Conformal Dynamics in 4D and Applications to LHC and Cosmology

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Qualifying Report
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Prelude

One of the most relevant and challenging questions in particle physics is to elucidate how the electroweak gauge symmetry is spontaneously broken. According to the Standard Model (SM), the answer is a fundamental scalar (Higgs) field.

However, the SM Higgs sector is plagued by the so-called *hierarchy problem* meaning that quantum corrections generate unprotected quadratic divergences requiring a huge fine-tuning if the models were to be true till the Planck scale.

Moreover, we still do not know whether a fundamental scalar field exists. Finding its replacement is one of the great campaigns, e.g. Technicolor (TC) and Supersymmetry (SUSY).

On the one hand, the main idea of TC is to introduce a new strongly coupled gauge theory in which Higgs sector of the SM is replaced by a composite field featuring only fermionic matter. On the other hand, one of the prominent motivations of SUSY is to deal with the same kind of untamed quantum (loop) corrections.

This qualifying report accounts for my work that I have been doing since the 1^{st} of September 2008. This report is divided into six chapters. A very short introduction to Standard Model has been mentioned in Chapter 1. In Chapter 2, the chiral symmetries and chiral symmetry breaking are precisely introduced.

The motivation of technicolor has been first taken into account in Chapter 3. In addition, more relevant details of technicolor theory have been mentioned afterwards. An introduction to supersymmetry is subsequantly presented in the beginning of Chapter 4. Later, the construction of the supersymmetry Lagrangian is also given in the same chapter.

After getting acquainted with technicolor and supersymmetry, the main route of Chapter 5 is to combine them. The resulting model is called a supersymmetric technicolor model. Besides, to go beyond this report, some specific plans for my future research are also introduced in the last chapter.

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Abstract

The main objective is the study of the mechanism of dynamical electroweak symmetry breaking (DESB) termed Technicolor (TC) and supersymmetric gauge theories (SUSY). After introducing TC and SUSY, this work subsequently aims to tie such two agenda. The resulting model is named a supersymmetric technicolor model.

Contents

CHAPTER 1

Standard Model (SM) In A Nutshell

1. Particle Contents

According to the current understanding of physics, the basic constituents of matter are a dozen of spin-1/2 particles (fermions). These are the three pairs of leptons (electron, muon, tau and their associated neutrinos) and three pairs of quarks (up, down, strange, charm, bottom and top) as shown below

Basic constituents of Matter

Fermion/Generation	1^{st}	2^{nd}	3^{rd}	charge
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	$\begin{pmatrix} +2/3 \\ -1/3 \end{pmatrix}$
Leptons	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

The members of each pair differ by 1 unit of electric charge as shown in the last column, i.e. charge 0 and -1 for the neutrinos and charged leptons and 2/3 and -1/3 for the up- and down-type quarks. This is relevant for their weak interaction. Apart from this electric charge, the quarks also possess a new kind of charge called *color charge*. This is relevant for their strong interaction which binds them together inside, for example, the proton (hadrons).

There are four basic interactions among these particles - strong, electromagnetic, weak and gravitational. Apart from gravity, which is too weak, the other three are all described by gauge interactions. They are all mediated by spin 1 particles, gauge bosons, whose interactions are completely specified by the corresponding gauge groups.

Basic Interactions

Interaction	Strong	EM	Weak
Carrier	g	γ	$W^{\pm}, Z^0$
Gauge Group	$SU(3)$	$U(1)$	$SU(2)$

From the unified gauge theory point of views, the weak, electromagnetic and strong interactions are given by a direct product of a weak and electromagnetic gauge group G_W and a strong gauge group G_S. We know that G_W is $SU(2) \times U(1)$ which is the gauge group of the original proposal of a unified gauge theory of weak and electromagnetic interactions, and the strong gauge group G_S is $SU(3)$ mentioned above. The strong gauge group G_S has also its gauge coupling constant assigned as g_s.

Roughly speaking, the theoretical version of the stamdard model (SM) is based on the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. The last two groups $SU(2)_L \otimes U(1)_Y$ define the theory of electroweak model, while $SU(3)_C$ characterizes the theory of strong interactions. The $SU(2)$ is called the weak interaction group, while $U(1)$ is called hypercharge. The $SU(3)$ is called the color group.

The strong interaction between quarks is mediated by the exchange of a gluon. This is analogous to the photon which mediates the electromagnetic interaction between charged particles (quarks or charged leptons). The gluon coupling is proportional to the color charge just like the photon coupling is proportional to electric charge.

Field content of the SM

Fields	Component Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Matter (Fermion) Fields	$L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L$	$(1, 2, -1/2)$
	E_R	$(1, 1, 1)$
	$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L$	$(3, 2, 1/6)$
	U_R	$(3, 1, -2/3)$
	D_R	$(3, 1, +1/3)$
Higgs Fields	Φ	$(1, 2, 1/2)$
Gauge Fields	W	$(1, 3, 0)$
	B	$(1, 1, 0)$
	G	$(8, 1, 0)$

In the minimal version, there is one complex (Higgs) scalar field which transforms as doublets of $SU(2)_L$ and singlets of $SU(3)_C$. The neutral component of such scalar develops a non-vanishing vacuum expectation value (VEV) so that the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ theory is spontaneously broken at a characteristic scale into $SU(3)_C \otimes U(1)_{EM}$, i.e. the color symmetry of strong interaction and the symmetry of electromagnetism. Here $U(1)_{EM}$ is the abelian gauge group of electromagnetism. As a consequence, three gauge bosons (W, Z) become massive by eating the three remaining Higgs bosons, while the photon A_μ is the gauge boson of the unbroken $U(1)_{EM}$ group and remains massless.

2. SM Lagrangian

In the SM, the fermion fields of the theory consist of three families of quarks and leptons, each family comprises left-handed doublets and right-handed singlets.

$$Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L = (3, 2, 1/6)\,, \qquad L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L = (1, 2, -1/2)\,, \tag{1.1.1}$$

and

$$U_R = (3,1,+2/3)\,, D_R = (3,1,-1/3)\,, E_R = (1,1,1)\,, \tag{1.1.2}$$

where $U = u, c, t, D = d, s, b, E = e, \mu, \tau$ and $N = \nu_e, \nu_\mu, \nu_\tau$. The first two entries in each parenthesis denote the dimensions of the $SU(3)$- and the $SU(2)$-representation, respectively, and the last entry denotes the $U(1)$ hypercharge. Suppose the real world was discribed by the standard $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ Lagrangian as follow [**3**]:

$$\begin{aligned}
\mathcal{L}_{SM} =& -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&+ \bar{L}i\gamma^\mu\mathcal{D}_\mu L + \bar{R}i\gamma^\mu\acute{\mathcal{D}}_\mu R + \bar{Q}i\gamma^\mu\tilde{\mathcal{D}}_\mu Q \\
&+ (\mathcal{D}_\mu\varphi)^* (\mathcal{D}^\mu\varphi) - V(\varphi^*, \varphi) - g_1\bar{L}\varphi R - g_2\bar{L}\tilde{\varphi}R + h.c \\
&+ \frac{1}{2}g_s\bar{\psi}^j_q\gamma^\mu T^A_{jk}\psi^k_q G^A_\mu,
\end{aligned} \tag{1.2}$$

where the terms in the first line correspond to $W^\pm,\ Z^0,\ \gamma$, gluon kinetics terms, and self interactions. The second line corresponds to fermionic kinetic terms and their interactions with $W^\pm,\ Z^0,\ \gamma$. The third line corresponds to masses and couplings of $W^\pm,\ Z^0,\ \gamma$ and Higgs bosons. Whereas the last line corresponds to quark-gluon coupling.

Here the covariant derivatives are defined as follows:

$$\mathcal{D}_\mu = \left(\partial_\mu + \frac{i}{2}g\tau^a W^a_\mu + \frac{i}{2}g' Y B_\mu\right), \tag{1.3.1}$$

$$\tilde{\mathcal{D}}_\mu = \left(\partial_\mu + \frac{i}{2}g\tau^a W^a_\mu + ig_s T_A G^A_\mu\right), \tag{1.3.2}$$

$$\acute{\mathcal{D}}_\mu = \left(\partial_\mu + \frac{i}{2}g' Y B_\mu\right), \tag{1.3.3}$$

and the field strength tensors as

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - ig\left[W_\nu, W_\mu\right], \tag{1.4.1}$$

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_s f_{ABC} G^B_\mu G^C_\nu, \tag{1.4.2}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{1.4.3}$$

where (W, B) are $SU(2)$ and $U(1)$ gauge fields defined respectively by

$$\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \end{pmatrix}, \quad B_\mu \tag{1.5}$$

The $\left(\varphi, \varphi^\dagger\right)$ are scalars with

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}; \varphi^\dagger = \left(\varphi^-, \varphi^0\right); \varphi^c = \tilde{\varphi} = i\sigma_2\varphi^* = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix}; \tag{1.6}$$

i.e. $\varphi(\tilde{\varphi})$ is a Higgs (its conjugate) doublet.

After spontaneous symmetry breaking, the Lagrangian is [**4**]

$$\begin{aligned}\mathcal{L}_{SM}^{SSB} =& \sum_i \bar{\psi}_i \left(i\partial\!\!\!/ - m_i - \frac{g m_i H}{2M_W} \right) \psi_i \\ & - \frac{g}{2\sqrt{2}} \sum_i \bar{\psi}_i \gamma^\mu \left(1 - \gamma^\mu\right) \left(T^+ W_\mu^+ + T^- W_\mu^-\right) \psi_i \\ & - e \sum_i Q_i \bar{\psi}_i \gamma^\mu \psi_i A_\mu \\ & - \frac{g}{2\cos\theta_w} \sum_i \bar{\psi}_i \gamma^\mu \left(g_V^i - g_A^i \gamma^5\right) \psi_i Z_\mu, \end{aligned} \tag{1.7}$$

where $\theta_w = \tan^{-1}(g'/g)$ is the weak angle defined by the ratio of the $SU(2)$ and $U(1)$ couplings; $e = g\sin\theta_w$ is the electric charge;

$$A_\mu = B_\mu \cos\theta_w + W_\mu^3 \sin\theta_w, \tag{1.8}$$

is the massless photon field;

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp W_\mu^2\right), \tag{1.9}$$

and

$$Z_\mu = -B_\mu \sin\theta_w + W_\mu^3 \cos\theta_w, \tag{1.10}$$

are the massive charged and neutral weak boson fields, respectively; $T^+ = \left(\tau^1 + i\tau^2\right)/2$ and $T^- = \left(\tau^1 - i\tau^2\right)/2$ are the weak isospin raising and lowering operators. The vector and axial vector couplings are defined as

$$\begin{aligned} g_V^i &= t_{3L}(i) - 2Q_i \sin^2\theta_w, \\ g_A^i &= t_{3L}(i), \end{aligned} \tag{1.11}$$

where $t_{3L}(i)$ is the weak isospin of fermion i ($+1/2$ for u_i and ν_i; $-1/2$ for d_i and e_i) and Q_i is the charge of ψ_i in units of e.

The second term in $\mathcal{L}_{SM}^{SSB}$ represents the charged current weak interaction [**1**]. For example, the coupling of W to electron and neutrino is

$$A_\mu = B_\mu \cos\theta_w + W_\mu^3 \sin\theta_w, \tag{1.12}$$

which is the massless photon field;

$$-\frac{e}{2\sqrt{2}\sin\theta_w} \left[W_\mu^- \bar{e}\gamma^\mu \left(1 - \gamma^5\right) \nu + W_\mu^+ \bar{\nu}\gamma^\mu \left(1 - \gamma^5\right) e\right]. \tag{1.13}$$

The left-handed fermion fields

$$\begin{pmatrix} \nu_i \\ l_i^- \end{pmatrix}, \quad \begin{pmatrix} u_i \\ \acute{d}_i \end{pmatrix}, \tag{1.14}$$

of the i^{th} fermion family transform as doublets under $SU(2)$, and $\acute{d_i} = \sum_j V_{ij} d_j$, where V_{ij} is the Cabibbo-Kobayashi-Maskawa mixing matrix, whereas the right-handed fields are $SU(2)$ singlets. Here γ^μ and γ^5 are defined respectively as follows:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{1.15}$$

where $\sigma^\mu = (1, \vec{\sigma})$ and $\bar{\sigma}^\mu = (1, -\vec{\sigma})$, and $\vec{\sigma}$ are just ordinary 2×2 Pauli matrices.

The third term in $\mathcal{L}_{SM}^{SSB}$ describes the electromagnetic interaction, and the last term is the weak neutral-current interaction. In Eq.(1.7), m_i is the mass of the i^{th} fermion ψ_i, while H is the physical neutral (Higgs) scalar which is the only remaining part of φ after spontaneous symmtery breaking. Here $gm_i/2M_W$ is the coupling of scalar H to fermion ψ_i so-called Yukawa coupling.

CHAPTER 2

Gauge Symmetries and Chiral Symmetry Breaking

One can recognize two basic spin-1/2 representations of the Lorentz group transforming in the following transformations:

$$\psi_L \to \left(1 - \vec{\alpha}\cdot\vec{\tau} - \vec{\beta}\cdot\vec{\tau}\right)\psi_L,$$
$$\psi_R \to \left(1 - \vec{\alpha}\cdot\vec{\tau} + \vec{\beta}\cdot\vec{\tau}\right)\psi_R, \tag{2.1}$$

where $\vec{\tau} = \vec{\sigma}/2$, and $\vec{\alpha}$ is an infinitesimal rotation angle and $\vec{\beta}$ is an infinitesimal boost. One can construct the four-component spinor from these representations. Thus we can write it as:

$$\Psi = \begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix}, \tag{2.2}$$

where ψ_L and ψ_R are respectively left-handed and right-handed 2-component Weyl spinors. Note here that this is just an usual 4-component Dirac spinor.

By defining the matrix $c = -i\sigma^2$, it is trivial to show from the first expression of Eq.(2.1) that $-c\psi_L^*$ transforms like ψ_R. As a result, hence, we can write all fermion fields in our theory with left-handed Weyl spinor. Therefore, the Dirac spinor takes the form

$$\Psi = \begin{pmatrix}\psi_{1L} \\ -c\psi_{2L}^*\end{pmatrix} \equiv \begin{pmatrix}\chi \\ \bar{\xi}\end{pmatrix}. \tag{2.3}$$

We know that the coupling constant g_s of QCD becomes larger at low energies (or at long distances), and then it becomes infinite at some significant value Λ_{QCD}. This value is always referred to be $\Lambda_{\mathrm{QCD}} \sim$ 200 MeV. The masses of the lightest quarks, u and d, are of the order of a few MeV, and therefore much smaller than Λ_{QCD}. We can begin with the approximation that the up and down quarks are massless. The mass of strange quark is also less than Λ_{QCD}, hence it is sometimes useful to treat the strange quark as massless.

Let us now consider QCD with $n_F = 2$ flavors of massless quarks. Before writing the Lagrangian, let us assign the color and flavor indices to two left-handed Weyl fields given in (2.3). We have the left-handed Weyl fields $\chi_{\alpha i}$, where $\alpha = 1, 2, 3$ is a color index for the 3 representation, and $i = 1, 2$ is a flavor index, and left-handed Weyl fields $\xi^{\alpha\bar{\imath}}$, where $\alpha = 1, 2, 3$ is a color index for the $\bar{3}$ representation, and $\bar{\imath} = 1, 2$ is a flavor index. In this section we will follow the conventions given in [**6**]. By putting a bar over it, we can distinguish the flavor index of ξ's, and a superscript is assigned to ξ for later notational convenience. For massless QCD Lagrangian, we have

$$\mathcal{L}_{\mathrm{QCD}} = \overline{\Psi}(x) i\gamma^\mu D_\mu \Psi(x) - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}, \tag{2.4}$$

where $\overline{\Psi} = \Psi^\dagger \gamma^0$. The gamma matrices in Eq.(2.2) are defined by

$$\gamma^\mu = \begin{pmatrix} 0_{2\times 2} & \sigma^\mu \\ \bar{\sigma}^\mu & 0_{2\times 2} \end{pmatrix}, \tag{2.5}$$

where I is 2×2 identity matrix and $\sigma^\mu = (1, \vec{\sigma})\,, \bar{\sigma}^\mu = (1, -\vec{\sigma})$, and

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.6}$$

are the ordinary 2×2 Pauli matrices.

After substituting the fields in (2.3) into Eq.(2.4), the Lagrangian then becomes [**6**]

$$\mathcal{L}_{\mathrm{QCD}} = i\bar{\chi}^{\alpha i} \bar{\sigma}^\mu \left(D_\mu\right)_\alpha^{\ \beta} \chi_{\beta i} + i\bar{\xi}_{\alpha\bar{\imath}} \bar{\sigma}^\mu \left(\bar{D}_\mu\right)^\alpha_{\ \beta} \xi^{\beta\bar{\imath}} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}, \tag{2.7}$$

where $D_\mu = \partial_\mu - igT^a_3 A^a_\mu$ and $\bar{D}_\mu = \partial_\mu - igT^a_{\bar{3}} A^a_\mu$, with $\left(T^a_{\bar{3}}\right)^\alpha_{\ \beta} = -\left(T^a_3\right)_\beta^{\ \alpha}$.

This Lagrangian has not only the $SU(3)$ color symmetry, but also a global $SU(2)_L \times SU(2)_R$ flavor symmetry. Hence this Lagrangian is invariant under the following transformations [**6**]:

$$\chi_{\alpha i} \rightarrow L_i^{\ j} \chi_{\alpha j}, \tag{2.8.1}$$

$$\xi^{\alpha\bar{\imath}} \rightarrow \left(R^*\right)^{\bar{\imath}}_{\ \bar{\jmath}} \xi^{\alpha\bar{\jmath}}, \tag{2.8.2}$$

where L and R^* are independent 2×2 unitary matrices. The complex conjugate over R is just a notational convention. Furthermore, these transformations can be also written in terms of the Dirac field given in (2.3). To do that, let us define left and right projection matrices as

$$P_L \equiv \frac{1}{2}\left(1 - \gamma^5\right) = \begin{pmatrix} \delta_i^{\ j} & 0 \\ 0 & 0 \end{pmatrix}, \tag{2.9.1}$$

$$P_L \equiv \frac{1}{2}\left(1 + \gamma^5\right) = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\bar{\imath}}_{\ \bar{\jmath}} \end{pmatrix}. \tag{2.9.2}$$

By applying the indices to the Dirac field,

$$\Psi_{\alpha i} = \begin{pmatrix} \chi_{\alpha i} \\ \bar{\xi}_{\alpha\bar{\imath}} \end{pmatrix}, \tag{2.10}$$

the equation (2.8.1) and (2.8.2) respectively become [**6**]

$$P_L \Psi_{\alpha i} \rightarrow L_i^{\ j} P_L \Psi_{\alpha i}, \tag{2.11.1}$$

$$P_R \Psi_{\alpha\bar{\imath}} \rightarrow R_{\bar{\imath}}^{\ \bar{\jmath}} P_R \Psi_{\alpha\bar{\jmath}}, \tag{2.11.2}$$

when we take the conjugate transpose of both sides of Eq.(2.11.2), then we recover the expression in Eq.(2.8.2). A symmetry that treats the left- and right-handed component of a Dirac field differently is named as "*chiral*".

However there is an anomaly in the *axial* $U(1)$ symmetry corresponding to $L = R^* = e^{i\alpha} I$ which is equivalent to

$$\Psi \rightarrow e^{-i\alpha\gamma^5}\Psi \tag{2.12}$$

for a Dirac field, or in terms of two Weyl fields:

$$\chi \rightarrow L\chi = e^{i\alpha}\chi, \tag{2.13.1}$$

$$\xi \rightarrow R^*\xi = e^{i\alpha}\xi, \tag{2.13.2}$$

this is often referred as $U(1)_A$ symmetry which does not correspond to a conserved quantity. With $L \neq R$, the axial currents can be defined as $j_A^{\mu 5} = \overline{\Psi}\gamma^\mu\gamma^5\Psi$, $j_A^{\mu 5a} = \overline{\Psi}\gamma^\mu\gamma^5\tau^a\Psi$. Without the anomaly, the global flavor symmetry of the Lagrangian is $SU(2)_L \times SU(2)_R \times U(1)_V$ where V stands for *vector*. The $U(1)_V$ transformation corresponds to $L = R = e^{-i\alpha}I$, or equivalently

$$\Psi \rightarrow e^{-i\alpha}\Psi \tag{2.14}$$

for Dirac field, or in terms of two Weyl fields:

$$\chi \rightarrow L\chi = e^{-i\alpha}\chi, \tag{2.15.1}$$

$$\xi \rightarrow R^*\xi = e^{i\alpha}\xi. \tag{2.15.2}$$

With $L = R$, the vector currents are defined as $j_V^\mu = \overline{\Psi}\gamma^\mu\Psi$, $j_V^{\mu a} = \overline{\Psi}\gamma^\mu\tau^a\Psi$, and the corresponding symmetry group is $U(1)_V$ and $SU(2)_V$ respectively. Hence this $U(1)_V$ defines the classification of *hadrons** by their baryon number. The remaining chiral symmetry $SU(2)_L \times SU(2)_R$ is spontaneouly broken into the vector subgroup $SU(2)_V$ determined by setting $L = R$ in Eq.(2.8.2). This vector subgroup is known as isospin symmetry†. However isospin is not an exact symmetry, thus we see small differences in the masses of these multiplets.

To spontaneouly break the axial part of the $SU(2)_L \times SU(2)_R$ symmetry, some operator must acquire a nonzero vacuum expectation value. When the axial generators are spontaneously broken, then the three pions identified as the corresponding Goldstone bosons immediately emerge. Since the $SU(2)_L \times SU(2)_R$ symmetry is not exact, the pions are not exactly massless.The constraint of the spontaneous breakdown is that this operator must be a Lorentz scalar, and to avoid the spontaneous breakdown of the $SU(3)$ gauge symmetry such an operator must also be a color singlet. Since we have no fundamental scalar fields which are color singlet, this field has to be *composite*. According to [**6**], the simplest candidate is $\chi^a_{\alpha i}\xi_a^{\alpha\bar{j}} = \overline{\Psi}^{\alpha\bar{j}} P_L \Psi_{\alpha i}$ with a an undotted spinor index. For the vacuum

*Note that there are two types of hadrons: *mesons* and *baryons*. Mesons are color-singlet bound states of quark-antiquark pairs, e.g. pions, while bayons are color-singlet bound states of three quarks, e.g. proton and neutron.

†We know that hadrons can be represented by the $SU(2)_V$ representations. Concretely, the lightest spin-1/2 hadrons, proton and neutron, form a doublet (or 2) representation, while the lightest spin-0 hadrons, pions, form a triplet (or 3) representation.

expectation value (in [**31**], this is called an order parameter) of this composite field, we presume that [**6**]

$$\langle 0| \chi^a_{\alpha i}\xi^{\alpha\bar{\jmath}}_a |0\rangle = -v^3\delta_i^{\ \bar{\jmath}}, \tag{2.16}$$

where v is a parameter with dimension of mass. A measured magnetude of v is at 200 MeV approximately corresponding to Λ_{QCD}. In the standard $SU(2)_L \otimes U(1)_Y$ electroweak theory, the order parameter of symmetry breaking is $\langle\varphi\rangle$-the expectation value of the neutral component of the Higgs doublet. Now let us work further to see precisely how the *fermion condensate* in Eq.(2.16) signals the breaking of the axial generators of $SU(2)_L \times SU(2)_R$, and how the vector generators are still preserved. By substituting the transformation of fields in Eq.(2.8.1) and Eq.(2.8.2), we obtain the transformation law of Eq.(2.16) as

$$\begin{aligned}\langle 0| \chi^a_{\alpha i}\xi^{\alpha\bar{\jmath}}_a |0\rangle &\rightarrow L_i^{\ k} \left(R^*\right)^{\bar{\jmath}}_{\ \bar{n}} \langle 0| \chi^a_{\alpha k}\xi^{\alpha\bar{n}}_a |0\rangle \\ &= -v^3\delta_k^{\ \bar{n}} L_i^{\ k} \left(R^*\right)^{\bar{\jmath}}_{\ \bar{n}} \\ &\rightarrow -v^3 \left(LR^\dagger\right)_i^{\ \bar{\jmath}}. \end{aligned} \tag{2.17}$$

The right-hand side of Eq.(2.17) is unchanged from the value in Eq.(2.16) if we take $L = R$ which corresponds to an $SU(2)_V$ transformation. This indicates that $SU(2)_V$ (and also $U(1)_V$) is unbroken. With $L = R$, we thus have

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_V \otimes U(1)_A \rightarrow SU(2)_V \otimes U(1)_V, \tag{2.18}$$

where $U(1)_V$ corresponds to the baryon number conservation. It is clear that there exist four spontaneouly broken symmetries associated with the four axial vector currents. The order parameter has the dimensions of a cubed mass ‡ whereas the Higgs field in the Weinberg-Salam theory has dimensions of mass§. The value of the order parameter is determined by the symmetry breaking scale. In QCD, $\langle 0| \chi^a_{\alpha i}\xi^{\alpha\bar{\jmath}}_a |0\rangle \equiv \left\langle \bar{u}_L u_R + \bar{d}_L d_R \right\rangle \sim$ (200 MeV)^3 and in the Weinberg-Salam theory $\langle\varphi\rangle \sim$ (246 GeV). Generally the masses or mass differnces resulting from the symmetry breaking will be in the order of the breaking scale or smaller [**9**][**31**].

The pions, which would be the Goldstone boson for an exact symmetry, are not exactly massless, so let us work further to calculate their masses. We first assume that the orientation in flavor space of the order parameter is a function of spacetime, so we can write [**6**]

$$\langle 0| \chi^a_{\alpha i}(x)\xi^{\alpha\bar{\jmath}}_a(x) |0\rangle = -v^3 U_i^{\ \bar{\jmath}}(x), \tag{2.19}$$

where $U(x)$ is a spacetime dependent unitary matrix. This unitary matrix can be written as

$$U(x) = exp[2i\pi^a(x)T^a/f_\pi], \tag{2.20}$$

where $T^a = \sigma^a/2$ with $a = 1, 2, 3$ are the generators of $SU(2), \pi^a(x)$ are real scalar fields identified as with the pions, and f_π is the pion decay constant with dimension of mass. By assuming that these pions can be associated with the axial isospin current, we can

‡ $[\chi] = [\xi] = 3/2$

§ $[\varphi] = 1$

define the pion decay constant via the matrix element of the axial currents $j^{\mu 5a}$ between the vacuum and on-shell pion states as [**2**]

$$\langle 0| j^{\mu 5a} \left|\pi^b(p)\right\rangle = -ip^\mu f_\pi \delta^{ab} e^{-ip\cdot x}, \tag{2.21}$$

where a, b are isospin indices.

If we take the conservation of axial currents into the account:

$$\langle 0| \partial_\mu j^{\mu 5a} \left|\pi^b(p)\right\rangle = p^2 f_\pi \delta^{ab} e^{-ip\cdot x} = 0, \tag{2.22}$$

then we find that an on-shell pion must satisfy $p^2 = 0$, and that it must be massless as required by Goldstone's theorem.

However the $U(1)_A$ symmetry is eliminated by the anomaly, so we do not include the fourth generator matrix proportional to the identity; it would be the Goldstone boson for the $U(1)_A$ symmetry. If we now restore the quark mass terms in Eq.(2.7); $-\left(m\chi_{\alpha i}\xi^{\alpha\bar{\jmath}} + h.c.\right)$, then the axial currents corresponding to the transformation laws in Eq.(2.13.1) and Eq.(2.13.2) are no longer conserved.

Let us now include the small masses for up and down quarks to the Lagrangian. The most general mass term we can have is of the form [**6**]

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= -\xi^{\alpha\bar{\jmath}} M_{\bar{\jmath}}{}^{i} \chi_{\alpha i} + h.c. \\ &= -M_{\bar{\jmath}}{}^{i} \chi_{\alpha i} \xi^{\alpha\bar{\jmath}} + h.c. \\ &= -\text{Tr}\left(M\chi\xi\right) + h.c.,\end{aligned} \tag{2.23}$$

where

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \tag{2.24}$$

is the quark mass matrix. Next, we replace $\chi_{\alpha i}\xi^{\alpha\bar{\jmath}}$ in Eq.(2.23) with the order parameter as a function of spacetime in Eq.(2.19); then Eq.(2.23) becomes [**6**]

$$\mathcal{L}_{\text{mass}} = v^3 \text{Tr}\left(MU + M^\dagger U^\dagger\right)\right) . \tag{2.25}$$

To keep $\text{Tr}(MU)$ invariant, we need the following transformations:

$$M \to RML^\dagger, \tag{2.26.1}$$

$$U \to LUR^\dagger, \tag{2.26.2}$$

precisely,

$$\begin{aligned}\text{Tr}\left(MU\right) &\to \text{Tr}\left(RM\underbrace{L^\dagger L}_{=1}UR^\dagger\right) \\ &= \text{Tr}\left(RMUR^\dagger\right) \\ &= \text{Tr}\left(MU\underbrace{R^\dagger R}_{=1}\right) \\ &= \text{Tr}\left(MU\right).\end{aligned} \tag{2.27}$$

Now let us expand the Eq.(2.25) by expanding the matrices $U(x)$ and $U^\dagger(x)$, and use $M^\dagger = M$. We obtain in inverse powers of f_π

$$\begin{aligned}
\mathcal{L}_{\text{mass}} &= -2\left(v^3/f_\pi^2\right)\text{Tr}\left(MT^aT^b\right)\pi^a\pi^b + ... \\
&= -\left(v^3/f_\pi^2\right)\text{Tr}\left(M\underbrace{\left\{T^a,T^b\right\}}_{=\delta^{ab}/2} + M\underbrace{\left[T^a,T^b\right]}_{=0\text{ under Tr}}\right)\pi^a\pi^b + ... \\
&= -\frac{v^3}{2f_\pi^2}\text{Tr}\left(M\delta^{ab}\right)\pi^a\pi^b + ... \\
&= -\frac{v^3}{2f_\pi^2}\text{Tr}\left(M\right)\pi^a\pi^a + ... \; .
\end{aligned} \tag{2.28}$$

We see that all three pions have the same mass

$$m_\pi^2 = \frac{v^3}{f_\pi^2}\text{Tr}\left(M\right) = \left(m_u + m_d\right)\frac{v^3}{f_\pi^2}. \tag{2.29}$$

By substituting the following numerical values: $v \sim 246$ MeV, $f_\pi \sim 93$ MeV, and $(m_u + m_d) \sim 10$ MeV, we obtain $m_\pi \sim 140$ MeV. However we know that the mass of the $\pi^\pm$ is slightly heavier than that of π^0 because of electromagnetic interactions.

CHAPTER 3

An Introduction to Technicolor

1. Motivation for Technicolor

The Lagrangian of a massless doublet of up and down quarks in ordinary QCD, already introduced in Chapter 2, has a $SU(2)_L \times SU(2)_R \times U(1)_B$ symmetry. The remaining symmetry of the theory we want to work with is the $SU(2)_L \times SU(2)_R$ symmetry, and that turns out to be spontaneouly broken down to the vector subgroup $SU(2)_V$ known as *isospin*. This spontaneous breakdown is associated with the non-zero expectation value of the fermion condensate

$$\langle 0| \chi^a_{\alpha i}\xi^{\alpha\bar{j}}_a |0\rangle \equiv \left\langle \bar{u}_L u_R + \bar{d}_L d_R \right\rangle \neq 0. \tag{3.1}$$

Note that this condensate is not invariant under $SU(2)_L$ or $SU(2)_R$, but it is invariant under only the group that treats left- and right-handed parts equally: $SU(2)_V$ or $SU(2)_{\text{isospin}}$. From the symmetry breaking by the condensate, there will be three massless spin-0 bosons identified as Goldstone bosons. In this case, this means that they are three massless pions, π_a, forming an isotriplet. As previously reviewed, these pions are associated with the axial isospin currents. However, the triplet of axial isospin currents may be written either in terms of the quark fields $q = (u, d)$ or pion fields π^a as follows **[7][8]**:

$$j^\mu_{5a} = \bar{q}\gamma^\mu\gamma^5 \frac{\tau^a}{2} q = f_\pi \partial^\mu \pi_a, \tag{3.2}$$

where τ^a with $a = 1, 2, 3$ are 2×2 Pauli matrices, and f_π is the pion decay constant in QCD, independent of weak interactions, defined by the following matrix element **[2][8]**:

$$\langle 0| j^\mu_{5a} |\pi_b(p)\rangle = -i f_\pi p^\mu \delta_{ab}. \tag{3.3}$$

With the following definitions:

$$j^{\mu\pm}_5 = j^\mu_{51} \pm i j^\mu_{52}, j^{\mu 0}_5 = j^\mu_{53}, \tag{3.4}$$

and

$$\pi^\pm = \pi_1 \pm i\pi_2, \pi_0 = \pi_3, \tag{3.5}$$

we obtain

$$j^{\mu+}_5 = \bar{d}\gamma^\mu\gamma^5 u = f_{\pi^+}\partial^\mu\pi^+, \tag{3.6.1}$$

$$j^{\mu-}_5 = \bar{u}\gamma^\mu\gamma^5 d = f_{\pi^-}\partial^\mu\pi^-, \tag{3.6.2}$$

$$j^{\mu 0}_5 = \frac{1}{2}\left(\bar{u}\gamma^\mu\gamma^5 u - \bar{d}\gamma^\mu\gamma^5 d\right) = f_{\pi^0}\partial^\mu\pi_0. \tag{3.6.3}$$

Now let us define the π^+ and π^- fields in terms of the creation and annihilation operators as:

$$\pi^-(x) \sim \int \frac{d^3p}{(2\pi)^3}\left(e^{-ip\cdot x}a_{\mathbf{p},-} + e^{+ip\cdot x}a^\dagger_{\mathbf{p},+}\right), \tag{3.7.1}$$

$$\pi^+(x) \sim \int \frac{d^3p}{(2\pi)^3}\left(e^{-ip\cdot x}a_{\mathbf{p},+} + e^{+ip\cdot x}a^\dagger_{\mathbf{p},-}\right), \tag{3.7.2}$$

$$\pi^0(x) \sim \int \frac{d^3p}{(2\pi)^3}\left(e^{-ip\cdot x}a_{\mathbf{p},\mathbf{0}} + e^{+ip\cdot x}a^\dagger_{\mathbf{p},\mathbf{0}}\right). \tag{3.7.3}$$

The operators $a^\dagger_{\mathbf{p},-}$ and $a_{\mathbf{p},-}$ are the creation and annihilation operators acting on the π^- state $|\pi^-;\mathbf{p}\rangle \equiv a^\dagger_{\mathbf{p},-}|0\rangle$, while $a^\dagger_{\mathbf{p},+}$ and $a_{\mathbf{p},+}$ are the creation and annihilation operators acting on the π^+ state $|\pi^+;\mathbf{p}\rangle \equiv a^\dagger_{\mathbf{p},+}|0\rangle$, and so on. After working further with the commutation relation $[a_{\mathbf{p};\pm,\mathbf{0}}, a^\dagger_{\acute{\mathbf{p}};\pm,0}] = (2\pi)^3\,\delta^3(\mathbf{p}-\acute{\mathbf{p}})$, we get

$$\langle 0|\, j_5^{\mu+}\left|\pi^+\right\rangle = f_{\pi^+}\langle 0|\,\partial^\mu\pi^+\left|\pi^+\right\rangle = -if_{\pi^+}p^\mu, \tag{3.8.1}$$

$$\langle 0|\, j_5^{\mu-}\left|\pi^-\right\rangle = f_{\pi^-}\langle 0|\,\partial^\mu\pi^-\left|\pi^-\right\rangle = -if_{\pi^-}p^\mu, \tag{3.8.2}$$

$$\langle 0|\, j_5^{\mu0}\left|\pi^0\right\rangle = f_{\pi^0}\langle 0|\,\partial^\mu\pi^0\left|\pi^0\right\rangle = -if_{\pi^0}p^\mu. \tag{3.8.3}$$

We assume first that the electroweak interaction is turned off for the time being. What happens if we take the electroweak interactions into account? In doing so. let us now turn on the ordinary $SU(2)_L\times U(1)_Y$ electroweak interactions and ignore the fundamental scalar fields which are usually introduced to give mass to $W^\pm$ and Z. In this case, the massless pions replace the conventional scalar fields, and then they are eaten by the gauge bosons becomimg the longitudinal components of massive bosons $W^\pm$ and Z.

Let us first consider the kinetic terms of the quark field Lagrangian,

$$\bar{q}_L\gamma^\mu D_\mu q_L + \bar{u}_R\gamma^\mu D_\mu u_R + \bar{d}_R\gamma^\mu D_\mu d_R, \tag{3.9}$$

where $D_\mu q_L = \left(\partial_\mu + igA^a_\mu\tau^a + i\acute{g}YB_\mu\right)q_L$ is the covariant derivative, and where τ^a and Y depend on the particular representation to which fermion field belongs. Using the above results, we can write the kinetic terms as

$$\frac{g}{2}f_{\pi^+}W^+_\mu\partial^\mu\pi^+ + \frac{g}{2}f_{\pi^-}W^-_\mu\partial^\mu\pi^- + \frac{g}{2}f_{\pi^0}W^0_\mu\partial^\mu\pi^0 + \frac{g'}{2}f_{\pi^0}B_\mu\partial^\mu\pi^0, \tag{3.10}$$

where the derivative couplings give us the factor of p^μ. The W's couple to the currents $j^{\pm,0}_\mu$ with strength $g/2$. The current $j^{\pm,0}$ couples to the $\pi^{\pm,0}$ with strength $f_{\pi^{\pm,0}}$ as in Eq.(3.9.1-3.9.3). Next let us consider the weak gauge boson propagator since one can find $W^\pm, Z$ mass as a position of the pole of the $W^\pm, Z$ propagator. Consider the vacuum polarization diagram contributing to the propagator given in Fig.3.1.

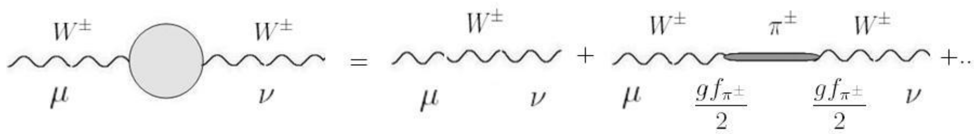

Fig.3.1: The charged gauge boson propagators.

Similar to QED, the propagator of gauge boson $W^{\pm}$ given by the geometric series can be determined as

$$\begin{aligned} D_{\mu\nu} &= \frac{-iP_{\mu\nu}}{p^2} + \frac{-iP_{\mu\nu}}{p^2}\left(gf_{\pi^\pm}/2\right)^2 \frac{-iP_{\mu\nu}}{p^2} + \ldots \\ &= \frac{-iP_{\mu\nu}}{p^2}\left[1 + \frac{\left(gf_{\pi^\pm}/2\right)^2}{p^2} + \frac{\left(gf_{\pi^\pm}/2\right)^4}{p^4}\ldots\right] \\ &= \frac{-iP_{\mu\nu}}{p^2}\left[1 - \frac{\left(gf_{\pi^\pm}/2\right)^2}{p^2}\right]^{-1} \\ &= \frac{-iP_{\mu\nu}}{p^2 - \left(\frac{gf_{\pi^\pm}}{2}\right)^2}, \end{aligned} \tag{3.11}$$

where $P_{\mu\nu} = g_{\mu\nu} - p_\mu p_\nu/p^2$ is the projection matrix in Feynman gauge. It is simple to see that the pole in the gauge boson propagator is shifted from zero to $p = gf_{\pi^\pm}/2$ which implies that the $W^{\pm}$ gauge boson has acquired mass:

$$M_{W^\pm} = \frac{gf_{\pi^\pm}}{2}. \tag{3.12}$$

However the neutral gauge boson propagator is slightly complicated since there is the mixing between the Z and γ. Consider the W^0 and B propagators including the mixing shown in Fig.3.2.

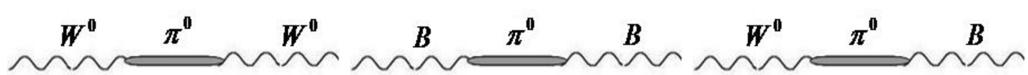

Fig.3.2: The neutral gauge boson propagators.

In this case, we can perform the calculation by introducing the following mass matrix **[7]** whose elements come from the diagrams in Fig.3.2:

$$\begin{pmatrix} W_\mu^0 & B_\mu \end{pmatrix} \underbrace{\begin{pmatrix} M^2_{W^0} & M^2_{W^0B} \\ M^2_{W^0B} & M^2_{BB} \end{pmatrix}}_{\text{mass matrix}} \begin{pmatrix} W^{\mu 0} \\ B^\mu \end{pmatrix} = \begin{pmatrix} W_\mu^0 & B_\mu \end{pmatrix} \underbrace{\frac{f^2_{\pi^0}}{4}\begin{pmatrix} g^2 & gg' \\ gg' & g'^2 \end{pmatrix}}_{\text{mass matrix}} \begin{pmatrix} W^{\mu 0} \\ B^\mu \end{pmatrix}. \tag{3.13}$$

Similary to the SM electroweak theory, we have to diagonalize this matrix by making a transformation into the physical fields (Z^μ, A^μ):

$$\begin{pmatrix} W^{\mu 0} \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}, \tag{3.14}$$

where θ_w is a mixing angle between Z and A. After diagonalizing such a matrix, the eigenvalues are

$$M_Z = \sqrt{g^2 + g'^2}\frac{f_{\pi^0}}{2}, \quad M_A = 0 \tag{3.15}$$

with corresponding eigenvectors,

$$Z^\mu = \frac{gW^{\mu 0} - g'B^\mu}{\sqrt{g^2 + g'^2}}, \tag{3.16.1}$$

$$A^\mu = \frac{g'W^{\mu 0} + gB^\mu}{\sqrt{g^2 + g'^2}}. \tag{3.16.2}$$

these eigenvectors are identified as weak neutral boson Z and usual photon A. Moreover, we see that the gauge boson mass ratio is in the form

$$\begin{aligned} \frac{M_{W^\pm}}{M_Z} &= \frac{g^2}{\sqrt{g^2 + g'^2}}\frac{f_{\pi^\pm}}{f_{\pi^0}} \\ &= \cos\theta_w \frac{f_{\pi^\pm}}{f_{\pi^0}} \\ &= \cos\theta_w. \end{aligned} \tag{3.17}$$

This shows that QCD induced *dynamical electroweak symmetry breaking* (DESW) produces the correct tree-level ratio of gauge boson mass $M_{W^\pm}/M_Z = \cos\theta_w$.

However, we can not recommend this as an accurate model of the real world. The reason is that the masses of the $W^\pm$ and Z obtained by substituting the known values of g, g' and f_π are in the tens of MeV, *viz*,

$$M_{W^\pm} = \frac{gf_\pi}{2} \simeq 29 \text{ MeV}, \tag{3.18}$$

whereas its measured value is much higher at $M_{W^\pm} \simeq 80.4$ GeV. As it is well known in SM, this f_π should be replaced by the VEV $v \sim 246$ GeV. If we can find some scheme for boosting f_π by a factor of 2,650, then the $W^\pm, Z$ masses would be shifted to the correct values. This leads to the empirical need of new scenario. Some of these model involve a new sector called Technicolor (TC) [**9**][**31**].

2. Technicolor

According to the electroweak theory, we know that one way for giving masses to the electroweak gauge bosons $W^\pm, Z$ is by spontaneous symmetry breaking through elementary scalar (Higgs) fields. Alternatively, we have just mentioned that we can achieve the same thing by the chiral symmetry breaking in QCD.

2.1. General Convention. To get acquainted with TC, let us suppose that an electroweak doublet of fermion fields engages in a new strong interaction called *technicolor*. We naively write this doublet as [**5**]

$$\mathrm{T} = \begin{pmatrix} \mathrm{U} \\ \mathrm{D} \end{pmatrix}, \tag{3.19}$$

where T_L is an $SU(2)_L$ doublet, while U_R and D_R are $SU(2)_L$ singlets. These fermions interact through the *techniforce*, and hence we call them *technifermions*. The interaction is almost like in QCD except that it is established on the scale of weak interactions. Here the (U, D) doublet is just like (u, d) except that color is replaced by the technicolor. TC provides a new strongly interacting gauge theory with new technifermions charged under the electroweak gauge group. We know that in fact the weak force carriers W, Z have non-zero masses, thus the electroweak symmetry of boson is spontaneously broken. This breakdown is due to the formation of a bilinear condensate of technifermions similar to the chiral condensate in ordinary QCD. Hence if the scale of symmetry breaking is the electroweak scale, the W and Z will gain the correct masses. To set up a TC model, we start defining an additional non-abelian strongly interaction gauge theory with an associated gauge group G_{TC}. By extending the SM gauge group, we have to include G_{TC} as follows:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow G_{\mathrm{TC}} \times SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{3.20}$$

Here G_{TC} is an arbitrary gauge group. It is conventional but not neccessary to identify the TC gauge group with unitary gauge group $SU(\mathrm{N})_{\mathrm{TC}}$. However, there are alternative gauge groups, e.g. $SO(\mathrm{N})_{\mathrm{TC}}$ and $SP(\mathrm{N})_{\mathrm{TC}}$, which can be viable candidates.

Now let us consider the single techni-family scenario based on the unitary gauge group:

$$SU(\mathrm{N})_{\mathrm{TC}} \times SU(3)_C \times SU(2)_L \times U(1)_Y \tag{3.20}$$

with the following field contents [**7**]

$$\begin{aligned} Q_L^{TC^\alpha} = \begin{pmatrix} U_L^{TC} \\ D_L^{TC} \end{pmatrix}^\alpha &= (N, 3, 2, +1/6)\,, \\ U_R^{TC^\alpha} &= (N, 3, 1, +2/3)\,, \\ D_R^{TC^\alpha} &= (N, 3, 1, -1/3)\,, \\ L_L^{TC^\alpha} = \begin{pmatrix} N_L^{TC} \\ E_L^{TC} \end{pmatrix}^\alpha &= (N, 1, 2, -1/2)\,, \\ N_R^{TC^\alpha} &= (N, 1, 1, +1)\,, \\ E_R^{TC^\alpha} &= (N, 1, 1, 0)\,. \end{aligned} \tag{3.21}$$

The first entry in each parenthesis denotes the dimension of the $SU(N)_{TC}$ representation, and the second and the third entries denote the $SU(3)$ and $SU(2)$ representation, respectively, while the last entry denotes the $U(1)$ hypercharge.

The technifermions which carry technicolor and QCD color are referred to as techniquarks, whereas the technifermions which carry only technicolor but not QCD color are called technileptons. However, the ordinary quarks and leptons are still technicolor singlets

$$\begin{aligned} Q_L^i = \begin{pmatrix} U_L^i \\ D_L^i \end{pmatrix}^i &= (1,3,2,+1/6)\,, \\ U_R^i &= (1,3,1,+2/3)\,, \\ D_R^i &= (1,3,1,-1/3)\,, \\ L_L^i = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}^i &= (1,1,2,-1/2)\,, \\ \nu_R^i &= (1,1,1,+1)\,, \\ e_R^i &= (1,1,1,0)\,, \end{aligned} \tag{3.22}$$

where $\alpha = 1,2,...,N$ is the $G_{\rm TC}$ index, and $i = 1,2,3$ labels the three families of quarks and leptons. As usual, the first entry stands for the $SU(N)_{\rm TC}$ group, the second and the third refer to $SU(3)_C$ and $SU(2)_L$, and the third is the hypercharge. The electroweak symmetry is broken by the condensates,

$$\langle \bar{U}_L^{TC} U_R^{TC} \rangle = \langle \bar{D}_L^{TC} D_R^{TC} \rangle = \langle \bar{N}_L^{TC} N_R^{TC} \rangle = \langle \bar{E}_L^{TC} E_R^{TC} \rangle \neq 0 \tag{3.23}$$

2.2. Minimal Version Of TC. Let us discuss further about the minimal version of TC model. It consists of a minimal set of technifermions:

$$\begin{aligned} \begin{pmatrix} p_L \\ q_L \end{pmatrix}^\alpha &= (N,1,2,0)\,, \\ p_R^\alpha &= (N,1,1,+1/2)\,, \\ q_R^\alpha &= (N,1,1,-1/2)\,, \end{aligned} \tag{3.24}$$

where $\alpha = 1,2,3$ labels TC. We assume here that each techniquark carries only TC but not ordinary color, and the left-handed techniquarks form an $SU(2)$ doublet just like ordinary quarks. By the definition of the electric charge operator $Q = T_3 + Y$, we will find that p and q have charges $+1/2$ and $-1/2$, respectively. Due to the requirement of anomaly freedom, the electric charges sum to zero.

Concretely, let us address the comparison of minimal TC and QCD with one quark doublet shown in Table.3.1.

QDC	TC
$SU(3)_C$	$SU(N)_{TC}$
Quarks	Techniquarks
$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} p \\ q \end{pmatrix}$
$\langle \bar{u}u + \bar{d}d \rangle \sim \Lambda_{QCD}^3$	$\langle \bar{p}p + \bar{q}q \rangle \sim \Lambda_{TC}^3$
$SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$	$SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$
3 QCD Pions	3 Technipions
$f_{\pi^\pm} = f_{\pi^0} \simeq 93$ MeV	$F_{\pi^\pm} = F_{\pi^0} \simeq 246$ GeV
$\Lambda_{QCD} \simeq 200$ MeV	$\Lambda_{TC} \simeq 500$ GeV

From the above table, there is a doublet of massless quarks and a doublet of massless techniquarks. The three pions and three technipions are massless. When $SU(2)_L \otimes U(1)_Y$ interactions are turned on, then three of the pions will be eaten by the Higgs mechanism and the other three pions will remain in the spectrum as physical states. The eaten three pions are mainly referred to technipions, while the three physical pions are mainly QCD pions. Next let us assume that quarks as well as technifermions interacts with the gauge bosons of $SU(2)_L \otimes U(1)_Y$. The pion states will result from a linear combination as follows [**5**]:

$$|\text{eaten pion}\rangle = \frac{F_\pi}{\sqrt{F_\pi^2 + f_\pi^2}} |\text{technipion}\rangle + \frac{f_\pi}{\sqrt{F_\pi^2 + f_\pi^2}} |\text{QCD pion}\rangle \,, \tag{3.25.1}$$

that will become the longitudinal gauge boson components, while

$$|\text{physical pion}\rangle = \frac{F_\pi}{\sqrt{F_\pi^2 + f_\pi^2}} |\text{QCD pion}\rangle - \frac{f_\pi}{\sqrt{F_\pi^2 + f_\pi^2}} |\text{technipion}\rangle \tag{3.25.2}$$

becomes the orthogonal combination and remains massless. Since $F_\pi \sim 246$ GeV $>> f_\pi \sim 93$ MeV, the physical pion is mostly a QCD pion, whereas the eaten pion is mostly a technipion. To see how these linear combinations signify, we then consider as usual

$$\langle 0 | \, j_{5a}^\mu \, |\text{QCD pion}\rangle = \langle 0 | \, j_{5a}^\mu \left| \pi_b^{\text{QCD}}(p) \right\rangle = -i f_\pi p^\mu \delta_{ab}, \tag{3.26}$$

and

$$\langle 0 | \, j_{5a}^\mu \, |\text{technipion}\rangle = \langle 0 | \, j_{5a}^\mu \left| \pi_b^{\text{technipion}}(p) \right\rangle = -i F_\pi p^\mu \delta_{ab}, \tag{3.27}$$

where j_{5a}^μ is the axial isospin currents. By substituting the linear combinations given above, we will obtain respectively

$$\langle 0 | \, j_{5a}^\mu \left| \pi_b^{\text{eaten pion}}(p) \right\rangle = -i \sqrt{F_\pi^2 + f_\pi^2} p^\mu \delta_{ab}, \tag{3.28}$$

and

$$\langle 0 | \, j_{5a}^\mu \left| \pi_b^{\text{physical pion}}(p) \right\rangle = 0. \tag{3.29}$$

As it is shown in Eq.(3.3), the weak gauge bosons couple to the pions through the axial isospin current. So the physical pion has no coupling, while the orthogonal combination is completly absorbed.

2.3. Weak Gauge Boson Masses. Now we are going to recover the correct mass of weak gauge bosons by considering a particularly simple [**10**] TC model. With this model, we take the gauge group as $SU(N)_{\text{TC}} \times SU(2)_L \times U(1)_Y$, where we choose here the technicolor group as $SU(N)_{\text{TC}}$. Now we assign the technifermions to be in the fundamental representions of the electroweak gauge group $SU(2)_L \times U(1)_Y$. For more simplicity, we shall restrict ourselves to work with one flavor doublet, and assign the techniquarks to be in the fundamental representation for both $SU(3)_C$ and $SU(N)_{\rm TC}$ groups,

$$\begin{pmatrix} U \\ D \end{pmatrix}_L^\alpha = (N, 2, +1/6)\,, \quad U_R^\alpha = (N, 1, +2/3)\,, \quad D_R^\alpha = (N, 1, -1/3)\,, \tag{3.30}$$

where $\alpha = 1, ..., N$ is the TC index. As usual, the first entry stands for the $SU(N)_{\rm TC}$ group, the second refers to $SU(2)_L$, and the third is the hypercharge. Let us start writing the Lagrangian for <u>two-flavor</u> massless techniquarks $q_f^\alpha = (U, D)^\alpha$ with the electroweak interaction switched off,

$$\mathcal{L}_{TC} = -\frac{1}{4}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \sum_{f,\acute{f}} \bar{q}_f \left(i\gamma^\mu D_\mu\right)_{f\acute{f}} q_{\acute{f}}, \tag{3.31}$$

where $F_{\mu\nu} = F^a_{\mu\nu}T^a$ is the field strength tensor of $SU(N)_{\rm TC}$, T^a are the group generators, $D_\mu = \partial_\mu - ig_{\rm TC}A^a_\mu T^a$ is the covariant derivative, and $g_{\rm TC}$ is the TC gauge coupling. Similary to QCD, this Lagrangian has a global symmetry [**10**]

$$U(2)_L \times U(2)_R \sim SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A. \tag{3.32}$$

The transformations of these various groups are in the form, respectively,

$$\begin{aligned} U(2)_L : q_f &\to \left\{\exp\left[-iP_L\left(\frac{\theta\cdot\tau}{2}\right)\right]\right\}_{f\acute{f}} q_{\acute{f}}, \\ U(2)_R : q_f &\to \left\{\exp\left[-iP_R\left(\frac{\tilde{\theta}\cdot\tau}{2}\right)\right]\right\}_{f\acute{f}} q_{\acute{f}}, \\ U(1)_V : q_f &\to \exp\left(-i\alpha\right) q_f \\ U(1)_A : q_f &\to \exp\left(-i\tilde{\alpha}\gamma_5\right) q_f. \end{aligned} \tag{3.33}$$

Due to an anomaly, the axial current $j^{\mu 5} = \sum_f \bar{q}_f\gamma_\mu\gamma_5 q_f$ is not conserved. To verify that, we now consider the coupling of the quark (axial vector) currents to the gluon fields of QCD. Here we expect that an axial vector current will receive an anomalous contribution from the diagrams [**2**] shown in Fig. 3.3.

Since we are working for non-abelian gauge fields, so the anomaly should be generalized from the abelian case. After obtaining such group factors from Fig 3.3, for axial isospin current $j^{\mu 5a} = \bar{q}_f\gamma_\mu\gamma_5\tau^a q_f$, we have [**2**]

$$\begin{aligned} \partial_\mu j^{\mu 5a} &= -\frac{g^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}\mathcal{F}^b_{\alpha\beta}\mathcal{F}^c_{\mu\nu}\cdot\mathrm{Tr}\left[\tau^a t^b t^c\right] \\ &= -\frac{g^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}\mathcal{F}^b_{\alpha\beta}\mathcal{F}^c_{\mu\nu}\cdot\mathrm{Tr}\left[\tau^a\right]\mathrm{Tr}\left[t^b t^c\right], \end{aligned} \tag{3.34}$$

where $\mathcal{F}^c_{\mu\nu}$ is a gluon field strength tensor, $\epsilon^{\alpha\beta\mu\nu}$ is an anti-symmetric tensor, τ^a is an isospin matrix, while t^b is a color matrix, and the trace is performed over colors and flavors. Here

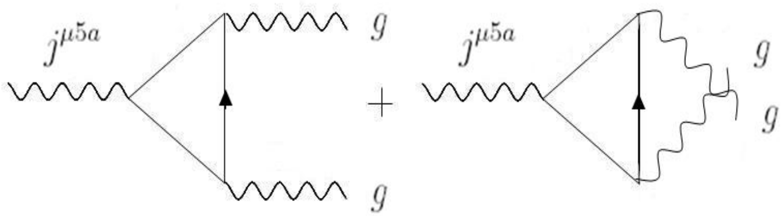

Fig.3.3: Diagrams that lead to an axial vector anomaly for a chiral current in QCD.

we find $\mathrm{Tr}\,[\tau^a]\left[t^b t^c\right] = 0$ since the trace of τ^a vanishes. Therefore the axial isospin currents are said to be an anomaly free.

However, in the case of the axial current, $j^{\mu 5} = \bar{q}_f \gamma_\mu \gamma_5 q_f$, the matrix τ^a will be replaced by the unity matrix, and we obtain

$$\partial_\mu j^{\mu 5} = -\frac{g^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}\mathcal{F}^b_{\alpha\beta}\mathcal{F}^c_{\mu\nu}. \tag{3.35}$$

Thus this expression tells us that the axial current is not conserved. The resulting chiral symmetry is $SU(2)_L \times SU(2)_R \times U(1)_V$. Now the situation is exactly parallel to the chiral limit of ordinary QCD. Thus the TC vacuum has only $SU(2)_{\text{isospin}}$ and baryon number conservation $U(1)_V$. Hence this concerns with the condensate

$$\langle \bar{U}U \rangle = \langle \bar{D}D \rangle \neq 0, \tag{3.36}$$

which breaks the resulting symmetry down to

$$SU(2)_L \times SU(2)_R \times U(1)_V \rightarrow SU(2)_{L+R} \times U(1)_V, \tag{3.37}$$

dynamically. There are three broken generators assigned to be technipions π_{TC}. Let us start considering the vacuum polarization given in Fig.3.4.

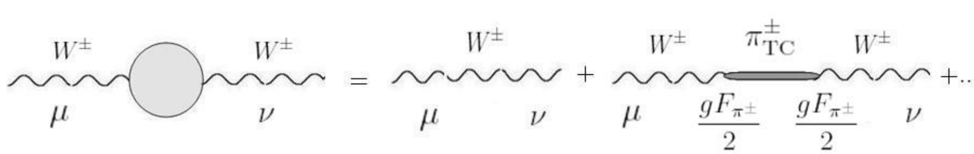

Fig.3.4: The charged gauge boson propagators.

The massless technipion couples to the axial isospin currents with the strength F_π defined in Eq.(3.27). In this case, we have technipions instead of QCD pions, and we can replace the pion decay constant f_π by the technipion decay constant F_π given in Eq.(3.27).

The propagator of gauge boson $W^{\pm}$ given by the geometric series can be determined as

$$\begin{aligned} D_{\mu\nu} &= \frac{-iP_{\mu\nu}}{p^2} + \frac{-iP_{\mu\nu}}{p^2}\left(gF_{\pi^\pm}/2\right)^2\frac{-iP_{\mu\nu}}{p^2} + ... \\ &= \frac{-iP_{\mu\nu}}{p^2}\left[1 + \frac{\left(gF_{\pi^\pm}/2\right)^2}{p^2} + \frac{\left(gF_{\pi^\pm}/2\right)^4}{p^4}...\right] \\ &= \frac{-iP_{\mu\nu}}{p^2}\left[1 - \frac{\left(gF_{\pi^\pm}/2\right)^2}{p^2}\right]^{-1} \\ &= \frac{-iP_{\mu\nu}}{p^2 - \left(\frac{gF_{\pi^\pm}}{2}\right)^2}. \end{aligned} \tag{3.38}$$

As one can see, the pole in the gauge boson propagator has shifted from zero to $p = gF_{\pi^\pm}/2$ which implies that the $W^{\pm}$ gauge boson has acquired a mass:

$$M_{W^\pm} = \frac{gF_\pi}{2}. \tag{3.39}$$

However there is a mixing between the Z and γ. We can determine the expression for the mass of Z by introducing a mass matrix similar to that of Eq.(3.13). After diagonalizing such a matrix, we obtain the following eigenvalues:

$$M_Z = \sqrt{g^2 + \acute{g}^2}\frac{F_\pi}{2}, \quad M_A = 0. \tag{3.40}$$

Here we see that these results fix the technipion decay constant as

$$F_\pi = \frac{2M_W}{g} = \left(\sqrt{2}G_F\right)^{-1/2} \simeq 246 \text{ GeV}, \tag{3.41}$$

which leads to the correct mass of weak gauge bosons $M_W \simeq 80.4$ GeV and $M_Z \simeq 91.2$ GeV.

However TC itself is not enough to provide the SM fermion masses. To generate the masses of the fundamental fermions, one needs to work further by embedding the TC gauge group into a larger gauge group known as Extended Technicolor (ETC) [**11**][**32**][**33**].

2.4. Minimal Walking Technicolor: A Brief Review. According to the theory of renormalization group [**2**], one suggests that the parameters, e.g. coupling constant, of a renormalizable field theory can be assigned to be scale-dependent quantities. The dependence of a coupling $\alpha(\mu)$ on the momentum (energy) scale is known as running of the coupling, e.g. the coupling constant of QCD-like theory is given in the left panel of Fig.(3.5).

However, it is possible for theories in which the technicolor gauge coupling as a function of the renormalization scale *walk* rather than *run* illustrated in the right panel of Fig 3.5. Besides, our situation is rather critical due to the fact that simple technicolor models are severely constrained by the limits on *flavor changing neutral currents* (FCNCs) and by the emergence of unwanted additional pseudo-Goldstone bosons.

Fortunately, these problems can be suppressed by introduing *walking technicolor theories* [**34**][**35**][**36**] in which the technicolor gauge coupling evolves slowly because of a near-conformal behavior illustated in Fig 3.6.

To achieve conformal behavior with much smaller particle content, higher matter field representations are impressively proposed [**14**][**37**][**38**]. According to [**15**][**39**], the phase diagrams for higher representations concerning walking technicolor model building were constructed and sensational candidates for minimal models of walking technicolor were also implemented.

With just two techniflavors in the adjoint representation of the $SU(2)_{\mathrm{TC}}$ gauge group, the theory is already close to, or even within, the conformal window. In particular, the theory with two-color and two-flavor is called the *Minimal Walking Technicolor* (MWT).

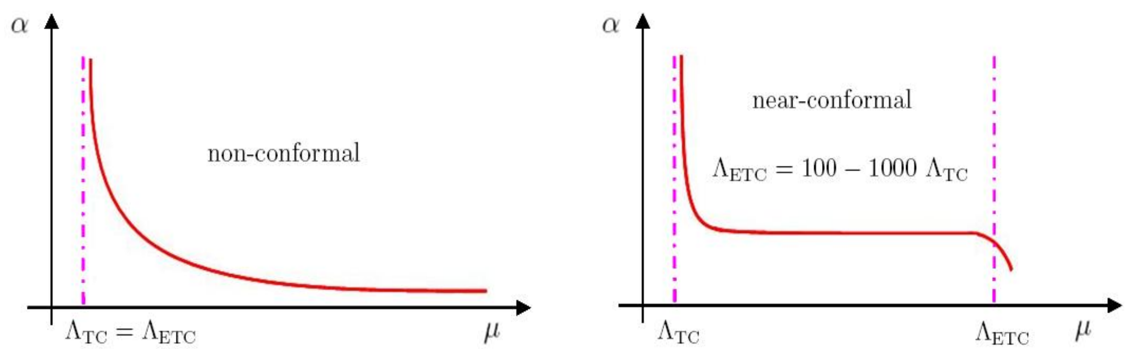

Fig.3.5: Left panel: the QCD-like behavior of the coupling constant α as a function of momentum μ (running). Right panel: Walking-like behavior of the coupling constant as a function of momentum (walking)

In Fig.3.6, a cartoon shows a β function associated to walking theory with a negative first coefficient that starts out in the UV region similary to QCD, due to the asymptotic freedom. However, contrary to QCD, the second coefficient is positive and large enough to turn the curve around at larger α. If the coupling is sufficiently large, the β function would hit the IR-fixed point α_*. However, before this happens, the coupling would get significantly large around α_c so that chiral symmetry is trigged.

Concretely let us review an $SU(2)_{\mathrm{TC}}$ gauge theory with two flavors belonging to an adjoint Dirac technifermions [**15**]. The two adjoint fermions may be written as follows:

$$Q_L^a = \begin{pmatrix} U^a \\ D^a \end{pmatrix}_L, \quad U_R^a,\ D_R^a, \quad a = 1, 2, 3, \tag{3.42}$$

where a is the adjoint technicolor index of $SU(2)_{\mathrm{TC}}$. U_L and D_L are respectively the left-handed techniup and left-handed technidown, while U_R and D_R are the corresponding right-handed particles. The left-handed are rearranged in three doublets of $SU(2)_L$ weak interactions in the standard fashion. The following condensate $\langle \bar{U}U + \bar{D}D \rangle$ breaks the electroweak symmetry.

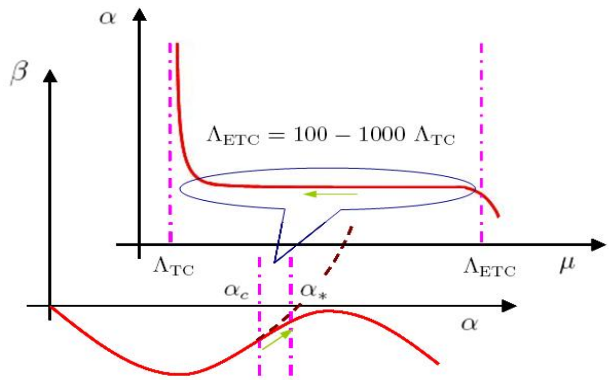

Fig.3.6: Cartoon of the beta function associated to a generic walking theory.

With all above field contents, the theory which has an odd number of the left-handed fermion (or EW) doublets (and no other representations) is mathematically inconsistent featuring a so-called Witten anomaly [**10**].

However, this can be simply solved by adding a new weakly charged fermion doublet which transforms as a technicolor singlet [**15**]:

$$L_L = \begin{pmatrix} N \\ E \end{pmatrix}_L, \quad N_R,\ E_R. \tag{3.43}$$

The gauge anomalies can be completely suppressed by the following hypercharge assignments:

$$\begin{aligned} Y\left(Q_L\right) &= \frac{y}{2}, \qquad & Y\left(U_R, D_R\right) &= \left(\frac{y+1}{2}, \frac{y-1}{2}\right), \\ Y\left(L_L\right) &= -3\frac{y}{2}, \qquad & Y\left(N_R, E_R\right) &= \left(\frac{-3y+1}{2}, \frac{-3y-1}{2}\right), \end{aligned} \tag{3.44}$$

where y can take arbitrary real value. In this convention, the electric charge operator is $Q = T_3 + Y$, where T_3 is weak isospin generator. Hence the choice $y = 1/3$ corresponds to the SM hypercharge assignment.

CHAPTER 4

Introducing Supersymmetry

All observable phenomena at low energy (weak) scale, $\Lambda_{\text{weak}} \sim 100$ GeV, can be impressively described by the Standard Model. However, up to now, there are some phenomena, e.g. the dark matter represented by a new stable weak-scale particle with weak coupling and the acceleration of the universe, which can not yet be described by such a model. Moreover, there is a problem in the scalar sector with the quantum (loop) corrections. To see this concretely, let us now begin by considering a theory with a single fermion, ψ, coupled to a massive Higgs scalar **[29]**,

$$\mathcal{L}_\varphi = \bar{\psi}\left(i\gamma^\mu \partial_\mu\right)\psi + \left|\partial_\mu \varphi\right|^2 - m_S^2 \left|\varphi\right|^2 - \left(\frac{\lambda_F}{\sqrt{2}}\bar{\psi}\psi\varphi + h.c.\right). \tag{4.1}$$

We will take $\varphi = (h+v)/\sqrt{2}$, with h the physical Higgs boson leading to spontaneous symmetry breaking. After spontaneous symmetry breaking, the fermion acquires a mass, $m_F = \lambda_F v/\sqrt{2}$.

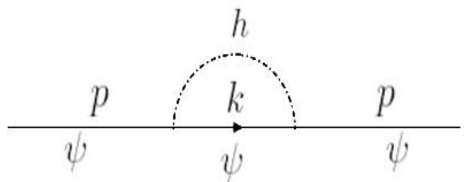

Fig.4.1: Fermion mass renormalization from a Higgs loop.

First of all, let us consider the fermion self-energy contributing from the scalar loop diagram illustrated in Fig. 4.1. From such a diagram, the propagator is given by

$$\text{Propagator} = i\frac{(\not{p}+m_F)}{p^2-m_F^2}\left[-i\Sigma_F(p)\right] i\frac{(\not{p}+m_F)}{p^2-m_F^2}, \tag{4.2}$$

where

$$\begin{aligned}-i\Sigma_F(p) &= \left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4}\left[\frac{i\left(\not{k}+m_F\right)}{[k^2-m_F^2]}\frac{i}{[(p-k)^2-m_S^2]}\right] \\ &= \frac{\lambda_F^2}{2}\int \frac{d^4k}{(2\pi)^4}\left[\frac{(\not{k}+m_F)}{[k^2-m_F^2]\,[(p-k)^2-m_S^2]}\right].\end{aligned} \tag{4.3}$$

Introducing a Feynmann parameter to combine the two denominators, we have

$$\frac{1}{\left[k^2-m_F^2\right]\left[(p-k)^2-m_S^2\right]}=\int_0^1 dx\frac{1}{\left[k^2-2x\,(p\cdot k)+xp^2-xm_S^2-(1-x)m_F^2\right]^2}. \tag{4.4}$$

Defining a shifted momentum $q\equiv k-xp$, we have

$$-i\Sigma_F(p)=\frac{\lambda_F^2}{2}\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{(\not{q}+x\not{p}+m_F)}{\left[q^2-xm_S^2-(1-x)m_F^2\right]^2}. \tag{4.5}$$

Dropping the linear term in q from the numerator, we obtain

$$-i\Sigma_F(p)=\frac{\lambda_F^2}{2}\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{(x\not{p}+m_F)}{\left[q^2-xm_S^2-(1-x)m_F^2\right]^2}. \tag{4.6}$$

The renormalized fermion mass is given by

$$m_F^r=m_F^{bare}+\delta m_F, \tag{4.7}$$

where

$$\begin{aligned}\delta m_F&=\Sigma_F(p)|_{\not{p}=m_F}\\&=i\frac{\lambda_F^2}{32\pi^4}\int\frac{d^4q}{(2\pi)^4}\int_0^1 dx\frac{m_F\,(x+1)}{\left[q^2-xm_S^2-(1-x)m_F^2\right]^2}.\end{aligned} \tag{4.8}$$

This integral can be simply determined by making the following transformation into Euclidean spcace

$$q_0=iq_E,\qquad d^4q=id^4q_E,\qquad q^2=-q_E^2. \tag{4.9}$$

Since the expression (4.8) depends only on q_E^2, we can determine such an expression by using the following expression:

$$\int d^4q_E f\left(q_E^2\right)=\pi^2\int_0^{\Lambda^2} ydyf\left(y\right), \tag{4.10}$$

where Λ is a high (UV) cut-off of order a GUT scale or even the Planck one.

So, the expression (4.8) becomes **[29]**

$$\begin{aligned}\delta m_F&=-\frac{\lambda_F^2 m_F}{32\pi^2}\int_0^1 dx(1+x)\int_0^{\Lambda_{\mathrm{UV}}^2}\frac{ydy}{\left[y+xm_S^2+(1-x)m_F^2\right]^2}\\&=-3\frac{\lambda_F^2 m_F}{64\pi^2}\log\left(\frac{\Lambda_{\mathrm{UV}}^2}{m_F^2}\right)+\text{(regular terms)},\end{aligned} \tag{4.11}$$

where the regular terms argue that such terms are cut-off independent or vanish when $\Lambda_{\mathrm{UV}}\to\infty$.

However, the situation is very interesting when we consider the renormalization of a Higgs boson mass from a fermion loop illustrated in Fig.4.2. Using the same Lagrangian

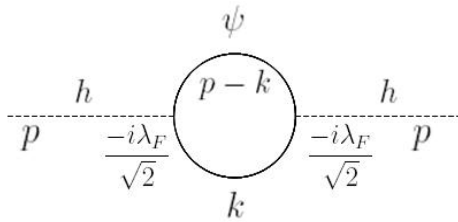

Fig.4.2: Higgs mass renormalization from a fermion loop.

given in Eq.(4.1), we have [**29**]

$$-i\Sigma_S(p) = (-1)\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\left(\frac{-i\lambda_F}{\sqrt{2}}\right)\frac{i}{(\not{k}-m_F)}\left(\frac{-i\lambda_F}{\sqrt{2}}\right)\frac{i}{((\not{p}-\not{k})-m_F)}\right]$$
$$= (-1)\left(\frac{-i\lambda_F}{\sqrt{2}}\right)^2 (i)^2\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\frac{(k+m_F)\left((\not{p}-\not{k})+m_F\right)}{(k^2-m_F^2)\left((p-k)^2-m_F^2\right)}\right]. \tag{4.12}$$

Integrating with a momentum space cut-off Λ as above, we obtain the contribution to the Higgs mass, $\delta M_h^2 \equiv \Sigma_S(m_s)$ [**29**],

$$\left(\delta M_h^2\right)_a = -\frac{\lambda_F^2}{8\pi^2}\left[\Lambda_{\mathrm{UV}}^2 + \left(m_S^2 - 6m_F^2\right)\log\left(\frac{\Lambda}{m_F}\right) + \left(2m_F^2 - \frac{m_S^2}{2}\right)(1+I_1(a))\right]$$
$$+ O\left(\frac{1}{\Lambda_{\mathrm{UV}}^2}\right), \tag{4.13}$$

where $I_1(a) \equiv \int_0^1 dx \log(1-ax(1-x))$, with $a = m_S^2/m_F^2$.

We see that the Higgs boson mass diverges quadratically. The correction is not proportional to the Higgs mass. Unfortunately, there is nothing that protects the Higgs mass from the large corrections.

Now we can write the renormalized (physical) Higgs mass, $M_{h,\mathrm{r}}$, as

$$M_{h,\mathrm{r}}^2 = M_{h,\mathrm{bare}}^2 + \delta M_h^2 + \text{counterterms}, \tag{4.14}$$

where the counterterms must be introduced to the theory in order to cancel the quadratically divergent contributions to δM_h^2. The adjustment must be made at each order in perturbation theory. This is so-called the **hierarchy problem**.

Fortunately, the quadratic divergence can be suppressed if one introduces additional two complex scalar fields, φ_1, φ_2, interacting with the physical (SM) Higgs boson, h. So, our Lagrangian can be written as [**29**]

$$\mathcal{L} = |\partial_\mu\varphi_1|^2 + |\partial_\mu\varphi_2|^2 - m_{S_1}^2|\varphi_1|^2 - m_{S_2}^2|\varphi_2|^2 + \lambda_S|\varphi|^2\left(|\varphi_1|^2+|\varphi_2|^2\right) + \mathscr{L}_\varphi, \tag{4.15}$$

where $\mathscr{L}_\varphi$ is given in Eq.(4.1). Here we consider the contribution to the Higgs mass [**29**] illustrated in Fig.4.3:

$$\begin{aligned}\left(\delta M_h^2\right)_b &= -\lambda_S \int \frac{d^4k}{(2\pi)^4}\left[\frac{i}{k^2-m_{S_1}^2}+\frac{i}{k^2-m_{S_2}^2}\right]\\ &= \frac{\lambda_S}{16\pi^2}\left[2\Lambda_{\rm UV}^2 - 2m_{S_1}^2\log\left(\frac{\Lambda_{\rm UV}}{m_{S_1}}\right) - 2m_{S_2}^2\log\left(\frac{\Lambda_{\rm UV}}{m_{S_2}}\right)\right]\\ &+ O\left(\frac{1}{\Lambda_{\rm UV}^2}\right). \end{aligned} \tag{4.16}$$

Comparing Eq.(4.13) with Eq.(4.16), we find that if $\lambda_S = \lambda_F^2$, the quadratic divergences coming from these two terms cancel each other. Note here that the cancellation appears to be independent of the masses, M_F and m_{S_i}, as well as of the magnitude of the couplings, λ_S and λ_F.

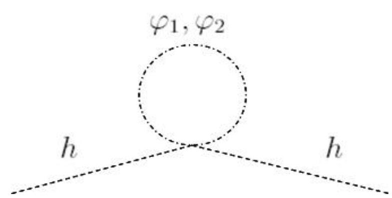

Fig.4.3: Higgs mass renormalization from scalar loops.

The attempts for solving such a problem lead to construct theories beyond the Standard Model such as Technicolor and Supersymmetry [**13**].

1. The Construction Of Supersymmetric Field Theories

To naturally maintain our model, the Higgs mass term $\mu^2 |\varphi|^2$ has to be generated through a well-defined mechanism. A prerequisite for this is that the term μ^2 should not receive the additive radiative corrections. Hence this requires that the emergence of a non-zero μ^2 in the Lagrangian should be forbidden by a symmetry of the theory. Therefore, alternatively, we can postulate a symmetry that connects φ to a fermion field ψ [**17**]:

$$\delta\varphi = \epsilon\cdot\psi. \tag{4.17}$$

The expression looks quite strange, but it leads to the construction of **supersymmetry** (SUSY). Roughly speaking, supersymmetry is a symmetry between bosonic degrees of freedom and fermionic degrees of freedom. Supersymmetry requires every type of particle to have an associated superpartner. For example, the superpartner of an **electron** (a fermion) is so-called a **selectron** (a boson). A symmetry that links a boson with a fermion is generated by a conserved charge operator Q_α so-called **supercharge** that carries spin 1/2 in such a way that

$$[Q_\alpha, \varphi] = \psi, \qquad [Q_\alpha, H] = 0, \tag{4.18}$$

with H being the Hamiltonain of the system.

In this chapter, we will review the supersymmetric field theory. The notations and conventions we used are almost based on the Wess and Bagger's book [**13**].

1.1. SUSY Algebra. Roughly speaking, SUSY is an extension of the Poincaré symmetries of space-time that relates bosons to fermions, i.e. Eq.(4.4). In doing so, it is consistent to express the supersymmetrically extended Poincaré algebra in terms of two -component Weyl spinor Q_α and its conjugate $\bar{Q}_{\dot{\alpha}}$. Since these generators are fermionic, their algebra can be written in terms of the anti-commutators such that

$$\left\{ {Q_\alpha}^A, {Q_\beta}^B \right\} = \left\{ \bar{Q}_{\dot{\alpha}A}, \bar{Q}_{\dot{\beta}B} \right\} = 0, \tag{4.19.1}$$

$$\left\{ {Q_\alpha}^A, \bar{Q}_{\dot{\beta}B} \right\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^A{}_B, \tag{4.19.2}$$

where P_μ is the usual momentum operator. Here the indices α, β of Q and $\dot{\alpha}, \dot{\beta}$ of $\overline{Q}$ run from 1 to 2 and denote two component Weyl spinors, $\sigma^\mu = (1, \sigma^i)$ with σ^i being the ordinary 2×2 Pauli matrices, the capital letters, A, B, refer to the internal space which run from 1 to the integer N, and μ identifies Lorentz four vectors.

To achieve the SUSY transformations, we have to extend our formalism from Minkowski space to **superspace**. The Elements of superspace are called **supercoordinates** which are composed of the usual four Minkowski space-time coordinates and four anticommuting Grassmann variables. In terms of two-component Weyl spinors, these variables are

$$\{\theta_\alpha\}, \quad \alpha = 1, 2 \quad \text{and} \quad \left\{\bar{\theta}_{\dot{\alpha}}\right\}, \quad \dot{\alpha} = 1, 2, \tag{4.20}$$

which transform under the self-representation of SL(2,C) and the complex conjugate self-representation of SL(2,C), respectively, and they are assigned to be independent. For these anti-commuting Grassmann variables, we commonly see that

$$\{\theta_\alpha, \theta_\beta\} = \left\{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\right\} = \left\{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\right\} = 0. \tag{4.21}$$

A finite SUSY transformation can be written in the form

$$\exp\left[i \left\{ \theta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} - x^\mu P_\mu \right\} \right]. \tag{4.22}$$

Hence this can be compared with a non-abelian gauge transformation with $\exp\left[i\theta_a T^a\right]$ being the gauge generators. Note that the SUSY transformations will act on the objects understood to be functions of both θ and $\bar{\theta}$ as well as the space-time coordinates x^μ. These objects are called **superfields**.

Furthermore, it is very useful to consider the infinitesimal SUSY transformations. These can be generally written as

$$\begin{aligned} \delta_S\left(\xi, \bar{\xi}\right) \Phi\left(x, \theta, \bar{\theta}\right) &= \left(\xi^\alpha Q_\alpha + \bar{\xi}_{\dot{\alpha}} Q^{\dot{\alpha}}\right) \Phi\left(x, \theta, \bar{\theta}\right) \\ &= \left[\xi \frac{\partial}{\partial \theta} + \bar{\xi} \frac{\partial}{\partial \bar{\theta}} - i \left(\xi \sigma^\mu \bar{\theta} - \theta \sigma^\mu \bar{\xi} \right) \frac{\partial}{\partial x^\mu} \right] \Phi\left(x, \theta, \bar{\theta}\right), \end{aligned} \tag{4.23}$$

where Φ is an arbitrary superfield and $\xi, \bar{\xi}$ are also Grassmann variables. This corresponds to the following explicit representation of the SUSY generators:

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot{\beta}} \bar{\theta}^{\dot{\beta}} \partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i\theta^\beta \sigma^\mu_{\beta\dot{\alpha}} \partial_\mu, \tag{4.24}$$

where, for later convenience, from now on we write ∂_μ instead of $\partial/\partial x^\mu$. With this notaion, the SUSY-covariant derivatives which anti-commute with the infinitesimal SUSY transformation in Eq.(4.23) are of the form

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu, \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu. \tag{4.25}$$

Note that all expressions given above imply that θ and $\bar{\theta}$ have mass dimension $-1/2$, while D and Q have mass dimension $+1/2$. Alternatively, we have two more slightly different (irreducible) representations often referred to as **chiral representations**. With the chiral representations, we have

L-representation:

$$\delta_S\left(\xi,\bar{\xi}\right)\Phi_L = \left[\xi\frac{\partial}{\partial\theta} + \bar{\xi}\frac{\partial}{\partial\bar{\theta}} + 2i\theta\sigma^\mu\bar{\xi}\partial_\mu\right]\Phi_L, \tag{4.26.1}$$

$$D_L = \frac{\partial}{\partial\theta} + 2i\sigma^\mu\bar{\theta}\partial_\mu; \quad \bar{D}_L = -\frac{\partial}{\partial\bar{\theta}}, \tag{4.26.2}$$

R-representation:

$$\delta_S\left(\xi,\bar{\xi}\right)\Phi_R = \left[\xi\frac{\partial}{\partial\theta} + \bar{\xi}\frac{\partial}{\partial\bar{\theta}} - 2i\xi\sigma^\mu\bar{\theta}\partial_\mu\right]\Phi_R, \tag{4.27.1}$$

$$\bar{D}_R = -\frac{\partial}{\partial\bar{\theta}} - 2i\theta\sigma^\mu\partial_\mu; \quad D_R = \frac{\partial}{\partial\theta}. \tag{4.27.2}$$

Here there are three different operators which lead to three different types of superfields Φ, Φ_L and Φ_R. Hence we can prove that they are related by the following identity

$$\Phi\left(x^\mu,\theta,\bar{\theta}\right) = \Phi_L\left(x^\mu + i\theta\sigma^\mu\bar{\theta},\theta,\bar{\theta}\right) = \Phi_R\left(x^\mu - i\theta\sigma^\mu\bar{\theta},\theta,\bar{\theta}\right). \tag{4.28}$$

An arbitrary superfield Φ is in general a **reducible representation**. However, we will only need these two special superfields which are in an **irreducible representation** of the SUSY algebra by imposing supersymmetric conditions on the superfield.

1.2. Chiral Superfield. The first type of superfield we have to work with is a **chiral superfields**. This superfield is named from the fact that the SM fermions are chiral which transforms differently under $SU(2) \times U(1)_Y$. Hence we need this superfield with only two physical fermionic degrees of freedom which can precisely describe the left- or right-handed component of SM fermions. In addition, this superfield itself will also contain bosonic partners, the sfermions.

The chiral superfield, Φ, is obtained by imposing either

$$\bar{D}\Phi_L = 0 \quad (\Phi_L \text{ is left-chiral}) \quad \text{or} \tag{4.29.1}$$

$$D\Phi_R = 0 \quad (\Phi_R \text{ is right-chiral})\,. \tag{4.29.2}$$

Clearly, these constraints show us that in the L-representation Φ_L is independent of $\bar{\theta}$ but depends on both x^μ and θ. Since θ is an anti-commuting Grassmann variable, we can then expand Φ_L in which its expansion is finite as follow:

$$\begin{aligned}\Phi_L(y,\theta) &= \phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^\alpha\theta^\beta\epsilon_{\alpha\beta}F(y)\\ &= \phi(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\phi(x)\\ &+ \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta F(x),\end{aligned} \tag{4.30}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$, summation over upper and lower indices is understood, and $\epsilon_{\alpha\beta}$ is the anti-symmetric tensor in two dimensions. Since θ has the mass dimension $-1/2$, we now assign the usual mass dimension $+1$ to the scalar field ϕ which gives the mass dimension $+3/2$ for the fermionic field ψ, and the unusual mass dimension $+2$ for the (auxiliary) scalar field F. Thus the superfield Φ itself has mass dimension $+1$. The expansion of Φ_L is exactly finite since θ has only two components. The F term in Eq.(4.25.1) has been taken into account for the off-shell degrees of freedom of the chiral multiplets.

We see in Eq.(4.21) that the square of any one component of θ vanishes. Hence the expansion of Φ_L can not have any terms with three or more factors of θ.

In addition, the field ϕ and F are in general complex scalars, while ψ is a two-component Weyl spinor. From the above information, we see that Φ_L contains four fermionic and bosonic degrees of freedom: just the off-shell degrees of freedom. Likely the expression for Φ_R in R-representation is very similar to the case of Φ_L in L-representation obtained by merely replacing θ by $\bar{\theta}$, i.e.,

$$\begin{aligned}\Phi_R(z,\theta) &= \phi^*(z) + \sqrt{2}\bar{\theta}\bar{\psi}(z) + \bar{\theta}\bar{\theta}F^*(z)\\ &= \phi^*(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi^*(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\phi^*(x)\\ &+ \sqrt{2}\bar{\theta}\bar{\psi}(x) + \frac{i}{\sqrt{2}}\bar{\theta}\theta\theta\sigma^\mu\partial_\mu\bar{\psi}(x) + \bar{\theta}\bar{\theta}F(x)^*,\end{aligned} \tag{4.31}$$

where $z^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$. Applying the infinitesimal SUSY transformation in Eq.(4.26.1) to the left-chiral superfield in Eq.(4.30), we have

$$\begin{aligned}\delta_S\Phi_L &= \left[\xi\frac{\partial}{\partial\theta} + \bar{\xi}\frac{\partial}{\partial\bar{\theta}} + 2i\theta\sigma^\mu\bar{\xi}\partial_\mu\right]\Phi_L\\ &= \left[\xi\frac{\partial}{\partial\theta} + \bar{\xi}\frac{\partial}{\partial\bar{\theta}} + 2i\theta\sigma^\mu\bar{\xi}\partial_\mu\right]\left[\phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^\alpha\theta^\beta\epsilon_{\alpha\beta}F(y)\right]\\ &= \sqrt{2}\xi^\alpha\psi_\alpha + 2\xi^\alpha\theta^\beta\epsilon_{\alpha\beta}F + 2i\theta^\alpha\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\phi + 2\sqrt{2}i\theta^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{\xi}^{\bar{\beta}}\theta^\beta\partial_\mu\psi_\beta\\ &\equiv \delta_S\phi + \sqrt{2}\theta\delta_S\psi + \theta\theta\delta_S F.\end{aligned} \tag{4.32}$$

The first two terms of the third line come from the application of the $\partial/\partial\theta$ part of δ_S, whereas the last two terms come from the ∂_μ part. Note that the ∂_μ part applied to the last term in Eq.(4.30) then vanishes since it will contains three factors of θ. The last line implies that a SUSY transformation applied to the left-chiral superfield should again give a left-chiral superfield. It is simple to see that this is precisely true by comparing the Eq.(4.30) with Eq.(4.32).

Thus by equating the coefficients in Eq.(4.32), we explicitly get

$$\delta_S\phi = \sqrt{2}\xi\psi \qquad \left(\text{boson} \overset{\delta_S}{\to} \text{fermion}\right) \tag{4.33.1}$$

$$\delta_S\psi = \sqrt{2}\xi F + i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\phi \qquad \left(\text{fermion} \overset{\delta_S}{\to} \text{boson}\right) \tag{4.33.2}$$

$$\delta_S F = -i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\xi} \qquad \left(\text{F} \overset{\delta_S}{\to} \text{total derivative}\right) \tag{4.33.3}$$

Note that the result of Eq.(4.33.3) implies that $\int d^4xF(x)$ is invariant under SUSY transformations obtained by assuming that boundary terms vanish.

1.3. Vector Superfield. In the previous section, we saw that the chiral superfield can be only used to describe spin-0 bosons and spin-1/2 fermions, e.g. the Higgs boson and the quarks and leptons of the SM. However, we have to also describe the spin-1 gauge bosons of the SM. To this end, we have to introduce some additional superfields called **vector superfields** V. The vector superfield is obtained by imposing the following condition:

$$V\left(x,\theta,\bar{\theta}\right) = V^\dagger\left(x,\theta,\bar{\theta}\right). \tag{4.34}$$

In terms of the power series expansion in θ and $\bar{\theta}$, we will obtain V in terms of the component fields as follow:

$$\begin{aligned} V\left(x,\theta,\bar{\theta}\right) = {} & C(x) + i\theta\chi - i\bar{\theta}\bar{\chi} \\ & + \frac{i}{2}\theta\theta\left[M(x)+iN(x)\right] - \frac{i}{2}\theta\theta\left[M(x)+iN(x)\right] \\ & - \theta\sigma^\mu\bar{\theta}A_\mu(x) + i\theta\theta\bar{\theta}\left[\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right] \\ & - i\bar{\theta}\bar{\theta}\theta\left[\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right] \\ & + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left[D(x) + \frac{1}{2}\Box C(x)\right], \end{aligned} \tag{4.35}$$

that we can rewrite as

$$\begin{aligned} V\left(x,\theta,\bar{\theta}\right) = {} & \left[1 + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\right]C(x) + \left[i\theta - \frac{1}{2}\theta\theta\sigma^\mu\bar{\theta}\partial_\mu\right]\chi(x) \\ & + \frac{i}{2}\theta\theta\left[M(x)+iN(x)\right] - \frac{i}{2}\bar{\theta}\bar{\theta}\left[M(x)-iN(x)\right] \\ & + \left[-i\bar{\theta} + \frac{1}{2}\bar{\theta}\bar{\theta}\sigma^\mu\theta\partial_\mu\right]\bar{\chi}(x) - \theta\sigma^\mu\bar{\theta}A_\mu(x) \\ & - i\bar{\theta}\bar{\theta}\theta\lambda(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \end{aligned} \tag{4.36}$$

where $\Box$ stands for $\partial_\mu\partial^\mu$. Here C, M, N and D are real scalars, χ and λ are Weyl spinors, and A_μ is a vector field. If we want A_μ to describe a gauge boson, hence V must transform as an adjoint representation of the gauge group. A non-abelian supersymmetric gauge transformation acting on V can be represented by

$$e^{gV} \to e^{-ig\Lambda} e^{gV} e^{ig\Lambda}, \tag{4.37}$$

where $\Lambda = \Lambda(x, \theta, \bar{\theta})$ is a chiral superfield and g is the gauge coupling. In the case of an abelian gauge theory, this transformation rule can be written as

$$V \to V + i\left(\Lambda - \Lambda^\dagger\right) \qquad \text{(abelian case)}. \tag{4.38}$$

Note that a chiral superfield contains four scalar (bosonic) degrees of freedom as well as two Weyl spinors. The special gauge is defined by

$$\chi(x) = C(x) = M(x) = N(x) = 0, \tag{4.39}$$

removes the unphysical degrees of freedom in Eq.(4.36), and is called "Wess-Zumino"(WZ) gauge. Note here in WZ gauge that we assign mass dimension +1 to A^μ, and this gives the mass dimension +3/2 for the fermionic field λ, while the field D has the mass dimension +2 just like the component field F of the chiral superfield in Eq.(4.30). As a result, the vector superfield V itself has no mass dimension.

In addition, we can easily compute powers of the vector superfield V in WZ gauge:

$$\begin{aligned}
V &= -\theta\sigma^\mu\bar{\theta}A_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \\
V^2 &= -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}A_\mu(x)A^\mu(x), \\
V^3 &= 0.
\end{aligned} \tag{4.40}$$

Applying a SUSY transformation in Eq.(4.23) to the vector superfield in Eq.(4.36), it gives us lengthier expressions than in case of chiral superfields. Let us show only the following result which is useful for constructing the Langrangian

$$\delta_S D(x) = -\xi\sigma^\mu\partial_\mu\bar{\lambda}(x) + \bar{\xi}\bar{\sigma}^\mu\partial_\mu\lambda(x). \tag{4.41}$$

This significantly shows that the D component of a vector superfield transform into a total derivative.

2. SUSY Lagrangian

After getting acquainted with the superfields introduced in the previous subsections, we now are ready to construct the Lagrangian of a supersymmetric field theory. We assume that, by definition, the action must be invariant under the SUSY transformation:

$$\delta_S \int d^4x \mathcal{L}_{\text{SUSY}}(x) = 0. \tag{4.42}$$

Hence this is satisfied if $\mathcal{L}_{\text{SUSY}}$ itself transforms into a total derivative. As experienced above, we saw that the highest components of the chiral and vector superfields obtained by looking at the largest number of θ and $\bar{\theta}$ factors satisfy this requirement. Therefore, they can be used to construct the required Lagrangian. Hence we can write the action S of the form:

$$S = \int d^4x \mathcal{L}_{\text{SUSY}}(x). \tag{4.43}$$

The following integrations over Grassmann variables provide useful later:

$$\int d\theta_\alpha = 0, \qquad \int \theta_\alpha d\theta_\alpha = 1, \tag{4.44.1}$$

$$\int d^2\theta\,(\theta\theta) = 1, \qquad \int d^2\bar{\theta}\,\left(\bar{\theta}\bar{\theta}\right) = 1, \tag{4.44.2}$$

$$d^2\theta := -\frac{1}{4} d\theta^\alpha d\theta^\beta \epsilon_{\alpha\beta}, \quad d^2\bar{\theta} := -\frac{1}{4} d\bar{\theta}_{\dot{\alpha}} d\bar{\theta}_{\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}}. \tag{4.44.3}$$

Before constructing the SUSY Lagrangian, it is useful to compute the product of two left-chiral superfields:

$$\begin{aligned} \Phi_{i,L}\Phi_{j,L} &= \Big(\phi_i(y) + \sqrt{2}\theta\psi_i(y) + \theta\theta F_i(y)\Big)\Big(\phi_j(y) + \sqrt{1}\theta\psi_j(y) + \theta\theta F_j(y)\Big) \\ &= \phi_i(y)\phi_j(y) + \sqrt{2}\theta\left[\psi_i(y)\phi_j(y) + \phi_i(y)\psi_j(y)\right] \\ &\quad + \theta\theta\left[\phi_i(y)F_j(y) + \phi_j(y)F_i(y) - \psi_i(y)\psi_j(y)\right]. \end{aligned} \tag{4.45}$$

$$\begin{aligned} \Phi_{i,L}\Phi_{j,L}\Phi_{k,L} &= \phi_i(y)\phi_j(y)\phi_k(y) \\ &\quad + \sqrt{2}\theta\left[\psi_i(y)\phi_j(y)\phi_k(y) + \psi_j(y)\phi_k(y)\phi_i(y) + \psi_k(y)\phi_i(y)\phi_j(y)\right] \\ &\quad + \theta\theta\left[F_i(y)\phi_j(y)\phi_k(y) + F_j(y)\phi_k(y)\phi_i(y) + F_k(y)\phi_i(y)\phi_j(y)\right] \\ &\quad - \theta\theta\left[\psi_i(y)\psi_j(y)\phi_k(y) + \psi_j(y)\psi_k(y)\phi_i(y) + \psi_k(y)\psi_i(y)\phi_j(y)\right]. \end{aligned} \tag{4.46}$$

Of course, we see precisely that the product of two and three left-chiral superfields is still a left-chiral superfield. Thus, we can now conclude that the same must be true for the products of any number of left-chiral superfields, i.e. $\Phi_{i,L}...\Phi_{i,L}$ is again a chiral superfield. By integrating over Grassmann variables for the highest component in the product of three left-chiral superfields, we then obtain

$$\begin{aligned} \int d^2\theta \Phi_{i,L}\Phi_{j,L}\Phi_{k,L} &= \int d^2\theta\,(\theta\theta)\left(F_i(y)\phi_j(y)\phi_k(y) + F_j(y)\phi_k(y)\phi_i(y)\right) \\ &\quad + \int d^2\theta\,(\theta\theta)\left(F_k(y)\phi_i(y)\phi_j(y) - \psi_i(y)\psi_j(y)\phi_k(y)\right) \\ &\quad - \int d^2\theta\,(\theta\theta)\left(\psi_j(y)\psi_k(y)\phi_i(y) - \psi_k(y)\psi_i(y)\phi_j(y)\right) \\ &= F_i(y)\phi_j(y)\phi_k(y) + F_j(y)\phi_k(y)\phi_i(y) + F_k(y)\phi_i(y)\phi_j(y) \\ &\quad - \psi_i(y)\psi_j(y)\phi_k(y) - \psi_j(y)\psi_k(y)\phi_i(y) - \psi_k(y)\psi_i(y)\phi_j(y). \end{aligned} \tag{4.47}$$

Note that the last three terms in Eq.(4.47) describes Yukawa interactions between one scalar and two fermions. We know that in SM these interactions give rise to quark and lepton masses. By assuming **[12]** ϕ_i to be the Higgs field, and ψ_j and ψ_k to be the left- and right-handed components of the top quark, respectively, Eq.(4.47) will not only produce the Higgs-antitop-top interaction but also interactions between a scalar top $\tilde{t}$, the fermionic "higgsino "$\tilde{h}$, and the top quark, with equal strength.

As it is mentioned above, there are some terms which can give rise to fermion masses, $\psi_j\psi_j$ in Eq.(4.45), and Yukawa interactions as in Eq.(4.46). Unfortunately, we have not yet discovered any terms with derivatives which lead to the kinetic energy terms. Alternatively we can get inspiration by looking at the products between the left- and right-chiral superfields.

Explicitly, the product $\Phi_L^\dagger\Phi_L$ is self-conjugate, i.e. $\left(\Phi_L^\dagger\Phi_L\right)^\dagger = \Phi_L^\dagger\Phi_L$, so it is a vector superfield. So far, there are lengthy expressions for such products. Here is part of the significant candidates contributing to the construction of the Lagrangian of Eq.(4.43)

$$\int d^2\theta d^2\bar{\theta}\Phi_L^\dagger\Phi_L = F^*(x)F(x) - \partial^\mu\phi_i^\dagger\partial_\mu\phi_i - i\bar{\psi}(x)\sigma^\mu\partial_\mu\psi(x). \tag{4.48}$$

Precisely, this contains kinetic energy terms not only for the scalar component ϕ, but also for the fermionic component ψ. In addition, the auxiliary field F can be integrated out by using the equation of motion, e.g. page 32 of [**13**].

To see exactly how to remove the F-field from the Lagrangian, we have to introduce the **superpotential** $P(\Phi_i)$:

$$P(\Phi_i) = \sum_i k_i\Phi_i + \frac{1}{2}\sum_{i,j} m_{ij}\Phi_i\Phi_j + \frac{1}{3}\sum_{i,j,k} g_{ijk}\Phi_i\Phi_j\Phi_k, \tag{4.49}$$

where the Φ_i are all the left-chiral superfields, and the k_i, m_{ij} and g_{ijk} are constants with mass dimension 2,1 and 0, respectively. We are now ready to construct our Lagrangian in which it can be written as

$$\begin{aligned}\mathcal{L} &= \sum_i \left\{\int d^2\theta d^2\bar{\theta}\Phi_i^\dagger\Phi_i + \left[\int d^2\theta P\left(\Phi_i\right) + h.c.\right]\right\} \\ &= F_i^*F_i - \partial^\mu\phi_i^\dagger\partial_\mu\phi_i - i\bar{\psi}_i(x)\sigma^\mu\partial_\mu\psi_i(x) \\ &\quad + W_iF_i + W_i^*F_i^* - \frac{1}{2}\left(W_{ij}\psi_i\psi_j + h.c.\right),\end{aligned} \tag{4.50}$$

where $W_i = \partial P/\partial\phi_i$ and $W_{ij} = \partial^2 P/\partial\phi_i\partial\phi_j$. As early mentioned, we can remove the auxilairy field F by looking at their equations of motions obtained by the following algebraic schemes:

$$\begin{aligned}\partial\mathcal{L}/\partial F = 0 &\rightarrow F_i^* + W_i = 0, \\ \partial\mathcal{L}/\partial F^* = 0 &\rightarrow F_i + W_i^* = 0.\end{aligned} \tag{4.51}$$

Substituting this result into Eq.(4.50) yields

$$\mathcal{L} = \mathcal{L}_{\text{kin}} - \frac{1}{2}\left(\frac{\partial^2 P}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + h.c.\right) - \left|\frac{\partial P}{\partial\phi_i}\right|^2, \tag{4.52}$$

where $\mathcal{L}_{\text{kin}}$ except $F_i^* F_i$ refers to the second line of Eq.(4.50). Furthermore the fermion masses and Yukawa interactions are described by the second term of Eq.(4.52), while the last term describes a scalar mass terms and also scalar interactions.

To complete the construction of a renormalizable supersymmetric Lagrangian, we need to introduce gauge interactions. The "minimal coupling"of the gauge superfields to the chiral matter superfields is automatically introduced by the following substitution:

$$\begin{aligned}\int d^2\theta d^2\bar{\theta}\Phi_i^\dagger \Phi_i \to \int d^2\theta d^2\bar{\theta}\Phi_i^\dagger e^{2gV}\Phi_i \\ = -D^\mu \phi_i^\dagger D_\mu \phi_i - i\bar{\psi}_i \sigma^\mu D_\mu \psi_i + g\phi_i^\dagger D\phi_i \\ + i\sqrt{2}g\left(\phi_i^\dagger \lambda\psi_i - \bar{\lambda}\bar{\psi}_i\phi_i\right) + F_i^* F_i.\end{aligned} \tag{4.53}$$

Here we have used the WZ gauge, $V = V_{WZ}$, and gauge-covariant derivative $D_\mu = \partial_\mu + igA_\mu^a T^a$ with T^a the group generators. Note that these pieces not only describes the interactions of the matter fields, fermions and bosons, with the gauge fields, but also contain gauge-strength Yukawa interactions between fermions (higgsinos) ψ, sfermions (or Higgs bosons) ϕ, and gauginos λ.

So, the last ingredient we need is the kinetic terms of the gauge field which can be constructes with the help of the following additional superfield:

$$\begin{aligned}W_\alpha &= -\frac{1}{8}\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}} e^{-2gV} D_\alpha e^{2gV} \\ &= -ig\lambda_\alpha(y) + g\theta_\alpha D(y) - g\sigma^{\mu\nu}{}_\alpha{}^\beta \theta_\beta F_{\mu\nu}(y) \\ &\quad + g\,(\theta\theta)\,\sigma^{\mu\nu}_{\alpha\dot{\beta}} D_\mu \bar{\lambda}^{\dot{\beta}}(y),\end{aligned} \tag{4.54}$$

and its conjugate

$$\begin{aligned}\overline{W}_{\dot{\alpha}} &= +\frac{1}{8}D^\alpha D_\alpha e^{2gV}\bar{D}_{\dot{\alpha}} e^{-2gV} \\ &= +ig\bar{\lambda}_{\dot{\alpha}}(z) + g\bar{\theta}_{\dot{\alpha}} D(z) + ig\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}^{\mu\nu}{}_{\dot{\beta}}{}^{\dot{\delta}}\bar{\theta}_{\dot{\delta}}\tilde{F}_{\mu\nu}(z) \\ &\quad - g\left(\bar{\theta}\bar{\theta}\right)\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}^{\mu\ \dot{\beta}\delta} D_\mu \lambda_\delta(z),\end{aligned} \tag{4.55}$$

where we can simply prove that $F_{\mu\nu}(y) = \tilde{F}_{\mu\nu}(z) = F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + ig\left[A_\mu(x), A_\nu(x)\right]$ which is just usual field strength tensor, and $D, \bar{D}$ are again SUSY-covariant derivatives which carry the spinor indices. In additon, we can recover the abelian symmetries by reducing the above expression, i.e. $W_\alpha = -1/4\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}} D_\alpha V$. However we find that $\bar{D}_{\dot{\alpha}} W_\alpha = 0$, so W_α itself is also a left-chiral superfield as well as $D_\alpha \bar{W}_{\dot{\alpha}} = 0$, so $\bar{W}_{\dot{\alpha}}$ itself is again a right-chiral superfield.

One can verify that some pieces which are useful to construct the Lagrangian are obtained from the following expression:

$$\begin{aligned}&\frac{1}{16kg^2}\int d^2\theta d^2\bar{\theta}\mathrm{Tr}\left[(W^\alpha W_\alpha)\,\delta^2(\bar{\theta})+\left(\bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}\right)\delta^2(\theta)\right]\\&=-\frac{1}{4}F^{(a)}_{\mu\nu}(x)F^{(a)\mu\nu}(x)+\frac{1}{2}D^a(x)D^a(x)-i\bar{\lambda}^{(a)}(x)\bar{\sigma}^\mu D_\mu\lambda^{(a)}(x).\end{aligned}\tag{4.56}$$

Now, we discover the kinetic terms for the gauge fields as well as the kinetic terms for the gauginos λ^a. Note again that we can integrate out the D_a fields from $\partial\mathcal{L}/\partial D^b=0$, so we obtain

$$D_a(x)=-g\phi_i^\dagger T^{(a)}_{ij}\phi_j.\tag{4.57}$$

In addition, combining the third term in the second line of Eq.(4.53) and the second term in Eq.(4.56), we have

$$\frac{1}{2}D^{(a)}(x)D^{(a)}(x)+g\phi^\dagger D(x)\phi=-\frac{1}{2}g^2\left(\phi_i^\dagger T^a_{ij}\phi_j\right)\left(\phi_i^\dagger T^a_{ij}\phi_j\right)\equiv -V_D.\tag{4.58}$$

This contributes to the scalar interactions in the Lagrangian. Therefore, now we have completely constructed the Lagrangian for the renormalizable supersymmetric field theory.

2.1. SUSY Breaking. For simplicity, now we would like to start by considering a single anti-commuting generator Q_α. The theories with single such a generator are referred to as $\mathcal{N}=1$ which reflects the fact that there is only one irreducible generator.

In this case, the Hamiltonian of any SUSY theory obtained from the Eq.(4.19.2) can be written as

$$\left\{Q_\alpha,\bar{Q}_{\dot{\beta}}\right\}=2\sigma^\mu_{\alpha\dot{\beta}}P_\mu.\tag{4.59}$$

Multiplying from the right by $\bar{\sigma}^{\nu\dot{\beta}\alpha}$ and summing over α and $\dot{\beta}$, we have

$$\begin{aligned}\left\{Q_\alpha,\bar{Q}_{\dot{\beta}}\right\}\bar{\sigma}^{\nu\dot{\beta}\alpha}&=2\sigma^\mu_{\alpha\dot{\beta}}\bar{\sigma}^{\nu\dot{\beta}\alpha}P_\mu\\&=2\mathrm{Tr}\left[\sigma^\mu\bar{\sigma}^\nu\right]P_\mu\\&=-4\eta^{\mu\nu}P_\mu=-4P^\nu.\end{aligned}\tag{4.60}$$

Setting $\nu=0$, we obtain

$$\begin{aligned}P^0=H&=-\frac{1}{4}\left\{Q_\alpha,\bar{Q}_{\dot{\beta}}\right\}\bar{\sigma}^{0\dot{\beta}\alpha}\\&=-\frac{1}{4}\left[Q_\alpha\bar{Q}_{\dot{\beta}}+\bar{Q}_{\dot{\beta}}Q_\alpha\right]\bar{\sigma}^{0\dot{\beta}\alpha},\quad \bar{\sigma}^0=-1_{2\times 2}\\&=\frac{1}{4}\left[Q_1\bar{Q}_{\dot{1}}+\bar{Q}_{\dot{1}}Q_1+Q_2\bar{Q}_{\dot{2}}+\bar{Q}_{\dot{2}}Q_2\right].\end{aligned}\tag{4.61}$$

The invariance of the theory with respect to SUSY transformation implies that the Hamiltonian H commutes with the generators of the SUSY transformation, i.e.

$$[H,Q_\alpha]=0=\left[H,\bar{Q}_{\dot{\alpha}}\right].\tag{4.62}$$

For an arbitrary state $|\psi\rangle$, the spectrum of the Hamiltonian is positive definite, i.e. $\langle\psi| H |\psi\rangle \geq 0$. If there exists some state $|0\rangle$ such that $E_{vac} \equiv \langle 0| H |0\rangle \geq 0$, i.e.

$$\begin{aligned} Q_\alpha |0\rangle &= 0, \\ \bar{Q}_{\dot{\alpha}} |0\rangle &= 0, \end{aligned} \tag{4.63}$$

then the vacuum state $|0\rangle$ is said to be supersymmetric. However, if the vacuum state is not supersymmetric meaning that at least one SUSY generator does not annihilate the vacuum, then the Eq.(4.61) implies that $E_{vac} > 0$. Concretely the global symmetry can be spontaneously broken if there exists a positive vacuum energy.

In addition, in SUSY field theory, we can write the scalar potential related to the superpotential as

$$-V = \underbrace{-\left|\frac{\partial P}{\partial \phi_i^{(a)}}\right|^2}_{-V_F} \underbrace{-\frac{1}{2}\sum_j g_j^2 \left|\phi_i^\dagger T_j^{(a)} \phi_i\right|^2}_{-V_D}, \tag{4.64}$$

where i runs over all the scalar field labels, while j runs over all the gauge group labels, and a is the gauge group index. In the case of $E_{vac} = 0$, this can be illustrated in Fig.4.2 for a theory with scalar superfields.

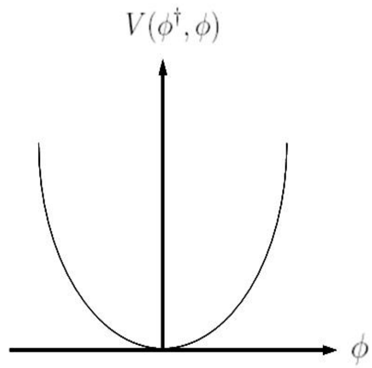

Fig.4.2: With $V = 0$. No breaking of SUSY and also an internal symmetry

According to Fig.4.2, supersymmetric ground states are at $E_{vac} = 0$.

However, supersymmetry is unbroken since the ground state energy, the minimum of the potentiaal, is still zero.

Alternatively, we can consider the case in which the vacuum state is not annihilated by the supercharge, i.e. $Q_\alpha |0\rangle \neq 0$, which implies in view of Eq.(4.47) that $E_{vac} = \langle 0| H |0\rangle \neq 0$.

Now we conclude that: If supersymmetry is not spontaneously broken, i.e. the vacuum is invariant under SUSY transformation, i.e.

$$\delta_S |0\rangle = \left(\xi^\alpha Q_\alpha + \bar{\xi}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}\right) |0\rangle = 0, \tag{4.65}$$

the energy of vacuum is still zero.

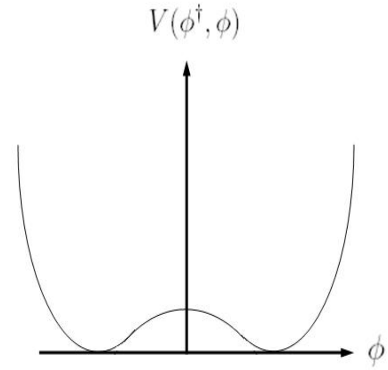

Fig.4.3: Degeneracy of the SUSY ground state. No breaking of SUSY but breaking of some internal symmetry

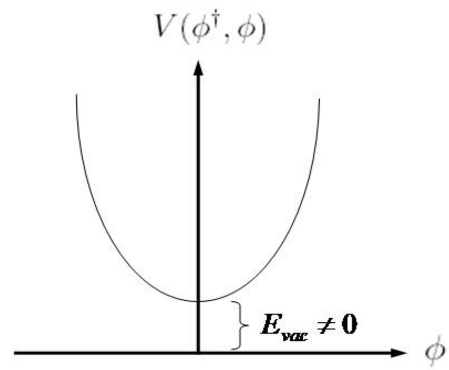

Fig.4.4: The case of broken supersymmetry but no breaking of internal symmetry.

However, if supersymmetry is spontaneously broken, the vacuum energy is positive $E_{vac} > 0$. The case of broken supersymmetry are illustrated in Fig.4.3 and Fig.4.4. In Fig.4.3, the expectation value of the scalar field is zero, but supersymmetry is spontaneously broken since the energy of the ground state is greater than zero.

However in Fig 4.5 we show that the case in which the expectation value of the scalar field is greater than zero and supersymmetry is spontaneously broken.

Precisely, We can break SUSY if either $\langle F_i \rangle = \langle \partial P / \partial \phi_i \rangle \neq 0$ for some i or if $\langle D_{j,a} \rangle = \left\langle \sum_j \phi_i^\dagger T_j^{(a)} \phi_i \right\rangle \neq 0$. In the latter case, some gauge symetries will also be broken.

For phenomenological purposes, however, we need to introduce some additional superfields into the theory. One of these objects is called soft-breaking terms which can be introduced simply using the physical components of the superfields. Such terms are the ones which do not introduce quadratic divergences.

We know that, at least at the one-loop order, quadratic divergencies still cancel if we introduce **[12]** (1) scalar mass terms, $-m_{\phi_i}^2 |\phi_i|^2$, and (2) trilinear scalar interactions

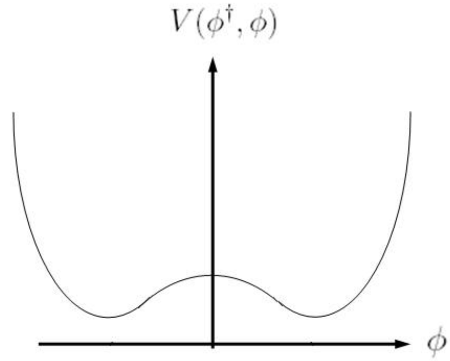

Fig.4.5: The case of broken supersymmetry and also some internal symmetry.

$-A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$ into the Lagrangian. However we need three additional types of soft-breaking terms (3) gaugino mass terms $-1/2m_l\lambda_l\lambda_l$ where l labels the group factor, (4) bilinear terms $-B_{ij}\phi_i\phi_j + \text{h.c.}$, and (5) linear terms $-C_i\phi_i$ (that are only allowed for gauge singlets).

2.2. R-Symmetry. There is an additive symmetry in supersymmetry: the so-called "R-symmetry". The R-symmetry does not commute with the supersymmetry generators. Such symmetries can be continuous or discrete. For the continuous case, the θ's can be transformed by the phase ($U(1)$) trasformation, $e^{i\alpha}$. Then the Lagrangian $\mathcal{L}$ is invariant under a new $U(1)$ symmetry which acts as follows [**17**][**18**]:

$$\Phi_k(x,\theta) \;\to\; e^{-ir_k\alpha}\Phi_k\left(x, e^{i\alpha}\theta\right), \quad V^a\left(x,\theta,\bar{\theta}\right) \;\to\; V^a\left(x, e^{i\alpha}\theta, e^{-i\alpha}\bar{\theta}\right), \tag{4.66}$$

or, in terms of the component fields,

$$\phi_k \to e^{-ir_k\alpha}\phi_k,\; \psi_k \to e^{i(1-r_k)\alpha}\psi_k,\; F_k \to e^{i(2-r_k)\alpha}F_k,\; \lambda^a \to e^{-i\alpha}\lambda^a. \tag{4.67}$$

In the minimal supersymmetric standard model, we can take $r_k = 2/3$ for all of the chiral fields. Since all left-handed fermions have the same charge under Eq.(4.67), then the R-symmetry will have an axial anomaly. To suppress such an anomaly, it is possible to combine the transformation laws in Eq.(4.66) with other $U(1)$ symmetries to define a non-anomalous $U(1)$ R-symmetry. Under such a symmetry, we will have

$$\Phi_k(x,\theta) \;\to\; e^{-i\beta_k}\Phi_k\left(x, e^{i\alpha}\theta\right) \text{such that } P\left(x,\phi\right) \;\to\; e^{2i\alpha}P\left(x, e^{i\alpha}\theta\right). \tag{4.68}$$

Here P is a superpotential which has dimensionful coefficients. In general, the case of $N > 1$, the R-symmetry has to be extended to $SU(2)$ for $N = 2$ and to $SU(4)$ for $N = 4$ supersymmetry.

3. The Minimal Supersymmetric Standard Model

3.1. Particle Contents. In this section, we will briefly discuss the **minimal supersymmetric standard model (MSSM)**. In particular, the word "minimal" means that one wants to maintain the number of superfields and interactions as small as possible. Now let us assign the matter fields of the SM to chiral supermultiplet and the vector fields in the SM to vector supermultiplets.

The vector supermultiplets correspond to the generators of $SU(3)\times SU(2)\times U(1)$ gauge group. To do this, we will refer to the gauge bosons of these groups as g^a_μ, W^a_μ and B_μ, respectively. The Weyl fermion partners of these fields will be represented as $\tilde{g}^a$, $\tilde{W}^a$ and $\tilde{B}$ so-called the "gluino,wino"and "bino", or collectively, "gauginos".

The quarks and leptons will be assigned as fermions in the chiral superfields, and we consider left-handed Weyl fermions as the usual particles and right-handed Weyl fermions as their antiparticles. In the SM, we have

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}, \qquad \bar{e}, \qquad Q = \begin{pmatrix} u \\ d \end{pmatrix}, \qquad \bar{u}, \qquad \bar{d}. \tag{4.69}$$

Here the field $\bar{e}$ is the left-handed positron; the fields $\bar{u}, \bar{d}$ are the left-handed antiquarks. The right-handed fermion fields are the conjugates of these fields. To make the supersymmetric Lagrangian, we have to double the number of fields in the theory by associating to each SM field a superpartner field, which we define by adding a "tilde"to the symbol of the SM field. For the SM fermions, therefore, we can write

$$\tilde{L}, \qquad \tilde{\bar{e}}, \qquad \tilde{Q}, \qquad \tilde{\bar{u}}, \qquad \tilde{\bar{d}}. \tag{4.70}$$

The particle contents given in Eq.(4.69) and Eq.(4.70) make up the physical conponents of the "chiral supermultiplets". Each supermultiplet contains both boson and fermion states. Note that the scalar particles in these supermultiplets are so-called "slepton"and "squarks", or collectively, "sfermions". What about the Higgs field? In SUSY, we have to include a Higgs supermultiplet with $Y = +1/2$ and a second Higgs chiral supermultiplet with $Y = -1/2$. Thus, for Higgs chiral supermultiplets, we have

$$H_u = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \qquad H' = \begin{pmatrix} H'^0 \\ H'^- \end{pmatrix}. \tag{4.71}$$

However, in MSSM, one adds a new symmetry which can eliminate the possibility of B and L violation terms in the superpotential. This symmetry is so-called "R=Parity"which can be defined for each particle as

$$R = (-1)^{3(B-L)+2S}, \tag{4.72}$$

where S is the spin of the particle. Hence all of the SM particles and Higgs bosons have even R-parity $R = +1$, while all of the squarks, slepton, gauginos, and higgsinos have odd R-parity $R = -1$. The R-parity odd particles are known as "sparticles". If R-parity is exactly conserved, then there can be no mixing between the sparticles and the $R = +1$ particle given in Table 4.1.

Particles	even R-parity	odd R-parity
Bosons	gauge bosons Higgs bosons graviton	sleptons squarks
Fermions	leptons quarks	gauginos higgsinos gravitino

Table 4.1: R-parities in the supersymmetric standard model.

The Weyl fermion partners of these fields are called "Higgsinos". Note that it is necessary to include both Higgs fields in order to obtain all the required couplings in the superpotential. In additon, there is another reason for which the axial vector anomaly of one $U(1)$ and two $SU(2)$ currents given in Fig.4.6 must vanish to maintain the gauge invariance. In SM, this anomaly cancels between the quarks and the leptons mentioned in [**2**]. However, in SUSY, each Higgsino makes a non-zero contribution to this anomaly. So these contributions cancel if we include a pair of Higgsinos with opposite hypercharge [**18**].

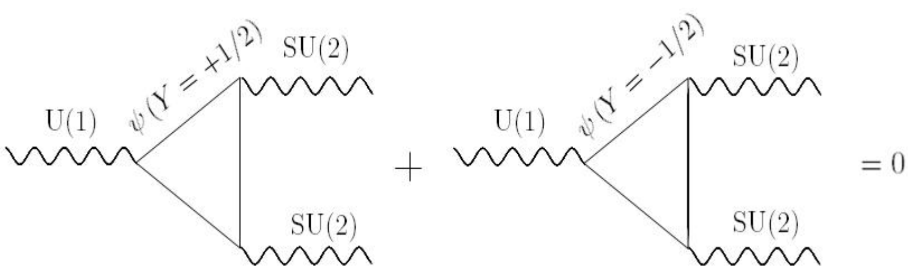

Fig.4.6: The anomaly cancellation that requires two doublets of Higgs fields in the MSSM.

Now let us emphasize that if there is only one Higgs chiral supermultiplet, the electroweak gauge symmetry will suffer a gauge anomaly. This is because the conditions for the cancellation of such anomaly requires [**17**] $\mathrm{Tr}\,[T_3^2 Y] = \mathrm{Tr}\,[Y^3] = 0$, where T_3 and Y are the weak isospin component and the weak hypercharge, respectively. Note that the trace runs over all of the left-handed Weyl fermionic degrees of freedom.

Super Fields	Comp.Fields	Name	spin-0	spin-1/2	$SU(3), SU(2), U(1)$
$Q=\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_L d_L $\tilde{u}_L$ $\tilde{d}_L$	Quark Quark Squark Squark	$\tilde{u}_L$ $\tilde{d}_L$	u_L d_L	$(3, 2, 1/6)$
$\bar{u}$	$u_R^\dagger$ $\tilde{u}_R^*$	R. Handed Upquark& Upsquark	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{3}, 1, -2/3)$
$\bar{d}$	$d_R^\dagger$ $\tilde{d}_R^*$	R. Handed Dquark& Dsquark	$\tilde{d}_R^*$	$d_R^\dagger$	$(\bar{3}, 1, 1/3)$
$L=\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	ν_L e_L $\tilde{\nu}_L$ $\tilde{e}_L$	Lepton Lepton Slepton Slepton	$\tilde{\nu}_L$ $\tilde{e}_L$	ν_L e_L	$(1, 2, -1/2)$
$\bar{e}$	$e_R^\dagger$ $\tilde{e}_R^*$	Antilepton Antislepton	$\tilde{e}_R^*$	$e_R^\dagger$	$(1, 1, 1)$
$H=\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	H^+ H^0 $\tilde{H}^+$ $\tilde{H}^0$	Higgs Higgs Higgsino Higgsino	H^+ H^0	$\tilde{H}^+$ $\tilde{H}^0$	$(1, 2, +1/2)$
$H'=\begin{pmatrix} H'^0 \\ H'^- \end{pmatrix}$	H'^0 H'^- $\tilde{H}'^0$ $\tilde{H}'^-$	Higgs Higgs Higgsino Higgsino	H'^0 H'^-	$\tilde{H}'^0$ $\tilde{H}'^-$	$(1, 2, -1/2)$

Table 4.2: Chiral supermultiplets in MSSM. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.

Notice that if we define each Higgs chiral supermultiplets to be a weak isodoublet with weak hypercharge either $Y = +1/2$ or $Y = -1/2$, we will get the cancellation of the anomaly contribution. Moreover, there is another requirement for including two Higgs supermultiplets. Such requirement is that only $Y = +1/2$ Higgs chiral supermultiplet can have the Yukawa couplings to give masses to up-type quarks, and only $Y = -1/2$ can have Yukawa couplings to give masses to down-type quarks and to charged leptons.

Now all of the chiral supermultiplets of the MSSM can be summarized in Table 4.2. Here Q stands for the $SU(2)_L$ -doublet chiral supermultiplet containing $u_L, \tilde{u}_L$ with weak isospin component $T_3 = +1/2$, and $d_L, \tilde{d}_L$ with weak isospin component $T_3 = -1/2$, while $\bar{u}$ stands for the $SU(2)_L$ -singlet chiral supermultiplet containing $u_R^\dagger, \tilde{u}_R^*$. The Higgs chiral supermultiplet H_d containing $\left(H'^0, H'^-, \tilde{H}'^0, \tilde{H}'^-\right)$ has the same quantum numbers as the left-handed sleptons and leptons L_i, e.g. $(\tilde{\nu}_L, \tilde{e}_L, \nu_L, e_L)$.

Super Fields	Comp.Fields	Name	spin-1/2	spin-1	$SU(3), SU(2), U(1)$
$g, \tilde{g}$	g $\tilde{g}$	Gluon Gluino	$\tilde{g}$	g	$(8, 1, 0)$
$W^{\pm}, W^0$ $\tilde{W}^{\pm}, \tilde{W}^0$	$W^{\pm}, W^0$ $\tilde{W}^{\pm}, \tilde{W}^0$	W bosons Winos	$\tilde{W}^{\pm}, \tilde{W}^0$	$W^{\pm}, W^0$	$(1, 3, 0)$
$B^0, \tilde{B}^0$	B^0 $\tilde{B}^0$	B boson Bino	$\tilde{B}^0$	B^0	$(1, 1, 0)$

Table 4.3: Vector supermultiplets in MSSM.

The vector bosons of the SM should be clearly placed in vector (gauge) supermultiplets. Their fermionic superpartners are generally called gauginos. We know well that the $SU(3)_C$ color gauge interactions have the gluon as mediators whose spin-1/2 superpartner is the gluino. Therefore we need eight gluinos $\tilde{g}$ as superpartners of the eight gluons g of QCD, three winos $\tilde{W}$ as superpartners of the $SU(2)$ gauge bosons W, and bino $\tilde{B}$ as $U(1)_Y$ gaugino. Since electroweak $SU(2) \times U(1)_Y$ is broken, the wino $\tilde{W}^3$ and the bino $\tilde{B}$ are not mass eigenstates. These eigenstates mix to give mass eigenstates $\tilde{Z}^0, \tilde{\gamma}$. If SUSY were unbroken, they would be mass eigenstates with masses m_Z and 0. We summarize the gauge supermultiplets in Table 4.3.

The chiral and gauge supermultiplets given in Tables 4.2 and 4.3 give rise to the particle content of the MSSM. Up to now, none of the superpartners of the SM particles has been discovered. Seemingly, supersymmetry should be a broken symmetry in a vacuum state ruled out by Nature. With the supersymmetry breaking, we now return, for the time being, to the motivation obtained by the gauge hierarchy problem. Supersymmetry forced us to include two complex scalar fields. This is just what we need to enable a cancellation of the quadratically divergent piece δM_h^2 of Eq.(4.14). This sort of cancellation also requires the relation between dimensionless couplings, e.g. $\lambda_S = \lambda_F^2$. In fact, the unbroken supersymmetry guarantees that the quadratic divergences in scalar sector must vanish to all order in perturbation theory.

To provide a solution to the gauge hierarchy problem, one introduces the so-called "soft" supersymmetry breaking term. Therefore the Lagrangian of the MSSM can be written in the form

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}, \tag{4.73}$$

where $\mathcal{L}_{\text{SUSY}}$, obtained by returning to the section 1.1 and 1.2, contains all of the gauge and Yukawa interactions and also preserve supersymmetry invariance. The $\mathcal{L}_{\text{soft}}$ term violates supersymmetry and contains only mass terms and coupling parameters with positive mass dimension. The soft supersymmetry-breaking terms in the Lagrangian can be in general written in the form [**17**]

$$\mathcal{L}_{\text{soft}} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + h.c. - \frac{1}{2}\left(m^2\right)^i_j \phi^{j*}\phi_i. \tag{4.74}$$

They consist of gaugino masses M_a for each gauge group, scalar squared-mass terms $(m^2)^i_j$ and b^{ij}, and $(\text{scalar})^3$ coupling a^{ijk}, and "tadpole"couplings t^i. The softly broken terms are obtained by re-writing the superpotential, given in Eq.(4.29), in terms of the scalar fields together with its Hermitian conjugate, and also the mass terms of gauginos and the mass terms of scalar field.

In the MSSM case, the superpotential is given by [**19**][**20**]

$$P_{\rm MSSM} = \epsilon_{\rho\sigma}\left[-\hat{H}^{\rho}\hat{Q}_i^{\sigma}y_u^{ij}\hat{\bar{U}}_j + \hat{H}'^{\rho}\hat{Q}_i^{\sigma}y_d^{ij}\hat{\bar{D}}_j + \hat{H}'^{\rho}\hat{L}_i^{\sigma}y_e^{ij}\hat{\bar{E}}_j - \mu\hat{H}'^{\alpha}\hat{H}^{\beta}\right], \tag{4.75}$$

where $\epsilon_{\rho\sigma} = -\epsilon_{\sigma\rho}$, and $\epsilon_{12} = 1$, and the superfields are defined in the following standard ways:

$$\begin{aligned}
\hat{Q}_i^{\rho} &= \begin{pmatrix}\hat{Q}_i^1\\ \hat{Q}_i^2\end{pmatrix} = \begin{pmatrix}\tilde{U}_{L_i},\ U_{L_i}\\ \tilde{D}_{L_i},\ D_{L_i}\end{pmatrix},\\
\hat{\bar{U}}_i &= \left(\tilde{\bar{U}}_{L_i},\ \bar{U}_{L_i}\right),\\
\hat{\bar{D}}_i &= \left(\tilde{\bar{D}}_{L_i},\ \bar{D}_{L_i}\right),\\
\hat{L}_i^{\rho} &= \begin{pmatrix}\hat{L}_i^1\\ \hat{L}_i^2\end{pmatrix} = \begin{pmatrix}\tilde{N}_{L_i},\ N_{L_i}\\ \tilde{E}_{L_i},\ E_{L_i}\end{pmatrix},\\
\hat{\bar{N}}_i &= \left(\tilde{\bar{N}}_{L_i},\ \bar{N}_{L_i}\right),\\
\hat{\bar{E}}_i &= \left(\tilde{\bar{E}}_{L_i},\ \bar{E}_{L_i}\right),\\
\hat{H}^{\rho} &= \begin{pmatrix}\hat{H}^1\\ \hat{H}^2\end{pmatrix} = \begin{pmatrix}H^+,\ \tilde{H}^+\\ H^0,\ \tilde{H}^0\end{pmatrix},\\
\hat{H}'^{\rho} &= \begin{pmatrix}\hat{H}'^1\\ \hat{H}'^2\end{pmatrix} = \begin{pmatrix}H'^0,\ \tilde{H}'^0\\ H'^-,\ \tilde{H}'^-\end{pmatrix},
\end{aligned} \tag{4.76}$$

where $i, j = 1, 2, 3$ lebel the family indices. The soft supersymmetry-breaking Lagrangian $\mathcal{L}_{\rm soft}$ obtained from the Eq.(4.74) takes the following form [**19**][**20**]:

$$\begin{aligned}
-\mathcal{L}_{\rm soft} =& \frac{1}{2}\left[M_3\tilde{g}^A\tilde{g}^A + M_2\tilde{W}^a\tilde{W}^a + M_1\tilde{B}\tilde{B} + h.c.\right]\\
&+ \epsilon_{\rho\sigma}\left[-H^{\rho}\tilde{Q}_i^{\sigma}\tilde{a}_{u_{ij}}\tilde{\bar{U}}_j + H'^{\rho}\tilde{Q}_i^{\sigma}\tilde{a}_{d_{ij}}\tilde{\bar{D}}_j + H'^{\rho}\tilde{L}_i^{\sigma}\tilde{a}_{e_{ij}}\tilde{\bar{E}}_j + h.c.\right]\\
&+ m_{H'}^2\left|H'\right|^2 + m_H^2\left|H\right|^2 - \left(b\mu\epsilon_{\rho\sigma}H'^{\rho}H^{\sigma} + h.c.\right)\\
&+ \tilde{Q}_i^{\rho}m_{Q_{ij}}^2\tilde{Q}_j^{*\rho} + \tilde{L}_i^{\rho}m_{L_{ij}}^2\tilde{L}_j^{*\rho} + \tilde{\bar{U}}_i^{*}m_{U_{ij}}^2\tilde{\bar{U}}_j^{\rho} + \tilde{\bar{D}}_i^{*}m_{D_{ij}}^2\tilde{\bar{D}}_j^{\rho} + \tilde{\bar{E}}_i^{*}m_{E_{ij}}^2\tilde{\bar{E}}_j^{\rho},
\end{aligned} \tag{4.77}$$

where A represents the $SU(3)$ indices, while a represents $SU(2)$ indices. The terms in the first line contain mass terms for the gauginos. The second line contributes to the potential for the Higgs bosons. The $SU(2)$ representations of the squark, slepton, and Higgs doublets in Eq.(4.77) can be expressed as follows:

$$\tilde{Q} = \begin{pmatrix}\tilde{U}_L\\ \tilde{D}_L\end{pmatrix}, \quad \tilde{L} = \begin{pmatrix}\tilde{N}_L\\ \tilde{E}_L\end{pmatrix}, \quad H = \begin{pmatrix}H^+\\ H^0\end{pmatrix}, \quad H' = \begin{pmatrix}H'^0\\ H'^-\end{pmatrix}. \tag{4.78}$$

The Higgs fields acquire the following non-zero vacuum expectation values:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \qquad \langle H' \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \tag{4.79}$$

where $v_u^2 + v_d^2 \equiv v^2$, $\tan\beta = v_u/v_d$, and $v^2 = (174 \text{ GeV})^2 = 2m_Z^2/\left(g^2 + g'^2\right)$. Here g and g' are respectively the $SU(2)$ and $U(1)_Y$ gauge couplings which commonly satisfy $e = g\sin\theta_w = g'\cos\theta_w$, where e is the electron charge and θ_w is the electroweak mixing angle.

The soft term couplings are also responsible for electroweak symmetry breaking. The scalar potential for the Higgs sector (further details are presented in Appendix A) at tree-level is given by [**20**]

$$\begin{aligned} V_H = {} & m_H^2 \left|H\right|^2 + m_{H'}^2 \left|H'\right|^2 - \left(b\mu\epsilon_{ij} H'^i H^j + h.c.\right) \\ & + \left|\mu\right|^2 \left|H\right|^2 + \left|\mu\right|^2 \left|H'\right|^2 + \frac{g^2 + g'^2}{8}\left(\left|H\right|^2 - \left|H'\right|^2\right)^2 + \frac{g^2}{2}\left|H^* \cdot H'\right|^2, \end{aligned} \tag{4.80}$$

Here, the first line comes from the soft SUSY breaking terms, the second line is obtained from the F-term and D-term potentials, respectively.

CHAPTER 5

Technicolor Meets Supersymmetry

The main idea of Technicolor (TC) is to introduce a new strongly coupled gauge theory replacing the Higgs as a fundamental degree of freedom. In TC models, the electroweak symmetry is dynamically broken by a bilinear condensate of techifermions similary to the chiral condensate in QCD. Furthermore, in TC, the scale of symmetry breaking is at the EW scale, so the W and Z gauge bosons will gain the correct masses.

On the other hand, naively, one of the main motivation of supersymmetry (SUSY) comes from the quadratically divergent contributions to the Higgs squared masses. For exact unbroken supersymmetry, one would predict that a particle and its superpartners would have the same mass. Unfortunately, up to now, the superpartners of the Standard Model particles have not yet been observed. This indicates that supersymmetry must be spontaneously broken at a scale of the order of the weak interactions.

According to [**24**], new types of theories, which combine supersymmetry and a new trong interaction which is referred to as **supercolor**, have been proposed. In [**24**], the authors constructed a specific example featuring the gauge group

$$SO(N)_{\mathrm{SC}} \otimes SU(4)_{\mathrm{PS}} \otimes SU(2)_{\mathrm{L}} \otimes T_{3\mathrm{R}}, \tag{5.1}$$

where $SO(N)_{\mathrm{SC}}$ is the supercolor group which becomes strong at the scale Λ_{SC}, $SU(4)_{\mathrm{PS}}$ is the Pati-Salam group, $T_{3\mathrm{R}}$ is the third component of right-handed isospin and $SU(2)_{\mathrm{L}}$ is the standard left-handed weak isospin. The weak hypercharge in such a model is a linear conbination of $T_{3\mathrm{R}}$ and the 15th component of $SU(4)_{\mathrm{PS}}$.

While in [**26**], the gauge group is of the form

$$SU(M)_{\mathrm{SC}} \otimes SU(N)_{\mathrm{TC}} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y, \tag{5.2}$$

with coupling constants respectively g_M, g_N, g_3, g_2 and g_1. The first two groups (supercolor and technicolor) have been taken into account for breaking supersymmetry and electroweak interactions, $SU(2)_L \otimes U(1)_Y$, respectively. They assume that the supercolor, technicolor and color interaction are all asymptotically free and characterized by scales $\Lambda_{\mathrm{SC}}, \Lambda_{\mathrm{TC}}$, and Λ_{C}, respectively, such that

$$\Lambda_{\mathrm{SC}} \gg \Lambda_{\mathrm{TC}} \gg \Lambda_{\mathrm{C}}, \tag{5.3}$$

Technically, this model has two weak doublet chiral superfields with supercolor interactions. The component fields of these doublets form condensates at a scale Λ_{SC} of order 10 TeV. These condensates break SUSY but not $SU(2)_L \otimes U(1)_Y$. The breaking of SUSY gives rise to masses for the scalar partners of quarks and leptons, as well as for fermionic partners of all gauge bosons.

In addition, the model contains a weak doublet and two weak singlet chiral superfields with technicolor. At a scale $\Lambda_{\rm TC}$, the fields in principle will form condensates which breaks $SU(2)_L \otimes U(1)_Y$ down to $U(1)_{EM}$ electromagnetism. Importantly, the model also contains two weak doublet (Higgs) superfields, neutral under all strong gauge groups, introduced to give rise to masses for quarks and leptons. Lastly, the model has weak Yukawa couplings to the technicolored fields as well as the quark and lepton superfields. As a result of these couplings, the techniquark condensates induce vacuum expectation values (VEV's) for the Higgs fields. These VEV's give masses to quarks and leptons.

Here we consider the case in which the technicolor group is SU(2). In this report, the $SU(2)$ technicolor gauge group is taken into account. Contrary to [**24**] and [**26**], the model features two weak doublet superfields charged under the technicolor gauge group. By combining the supersymmetric technicolor sector with the traditional MSSM we define the super technicolor models [**21**].

Clearly these type of models are still a highly natural extension of the SM while possessing further advantages with respect to the traditional technicolor models:

i) It is possible to give masses to the standard model fermions without introducing extended technicolor interactions which are typically hard to construct.

ii) The top mass can be easily reproduced.

iii) One can interpolate, depending on the way that supersymmetry is broken, between MSSM and ordinary technicolor.

We will also see that the simplest incarnation of the supertechnicolor model feature, for the first time, the maximal supersymmetry in four dimensions.

1. The MSSM Sector

In general, we can write the supersymmetric Lagrangian in the following form:

$$\begin{aligned}\mathcal{L}_{\rm SUSY} = & \frac{1}{16kg^2}\mathrm{Tr}\left[\int d^2\theta d^2\bar{\theta}\left(W^\alpha W_\alpha \delta^2\bar{\theta} + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}\delta^2\theta\right)\right] \\ & + \int d^2\theta d^2\bar{\theta}\Phi_i^\dagger \exp\left(2gV\right)\Phi_i \\ & + \int d^2\theta d^2\bar{\theta}\left[P\delta^2\bar{\theta} + h.c.\right], \end{aligned} \tag{5.4}$$

where

$$W_\alpha = -\frac{1}{8}\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}e^{-2gV}D_\alpha e^{+2gV}, \tag{5.5.1}$$

$$\bar{W}_{\dot{\alpha}} = +\frac{1}{8}D^\alpha D_\alpha e^{+2gV}\bar{D}_{\dot{\alpha}}e^{-2gV}. \tag{5.5.2}$$

Here $V = V^a T^a_A$ is the vector super field in the Wess-Zumino gauge $V_{WZ}(x, \theta, \dot{\theta})$ given already in chapter 4.

Applying some results given in chapter 4, the defined Lagrangian can be written as follow:

$$\mathcal{L}_{\rm SUSY} = \mathscr{L}_{\rm kin} + \mathscr{L}_{g-{\rm Yuk}} + \mathscr{L}_D + \mathscr{L}_F + \mathscr{L}_{\text{P-Yuk}} + \mathscr{L}_{\rm soft}, \tag{5.6.1}$$

or, in terms of component fields,

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}F^{(a)}_{j\mu\nu}F^{(a)\mu\nu}_j - i\bar{\lambda}^{(a)}_j\bar{\sigma}^\mu D_\mu\lambda^{(a)}_j - D^\mu\phi^{(a)\dagger}_i D_\mu\phi^{(a)}_i - i\bar{\psi}^{(a)}_i\bar{\sigma}^\mu D_\mu\psi^{(a)}_i, \tag{5.6.2}$$

$$\mathcal{L}_{g-{\rm Yuk}} = \sum_j i\sqrt{2}g_j\left[\phi^\dagger_i T^{(a)}_j\psi_i\lambda^{(a)}_j - \bar{\lambda}^{(a)}_j\bar{\psi}_i T^{(a)}_j\phi_i\right], \tag{5.6.3}$$

$$\mathcal{L}_F = -\left|\frac{\partial P}{\partial\phi^{(a)}_i}\right|^2, \tag{5.6.4}$$

$$\mathcal{L}_D = -\frac{1}{2}\sum_j g^2_j\left(\phi^\dagger_i T^{(a)}_j\phi_i\right)^2, \tag{5.6.5}$$

$$\mathcal{L}_{\text{P-Yuk}} = -\frac{1}{2}\left[\frac{\partial^2 P}{\partial\phi^{(a)}_i\partial\phi^{(b)}_k}\psi^{(a)}_i\psi^{(b)}_k + h.c.\right], \tag{5.6.6}$$

where the first expression, Eq.(5.6.2), represents the kinetic terms, the second one corresponds to the Yukawa terms obtained from gauge and superpotential interactions, while the third and the forth ones are the F and D scalar interaction terms, respectively, and the soft SUSY breaking Lagrangian is given in Chapter 4.

We now begin by considering $\mathcal{N}=4$ supersymmetric Yang-Mills theory with gauge group $SU(N)$ in four dimensions. The $\mathcal{N}=4$ supersymmetric Lagrangian for an $SU(N)$ gauge theory can be written in terms of three $\mathcal{N}=1$ chiral superfields $\Phi_i, i=1,2,3$ and one $\mathcal{N}=1$ vector superfield V, in which all such superfields fall in the adjoint representation of $SU(N)$. Here we introduce the superpotential for the $\mathcal{N}=4$ Lagrangian in the following form [**22**]:

$$P = -\frac{g}{3\sqrt{2}}\epsilon_{ijk}f^{abc}\Phi^a_i\Phi^b_j\Phi^c_k, \quad j,k=1,\ 2,\ 3; \quad a,b,c,=1,\ 2,...,\ N^2-1, \tag{5.7}$$

where g is the coupling constant, and f^{abc} is the structure constant. After substituting the superpotential in Eq.(5.7) into Eq.(5.6), we obtain

$$\begin{aligned}\mathcal{L}_{\rm SUSY} = &-\frac{1}{4}F^{(a)}_{j\mu\nu}F^{(a)\mu\nu}_j - i\bar{\lambda}^{(a)}_j\bar{\sigma}^\mu D_\mu\lambda^{(a)}_j - D^\mu\phi^{(a)\dagger}_i D_\mu\phi^{(a)}_i - i\bar{\psi}^{(a)}_i\bar{\sigma}^\mu D_\mu\psi^{(a)}_i\\ &+\sqrt{2}gf^{abc}\left(\phi^\dagger_i\psi^b_i\lambda^a - \bar{\lambda}^a\bar{\psi}^b_i\phi^a_i\right) + \frac{g}{\sqrt{2}}\epsilon_{ijk}f^{abc}\left(\phi^a_i\psi^b_j\psi^c_k + \bar{\psi}^c_k\bar{\psi}^b_j\phi^{a\dagger}_i\right)\\ &-\frac{1}{2}g^2\left(f^{abd}f^{ace} + f^{abe}f^{acd}\right)\phi^{b\dagger}_i\phi^c_i\phi^{d\dagger}_j\phi^e_j,\end{aligned} \tag{5.8}$$

where λ is the gaugino, while ψ_i and ϕ_i are the fermionic and bosonic components of the chiral superfield Φ_i, respectively. Further details of deriving Eq.(5.8) are presented in Appendix B. Here the field strength tensor $F^a_{\mu\nu}$ and covariant derivative D_μ are respectively defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu, \tag{5.9.1}$$

$$D_\mu \zeta^a = \partial_\mu \zeta^a - g f^{abc} A^b_\mu \zeta^c, \quad \zeta^c = \lambda^c,\ \psi^c,\ \phi^c. \tag{5.9.2}$$

Furthermore, to make explicit the $SU(4)$ R-symmetry of the above Lagrangian, we can use the following change of variables:

$$\varphi^a_{rs} = -\varphi^a_{sr},\ \varphi^a_{i4} = \frac{1}{2}\phi^a_i,\ \varphi^a_{ij} = \frac{1}{2}\epsilon_{ijk}\phi^{a\dagger}_k,\ \eta^a_i = \psi^a_i,\ \eta^a_4 = \lambda^a;,\quad r,\ s = 1,\ 2,\ 3,\ 4, \tag{5.10}$$

or precisely,

$$\varphi^a_{rs} = \frac{1}{2}\begin{pmatrix} 0 & \phi^{a\dagger}_3 & -\phi^{a\dagger}_2 & \phi^a_1 \\ -\phi^{a\dagger}_3 & 0 & \phi^{a\dagger}_1 & \phi^a_2 \\ \phi^{a\dagger}_2 & -\phi^{a\dagger}_1 & 0 & \phi^a_3 \\ -\phi^a_1 & -\phi^a_2 & -\phi^a_3 & 0 \end{pmatrix}, \quad \eta^a_r = \begin{pmatrix} \psi^a_1 \\ \psi^a_2 \\ \psi^a_3 \\ \lambda^a \end{pmatrix}. \tag{5.11}$$

The Lagrangian in Eq.(5.8) can be rewritten in the form

$$\begin{aligned}\mathcal{L}_{\text{SUSY}} = &-\frac{1}{4}F^{(a)}_{\mu\nu}F^{(a)\mu\nu} - \text{Tr}D^\mu\varphi^{a\dagger}D_\mu\varphi^a - i\bar{\eta}^a_r\bar{\sigma}^\mu D_\mu\eta^a_r \\ &- \sqrt{2}g f^{abc}\left(\varphi^{a\dagger}_{rs}\eta^b_r\eta^c_s + \bar{\eta}^c_r\bar{\eta}^b_s\varphi^a_{rs}\right) \\ &- \frac{1}{2}g^2\left(f^{abd}f^{ace} + f^{abe}f^{acd}\right)\text{Tr}\varphi^{b\dagger}\varphi^c\text{Tr}\varphi^{d\dagger}\varphi^e.\end{aligned} \tag{5.12}$$

Further details of deriving Eq.(5.12) are presented in Appendix B. Under $SU(4)$, we see that φ^a_{rs} transform as **6**, ξ^a_r transforms as a **4**, while A^a_μ transforms as a **1** which leave the Lagrangian in Eq.(5.12) unchanged.

2. Higgs Sector

In order to cancel the gauge anomaly, contrary to the SM, a second Higgs doublet has to be introduced. Such two Higgs doublets have the following charge assignment:

$$\hat{H} \sim \left(1, 1, 2, +\frac{1}{2}\right), \hat{H}' \sim \left(1, 1, 2, -\frac{1}{2}\right). \tag{5.13}$$

One Higgs doublet, H, gives mass to the up-tye fermions (with weak isospin $+1/2$), while the another doublet, H', gives mass to the down-type fermions (with weak isospin $-1/2$) and charged leptons.

Recalling the trilinear superpotential given in chapter 4

$$P_{\text{MSSM}} = \epsilon_{\rho\sigma}\left[-\hat{H}^\rho\hat{Q}^\sigma_i y^{ij}_u\hat{\bar{U}}_j + \hat{H}'^\rho\hat{Q}^\sigma_i y^{ij}_d\hat{\bar{D}}_j + \hat{H}'^\rho\hat{L}^\sigma_i y^{ij}_e\hat{\bar{E}}_j\right], \tag{5.14}$$

concretely, the vacuum expectation values of two Higgs doublet described by $H \equiv H_u$ and $H' \equiv H_d$ generate up-quark, and down-quark and charged-lepton masses, given by $m_u = y_u v_u/2$, $m_d = y_d v_d/2$ and $m_e = y_e v_d/2$, respectively.

3. The M4ST Sector

The gauge group we study here is $SU(2)_{\rm TC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. In the technicolor sector, the field content is constituted by four techni-fermions and also one techni-gluon in which all field contents fall in the adjoint representation of $SU(2)_{\rm TC}$. The only source of electroweak symmetry breaking is the vacuum expectation value of the techni-fermion bilinear.

Now let us upgrade minimal walking technicolor, presented briefly in chapter 3, to $\mathcal{N} = 4$ super Yang-Mills (4SYM). To couple the new supersymmetric sector to the weak and hypercharge interactions of the SM, we need to gauge part of the $SU(4)_R$ global symmetry of the supertechnicolor theory. In doing so, one of the four Weyl technifermions is identified as the techni-gaugino and is assigned to be singlet under the SM gauge group. Here, the only possible choice for this role is $\bar{D}_R$ for $y = +1$, whereas the alternative candidate is $\bar{U}_R$ in the case of $y = -1$.

Superfields	$SU(2)_{\rm TC}$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\hat{Q}_L = \begin{pmatrix}\Phi_1 \\ \Phi_2\end{pmatrix}$	$\square\!\square$	1	$\square$	+1/2
$\Phi_3 = \boxed{\varphi_{\bar{U}_R}, \bar{U}_R}$	$\square\!\square$	1	1	-1
$V = \boxed{G, \bar{D}_R}$	$\square\!\square$	1	1	$\emptyset$
$\hat{L}_L = \begin{pmatrix}\hat{\Lambda}_1 \\ \hat{\Lambda}_2\end{pmatrix}$	1	1	$\square$	-3/2
$\hat{N} = (\varphi_{\bar{N}_R}, \bar{N}_R)$	1	1	1	+1
$\hat{E} = (\varphi_{\bar{E}_R}, \bar{E}_R)$	1	1	1	+2
$\hat{H} = \left(H, \tilde{H}\right)$	1	1	$\square$	+1/2
$\hat{H}' = \left(H', \tilde{H}'\right)$	1	1	$\square$	-1/2

Table 5.1: Vector superfield, chiral superfield and Higgs superfield properties are summarized in terms of Young Tableaux

Here, we will choose $y = +1$ and identify $\bar{D}_R$ as the techni-gaugino. The charges of the technicolored particles are implemented by imposing hypercharge conservation as well as the cancellation of the gauge anomalies. The technicolored chiral superfields and their $SU(2)_{\rm TC} \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ representations are listed below [**21**]:

$$
\begin{aligned}
Q_L &\sim \left(3, 1, 2, +\frac{1}{2}\right), \quad \bar{U}_R \sim (3, 1, 1, -1), \quad \bar{D}_R \sim (3, 1, 1, 0), \\
L_L &\sim \left(1, 1, 2, -\frac{3}{2}\right), \quad \bar{N}_R \sim (1, 1, 1, +1), \quad \bar{E}_R \sim (1, 1, 1, +2).
\end{aligned} \tag{5.15}
$$

Based on these assignments, we can define the scalar and fermion components of the $\mathcal{N}=4$ superfields, presented in terms of three $\mathcal{N}=1$ chiral superfields Φ_i and one $\mathcal{N}=1$ vector superfield, as follows:

$$\Phi_1 \ni \left(\tilde{U}_L, U_L\right), \quad \Phi_2 \ni \left(\tilde{D}_L, D_L\right), \quad \Phi_3 \ni \left(\tilde{\bar{U}}_R, \bar{U}_R\right), \quad V \ni \left(G, \bar{D}_R\right), \tag{5.16}$$

with a tilde labeling the scalar superpartner of the corresponding fermion. Here, $\Phi_i,\ i=1,2,3$ represent the three chiral superfield of 4SYM and V represents the vector superfield. Furthermore, there are four more chiral superfields needed to supersymmetrize the MWT model. These additional chiral superfields are

$$\hat{\Lambda}_1 \ni \left(\tilde{N}_L, N_L\right), \quad \hat{\Lambda}_2 \ni \left(\tilde{E}_L, E_L\right), \quad \hat{N} \ni \left(\tilde{\bar{N}}_R, \bar{N}_R\right), \quad \hat{E} \ni \left(\tilde{\bar{E}}_R, \bar{E}_R\right). \tag{5.17}$$

3.1. The M4ST Superpotential. The M4ST superpotential can be written as

$$P = P_{\text{MSSM}} + P_{\text{TC}}, \tag{5.18}$$

where P_{MSSM} is the MSSM superpotential given in Eq.(5.7), and

$$P_{\text{TC}} = \underbrace{-\frac{g_{\text{TC}}}{3\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\Phi_i^a\Phi_j^b\Phi_k^c}_{P_{\text{TC}}^{\text{pure}}} + \underbrace{y_U\epsilon_{ij3}\Phi_i^a\hat{H}_j\Phi_3^a + y_N\epsilon_{ij3}\hat{\Lambda}_i\hat{H}_j\hat{N} + y_E\epsilon_{ij3}\hat{\Lambda}_i\hat{H}'_j\hat{E}}_{P_{\text{mix}}}, \tag{5.19}$$

where P_{mix} contains both TC fields and MSSM (Higgs) ones. To recover the $\mathcal{N}=4$ invariance, one takes the limit when g_{TC} is much greater than the gauge coupling constants g_L, g_Y, and Yukawa coupling constants y_U, y_N, y_E. Note that the gauge invariance of the $P_{\text{TC}}^{\text{pure}}$ in Eq.(5.19) is guaranteed by the unbroken $SU(4)$ flavor symmetry and the need of gauge anomaly cancellation.

3.2. The M4ST Lagrangian. The Lagrangian of the M4ST can be written as

$$\mathcal{L}_{\text{M4ST}} = \mathcal{L}_{\text{MSSM}} + \mathcal{L}_{\text{TC}}, \tag{5.20}$$

where $\mathcal{L}_{\text{MSSM}}$ is the MSSM Lagrangian, and

$$\begin{aligned}\mathcal{L}_{\text{TC}} = &\frac{1}{2}\text{Tr}\left[\int d^2\theta d^2\bar{\theta}\left(W^\alpha W_\alpha\delta^2\bar{\theta} + \overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}\delta^2\theta\right)\right]\\ &+ \int d^2\theta d^2\bar{\theta}\Phi_f^\dagger \exp\left(2g_X V_X\right)\Phi_f\\ &+ \int d^2\theta d^2\bar{\theta}\left[P_{\text{TC}}\delta^2\bar{\theta} + h.c.\right],\end{aligned} \tag{5.21}$$

where

$$\Phi_f = \hat{Q},\ \Phi_3,\ \hat{\Lambda},\ \hat{N},\ \hat{E}; \quad X = TC,\ C,\ L,\ Y, \tag{5.22}$$

where $\hat{Q}$ and $\hat{\Lambda}$ are defined as the weak doublet superfields with components $\Phi_1,\ \Phi_2$, and $\hat{\Lambda}_1,\ \hat{\Lambda}_2$, respectively. The superpotential P_{TC} is given already in Eq.(5.19). In the same manner as MSSM, the first line in Eq.(5.21) gives rise to the kinetic terms of the Lagrangian for the self-interacting techni-gluon and techni-gaugino, while the second line gives rise to the kinetic terms for the TC superfields. Finally, the last line corresponds to the mixing between TC and MSSM superfields.

4. Technicolor Meets Supersymmetry

Using all given equations in Eqs.(5.6), it is rather straightforward to precisely write the M4ST Lagrangian. Here, let us present the result only for the $\mathcal{L}_{\rm TC}$ sector. First of all, the kinetic terms are rather trivial:

$$\begin{aligned}\mathcal{L}_{\rm TC,kin} =& -\frac{1}{4}\mathcal{F}^a_{\mu\nu,\rm TC}\mathcal{F}^{a\mu\nu}_{\rm TC} - iD^a_R\bar{\sigma}^\mu D_\mu \bar{D}^a_R \\ & - D^\mu \Phi^{a\dagger}_i D_\mu \Phi^a_i \quad \left(\Phi^a_i = \tilde{Q}_L, \tilde{\bar{U}}_R, \tilde{L}_L, \tilde{\bar{N}}_R, \tilde{\bar{E}}_R\right) \\ & - i\bar{\Psi}^a_i \bar{\sigma}^\mu D_\mu \Psi^a_i, \quad \left(\Psi^a_i = Q_L, \bar{U}_R, L_L, \bar{N}_R, \bar{E}_R\right)\end{aligned} \tag{5.23}$$

where $\mathcal{F}^a_{\mu\nu,\rm TC} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g_{TC}\epsilon^{abc}\mathcal{A}^b_\mu \mathcal{A}^c_\nu$ is the TC field strength tensor, $a, b, c = 1, 2, 3$. For the left-handed techni-fermions, the covariant derivative takes the form

$$D_\mu Q^a_L = \left(\delta^{ac}\partial_\mu + g_{TC}\mathcal{A}^b_\mu \epsilon^{abc} - \frac{ig}{2}\vec{W}_\mu \cdot \vec{\tau}\delta^{ac} - ig'\frac{y}{2}B_\mu \delta^{ac}\right) Q^c_L, \tag{5.24}$$

where $\mathcal{A}_\mu$ are the techni-gauge bosons, while W_μ are the gauge bosons associated to $SU(2)_L$, and B_μ is the gauge boson associated to the hypercharge assiments. Here, τ^a are the ordinary Pauli matrices and ϵ^{abc} is the anti-symmetric tensor. There is no weak interaction (the third term) in the case of right-handed techni-fermions, and the hypercharge assignment $y/2$ has to be replaced according to whether it is an up- or down-type techni-quarks. In the same manner, the left-handed leptons do not contain the second terms (technicolor interactions), and the hypercharge $y/2$ has to be replaced by $-3y/2$. Finally, only the last term is present for the right-handed leptons with an appropriate hypercharge assignment. Here, we have herein selected $y = +1$ for our theory.

The gauge Yukawa terms are given by

$$\begin{aligned}\mathcal{L}_{g-\rm Yuk} =& \sqrt{2}g_{TC}\times \\ & \left[\tilde{\bar{U}}^b_L U^c_L \bar{D}^a_R - D^a_R \bar{U}^b_L \tilde{U}^c_L + \tilde{\bar{D}}^b_L D^c_L \bar{D}^a_R - D^a_R \bar{D}^b_L \tilde{D}^c_L + \tilde{U}^b_L \bar{U}^c_R \bar{D}^a_R - D^a_R U^b_L \tilde{\bar{U}}^c_R\right]\epsilon^{abc} \\ & + \frac{i}{\sqrt{2}}g_L\left[\tilde{\bar{Q}}^i_L Q^j_L \tilde{W}^k - \tilde{\bar{W}}^k \bar{Q}^i_L \tilde{Q}^j_L + \tilde{\bar{L}}^i_L L^j_L \tilde{W}^k - \tilde{\bar{W}}^k \bar{L}^i_L \tilde{L}^j_L\right]\sigma^k_{ij} \\ & + i\sqrt{2}g_Y \sum_p Y_p \left[\tilde{\bar{\psi}}_p \psi_p \tilde{B} - \tilde{\bar{B}}\bar{\psi}_p \tilde{\psi}_p\right], \quad \psi_p = U^a_L, D^a_L, \bar{U}^a_R, N_L, E_L, \bar{N}_R, \bar{E}_R,\end{aligned} \tag{5.25}$$

where $\tilde{W}^k$ and $\tilde{B}$ are the wino and the bino, respectively, σ^k is the 2×2 Pauli matrices, $i, j = 1, 2; k, a, b, c = 1, 2, 3$, and each field ψ_p has the hypercharge Y_p presented in Eq.(5.15).

The D terms are given by

$$\mathcal{L}_D = -\frac{1}{2}\left[g^2_{TC}D^a_{TC}D^a_{TC} + g^2_L D^k_L D^k_L + g^2_Y D_Y D_Y\right] + \frac{1}{2}\left[g^2_L D^k_L D^k_L + g^2_Y D_Y D_Y\right]_{\rm MSSM}, \tag{5.26}$$

where

$$
\begin{aligned}
&D^a_{TC} = -i\epsilon^{abc}\left(\bar{\tilde{U}}^b_L \tilde{U}^c_L + \bar{\tilde{D}}^b_L \tilde{D}^c_L + \tilde{U}^b_R \bar{\tilde{U}}^c_R\right), \; D^k_L = \frac{1}{2}\sigma^k_{ij}\left(\bar{\tilde{Q}}^i_L \tilde{Q}^j_L + \bar{\tilde{L}}^i_L \tilde{L}^j_L\right) + D^k_{\text{MSSM}}, \\
&D_Y = \sum_p Y_p \bar{\tilde{\psi}}_p \tilde{\psi}_p + D_{Y,\text{MSSM}}.
\end{aligned} \tag{5.27}
$$

The rest of the scalar interaction terms is given by

$$
\begin{aligned}
\mathcal{L}_F = &- g^2_{TC}\left(\tilde{U}^b_L \bar{\tilde{U}}^b_L + \tilde{D}^b_L \bar{\tilde{D}}^b_L + \bar{\tilde{U}}^b_R \tilde{U}^b_R\right)^2 \\
&+ g^2_{TC}\left(\tilde{U}^b_L \bar{\tilde{U}}^c_L + \tilde{D}^b_L \bar{\tilde{D}}^c_L + \bar{\tilde{U}}^b_R \tilde{U}^c_R\right)\left(\bar{\tilde{U}}^b_L \tilde{U}^c_L + \bar{\tilde{D}}^b_L \tilde{D}^c_L + \tilde{U}^b_R \bar{\tilde{U}}^c_R\right) \\
&- y^2_U\left(\tilde{H}_1 \tilde{D}^a_L - \tilde{H}_2 \tilde{U}^a_L\right)\left(\bar{\tilde{H}}_1 \bar{\tilde{D}}^a_L - \bar{\tilde{H}}_2 \bar{\tilde{U}}^a_L\right) + \tilde{U}^a_R \bar{\tilde{U}}^a_R\left(\tilde{H}_1 \bar{\tilde{H}}_1 + \tilde{H}_2 \bar{\tilde{H}}_2\right) \\
&- y^2_U \tilde{U}^a_R \bar{\tilde{U}}^b_R\left(\bar{\tilde{U}}^a_L U^b_L + \bar{\tilde{D}}^a_L \tilde{D}^b_L\right) - y^2_N\left(\bar{\tilde{N}}_L \bar{\tilde{H}}_2 - \bar{\tilde{E}}_L \bar{\tilde{H}}_1\right)\left(\tilde{N}_L \tilde{H}_2 - \tilde{E}_L \tilde{H}_1\right) \\
&- y^2_N \tilde{N}_R \bar{\tilde{N}}_R\left(\tilde{H}_1 \bar{\tilde{H}}_1 + \tilde{H}_2 \bar{\tilde{H}}_2 + \tilde{N}_L \bar{\tilde{N}}_L + \tilde{E}_L \bar{\tilde{E}}_L\right) \\
&- y^2_E\left(\bar{\tilde{N}}_L \bar{\tilde{H}}'_2 - \bar{\tilde{E}}_L \bar{\tilde{H}}'_1\right)\left(\tilde{N}_L \tilde{H}'_2 - \tilde{E}_L \tilde{H}'_1\right) \\
&- y^2_E \tilde{N}_R \bar{\tilde{N}}_R\left(\tilde{H}'_1 \bar{\tilde{H}}'_1 + \tilde{H}'_2 \bar{\tilde{H}}'_2 + \tilde{E}_L \bar{\tilde{E}}_L + \tilde{E}_L \bar{\tilde{E}}_L\right) \\
&+ \{\sqrt{2} y_U g_{TC} \epsilon^{abc}\left[\tilde{U}^b_L \tilde{D}^c_L\left(\bar{\tilde{H}}_1 \bar{\tilde{D}}^a_L - \bar{\tilde{H}}_2 \bar{\tilde{U}}^a_L\right) + \bar{\tilde{U}}^b_R \tilde{U}^c_L \bar{\tilde{H}}_1 \tilde{U}^a_R + \bar{\tilde{U}}^b_R \tilde{D}^c_L \bar{\tilde{H}}_2 \tilde{U}^a_R\right] \\
&- y_U y_N \tilde{U}^a_R \bar{\tilde{N}}_R\left(\bar{\tilde{U}}^a_L \tilde{N}_L + \bar{\tilde{D}}^a_L \tilde{E}_L\right) - y_N y_E \tilde{N}_R \bar{\tilde{E}}_R\left(\bar{\tilde{H}}_1 \tilde{H}'_1 + \bar{\tilde{H}}_2 \tilde{H}'_2\right) + h.c.\} \\
&+ \mathcal{L}_{\text{mix}},
\end{aligned} \tag{5.28}
$$

where $\mathcal{L}_{\text{mix}}$ is defined as a function of the auxiliary fields F associated with the MSSM two Higgs super-doublets:

$$
\mathcal{L}_{\text{mix}} = -\sum_{\phi_p} F_{\phi_p,TC} F^\dagger_{\phi_p,MSSM}, \quad \phi_p = H_1, H_2, H'_1, H'_2, \tag{5.29}
$$

where

$$
\begin{aligned}
&F_{H_1,TC} = -y_U \tilde{D}^a_L \bar{\tilde{U}}^a_R - y_N \tilde{E}_L \bar{\tilde{N}}_R, \\
&F_{H_2,TC} = y_U \tilde{U}^a_L \bar{\tilde{U}}^a_R + y_N \tilde{N}_L \bar{\tilde{N}}_R, \\
&F_{H'_1,TC} = -y_E \tilde{E}_L \bar{\tilde{E}}_R, \quad F_{H'_2,TC} = y_E \tilde{N}_L \bar{\tilde{E}}_R.
\end{aligned} \tag{5.30}
$$

The remaining Yukawa interaction terms determined by the superpotential terms are given by

$$\begin{aligned}\mathcal{L}_{\text{P-Yuk}} &= \sqrt{2} g_{TC} \epsilon^{abc} \left[U_L^a D_L^a \tilde{\bar{U}}_R^c + U_L^a \tilde{D}_L^b \bar{U}_R^c + \tilde{U}_L^a U_L^b \bar{U}_R^c \right] \\ &+ y_U \left[\left(H_1 D_L^a - H_2 U_L^a \right) \tilde{\bar{U}}_R^a + \left(\tilde{H}_1 D_L^a - \tilde{H}_2 U_L^a \right) \bar{U}_R^a + \left(H_1 \tilde{D}_L^a - H_2 \tilde{U}_L^a \right) \bar{U}_R^a \right] \\ &+ y_N \left[\left(H_1 E_L - H_2 N_L \right) \tilde{\bar{N}}_R + \left(H_1 \tilde{E}_L - H_2 \tilde{N}_L \right) \bar{N}_R + \left(\tilde{H}_1 E_L - \tilde{H}_2 N_L \right) \bar{N}_R \right] \\ &+ y_E \left[\left(H_1' E_L - H_2' N_L \right) \tilde{\bar{E}}_R + \left(H_1' \tilde{E}_L - H_2' \tilde{N}_L \right) \bar{E}_R + \left(\tilde{H}_1' E_L - \tilde{H}_2' N_L \right) \bar{E}_R \right] \\ &+ h.c. \end{aligned} \tag{5.31}$$

By adding the techni-gaugino and scalar mass terms into the superpotential in Eq.(5.19), i.e. $-1/2 M_\lambda^2 \lambda^a \lambda^a + h.c$ and $-M_{\Phi_i}^2 \Phi_i^\dagger \Phi_i$, the soft SUSY breaking terms are given by

$$\begin{aligned}\mathcal{L}_{\text{soft}} &= -a_{\text{TC}} \epsilon^{abc} \Phi_1^a \Phi_2^b \Phi_3^c + a_U \epsilon_{ij3} \Phi_i^a \hat{H}_j \Phi_3^a + a_N \epsilon_{ij3} \hat{\Lambda}_i \hat{H}_j \hat{N} + a_E \epsilon_{ij3} \hat{\Lambda}_i \hat{H}_j' \hat{E} \\ &\quad - \frac{1}{2} M_\lambda^2 \lambda^a \lambda^a + h.c \\ &\quad - M_{\Phi_i}^2 \Phi_i^\dagger \Phi_i \\ &= -a_{\text{TC}} \epsilon^{abc} \Phi_1^a \Phi_2^b \Phi_3^c + a_U \left[\Phi_1^a \hat{H}_2 - \Phi_2^a \hat{H}_1 \right] \Phi_3^a \\ &\quad + a_N \left[\hat{\Lambda}_1 \hat{H}_2 - \hat{\Lambda}_2 \hat{H}_1 \right] \hat{N} + a_E \left[\hat{\Lambda}_1 \hat{H}_2' - \hat{\Lambda}_2 \hat{H}_1' \right] \hat{E} \\ &\quad - \frac{1}{2} M_\lambda^2 \lambda^a \lambda^a + h.c \\ &\quad - M_{\Phi_i}^2 \Phi_i^\dagger \Phi_i, \end{aligned} \tag{5.32.1}$$

or

$$\begin{aligned}\mathcal{L}_{\text{soft}} &= - \left[a_{TC} \epsilon^{abc} \tilde{U}_L^a \tilde{D}_L^b \tilde{\bar{U}}_R^c + a_U \left(\tilde{H}_1 \tilde{D}_L^a - \tilde{H}_2 \tilde{U}_L^a \right) \tilde{\bar{U}}_R^a + a_N \left(\tilde{H}_1 \tilde{E}_L - \tilde{H}_2 \tilde{N}_L \right) \tilde{\bar{N}}_R^a \right. \\ &\quad \left. + a_E \left(\tilde{H}_1' \tilde{E}_L - \tilde{H}_2' \tilde{N}_L \right) \tilde{\bar{E}}_R^a + \frac{1}{2} M_D \bar{D}_R^a \bar{D}_R^a + h.c. \right] - M_Q^2 \tilde{\bar{Q}}_L^a \tilde{Q}_L^a - M_U^2 \tilde{\bar{U}}_R^a \tilde{U}_R^a \\ &\quad - M_L^2 \tilde{\bar{L}}_L \tilde{L}_L - M_N^2 \tilde{\bar{N}}_R \tilde{N}_R - M_E^2 \tilde{\bar{E}}_R \tilde{E}_R. \end{aligned} \tag{5.32.2}$$

Note here that more details of deriving for Eq.(5.31) are given in Appendix E.

CHAPTER 6

Future Research Plans

1. Novel Candidates for Dark Matter

One of the significant flaws of the Standard Model (SM) is that the SM itself does not account for dark matter (DM). One suggests that DM, for example, can be represented in the form of "Weakly Interacting Massive Particles (WIMPs)". The WIMP paradigm argues that DM consists of particles which interact very weakly with SM charged particles.

Now the WIMP paradigm has being attractive since several WIMP candidates arise naturally in the context of some theories beyond the SM such as supersymmetry, e.g. [**30**]. In the context of MSSM, there are four neutralinos, $\tilde{\chi}_i^0$, which may yield the DM candidates.

Based on [**21**], I would like to study further the WIMPs motivated by supersymmetric technicolor model, supertechnicolor WIMPs. I do expect here that there would be several viable supertechnicolor WIMPs which may yield novel candidates for DM.

2. Different Matter Representations

Having proposed the $\mathcal{N} = 1$ extensions of the Minimal Walking Technicolor, called Minimal $\mathcal{N} = 1$ Super Technicolor (MST) [**21**], they have taken into account the chiral (matter) superfields transforming according to the adjoint representation of the $SU(2)_{\mathrm{TC}}$ gauge group.

However, one can generalize to TC gauge group from $SU(2)_{\mathrm{TC}}$ to $SU(N)_{\mathrm{TC}}$, and, forthermore, define the matter superfields in a representation different from the adjoint one, for example the fundamental representation and two-index antisymmetric representation, which can be phenomenologically viable interesting extensions of the SM. In looking for such viable, alternative to the M4ST or MST, supertechnicolor models, the $SU(N)$ supersymmetric phase diagram for matter fields in different representations provides useful [**25**].

It would be of great interesting for me to explore these different scenarios. In particular, I would like to study the technisuperfield belonging to the fundamental representation of the $SU(3)_{\mathrm{TC}}$ gauge group.

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APPENDIX A

Details of the derivation of the expression (4.82) in chapter 4

Now let us start by considering the first contribution to the Higgs potential so-called the D-term, V_D^H:

$$V_D^H = \sum_A \frac{1}{2} D^A D^A, \qquad D^A \equiv -g_A \phi_i^* T_{ij}^A \phi_j. \tag{A1}$$

For scalar components of Higgs superfileds, $H\,(H')$, we have $Y = 1/2\,(-1/2)$ and so the D-terms are given by,

$$U(1)_Y \quad : \quad D^1 = -\frac{g'}{2}\left(|H|^2 - |H'|^2\right), \tag{A2.1}$$

$$SU(2)_L \quad : \quad D^a = -\frac{g}{2}\left(H^i \tau_{ij}^a H^j + H'^i \tau_{ij}^a H'^j\right), \tag{A2.2}$$

where $T^a = \tau^a/2$. The D-terms contribute to the scalar potential:

$$V_D^H = \frac{g'^2}{8}\left(|H|^2 - |H'|^2\right) + \frac{g^2}{8}\left(H^i \tau_{ij}^a H^j + H'^i \tau_{ij}^a H'^j\right)^2. \tag{A3}$$

Using the following $SU(2)$ identity,

$$\vec{\tau} \cdot \vec{\tau} = \sum_a \tau_{ij}^a \tau_{kl}^a = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}, \tag{A4}$$

we find

$$\begin{aligned}
\left(H^i \tau_{ij}^a H^j + H'^i \tau_{ij}^a H'^j\right)^2 &= \left(H^{i*} \tau_{ij}^a H^j + H'^{i*} \tau_{ij}^a H'^j\right)\left(H^{k*} \tau_{kl}^a H^l + H'^{k*} \tau_{kl}^a H'^l\right) \\
&= 2H^{i*} H^j \delta_{il}\delta_{jk} H^{k*} H^l - H^{i*} H^j \delta_{ij}\delta_{kl} H^{k*} H^l \\
&+ 2H^{i*} H^j \delta_{il}\delta_{jk} H'^{k*} H'^l - H^{i*} H^j \delta_{ij}\delta_{kl} H'^{k*} H'^l \\
&+ 2H'^{i*} H'^j \delta_{il}\delta_{jk} H^{k*} H^l - H'^{i*} H'^j \delta_{ij}\delta_{kl} H^{k*} H^l \\
&+ 2H'^{i*} H'^j \delta_{il}\delta_{jk} H'^{k*} H'^l - H'^{i*} H'^j \delta_{ij}\delta_{kl} H'^{k*} H'^l.
\end{aligned} \tag{A5}$$

After matching all indices of the above expression, we obtain

$$\begin{aligned}\left(H^i\tau^a_{ij}H^j + H'^i\tau^a_{ij}H'^j\right)^2 &= 2H^{i*}H^j\delta_{il}\delta_{jk}H^{k*}H^l - H^{i*}H^j\delta_{ij}\delta_{kl}H^{k*}H^l \\ &+ 2H^{i*}H^j\delta_{il}\delta_{jk}H'^{k*}H'^l - H^{i*}H^j\delta_{ij}\delta_{kl}H'^{k*}H'^l \\ &+ 2H'^{i*}H'^j\delta_{il}\delta_{jk}H^{k*}H^l - H'^{i*}H'^j\delta_{ij}\delta_{kl}H^{k*}H^l \\ &+ 2H'^{i*}H'^j\delta_{il}\delta_{jk}H'^{k*}H'^l - H'^{i*}H'^j\delta_{ij}\delta_{kl}H'^{k*}H'^l \\ &= 4\left|H^*\cdot H'\right|^2 + \left(|H|^2 - |H'|^2\right)^2. \end{aligned} \tag{A6}$$

Therefore for the D-term, we obtain

$$V_D^H = \frac{(g^2+g'^2)}{8}\left[|H|^2 - |H'|^2\right]^2 + \frac{g^2}{2}\left|H^*\cdot H'\right|^2. \tag{A7}$$

Now the last contribution to the Higgs potential is the F-term. We can write such a term as

$$V_F^H = \left|\frac{\partial P}{\partial\phi_i}\right|^2, \tag{A8}$$

where ϕ_i is a scalar component of i^{th} superfield. For Higgs superfields, P is given as

$$P_H = -\mu\epsilon_{ij}H'^iH^j. \tag{A9}$$

After plugging the expression (A9) into (A8), thus for F term, we obtain

$$V_F^H = |\mu|^2\left(|H|^2 + |H'|^2\right). \tag{A10}$$

Thus we find the potential in the following form:

$$\begin{aligned} V &= V_D^H + V_F^H \\ &= |\mu|^2\left(|H|^2+|H'|^2\right) + \frac{(g^2+g'^2)}{8}\left[|H|^2 - |H'|^2\right]^2 + \frac{g^2}{2}\left|H^*\cdot H'\right|^2. \end{aligned} \tag{A11}$$

This potential has its minimum at $\langle H^0\rangle = 0 = \langle H'^0\rangle$ which give $\langle V\rangle = 0$. This represents a model with no electroweak symmetry breaking as well as no SUSY breaking.

Adding all possible soft SUSY breaking terms given in Chapter 4, we obtain the scalar potential involving the Higgs fields in the following form:

$$\begin{aligned} V_H &= V_{\text{soft}} + V_F + V_D \\ &= \left(|\mu|^2 + m_H^2\right)|H|^2 + \left(|\mu|^2 + m_{H'}^2\right)|H'|^2 - \left(b\mu\epsilon_{ij}H'^iH^j + h.c.\right) \\ &+ \frac{g^2+g'^2}{8}\left(|H|^2 - |H'|^2\right)^2 + \frac{g^2}{2}\left|H^*\cdot H'\right|^2, \end{aligned} \tag{A12}$$

where b is a new mass parameter.

APPENDIX B

Details of the derivation of the expression (5.8) in chapter 5

First of all, let us suppress, for a while, all indices of the superpotential given by the expression (5.4) in chapter 5. By replacing the chiral superfield with its scalar componet, we now onsider

$$\frac{\partial P}{\partial \phi} = -\frac{g}{\sqrt{2}}\epsilon f \phi\phi, \qquad \frac{\partial^2 P}{\partial\phi\partial\phi} = -\sqrt{2} g \epsilon f \phi. \tag{B1}$$

Restoring all indices, we have

$$\begin{aligned}
\mathcal{L}_{P-Yuk} &= \frac{1}{\sqrt{2}} g \left(\epsilon_{ijk} f^{abc} \phi^c_k \psi^a_i \psi^b_j + h.c.\right) = \frac{1}{\sqrt{2}} g \left(-\epsilon_{kji} f^{abc} \phi^c_i \psi^a_k \psi^b_j + h.c.\right) \\
&= \frac{1}{\sqrt{2}} g \left(\epsilon_{jki} f^{abc} \phi^c_i \psi^a_j \psi^b_k + h.c.\right) = \frac{1}{\sqrt{2}} g \left(\epsilon_{ijk} f^{abc} \phi^c_i \psi^a_j \psi^b_k + h.c.\right) \\
&= \frac{1}{\sqrt{2}} g \left(-\epsilon_{ijk} f^{cba} \phi^a_i \psi^c_j \psi^b_k + h.c.\right) = \frac{1}{\sqrt{2}} g \left(\epsilon_{ijk} f^{abc} \phi^a_i \psi^b_j \psi^c_k + h.c.\right),
\end{aligned} \tag{B2.1}$$

and

$$\begin{aligned}
\mathcal{L}_F &= -\frac{1}{2} g^2 \left(\epsilon f \phi\phi\right)\left(\epsilon f \phi^\dagger \phi^\dagger\right) = -\frac{1}{2} g^2 \left(\epsilon_{ijk} f^{abc} \phi^b_j \phi^c_k\right)\left(\epsilon_{ilm} f^{ade} \phi^{d\dagger}_l \phi^{e\dagger}_m\right) \\
&= -\frac{1}{2} g^2 f^{abc} f^{ade} \epsilon_{ijk}\epsilon_{ilm} \phi^b_j \phi^c_k \phi^{d\dagger}_l \phi^{e\dagger}_m = -\frac{1}{2} g^2 f^{abc} f^{ade} \left(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}\right) \phi^b_j \phi^c_k \phi^{d\dagger}_l \phi^{e\dagger}_m \\
&= -\frac{1}{2} g^2 f^{abc} f^{ade} \left\{\phi^b_j \phi^c_k \phi^{d\dagger}_j \phi^{e\dagger}_k - \phi^b_j \phi^c_k \phi^{d\dagger}_k \phi^{e\dagger}_j\right\} \\
&= -\frac{1}{2} g^2 \left\{f^{abc} f^{ade} \phi^b_j \phi^c_k \phi^{d\dagger}_j \phi^{e\dagger}_k - f^{abc} f^{ade} \phi^b_j \phi^c_k \phi^{d\dagger}_k \phi^{e\dagger}_j\right\} \\
&= -\frac{1}{2} g^2 \left\{f^{abc} f^{ade} \underbrace{\phi^b_j \phi^c_k \phi^{d\dagger}_j \phi^{e\dagger}_k}_{b\longleftrightarrow e} - f^{abc} f^{ade} \underbrace{\phi^b_j \phi^c_k \phi^{d\dagger}_k \phi^{e\dagger}_j}_{b\longleftrightarrow d}\right\} \\
&= -\frac{1}{2} g^2 \left\{f^{aec} f^{adb} \phi^e_j \phi^c_k \phi^{d\dagger}_j \phi^{b\dagger}_k - f^{adc} f^{abe} \underbrace{\phi^d_j \phi^c_k \phi^{b\dagger}_k \phi^{e\dagger}_j}_{d\longleftrightarrow e}\right\} \\
&= -\frac{1}{2} g^2 \left\{-f^{aec} f^{adb} \phi^{b\dagger}_i \phi^c_i \phi^{d\dagger}_j \phi^e_j - f^{aec} f^{abd} \phi^{b\dagger}_i \phi^c_i \phi^{d\dagger}_j \phi^e_j\right\} \\
&= +g^2 f^{ace} f^{adb} \phi^{b\dagger}_i \phi^c_i \phi^{d\dagger}_j \phi^e_j,
\end{aligned} \tag{B2.2}$$

and

$$\begin{aligned}\mathcal{L}_D &= -\frac{1}{2}g^2\phi_i^{b\dagger}\left(-if^{abc}\right)\phi_i^c\phi_j^{d\dagger}\left(-if^{ade}\right)\phi_j^e \\ &= \frac{1}{2}g^2 f^{abc}f^{ade}\phi_i^{b\dagger}\phi_i^c\phi_j^{d\dagger}\phi_j^e. \end{aligned} \tag{B2.3}$$

So we obtain

$$\begin{aligned}\mathcal{L}_F + \mathcal{L}_D &= \frac{1}{2}g^2\left(f^{abc}f^{ade} + 2f^{ace}f^{adb}\right)\phi_i^{b\dagger}\phi_i^c\phi_j^{d\dagger}\phi_j^e \\ &= \frac{1}{2}g^2\left(f^{abc}f^{ade} - 2f^{ace}f^{abd}\right)\phi_i^{b\dagger}\phi_i^c\phi_j^{d\dagger}\phi_j^e. \end{aligned} \tag{B3}$$

Considering the following Jacobi identity:

$$\left[T^d,\left[T^b,T^c\right]\right] + \left[T^b,\left[T^c,T^d\right]\right] + \left[T^c,\left[T^d,T^b\right]\right] = 0, \tag{B4}$$

we have

$$f^{abc}f^{eda} + f^{acd}f^{eba} + f^{adb}f^{eca} = 0. \tag{B5}$$

Consider

$$\begin{aligned}f^{abc}f^{ade} - 2f^{ace}f^{abd} &= f^{acd}F^{eba} + f^{adb}f^{eca} - 2f^{ace}f^{abd} \\ &= -f^{acd}f^{abe} - f^{ace}f^{abd}. \end{aligned} \tag{B6}$$

Therefore, the Eq.(B3) finally becomes

$$\begin{aligned}\mathcal{L}_F + \mathcal{L}_D &= \frac{1}{2}g^2\left(-f^{acd}f^{abe} - f^{ace}f^{abd}\right)\phi_i^{b\dagger}\phi_i^c\phi_j^{d\dagger}\phi_j^e \\ &= -\frac{1}{2}g^2\left(f^{acd}f^{abe} + f^{ace}f^{abd}\right)\phi_i^{b\dagger}\phi_i^c\phi_j^{d\dagger}\phi_j^e. \end{aligned} \tag{B7}$$

The last piece is the $\mathcal{L}_{g-Yuk}$ term. To this end, we now consider

$$\begin{aligned}\mathcal{L}_{g-Yuk} &= i\sqrt{2}g\left[\phi_i^{a\dagger}\left(-if^{abc}\right)\psi_i^b\lambda^c - \bar{\lambda}^c\bar{\psi}_i^b\left(-if^{abc}\right)\phi_i^a\right] \\ &= i\sqrt{2}g\left[-if^{abc}\phi_i^{a\dagger}\psi_i^b\lambda^c + if^{abc}\bar{\lambda}^c\bar{\psi}_i^b\phi_i^a\right] \\ &= \sqrt{2}gf^{abc}\left[\phi_i^{a\dagger}\psi_i^b\lambda^c - \bar{\lambda}^c\bar{\psi}_i^b\phi_i^a\right]. \end{aligned} \tag{B8}$$

APPENDIX C

Details of the derivation of the expression (5.12) in chapter 5

Now we will start here by using the following change of variables:

$$\begin{aligned}
&\varphi^a_{rs} = -\varphi^a_{sr};\ \varphi^a_{i4} = \frac{1}{2}\phi^a_i, \\
&\varphi^a_{ij} = \frac{1}{2}\epsilon_{ijk}\phi^{a\dagger}_k;\ \eta^a_i = \psi^a_i;\quad i,\ j,\ k = 1,\ 2,\ 3, \\
&\eta^a_4 = \lambda^a;\quad r,\ s = 1,\ 2,\ 3,\ 4,
\end{aligned} \tag{C1}$$

Then we have

$$\begin{aligned}
&\phi^a_i = 2\varphi^a_{i4};\ \phi^{a\dagger}_i = -2\varphi^{a\dagger}_{i4}, \\
&\phi^{a\dagger}_i = \epsilon_{ijk}\varphi^a_{jk};\ \phi^a_i = -\epsilon_{ijk}\varphi^{a\dagger}_{jk}, \\
&\bar{\eta}^a_4 = \bar{\lambda}^a;\quad \bar{\eta}^a_i = \bar{\psi}^a_i.
\end{aligned} \tag{C2}$$

So we then obtain

$$\begin{aligned}
-i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - i\bar{\psi}^a_i\bar{\sigma}^\mu D_\mu\psi^a_i &= -i\bar{\eta}^a_4\bar{\sigma}^\mu D_\mu\eta^a_4 - i\bar{\eta}^a_i\bar{\sigma}^\mu D_\mu\eta^a_i \\
&= -i\bar{\eta}^a_r\bar{\sigma}^\mu D_\mu\eta^a_r,
\end{aligned} \tag{C3.1}$$

$$\begin{aligned}
-D^\mu\phi^{a\dagger}_i D_\mu\phi^a_i &= -D^\mu\left(\epsilon_{ijk}\varphi^a_{jk}\right)D_\mu\left(-\epsilon_{ilm}\varphi^{a\dagger}_{lm}\right) \\
&= \epsilon_{ijk}\epsilon_{ilm}D^\mu\varphi^a_{jk}D_\mu\varphi^{a\dagger}_{lm} \\
&= \left(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}\right)D^\mu\varphi^a_{jk}D_\mu\varphi^{a\dagger}_{lm} \\
&= 2D^\mu\varphi^a_{lm}D_\mu\varphi^{a\dagger}_{ml} \\
&= 2\mathrm{Tr}\left[D^\mu\varphi^{a\dagger}D_\mu\varphi^a\right],\quad l,\ m = 1,\ 2,\ 3,
\end{aligned} \tag{C3.2}$$

$$
\begin{aligned}
-D^{\mu}\phi_i^{a\dagger}D_{\mu}\phi_i^a &= -\frac{1}{2}\left[D^{\mu}\phi_i^{a\dagger}D_{\mu}\phi_i^a + D^{\mu}\phi_i^{a\dagger}D_{\mu}\phi_i^a\right]\\
&= -\frac{1}{2}D^{\mu}\phi_i^{a\dagger}D_{\mu}\phi_i^a - D^{\mu}\varphi_{lm}^a D_{\mu}\varphi_{ml}^{a\dagger}\\
&= -\frac{1}{4}\left[D^{\mu}\phi_i^{a\dagger}D_{\mu}\phi_i^a + D^{\mu}\phi_i^{a\dagger}D_{\mu}\phi_i^a\right] - D^{\mu}\varphi_{lm}^a D_{\mu}\varphi_{ml}^{a\dagger}\\
&= -\frac{1}{4}\left[D^{\mu}\left(-2\varphi_{i4}^{a\dagger}\right)D_{\mu}\left(2\varphi_{i4}^a\right) + D^{\mu}\left(2\varphi_{4i}^{a\dagger}\right)D_{\mu}\left(2\varphi_{i4}^a\right)\right] - D^{\mu}\varphi_{lm}^a D_{\mu}\varphi_{ml}^{a\dagger}\\
&= -D^{\mu}\varphi_{i4}^{a\dagger}D_{\mu}\varphi_{4i}^a - D^{\mu}\varphi_{4i}^{a\dagger}D_{\mu}\varphi_{i4}^a - D^{\mu}\varphi_{lm}^a D_{\mu}\varphi_{ml}^{a\dagger}\\
&= -D^{\mu}\varphi_{rs}^a D_{\mu}\varphi_{sr}^{a\dagger}\\
&= -\mathrm{Tr}\left[D^{\mu}\varphi^a D_{\mu}\varphi^{a\dagger}\right] \qquad \text{(C3.3)}
\end{aligned}
$$

$$
\begin{aligned}
\phi_i^{b\dagger}\phi_i^c = \frac{1}{2}\left[\phi_i^{b\dagger}\phi_i^c + \phi_i^{b\dagger}\phi_i^c\right] &= \frac{1}{2}\phi_i^{b\dagger}\phi_i^c - \frac{1}{2}\epsilon_{ijk}\epsilon_{ilm}\varphi_{ij}^b\varphi_{lm}^{c\dagger}\\
&= \frac{1}{2}\phi_i^{b\dagger}\phi_i^c + \varphi_{ml}^b\varphi_{lm}^{c\dagger}\\
&= \frac{1}{4}\left[\phi_i^{b\dagger}\phi_i^c + \phi_i^{b\dagger}\phi_i^c\right] + \varphi_{ml}^b\varphi_{lm}^{c\dagger}\\
&= \frac{1}{4}\left[\left(-2\varphi_{i4}^{b\dagger}\right)\left(2\varphi_{i4}^c\right) + \left(2\varphi_{4i}^{b\dagger}\right)\left(-2\varphi_{i4}^c\right)\right] + \varphi_{ml}^b\varphi_{lm}^{c\dagger}\\
&= \varphi_{4i}^{b\dagger}\varphi_{i4}^c + \varphi_{i4}^{b\dagger}\varphi_{4i}^c + \varphi_{ml}^b\varphi_{lm}^{c\dagger}\\
&= \varphi_{rs}^{b\dagger}\varphi_{sr}^c\\
&= \mathrm{Tr}\left[\varphi^{b\dagger}\varphi^c\right], \qquad \text{(C3.4)}
\end{aligned}
$$

$$
\phi_i^{b\dagger}\phi_i^c\phi_j^{d\dagger}\phi_j^e = \mathrm{Tr}\left[\varphi^{b\dagger}\varphi^c\right]\mathrm{Tr}\left[\varphi^{d\dagger}\varphi^e\right], \qquad \text{(C3.5)}
$$

$$
\begin{aligned}
\sqrt{2}gf^{abc}\left[\phi_i^{a\dagger}\psi_i^b\lambda^c + \bar{\lambda}^c\bar{\psi}_i^b\phi_i^a\right] &= 2\sqrt{2}gf^{abc}\left[-\varphi_{i4}^{a\dagger}\eta_i^b\eta_4^c + \bar{\eta}_4^c\bar{\eta}_i^b\varphi_{i4}^a\right]\\
&= 2\sqrt{2}gf^{abc}\left(\frac{1}{2}\right)\left[\left(-\varphi_{i4}^{a\dagger}\right)\eta_i^b\eta_4^c + \left(-\varphi_{4i}^{a\dagger}\right)\eta_4^b\eta_i^c + h.c.\right]\\
&= \sqrt{2}gf^{abc}\left[\left(-\varphi_{i4}^{a\dagger}\right)\eta_i^b\eta_4^c + \left(-\varphi_{4i}^{a\dagger}\right)\eta_4^b\eta_i^c + h.c.\right], \qquad \text{(C3.6)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{g}{\sqrt{2}}\left[\epsilon_{ijk}f^{abc}\phi_i^a\psi_j^b\psi_k^c + h.c.\right] &= \frac{g}{\sqrt{2}}f^{abc}\left[\epsilon_{ijk}\epsilon_{ilm}\left(-\varphi_{lm}^{a\dagger}\right)\eta_i^b\eta_j^c + h.c.\right] \\
&= \frac{g}{\sqrt{2}}f^{abc}\left[\left(\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}\right)\left(-\varphi_{lm}^{a\dagger}\right)\eta_i^b\eta_j^c + h.c.\right] \\
&= \sqrt{2}gf^{abc}\left[\left(-\varphi_{lm}^{a\dagger}\right)\eta_l^b\eta_m^c + h.c.\right], \qquad \text{(C3.7)}
\end{aligned}
$$

$$
\begin{aligned}
&\sqrt{2}gf^{abc}\left[\phi_i^{a\dagger}\psi_i^b\lambda^a + \bar{\lambda}^c\bar{\psi}_i^b\phi_i^a\right] + \frac{g}{\sqrt{2}}\left[\epsilon_{ijk}f^{abc}\phi_k^c\psi_i^b\psi_j^a + h.c.\right] \\
&= \sqrt{2}gf^{abc}\left[\left(-\varphi_{i4}^{a\dagger}\right)\eta_i^b\eta_4^c + \left(-\varphi_{4i}^{a\dagger}\right)\eta_4^b\eta_i^c + h.c.\right] + \sqrt{2}gf^{abc}\left[\left(-\varphi_{lm}^{a\dagger}\right)\eta_l^b\eta_m^c + h.c.\right] \\
&= -\sqrt{2}gf^{abc}\left[\varphi_{rs}^{a\dagger}\eta_r^b\eta_s^c + h.c.\right]. \qquad \text{(C3.8)}
\end{aligned}
$$

Collecting all terms we just derived, therefore the Lagrangian of expression (5.12) in chapter 5 becomes,

$$
\begin{aligned}
\mathcal{L}_{\text{SUSY}} = &-\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} - \text{Tr}D^\mu\varphi^{a\dagger}D_\mu\varphi^a - i\bar{\eta}_r^a\bar{\sigma}^\mu D_\mu\eta_r^a \\
&- \sqrt{2}gf^{abc}\left(\varphi_{rs}^{a\dagger}\eta_r^b\eta_s^c - \bar{\eta}_r^c\bar{\eta}_s^b\varphi_{rs}^a\right) \\
&- \frac{1}{2}g^2\left(f^{abd}f^{ace} + f^{abe}f^{acd}\right)\text{Tr}\varphi^{b\dagger}\varphi^c\text{Tr}\varphi^{d\dagger}\varphi^e, \qquad \text{(C4)}
\end{aligned}
$$

APPENDIX D

Details of the derivation of the expression (5.28) in chapter 5

Let us first consider the following expression:

$$\mathcal{L}_F = -\left|\frac{\partial P}{\partial \phi_i^a}\right|^2, \tag{D1}$$

where the superpotential P is given by

$$\begin{aligned} P &= P_{\rm TC} + P_{\rm MSSM} \\ &= P_{\rm TC}^{\rm pure} \\ &\quad + P_{\rm mix} \\ &\quad + P_{\rm MSSM} \\ &= -\frac{g_{\rm TC}}{3\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\Phi_i^a\Phi_j^b\Phi_k^c \\ &\quad + y_U\epsilon_{ij3}\Phi_i^a\hat{H}_j\Phi_3^a + y_N\epsilon_{ij3}\hat{\Lambda}_i\hat{H}_j\hat{N} + y_E\epsilon_{ij3}\hat{\Lambda}_i\hat{H}'_j\hat{E} \\ &\quad + P_{\rm MSSM}. \end{aligned} \tag{D2}$$

Note here that ϵ^{abc} is the fully antisymmetric symbol. In terms of component fields, we have

$$\begin{aligned} \mathcal{L}_F = &-\left|\frac{\partial P}{\partial \phi_1^a}\right|^2 - \left|\frac{\partial P}{\partial \phi_2^a}\right|^2 - \left|\frac{\partial P}{\partial \phi_3^a}\right|^2 \\ &-\left|\frac{\partial P}{\partial \phi_{\hat{\Lambda}_1}}\right|^2 - \left|\frac{\partial P}{\partial \phi_{\hat{\Lambda}_2}}\right|^2 - \left|\frac{\partial P}{\partial \phi_{\hat{N}}}\right|^2 \\ &-\left|\frac{\partial P}{\partial \phi_{\hat{E}}}\right|^2 - \left|\frac{\partial P}{\partial \phi_{\hat{H}_1}}\right|^2 - \left|\frac{\partial P}{\partial \phi_{\hat{H}_2}}\right|^2 \\ &-\left|\frac{\partial P}{\partial \phi_{\hat{H}'_1}}\right|^2 - \left|\frac{\partial P}{\partial \phi_{\hat{H}'_2}}\right|^2, \end{aligned} \tag{D3}$$

and

$$\begin{aligned} P = & -\frac{g_{\text{TC}}}{3\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\Phi_i^a\Phi_j^b\Phi_k^c \\ & + y_U\left(\Phi_1^a\hat{H}_2 - \Phi_2^a\hat{H}_1\right)\Phi_3^a \\ & + y_N\left(\hat{\Lambda}_1\hat{H}_2 - \hat{\Lambda}_2\hat{H}_1\right)\hat{N} \\ & + y_E\left(\hat{\Lambda}_1\hat{H}_2' - \hat{\Lambda}_2\hat{H}_1'\right)\hat{E} \\ & + P_{\text{MSSM}}. \end{aligned} \tag{D4}$$

By the definition of $\mathcal{L}_F$, hence we have to write the superpotential in terms of all its scalar fields. Taking derivative to such superpotential with respect to scalar fields ϕ, it yields

$$-\left|\frac{\partial P}{\partial \phi_i^a}\right|^2 = -\left|-\frac{g_{TC}}{\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\phi_j^b\phi_k^c + y_U\epsilon_{ij3}\phi_{\hat{H}_j}\phi_3^a\right|^2, \tag{D5}$$

or in terms of component fields,

$$-\left|\frac{\partial P}{\partial \phi_1^a}\right|^2 = -\left|-\frac{g_{TC}}{\sqrt{2}}\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c + y_U\phi_{\hat{H}_2}\phi_3^a\right|^2, \tag{D6.1}$$

$$-\left|\frac{\partial P}{\partial \phi_2^a}\right|^2 = -\left|-\frac{g_{TC}}{\sqrt{2}}\epsilon_{2jk}\epsilon^{abc}\phi_j^b\phi_k^c - y_U\phi_{\hat{H}_1}\phi_3^a\right|^2, \tag{D6.2}$$

$$-\left|\frac{\partial P}{\partial \phi_3^a}\right|^2 = -\left|-\frac{g_{TC}}{\sqrt{2}}\epsilon_{3jk}\epsilon^{abc}\phi_j^b\phi_k^c + y_U\left(\phi_1^a\phi_{\hat{H}_2} - \phi_2^a\phi_{\hat{H}_1}\right)\right|^2. \tag{D6.3}$$

The remaining expressions we can have are given by

$$-\left|\frac{\partial P}{\partial \phi_{\hat{N}}}\right|^2 = -\left|y_N\left(\phi_{\hat{\Lambda}_1}\phi_{\hat{H}_2} - \phi_{\hat{\Lambda}_2}\phi_{\hat{H}_1}\right)\right|^2, \tag{D7.1}$$

$$-\left|\frac{\partial P}{\partial \phi_{\hat{E}}}\right|^2 = -\left|y_E\left(\phi_{\hat{\Lambda}_1}\phi_{\hat{H}_2'} - \phi_{\hat{\Lambda}_2}\phi_{\hat{H}_1'}\right)\right|^2, \tag{D7.2}$$

$$-\left|\frac{\partial P}{\partial \phi_{\hat{\Lambda}_1}}\right|^2 = -\left|y_N\phi_{\hat{H}_2}\phi_{\hat{N}} + y_E\phi_{\hat{H}_2'}\phi_{\hat{E}}\right|^2, \tag{D7.3}$$

$$-\left|\frac{\partial P}{\partial \phi_{\hat{\Lambda}_2}}\right|^2 = -\left|-y_N\phi_{\hat{H}_1}\phi_{\hat{N}} - y_E\phi_{\hat{H}_1'}\phi_{\hat{E}}\right|^2, \tag{D7.4}$$

$$-\left|\frac{\partial P}{\partial \phi_{\hat{H}_1}}\right|^2 = -\left|-y_N\phi_{\hat{\Lambda}_2}\phi_{\hat{N}} - y_U\phi_2^a\phi_3^a + \frac{\partial P_{\text{MSSM}}}{\partial \phi_{\hat{H}_1}}\right|^2, \tag{D7.5}$$

$$-\left|\frac{\partial P}{\partial\phi_{\hat{H}_2}}\right|^2 = -\left|y_N\phi_{\hat{\Lambda}_1}\phi_{\hat{N}} + y_U\phi_1^a\phi_3^a + \frac{\partial P_{\text{MSSM}}}{\partial\phi_{\hat{H}_2}}\right|^2, \tag{D7.6}$$

$$-\left|\frac{\partial P}{\partial\phi_{\hat{H}'_1}}\right|^2 = -\left|-y_E\phi_{\hat{\Lambda}_2}\phi_{\hat{E}} + \frac{\partial P_{\text{MSSM}}}{\partial\phi_{\hat{H}'_1}}\right|^2, \tag{D7.7}$$

$$-\left|\frac{\partial P}{\partial\phi_{\hat{H}'_2}}\right|^2 = -\left|y_E\phi_{\hat{\Lambda}_1}\phi_{\hat{E}} + \frac{\partial P_{\text{MSSM}}}{\partial\phi_{\hat{H}'_2}}\right|^2. \tag{D7.8}$$

For more useful later, let us define the following expressions:

$$F^{\text{TC}}_{H_1} = -y_N\phi_{\hat{\Lambda}_2}\phi_{\hat{N}} - y_U\phi_2^a\phi_3^a, \qquad F^{\text{MSSM}}_{H_1} = \frac{\partial P_{\text{MSSM}}}{\partial\phi_{\hat{H}_1}}, \tag{D8.1}$$

$$F^{\text{TC}}_{H_2} = y_N\phi_{\hat{\Lambda}_1}\phi_{\hat{N}} + y_U\phi_1^a\phi_3^a, \qquad F^{\text{MSSM}}_{H_2} = \frac{\partial P_{\text{MSSM}}}{\partial\phi_{\hat{H}_2}}, \tag{D8.2}$$

$$F^{\text{TC}}_{H'_1} = -y_E\phi_{\hat{\Lambda}_2}\phi_{\hat{E}}, \qquad F^{\text{MSSM}}_{H'_1} = \frac{\partial P_{\text{MSSM}}}{\partial\phi_{\hat{H}'_1}}, \tag{D8.3}$$

$$F^{\text{TC}}_{H'_2} = y_E\phi_{\hat{\Lambda}_1}\phi_{\hat{E}}, \qquad F^{\text{MSSM}}_{H'_2} = \frac{\partial P_{\text{MSSM}}}{\partial\phi_{\hat{H}'_2}}. \tag{D8.4}$$

Now let us calculate term by term of Eq.(D6.1)-Eq.(D7.8)

$$-\left|\frac{\partial P}{\partial\phi_{\hat{N}}}\right|^2 = -y_N^2\left(\phi_{\hat{\Lambda}_1}\phi_{\hat{H}_2} - \phi_{\hat{\Lambda}_2}\phi_{\hat{H}_1}\right)\left(\bar{\phi}_{\hat{\Lambda}_1}\bar{\phi}_{\hat{H}_2} - \bar{\phi}_{\hat{\Lambda}_2}\bar{\phi}_{\hat{H}_1}\right), \tag{D9.1}$$

$$-\left|\frac{\partial P}{\partial\phi_{\hat{E}}}\right|^2 = -y_E^2\left(\phi_{\hat{\Lambda}_1}\phi_{\hat{H}'_2} - \phi_{\hat{\Lambda}_2}\phi_{\hat{H}'_1}\right)\left(\bar{\phi}_{\hat{\Lambda}_1}\bar{\phi}_{\hat{H}'_2} - \bar{\phi}_{\hat{\Lambda}_2}\bar{\phi}_{\hat{H}'_1}\right), \tag{D9.2}$$

$$\begin{aligned}-\left|\frac{\partial P}{\partial\phi_{\hat{\Lambda}_1}}\right|^2 &= -\left(y_N\phi_{\hat{H}_2}\phi_{\hat{N}} + y_E\phi_{\hat{H}'_2}\phi_{\hat{E}}\right)\left(\bar{\phi}_{\hat{H}_2}\bar{\phi}_{\hat{N}} + y_E\bar{\phi}_{\hat{H}'_2}\bar{\phi}_{\hat{E}}\right)\\ &= -y_N^2\phi_{\hat{H}_2}\phi_{\hat{N}}\bar{\phi}_{\hat{H}_2}\bar{\phi}_{\hat{N}} - y_E^2\phi_{\hat{H}'_2}\phi_{\hat{E}}\bar{\phi}_{\hat{H}'_2}\bar{\phi}_{\hat{E}}\\ &- \left(y_Ey_N\phi_{\hat{H}_2}\phi_{\hat{N}}\bar{\phi}_{\hat{H}'_2}\bar{\phi}_{\hat{E}} + h.c.\right),\end{aligned} \tag{D9.3}$$

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{\Lambda}_2}}\right|^2 &= -\left(-y_N \phi_{\hat{H}_1}\phi_{\hat{N}} - y_E \phi_{\hat{H}_1'}\phi_{\hat{E}}\right)\left(\bar{\phi}_{\hat{H}_1}\bar{\phi}_{\hat{N}} + y_E \bar{\phi}_{\hat{H}_1'}\bar{\phi}_{\hat{E}}\right) \\
&= -y_N^2 \phi_{\hat{H}_1}\phi_{\hat{N}}\bar{\phi}_{\hat{H}_1}\bar{\phi}_{\hat{N}} - y_E^2 \phi_{\hat{H}_1'}\phi_{\hat{E}}\bar{\phi}_{\hat{H}_1'}\bar{\phi}_{\hat{E}} \\
&\quad - \left(y_E y_N \phi_{\hat{H}_1}\phi_{\hat{N}}\bar{\phi}_{\hat{H}_1'}\bar{\phi}_{\hat{E}} + h.c.\right),
\end{aligned} \tag{D9.4}
$$

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{H}_1}}\right|^2 &= -\left|F^{\mathrm{TC}}_{H_1} + F^{\mathrm{MSSM}}_{H_1}\right|^2 \\
&= -F^{\mathrm{TC}}_{H_1}F^{\dagger\mathrm{TC}}_{H_1} - F^{\mathrm{MSSM}}_{H_1}F^{\dagger\mathrm{MSSM}}_{H_1} - \left(F^{\mathrm{TC}}_{H_1}F^{\dagger\mathrm{MSSM}}_{H_1} + h.c.\right),
\end{aligned} \tag{D9.5}
$$

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{H}_2}}\right|^2 &= -\left|F^{\mathrm{TC}}_{H_2} + F^{\mathrm{MSSM}}_{H_2}\right|^2 \\
&= -F^{\mathrm{TC}}_{H_2}F^{\dagger\mathrm{TC}}_{H_2} - F^{\mathrm{MSSM}}_{H_2}F^{\dagger\mathrm{MSSM}}_{H_2} - \left(F^{\mathrm{TC}}_{H_2}F^{\dagger\mathrm{MSSM}}_{H_2} + h.c.\right),
\end{aligned} \tag{D9.6}
$$

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{H}_1'}}\right|^2 &= -\left|F^{\mathrm{TC}}_{H_1'} + F^{\mathrm{MSSM}}_{H_1'}\right|^2 \\
&= -F^{\mathrm{TC}}_{H_1'}F^{\dagger\mathrm{TC}}_{H_1'} - F^{\mathrm{MSSM}}_{H_1'}F^{\dagger\mathrm{MSSM}}_{H_1'} - \left(F^{\mathrm{TC}}_{H_1'}F^{\dagger\mathrm{MSSM}}_{H_1'} + h.c.\right),
\end{aligned} \tag{D9.7}
$$

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{H}_2'}}\right|^2 &= -\left|F^{\mathrm{TC}}_{H_2'} + F^{\mathrm{MSSM}}_{H_2'}\right|^2 \\
&= -F^{\mathrm{TC}}_{H_2'}F^{\dagger\mathrm{TC}}_{H_2'} - F^{\mathrm{MSSM}}_{H_2'}F^{\dagger\mathrm{MSSM}}_{H_2'} - \left(F^{\mathrm{TC}}_{H_2'}F^{\dagger\mathrm{MSSM}}_{H_2'} + h.c.\right).
\end{aligned} \tag{D9.8}
$$

Here for F-terms, we have

$$
\begin{aligned}
&\left|F^{\mathrm{TC}}_{H_1}\right|^2 = \left(-y_N \phi_{\hat{\Lambda}_2}\phi_{\hat{N}} - y_U \phi_2^a\phi_3^a\right)\left(-y_N \bar{\phi}_{\hat{\Lambda}_2}\bar{\phi}_{\hat{N}} - y_U \bar{\phi}_2^b\bar{\phi}_3^b\right) \\
&= y_U^2 \phi_2^a\phi_3^a\bar{\phi}_2^b\bar{\phi}_3^b + y_N^2 \phi_{\hat{\Lambda}_2}\phi_{\hat{N}}\bar{\phi}_{\hat{\Lambda}_2}\bar{\phi}_{\hat{N}} + \left(y_U y_N \phi_{\hat{\Lambda}_2}\phi_{\hat{N}}\bar{\phi}_2^a\bar{\phi}_3^a + h.c.\right),
\end{aligned} \tag{D10.1}
$$

$$
\begin{aligned}
&\left|F^{\mathrm{TC}}_{H_2}\right|^2 = \left(y_N \phi_{\hat{\Lambda}_2}\phi_{\hat{N}} + y_U \phi_1^a\phi_3^a\right)\left(y_N \bar{\phi}_{\hat{\Lambda}_1}\bar{\phi}_{\hat{N}} + y_U \bar{\phi}_1^b\bar{\phi}_3^b\right) \\
&= y_U^2 \phi_1^a\phi_3^a\bar{\phi}_1^b\bar{\phi}_3^b + y_N^2 \phi_{\hat{\Lambda}_1}\phi_{\hat{N}}\bar{\phi}_{\hat{\Lambda}_1}\bar{\phi}_{\hat{N}} + \left(y_U y_N \phi_{\hat{\Lambda}_1}\phi_{\hat{N}}\bar{\phi}_1^a\bar{\phi}_3^a + h.c.\right),
\end{aligned} \tag{D10.2}
$$

$$
\left|F^{\mathrm{TC}}_{H_1'}\right|^2 = y_E^2 \phi_{\hat{\Lambda}_2}\phi_{\hat{E}}\bar{\phi}_{\hat{\Lambda}_2}\bar{\phi}_{\hat{E}}, \tag{D10.3}
$$

$$\left|F^{\mathrm{TC}}_{H_2'}\right|^2 = y_E^2 \phi_{\hat{\Lambda}_1}\phi_{\hat{E}}\bar{\phi}_{\hat{\Lambda}_1}\bar{\phi}_{\hat{E}}. \tag{D10.4}$$

Collecting all F-terms, we obtain

$$\begin{aligned}
&-\left|F^{\mathrm{TC}}_{H_1}\right|^2 - \left|F^{\mathrm{TC}}_{H_2}\right|^2 - \left|F^{\mathrm{TC}}_{H_1'}\right|^2 - \left|F^{\mathrm{TC}}_{H_2'}\right|^2 \\
&\quad = -y_U^2 \phi_3^a \bar{\phi}_3^b \left(\phi_1^a \bar{\phi}_1^b + \phi_2^a \bar{\phi}_2^b\right) - y_N^2 \phi_{\hat{N}} \bar{\phi}_{\hat{N}} \left(\phi_{\hat{\Lambda}_1}\bar{\phi}_{\hat{\Lambda}_1} + \phi_{\hat{\Lambda}_2}\bar{\phi}_{\hat{\Lambda}_2}\right) \\
&\quad - \left[y_U y_N \phi_{\hat{N}} \bar{\phi}_3^a \left(\phi_{\hat{\Lambda}_1}\bar{\phi}_1^a + \phi_{\hat{\Lambda}_2}\bar{\phi}_2^a\right) + h.c.\right] \\
&\quad - y_E^2 \phi_{\hat{E}} \bar{\phi}_{\hat{E}} \left(\phi_{\hat{\Lambda}_1}\bar{\phi}_{\hat{\Lambda}_1} + \phi_{\hat{\Lambda}_2}\bar{\phi}_{\hat{\Lambda}_2}\right),
\end{aligned} \tag{D11.1}$$

or in terms of component fields,

$$\begin{aligned}
&-\left|F^{\mathrm{TC}}_{H_1}\right|^2 - \left|F^{\mathrm{TC}}_{H_2}\right|^2 - \left|F^{\mathrm{TC}}_{H_1'}\right|^2 - \left|F^{\mathrm{TC}}_{H_2'}\right|^2 \\
&\quad = -y_U^2 \bar{\tilde{U}}_R^a \tilde{U}_R^b \left(\tilde{U}_L^a \bar{\tilde{U}}_L^b + \tilde{D}_L^a \bar{\tilde{D}}_L^b\right) - y_N^2 \bar{\tilde{N}}_R \tilde{N}_R \left(\tilde{N}_L \bar{\tilde{N}}_L + \tilde{E}_L \bar{\tilde{E}}_L\right) \\
&\quad - \left[y_U y_N \bar{\tilde{N}}_R \tilde{U}_R^a \left(\tilde{N}_L \bar{\tilde{U}}_L^a + \tilde{E}_L \bar{\tilde{D}}_L^a\right) + h.c.\right] \\
&\quad - y_E^2 \bar{\tilde{E}}_R \tilde{E}_R \left(\tilde{N}_L \bar{\tilde{N}}_L + \tilde{E}_L \bar{\tilde{E}}_L\right).
\end{aligned} \tag{D11.2}$$

Taking into account the expression (D7.1) and (D7.2), we have

$$\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{N}}}\right|^2 &= -y_N^2 \left(\phi_{\hat{\Lambda}_1}\phi_{\hat{H}_2} - \phi_{\hat{\Lambda}_2}\phi_{\hat{H}_1}\right)\left(\bar{\phi}_{\hat{\Lambda}_1}\bar{\phi}_{\hat{H}_2} - \bar{\phi}_{\hat{\Lambda}_2}\bar{\phi}_{\hat{H}_1}\right) \\
&= -y_N^2 \left(\tilde{N}_L \tilde{H}_2 - \tilde{E}_L \tilde{H}_1\right)\left(\bar{\tilde{N}}_L \bar{\tilde{H}}_2 - \bar{\tilde{E}}_L \bar{\tilde{H}}_1\right),
\end{aligned} \tag{D12.1}$$

$$\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{E}}}\right|^2 &= -y_E^2 \left(\phi_{\hat{\Lambda}_1}\phi_{\hat{H}_2'} - \phi_{\hat{\Lambda}_2}\phi_{\hat{H}_1'}\right)\left(\bar{\phi}_{\hat{\Lambda}_1}\bar{\phi}_{\hat{H}_2'} - \bar{\phi}_{\hat{\Lambda}_2}\bar{\phi}_{\hat{H}_1'}\right) \\
&= -y_E^2 \left(\tilde{N}_L \tilde{H}_2' - \tilde{E}_L \tilde{H}_1'\right)\left(\bar{\tilde{N}}_L \bar{\tilde{H}}_2' - \bar{\tilde{E}}_L \bar{\tilde{H}}_1'\right).
\end{aligned} \tag{D12.2}$$

Next we consider the expression (D7.3) and (D7.4)

$$\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{\Lambda}_1}}\right|^2 &= -\left(y_N \phi_{\hat{H}_2}\phi_{\hat{N}} + y_E \phi_{\hat{H}_2'}\phi_{\hat{E}}\right)\left(y_N \bar{\phi}_{\hat{H}_2}\bar{\phi}_{\hat{N}} + y_E \bar{\phi}_{\hat{H}_2'}\bar{\phi}_{\hat{E}}\right) \\
&= -y_N^2 \phi_{\hat{H}_2}\phi_{\hat{N}}\bar{\phi}_{\hat{H}_2}\bar{\phi}_{\hat{N}} - y_E^2 \phi_{\hat{H}_2'}\phi_{\hat{E}}\bar{\phi}_{\hat{H}_2'}\bar{\phi}_{\hat{E}} - \left(y_E y_N \phi_{\hat{H}_2}\phi_{\hat{N}}\bar{\phi}_{\hat{H}_2'}\bar{\phi}_{\hat{E}} + h.c.\right) \\
&= -y_N^2 \tilde{H}_2 \bar{\tilde{N}}_R \bar{\tilde{H}}_2 \tilde{N}_R - y_E^2 \tilde{H}_2' \bar{\tilde{E}}_R \bar{\tilde{H}}_2' \tilde{E}_R - \left(y_E y_N \tilde{H}_2 \bar{\tilde{N}}_R \bar{\tilde{H}}_2' \tilde{E}_R + h.c.\right),
\end{aligned} \tag{D13.1}$$

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_{\hat{\Lambda}_2}}\right|^2 &= -\left(-y_N\phi_{\hat{H}_1}\phi_{\hat{N}} - y_E\phi_{\hat{H}_1'}\phi_{\hat{E}}\right)\left(-y_N\bar{\phi}_{\hat{H}_1}\bar{\phi}_{\hat{N}} - y_E\bar{\phi}_{\hat{H}_1'}\bar{\phi}_{\hat{E}}\right) \\
&= -y_N^2\phi_{\hat{H}_1}\phi_{\hat{N}}\bar{\phi}_{\hat{H}_1}\bar{\phi}_{\hat{N}} - y_E^2\phi_{\hat{H}_1'}\phi_{\hat{E}}\bar{\phi}_{\hat{H}_1'}\bar{\phi}_{\hat{E}} - \left(y_E y_N\phi_{\hat{H}_1}\phi_{\hat{N}}\bar{\phi}_{\hat{H}_1'}\bar{\phi}_{\hat{E}} + h.c.\right) \\
&= -y_N^2\tilde{H}_1\bar{\tilde{N}}_R\bar{\tilde{H}}_1\tilde{N}_R - y_E^2\tilde{H}_1'\bar{\tilde{E}}_R\bar{\tilde{H}}_1'\tilde{E}_R - \left(y_E y_N\tilde{H}_1\bar{\tilde{N}}_R\bar{\tilde{H}}_1'\tilde{E}_R + h.c.\right). \qquad \text{(D13.2)}
\end{aligned}
$$

Finally, let us consider the expression (D6.1)-(D6.3)

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_1^a}\right|^2 &= -\left|-\frac{g_{TC}}{\sqrt{2}}\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c + y_U\phi_{\hat{H}_2}\phi_3^a\right|^2 \\
&= -\left(-\frac{g_{TC}}{\sqrt{2}}\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c + y_U\phi_{\hat{H}_2}\phi_3^a\right)\left(-\frac{g_{TC}}{\sqrt{2}}\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c + y_U\phi_{\hat{H}_2}\phi_3^a\right)^\dagger \\
&= -\frac{g_{TC}^2}{2}\left(\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c\right)^2 - y_U^2\phi_{\hat{H}_2}\phi_3^a\bar{\phi}_{\hat{H}_2}\bar{\phi}_3^a + \left\{\frac{g_{TC}}{\sqrt{2}}\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c y_U\bar{\phi}_{\hat{H}_2}\bar{\phi}_3^a + h.c.\right\}, \qquad \text{(D14.1)}
\end{aligned}
$$

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_2^a}\right|^2 &= -\left|-\frac{g_{TC}}{\sqrt{2}}\epsilon_{2jk}\epsilon^{abc}\phi_j^b\phi_k^c - y_U\phi_{\hat{H}_1}\phi_3^a\right|^2 \\
&= -\frac{g_{TC}^2}{2}\left(\epsilon_{2jk}\epsilon^{abc}\phi_j^b\phi_k^c\right)^2 - y_U^2\phi_{\hat{H}_1}\phi_3^a\bar{\phi}_{\hat{H}_1}\bar{\phi}_3^a - \left\{\frac{g_{TC}}{\sqrt{2}}\epsilon_{2jk}\epsilon^{abc}\phi_j^b\phi_k^c y_U\bar{\phi}_{\hat{H}_1}\bar{\phi}_3^a + h.c.\right\}, \qquad \text{(D14.2)}
\end{aligned}
$$

$$
\begin{aligned}
-\left|\frac{\partial P}{\partial \phi_3^a}\right|^2 &= -\left|-\frac{g_{TC}}{\sqrt{2}}\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c + y_U\left(\phi_1^a\phi_{\hat{H}_2} - \phi_2^a\phi_{\hat{H}_1}\right)\right|^2 \\
&= -\frac{g_{TC}^2}{2}\left(\epsilon_{3jk}\epsilon^{abc}\phi_j^b\phi_k^c\right)^2 - y_U^2\left(\phi_1^a\phi_{\hat{H}_2} - \phi_2^a\phi_{\hat{H}_1}\right)\left(\bar{\phi}_1^a\bar{\phi}_{\hat{H}_2} - \bar{\phi}_2^a\bar{\phi}_{\hat{H}_1}\right) \\
&\quad + \left\{\frac{g_{TC}^2}{2}\left(\epsilon_{3jk}\epsilon^{abc}\phi_j^b\phi_k^c\right)y_U\left(\bar{\phi}_1^a\bar{\phi}_{\hat{H}_2} - \bar{\phi}_2^a\bar{\phi}_{\hat{H}_1}\right) + h.c.\right\}. \qquad \text{(D14.3)}
\end{aligned}
$$

Collecting the second term in each above expression, we have

$$
\begin{aligned}
&- y_U^2\phi_{\hat{H}_2}\phi_3^a\bar{\phi}_{\hat{H}_2}\bar{\phi}_3^a - y_U^2\phi_{\hat{H}_1}\phi_3^a\bar{\phi}_{\hat{H}_1}\bar{\phi}_3^a - y_U^2\left(\phi_1^a\phi_{\hat{H}_2} - \phi_2^a\phi_{\hat{H}_1}\right)\left(\bar{\phi}_1^a\bar{\phi}_{\hat{H}_2} - \bar{\phi}_2^a\bar{\phi}_{\hat{H}_1}\right) \\
&= -y_U^2\left[\bar{\tilde{U}}_R^a\tilde{U}_R^a\left(\tilde{H}_1\bar{\tilde{H}}_1 + \tilde{H}_2\bar{\tilde{H}}_2\right) + \left(\tilde{U}_L^a\tilde{H}_2 - \tilde{D}_L^a\tilde{H}_1\right)\left(\bar{\tilde{U}}_L^a\bar{\tilde{H}}_2 - \bar{\tilde{D}}_L^a\bar{\tilde{H}}_1\right)\right]. \qquad \text{(D15)}
\end{aligned}
$$

Here we collect the first term in each expression

$$
\begin{aligned}
&-\frac{g_{TC}^2}{2}\left(\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c\right)^2-\frac{g_{TC}^2}{2}\left(\epsilon_{2jk}\epsilon^{abc}\phi_j^b\phi_k^c\right)^2-\frac{g_{TC}^2}{2}\left(\epsilon_{3jk}\epsilon^{abc}\phi_j^b\phi_k^c\right)^2\\
&=-\frac{g_{TC}^2}{2}\left(\epsilon_{ijk}\epsilon^{abc}\phi_j^b\phi_k^c\right)^2\\
&=g_{TC}^2\epsilon^{ace}\epsilon^{adb}\phi_i^{\dagger b}\phi_i^c\phi_j^{\dagger d}\phi_j^e\\
&=g_{TC}^2\left(\delta^{cd}\delta^{eb}-\delta^{cb}\delta^{ed}\right)\phi_i^{\dagger b}\phi_i^c\phi_j^{\dagger d}\phi_j^e\\
&=g_{TC}^2\phi_i^{\dagger b}\phi_i^c\phi_j^{\dagger c}\phi_j^b-g_{TC}^2\phi_i^{\dagger b}\phi_i^b\phi_j^{\dagger c}\phi_j^c,
\end{aligned}\tag{D16}
$$

or in terms of component fields,

$$
\begin{aligned}
&-g_{TC}^2\phi_i^{\dagger b}\phi_i^b\phi_j^{\dagger c}\phi_j^c+g_{TC}^2\phi_i^{\dagger b}\phi_i^c\phi_j^{\dagger c}\phi_j^b\\
&=-g_{TC}^2\left(\bar{\tilde{U}}_L^b\tilde{U}_L^b+\bar{\tilde{D}}_L^b\tilde{D}_L^b+\tilde{U}_R^b\bar{\tilde{U}}_R^b\right)^2\\
&\quad+g_{TC}^2\left(\bar{\tilde{U}}_L^b\tilde{U}_L^c+\bar{\tilde{D}}_L^b\tilde{D}_L^c+\tilde{U}_L^b\bar{\tilde{U}}_L^c\right)\left(\tilde{U}_L^b\bar{\tilde{U}}_L^c+\tilde{D}_L^b\bar{\tilde{D}}_L^c+\bar{\tilde{U}}_L^b\tilde{U}_L^c\right).
\end{aligned}\tag{D17}
$$

Collecting the last term in each expression of (D14.1)-(D14.3), we have

$$
\begin{aligned}
&\frac{y_U g_{TC}}{\sqrt{2}}\left[\epsilon_{1jk}\epsilon^{abc}\phi_j^b\phi_k^c\bar{\phi}_{\hat{H}_2}\bar{\phi}_3^a-\epsilon_{2jk}\epsilon^{abc}\phi_j^b\phi_k^c\bar{\phi}_{\hat{H}_1}\bar{\phi}_3^a+\left(\epsilon_{3jk}\epsilon^{abc}\phi_j^b\phi_k^c\right)\left(\bar{\phi}_1^a\bar{\phi}_{\hat{H}_2}-\bar{\phi}_2^a\bar{\phi}_{\hat{H}_1}\right)+h.c.\right]\\
&=\frac{y_U g_{TC}}{\sqrt{2}}\left(\epsilon^{abc}\phi_2^b\phi_3^c-\epsilon^{abc}\phi_3^b\phi_2^c\right)\bar{\phi}_{\hat{H}_2}\bar{\phi}_3^a-\frac{y_U g_{TC}}{\sqrt{2}}\left(\epsilon^{abc}\phi_3^b\phi_1^c-\epsilon^{abc}\phi_1^b\phi_3^c\right)\bar{\phi}_{\hat{H}_1}\bar{\phi}_3^a\\
&+\frac{y_U g_{TC}}{\sqrt{2}}\left(\epsilon^{abc}\phi_1^b\phi_2^c-\epsilon^{abc}\phi_2^b\phi_1^c\right)\left(\bar{\phi}_1^a\bar{\phi}_{\hat{H}_2}-\bar{\phi}_2^a\bar{\phi}_{\hat{H}_1}\right)+h.c.\\
&=\frac{y_U g_{TC}}{\sqrt{2}}\left(2\epsilon^{abc}\phi_2^b\phi_3^c\right)\bar{\phi}_{\hat{H}_2}\bar{\phi}_3^a-\frac{y_U g_{TC}}{\sqrt{2}}\left(-2\epsilon^{abc}\phi_1^b\phi_3^c\right)\bar{\phi}_{\hat{H}_1}\bar{\phi}_3^a\\
&+\frac{y_U g_{TC}}{\sqrt{2}}\left(2\epsilon^{abc}\phi_1^b\phi_2^c\right)\left(\bar{\phi}_1^a\bar{\phi}_{\hat{H}_2}-\bar{\phi}_2^a\bar{\phi}_{\hat{H}_1}\right)+h.c.\\
&=\sqrt{2}y_U g_{TC}\epsilon^{abc}[\left(\phi_2^b\phi_3^c\right)\bar{\phi}_{\hat{H}_2}\bar{\phi}_3^a+\left(\phi_1^b\phi_3^c\right)\bar{\phi}_{\hat{H}_1}\bar{\phi}_3^a+\left(\phi_1^b\phi_2^c\right)\left(\bar{\phi}_1^a\bar{\phi}_{\hat{H}_2}-\bar{\phi}_2^a\bar{\phi}_{\hat{H}_1}\right)+h.c.],
\end{aligned}\tag{D18}
$$

or in terms of component fields,

$$
\begin{aligned}
&\sqrt{2}y_U g_{TC}\epsilon^{abc}[\left(\phi_2^b\phi_3^c\right)\bar{\phi}_{\hat{H}_2}\bar{\phi}_3^a+\left(\phi_1^b\phi_3^c\right)\bar{\phi}_{\hat{H}_1}\bar{\phi}_3^a+\left(\phi_1^b\phi_2^c\right)\left(\bar{\phi}_1^a\bar{\phi}_{\hat{H}_2}-\bar{\phi}_2^a\bar{\phi}_{\hat{H}_1}\right)+h.c.]\\
&=\sqrt{2}y_U g_{TC}\epsilon^{abc}[\tilde{D}_L^b\bar{\tilde{U}}_R^c\bar{\tilde{H}}_2\tilde{U}_R^a+\tilde{U}_L^b\bar{\tilde{U}}_R^c\bar{\tilde{H}}_1\tilde{U}_R^a+\tilde{U}_L^b\tilde{D}_L^c\left(\bar{\tilde{U}}_L^a\bar{\tilde{H}}_2-\bar{\tilde{D}}_L^a\bar{\tilde{H}}_1\right)+h.c.].
\end{aligned}\tag{D19}
$$

Collecting all terms, we exactly obtain the expression (5.28) in chapter 5.

APPENDIX E

Details of the derivation of the expression (5.31) in chapter 5

Let us consider first the TC superpotential given in Chapter 5

$$P_{\rm TC} = \underbrace{-\frac{g_{\rm TC}}{3\sqrt{2}}\epsilon_{ijk}\epsilon^{abc}\Phi_i^a\Phi_j^b\Phi_k^c}_{i} + \underbrace{y_U\epsilon_{ij3}\Phi_i^a\hat{H}_j\Phi_3^a}_{ii} + \underbrace{y_N\epsilon_{ij3}\hat{\Lambda}_i\hat{H}_j\hat{N}}_{iii} + \underbrace{y_E\epsilon_{ij3}\hat{\Lambda}_i\hat{H}'_j\hat{E}}_{iv}. \quad \text{(E1)}$$

The Yukawa interaction terms here can be determined by the above superpotential. Next, let us consider the following Yukawa interaction Lagrangian given in the last line of expression (5.8) of Chapter 5:

$$\mathcal{L}_{\text{P-Yuk}} = -\frac{1}{2}\left[\frac{\partial^2 P}{\partial\phi_i^a\partial\phi_l^b}\psi_i^a\psi_l^b + h.c.\right], \quad \text{(E2)}$$

where i, j run over all scalar field labels. Here let us consider the expression (i) in Eq.(E1). We start by taking derivative with respect to the scalar fields, i.e.,

$$\frac{\partial^2 P_{\rm TC}^{\rm pure}}{\partial\phi\partial\phi} = -\sqrt{2}g_{TC}\epsilon\epsilon\phi, \quad \text{(E3)}$$

or

$$\frac{\partial^2 P_{\rm TC}^{\rm pure}}{\partial\phi_i^a\partial\phi_j^b} = -\sqrt{2}g_{TC}\epsilon_{ijk}\epsilon^{abc}\phi_k^c. \quad \text{(E4)}$$

Substituting Eq.(E4) into Eq.(E2), it yields

$$\begin{aligned}
\mathcal{L}_i &= \frac{\sqrt{2}}{2}\left[g_{TC}\epsilon_{ijk}\epsilon^{abc}\psi_i^a\psi_l^b\phi_k^c + h.c.\right] \\
&= \frac{\sqrt{2}}{2}g_{TC}[\epsilon^{abc}\psi_1^a\psi_2^b\phi_3^c + \epsilon^{abc}\psi_2^a\psi_3^b\phi_1^c + \epsilon^{abc}\psi_3^a\psi_1^b\phi_2^c \\
&\quad - \epsilon^{abc}\psi_1^a\psi_3^b\phi_2^c - \epsilon^{abc}\psi_2^a\psi_1^b\phi_3^c - \epsilon^{abc}\psi_3^a\psi_2^b\phi_1^c + h.c.] \\
&= \frac{\sqrt{2}}{2}g_{TC}[\epsilon^{abc}\psi_1^a\psi_2^b\phi_3^c + \epsilon^{bca}\phi_1^a\psi_2^b\psi_3^c + \epsilon^{cab}\psi_1^a\phi_2^b\psi_3^c \\
&\quad - \epsilon^{acb}\psi_1^a\phi_2^b\psi_3^c - \epsilon^{bac}\psi_1^a\psi_2^b\phi_3^c - \epsilon^{cba}\phi_1^a\psi_2^2\psi_3^c + h.c.] \\
&= \frac{\sqrt{2}}{2}g_{TC}[\epsilon^{abc}\psi_1^a\psi_2^b\phi_3^c + \epsilon^{abc}\phi_1^a\psi_2^b\psi_3^c + \epsilon^{abc}\psi_1^a\phi_2^b\psi_3^c \\
&\quad + \epsilon^{abc}\psi_1^a\phi_2^b\psi_3^c + \epsilon^{abc}\psi_1^a\psi_2^b\phi_3^c + \epsilon^{abc}\phi_1^a\psi_2^2\psi_3^c + h.c.] \\
&= \sqrt{2}g_{TC}[\epsilon^{abc}\psi_1^a\psi_2^b\phi_3^c + \epsilon^{abc}\phi_1^a\psi_2^b\psi_3^c + \epsilon^{abc}\psi_1^a\phi_2^b\psi_3^c + h.c.], \quad \text{(E5)}
\end{aligned}$$

or in component fields,

$$\mathcal{L}_i = \sqrt{2} g_{TC} \epsilon^{abc} \left[U_L^a D_L^a \tilde{\bar{U}}_R^c + \tilde{U}_L^a U_L^b \bar{U}_R^c + U_L^a \tilde{D}_L^b \bar{U}_R^c \right] + h.c. \tag{E6}$$

Next let us consider the expression (ii). We start by taking derivative with respect to the scalar fields, i.e.,

$$\begin{aligned} \frac{\partial^2 P_{ii}^{\text{mix}}}{\partial\phi\partial\phi} &= y_U \frac{\partial^2}{\partial\phi\partial\phi} \epsilon_{ij3} \Phi_i^a \hat{H}_j \Phi_3^a \\ &= y_U \epsilon_{ij3} [\frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \breve{\Phi}_i^a \mathring{\hat{H}}_j \Phi_3^a - \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \mathring{\Phi}_i^a \breve{\hat{H}}_j \Phi_3^a \\ &\quad + \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \Phi_i^a \breve{\hat{H}}_j \mathring{\Phi}_3^a - \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \Phi_i^a \mathring{\hat{H}}_j \breve{\Phi}^a{}_3 \\ &\quad + \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \breve{\Phi}_i^a \hat{H}_j \mathring{\Phi}_3^a - \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \mathring{\Phi}_i^a \hat{H}_j \breve{\Phi}_3^a], \end{aligned} \tag{E7}$$

where

$$\frac{\partial^2}{\partial\breve{A}\partial\mathring{B}} \breve{C}\mathring{D} \equiv \frac{\partial}{\partial\breve{A}} \breve{C} \frac{\partial}{\partial\mathring{B}} \mathring{D}, \tag{E8.1}$$

$$\frac{\partial^2}{\partial\breve{A}\partial\mathring{B}} \mathring{C}\breve{D} \equiv \frac{\partial}{\partial\breve{A}} \breve{D} \frac{\partial}{\partial\mathring{B}} \mathring{C}. \tag{E8.2}$$

Plugging Eq.(E7) into Eq.(E2), we obtain

$$\begin{aligned} \mathcal{L}_{ii} &= -\frac{1}{2} [y_U \epsilon_{ij3} [\frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \breve{\Phi}_i^a \mathring{\hat{H}}_j \Phi_3^a - \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \mathring{\Phi}_i^a \breve{\hat{H}}_j \Phi_3^a \\ &\quad + \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \Phi_i^a \breve{\hat{H}}_j \mathring{\Phi}_3^a - \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \Phi_i^a \mathring{\hat{H}}_j \breve{\Phi}^a{}_3 \\ &\quad + \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \breve{\Phi}_i^a \hat{H}_j \mathring{\Phi}_3^a - \frac{\partial^2}{\partial\breve{\phi}\partial\mathring{\phi}} \mathring{\Phi}_i^a \hat{H}_j \breve{\Phi}_3^a] \psi_i \psi_j + h.c.] \\ &= -\frac{1}{2} [y_U \epsilon_{ij3} [[\psi_i^a \psi_{\hat{H}_j} - \psi_j^a \psi_{\hat{H}_i}] \phi_3^a \\ &\quad + [\phi_i^a \psi_{\hat{H}_j} - \phi_j^a \psi_{\hat{H}_i}] \psi_3^a \\ &\quad + [\psi_i^a \phi_{\hat{H}_j} - \psi_j^a \phi_{\hat{H}_i}] \psi_3^a] + h.c.], \end{aligned} \tag{E9}$$

or in component fields,

$$\begin{aligned} \mathcal{L}_{ii} = y_U & \left[(H_1 D_L^a - H_2 U_L^a) \tilde{\bar{U}}_R^a + \left(H_1 \tilde{D}_L^a - H_2 \tilde{U}_L^a \right) \bar{U}_R^a + \left(\tilde{H}_1 D_L^a - \tilde{H}_2 U_L^a \right) \bar{U}_R^a \right] \\ & + h.c. \end{aligned} \tag{E10}$$

Here it holds for the third and fourth terms in Eq.(E2). For the third term, namely, we just replace Φ_i^a with $\hat{\Lambda}_i$, and Φ_3^a with $\hat{N}$ such that

$$\begin{aligned}\mathcal{L}_{iii} = -\frac{1}{2}[y_N\epsilon_{ij3}[[\psi_{\hat{\Lambda}_i}\psi_{\hat{H}_j} - \psi_{\hat{\Lambda}_j}\psi_{\hat{H}_i}]\phi_{\hat{N}} \\ + [\phi_{\hat{\Lambda}i}\psi_{\hat{H}_j} - \phi_{\hat{\Lambda}j}\psi_{\hat{H}_i}]\psi_{\hat{N}} \\ + [\psi_{\hat{\Lambda}_i}\phi_{\hat{H}_j} - \psi_{\hat{\Lambda}j}\phi_{\hat{H}_i}]\psi_{\hat{N}}] + h.c.],\end{aligned} \tag{E11}$$

or in component fields,

$$\begin{aligned}\mathcal{L}_{iii} = y_N\left[(H_1E_L - H_2N_L)\tilde{\bar{N}}_R + \left(H_1\tilde{E}_L - H_2\tilde{N}_L\right)\bar{N}_R + \left(\tilde{H}_1E_L - \tilde{H}_2N_L\right)\bar{N}_R\right] \\ + h.c.\end{aligned} \tag{E12}$$

Finally, for the last terms, we just replace Φ_i^a with $\hat{\Lambda}_i$, $\hat{H}_i$ with $\hat{H}'_i$, and $\hat{N}$ with $\hat{E}$ such that

$$\begin{aligned}\mathcal{L}_{iv} = -\frac{1}{2}[y_E\epsilon_{ij3}[[\psi_{\hat{\Lambda}_i}\psi_{\hat{H}'_j} - \psi_{\hat{\Lambda}_j}\psi_{\hat{H}'_i}]\phi_{\hat{E}} \\ + [\phi_{\hat{\Lambda}i}\psi_{\hat{H}'_j} - \phi_{\hat{\Lambda}j}\psi_{\hat{H}'_i}]\psi_{\hat{E}} \\ + [\psi_{\hat{\Lambda}_i}\phi_{\hat{H}'_j} - \psi_{\hat{\Lambda}j}\phi_{\hat{H}'_i}]\psi_{\hat{E}}] + h.c.],\end{aligned} \tag{E13}$$

or in component fields,

$$\begin{aligned}\mathcal{L}_{iv} = y_E\left[(H'_1E_L - H'_2N_L)\tilde{\bar{E}}_R + \left(H'_1\tilde{E}_L - H'_2\tilde{N}_L\right)\bar{E}_R + \left(\tilde{H}'_1E_L - \tilde{H}'_2N_L\right)\bar{E}_R\right] \\ + h.c.\end{aligned} \tag{E14}$$

Collecting all expressions just derived above, we exactly obtain the expression (5.31) given in Chapter 5.