int dx^+ A^-_a(x^+, x^-, x_\perp)T_a\right] \quad (6)$$

Apart from the standard QCD action F^2, it contains the nonabelian analog of the eikonal interaction of the quantum fluctuations with the colour charge density due to the background field $\rho = \partial_i b^i$.

Integrating out the fluctuations yields the evolution equation[b]

$$\frac{d}{d\ln 1/x}Z = \alpha_s\left\{\frac{1}{2}\frac{\delta^2}{\delta b^i(u)\delta b^j(v)}\left[Z\chi^{ij}(u,v)\right] - \frac{\delta}{\delta b^i(u)}\left[Z\sigma^i(u)\right]\right\} \quad (7)$$

The quantities χ^i and σ^i are nonlinear functionals of the background field b. Their explicit form was calculated in [1]. Their physical meaning is rather simple. As explained above, the quantum fluctuations induce additional background field on top of preexisting b^i. χ^{ij} and σ^i are the mean fluctuation and the average value of this induced field.

Equation (7) generates a sequence of evolution equations for the correlators of the chromo-electric field. The simplest one is the equation for the two point function.

$$\frac{d}{d\ln 1/x} < b^i(u)b^j(v) >= \alpha_s\left[< \chi^{ij}(u,v) > + < \sigma^i(u)b^j(v) + b^i(u)\sigma^j(v) > \right] \quad (8)$$

where $<>$ denotes averaging over the hadronic ensemble with the weight $Z[b]$.

3 Saturation

This is still a complicated functional equation and is not easily amenable to analysis. One can however make some headway by considering a simpler limit when the background field is slowly varying in the coordinate space. This corresponds to the doubly logarithmic regime where the transverse momentum is strictly ordered during the evolution. In this limit the formulae simplify very much and eq.(8) becomes (when transformed to momentum space)[c]

$$\frac{d}{d\ln 1/x} < b(k)b(-k) >= 4\alpha_s < \frac{b^2}{k^2 + 8\pi\alpha_s b^2} > \quad (9)$$

The striking feature of this equation is appearance of the "effective mass" proportional to the intensity of the background field. In the language of emission probability this means that when the field becomes large (the system is very dense), the emission probability of an extra gluon is *inversely proportional* to the field intensity. In this large field limit the solution of eq.(9) for the physical gluon density $\frac{dN}{d^2a d^2k} \equiv g(k) \propto < b(k)b(-k) >$ (where a is the impact parameter) is

$$g(k,x) = \frac{N_c^2}{\pi^3}\ln\frac{x_0(k^2)}{x} \quad (10)$$

[b]Henceforth we drop the subscript $\perp$ for brevity. All coordinates and momenta in the rest of this paper are transverse.

[c]For simplicity we take trace of eq.(8) over colour and Lorentz indices.

The value of x_0 should be determined from the condition that the strong field solution is valid in the region where the k^2 term in eq.(9) can be neglected. That is

$$\frac{dN(k^2,x_0)}{d^2a}=k^2 \tag{11}$$

Although careful determination of x_0 is not easy, one can roughly estimate it by the following simple argument[d]. In the region where the field is weak eq.(9) reduces to the DLA limit of the standard DGLAP evolution. We assume that it is valid all the way up to the point where the field reaches the limiting value eq.(11). From that point onwards we approximate the solution by eq.(10). In that case in the weak field region g is given by the asymptotic solution of the DLA DGLAP equation

$$\frac{dN(k^2,x)}{d^2a}\sim\exp\{\sqrt{\frac{4\alpha_sN_c}{\pi}\ln k^2\ln 1/x}\} \tag{12}$$

Using eqs.(12,11) we find $1/x_0(k)\sim(k^2)^{\frac{\pi}{4\alpha_sN_c}}$ With this we can rewrite eq.(10) as

$$g(k,x)=\frac{N_c}{4\pi^2}\frac{1}{\alpha_s}\ln\frac{k_s^2(x)}{k^2} \tag{13}$$

with $k_s^2(x)\sim(1/x)^{\frac{4\alpha_sN_c}{\pi}}$ Eqs.(10) and (13) are "complementary" manifestations of the gluon saturation. Eq.(10) tells us that in the strong field regime the gluon density at fixed transverse momentum grows with x only logarithmically rather than a much sharper growth in the weak field region governed by the linear QCD evolution. This slow growth is perfectly consistent with the unitarity. Eq. (13) shows a very similar behavior at fixed x as a function of k^2. At $k^2>k_s^2$ the gluon density is purely perturbative and behaves as $1/k^2$. However below k_s^2 the growth at low momentum is very significantly slowed down and is only logarithmic. Again this is not a complete saturation just like in the case of the x - dependence. Finally I want to mention that recent "saturation" fit to HERA data[3] suggests that the saturation momentum[e] k_s^2 is in the range $1-2$ Gev^2 at $x\sim10^{-4}$. If this is indeed the case we may have reached the (semi)perturbative large field regime already.

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[d]This argument is essentially due to A. Mueller[2].

[e]Although the fit does not determine directly the saturation scale in the gluon sector but rather in the quark sector, one hopes that the two are close.