FSI Phases and CP Asymmetries: QCD Approach to Inclusive B Decays

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Abstract

We consider the inclusive width difference $\Gamma(\bar{b} \to \bar{s} + charmless) - \Gamma(b \to s + charmless)$ in the Leading Log Approximation. Contrary to the popular opinion higher loops do not cancel the CP odd asymmetry but rather decrease it by $\leq 15\%$. The asymmetry is about -10^{-2} and $Br(b \to s + charmless) \simeq 2.5 \cdot 10^{-3}$ at $|V_{ub}/V_{cb}| = 0.1$ and the CP odd phase $\sin\alpha = 0.28$. Similar effects for the $b \to s\bar{s}s$ and $b \to d\bar{s}s$ are also calculated. We discuss briefly the physical conclusions of our analysis.

At present the problem of the CP non-conservation in beauty particles is rather urgent from both theoretical and experimental points of view. The standard strategy for the direct search of it in the near future in the Standard Model in e^+e^- annihilation is more or less clear as well as the general pattern of the expected effects and their size[1]. The prospects are based on the search for the CP odd effects in B mesons, which appear due to $B^0-\bar{B}^0$ transitions. Also of great interest is another kind of effects where the $B^0-\bar{B}^0$ mixing is not important, say in decays of $B^{\pm}$ mesons or Λ_b barions. Such reactions seem to be especially interesting for hadronic collisions as they do not require flavour tagging. However the simplest CP odd asymmetries in the decays not involving mixing are rather obscure as they are not determined only by a CP odd phase difference of the particular products of the KM matrix elements for the interfering amplitudes. In fact the sine of the difference of the CP even phases generated by the strong interactions in the final state (the FSI phases) for the amplitudes enters the asymmetries as an explicit factor. The values of FSI phases are completely unknown, so it is difficult to predict accurately not only the magnitude of the effects but their sign also. Provided some of the effects are measured, it will be difficult to interpret them unambiguously in terms of the fundamental parameters of the underlying theory.

The decays corresponding to the $b \to u\bar{u}s$ quark transitions seem to be the most interesting ones. The main reason is that for such processes the "Penguin" amplitudes due to the $b \to s + (c\bar{c}, t\bar{t})_{virt} \to s + q\bar{q}$ chain also contribute, and their magnitude appears to be close to that of the above tree level doubly suppressed amplitude. In the one loop approximation the corresponding Penguin amplitude has literally a CP even phase δ_P,

$$\tan\delta_P \simeq \pi/\log\frac{m_t^2, M_W^2}{m_b^2}.$$

Numerically it gives $\delta_P \simeq 0.5$, however the account for the relatively large mass of the c quark leads to the estimate $\delta_P \simeq 0.1$ and to the fact that δ_P is not actually local.

There is popular opinion that other FSI phases coming, for example, from large distances are negligible and for estimates of the effects one can rely on the Penguin phase δ_P. At the same time the opposite point of view can be found in the literature that in spite of the significant mass of b quark and independently of the large energy release in heavy quarks decays the FSI phases are generally large. In such case however it is impossible to predict theoretically even the sign of the effect.

We believe that *a priori* there are no real grounds to consider FSI phases as small, at least as compared to δ_P . This should be especially the case for exclusive processes where the result depends crucially on the actual dynamics of the formation of the particular final state, for example, on its "hard" stage. For instance for colour suppressed decays the hard part of the process can naturally contain a hard gluon exchange, and if so one could expect the FSI phase to be of the order of $\pi/2$. Moreover, even from the perturbative QCD point of view the expansion parameter for few body exclusive processes is $\alpha_s(m_b m_{\mu_{hadr}})$ rather than $\alpha_s(m_b^2)$; the so-called "hybrid" logs[2] appear here in many cases, that are certainly infrared sensitive. On the contrary from the theoretical point of view, for inclusive processes the QCD language of quarks and gluons is adequate; perturbative corrections are governed by the parameter $\alpha_s(m_b^2)$ and nonperturbative effects are small for sufficiently heavy b quark. Therefore, for inclusive processes the estimate based on the δ_P value is a reasonable first approximation.

In this paper we consider the CP odd inclusive width difference for the decays of b and $\bar{b}$ quarks into the states without heavy quarks. We present here the main points of our calculations and cite the results. We have calculated higher order QCD effects and found them to reduce the asymmetry only slightly (by 10-20%). This disagrees with the result of the paper [3] where the strong cancellation of the effect was claimed.

The main equation for the CP odd width difference has the form

$$\Delta\Gamma \equiv \Gamma(\bar{b} \to \bar{s}q\bar{q}) - \Gamma(b \to sq\bar{q}) = -4Im(\lambda_i\lambda_j^*) \cdot \sum_F Im(A_i(b \to F) \cdot A_j^*(b \to F)), \tag{1}$$

where F are the final states included for the process, $A_{i,j}$ are the decay amplitudes with the KM factors factored out and $\lambda_{i,j}$ are the corresponding KM factors. The S matrix unitarity enables one to write this equation in a more detailed way as a sum over two sets of real intermediate states for the $b \to b$ forward transition,

$$\Delta\Gamma = -2Im(\lambda_i\lambda_j^*) \cdot \sum_F \sum_I \{A_i(b \to I)A^*(F \to I)ReA_j(b \to F) -$$

$$- A_j(b \to I)A^*(F \to I)ReA_i(b \to F)\}. \tag{2}$$

Here I are real intermediate states in the decays $b \to F$, that lead to nontrivial FSI phases of the amplitudes A_i,A_j; $A(F \to I)$ are generated by strong interaction.

It can be shown using the S matrix unitarity condition that in any process at fixed i,j the second imaginary part in eq.(1) vanishes if one sums up the contributions over all possible final states F. Therefore the unitarity of Feynman diagrams leads us to the fact that the usual CPT identities hold even if one accounts for all possible cuts of any particular graph, and not only after summing the contributions of all graphs. Using this observation we can calculate the width difference for the decays into states with charm, $\Delta\Gamma_c \equiv \Delta\Gamma(\bar{b} \to \bar{s}c\bar{c} + X)$ instead of the original $\Delta\Gamma$. This is convenient because in such approach the suppression of the effect by the $c\bar{c}$ phase space factor is obvious in any contribution and never appears as some accidental cancellation of the resulting FSI phase for certain class of diagrams.

In the QCD perturbative expansion $\Delta\Gamma$ appears in the α_s order and does not contain $\log(m_t^2/m_b^2)$. In the spirit of the standard LLA we calculate here all the corrections of the form $\alpha_s^{n+1}\log^n(m_t^2/m_b^2)$. Our final result appears to be very simple. To calculate $\Delta\Gamma$ it is sufficient to account only for one-gluon rescattering amplitude $c\bar{c}X \to q\bar{q}X'$, and this effectively is just what one does in the lowest order estimate using the Penguin phase δ_P. To account for the higher orders one should, however, obtain the weak decay amplitudes $A_{i,j}(b \to F, I)$ using the effective $\Delta B = 1$ weak interaction Lagrangian normalized at the scale $q^2 = -m_b^2$ rather than the bare four-fermion one specified at $q^2 = -M_W^2$. It is worth to note that this result **cannot** be obtained by merely putting the FSI phase to be equal to the phase of the $b \to sq\bar{q}$ amplitude given by the extrapolation of the renormalization group expressions to the Minkowski region $q^2 = +m_b^2$.

To prove our prescription one can consider all possible states F and I (such as $s+g$, $sq\bar{q}$, $sq\bar{q}+g$ ...) step by step and estimate the corresponding 'strong' amplitudes $A(F \to I)$, namely their orders in powers of g_s. The important point here is that owing to the on-shellness of both F and I the strong amplitude $A(F \to I)$ cannot contain large logs, $\log\frac{m_t^2, M_W^2}{m_b^2}$, provided it is expressed in terms of the α_s normalized at $q^2 = -m_b^2$ [1]. In fact this statement is not more but the renormalizability of QCD.

It is not difficult to see that for states I containing a $c\bar{c}$ pair the two terms in eq.(2) cancel. This also naturally follows from CPT. In all other cases $A(F \to I)$ contains at least one power of g_s. The sequential analysis of all potential kinds of F and I states shows that the minimal power of α_s is obtained only in the one gluon annihilation $c\bar{c} \to g_{virt} \to q\bar{q}$. Here we use another crucial point—the absence in the LLA of the states $gg+s$ and $ggg+s$ among the intermediate states I.

Indeed, the Penguin operator which appear in the effective Lagrangian due to the integration over the virtual states with $q^2 \gg m_b^2$ has the form $\bar{b}\gamma_\mu\frac{\lambda^a}{2}s \cdot \nabla_\nu G^a_{\mu\nu}$, and in the absence of a light quark current the equation of motion $\nabla_\nu G^a_{\mu\nu} = 0$ ensures the vanishing of the matrix element for purely gluon final states in the leading order in α_s. For the $gg+s$ states, in particular, this vanishing corresponds to the cancellation in the LLA of the contributions of two graphs Figs. a) and b) for real gluons. As for the operator $m_b\bar{b}_R(\sigma G)s_L$ induced (with the small coefficient) by the renormalization, it also does not contribute obviously to eq.(2)

[1] For separate states F and I amplitudes may have infrared singularities which disappear after summing over the states with arbitrary number of soft and collinear gluons. In any case such singularities have nothing to do with and can be easily separated from the "ultraviolet" logs of the form $\log\frac{m_t^2, M_W^2}{m_b^2}$ we are interested in.

in the considered order in α_s.

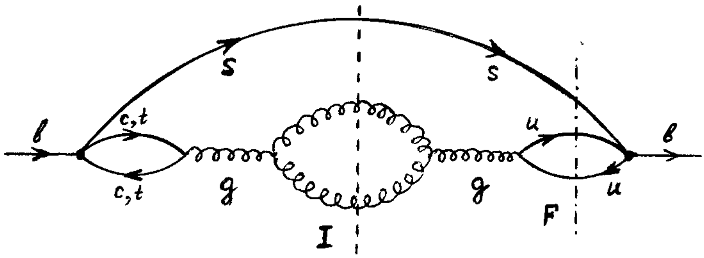

Fig. a)

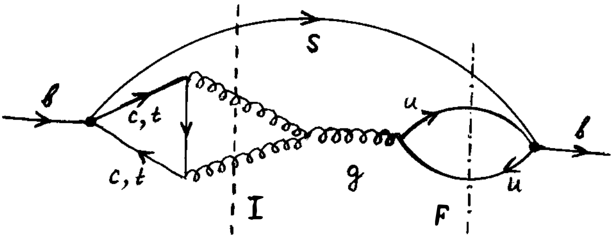

Fig. b)

As a result the first nontrivial correction to $\Delta\Gamma$ generated in two loops can be obtained by the substitution

$$\begin{aligned}\Delta\Gamma \quad &\propto \quad \frac{\alpha_s}{3\pi} Im[\log\frac{m_t^2}{m_b^2} + i\pi\zeta(m_c^2/m_b^2)] \\ &\rightarrow \quad \frac{\alpha_s(m_b^2)}{3\pi} Im[\log\frac{m_t^2}{m_b^2} + i\pi\zeta(m_c^2/m_b^2)\cdot(1-\frac{\alpha_s}{3\pi}(n_f+1)\log\frac{m_t^2}{m_b^2} + \kappa(c_\pm))],\end{aligned} \tag{3}$$

where n_f is the number of light flavours excluding c quark, $\zeta \simeq 0.2$ is the phase space suppression factor, $\kappa \simeq 2\alpha_s/4\pi \cdot \log M_W^2/m_b^2$ is the 'ordinary' correction factor due to the deviation of the standard factors $c_\pm$ in the effective Hamiltonian, H_{eff} , from unity. (In a very nonstrict sense the last term κ in the eq.(3) together with the unity added to n_f can be attributed to the corrections to modules of the interfering amplitudes rather than to the phases). For $n_f = 3$ and $\alpha_s(m_b^2) = 0.18$ the first term representing less trivial correction is about -0.17, however it is strongly canceled by the trivial corrections, $\kappa \simeq 0.13$. The summation of all orders in the LLA practically does not change the conclusion: the total correction to $\Delta\Gamma$ if taken literally appears to be near -2% for $\Lambda_{QCD} = 0.1 \div 0.3 GeV$. Of course there are some non-leading corrections which can be larger than the above LLA contribution due to the cancellation in the latter. We would estimate their natural size as $5 \div 10\%$. The electroweak corrections to H_{eff} which should be taken into account for $m_t \geq M_W$ are also small even at $m_t \simeq 250GeV$. Therefore numerically for the inclusive asymmetry we have

$$\frac{\Gamma(\bar{b} \to \bar{s} + charmless) - \Gamma(b \to s + charmless)}{\Gamma(\bar{b} \to \bar{s} + charmless) + \Gamma(b \to s + charmless)} \simeq -1.9 \cdot \zeta \cdot \left|\frac{V_{ub}}{V_{cb}}\right| \cdot \sin\alpha \sim \quad (4)$$

where $\alpha = arg(V_{cb}^* V_{cd} V_{ub} V_{ud}^*)$ is one of the angles of the Unitarity Triangle (see, e.g. ref.[1]), $\alpha \simeq 0.28$ at $|V_{ub}/V_{cb}| = 0.1$. The total probability of such decays $Br(b \to s + charmless)$ is in this case about $2.5 \cdot 10^{-3}$.

The above analysis disagrees essentially with the results of the ref.[3] where the QCD corrections were stated to nearly cancel the discussed width difference. In fact in that paper the authors accounted only for the (gauge dependent!) part of the second loop QCD corrections given by Fig. a) but not by Fig. b). We can guess also that the authors did not take into account certain cuts in the diagrams they considered assuming probably that the gg final states could be distinguished from those produced from the light quarks.

In the same way one can easily calculate also the CP odd inclusive width differences in the decays $b \to qs\bar{s}$, which are nonzero owing to the different phase space for the $c\bar{c}$ and $u\bar{u}$ intermediate states. Under the same numerical assumptions the similar to eq.(4) asymmetry for the decays $b \to ss\bar{s}$ is about $-9 \cdot 10^{-3}$ at $Br(b \to ss\bar{s}) \simeq 5 \cdot 10^{-4}$ and for the channel $b \to ds\bar{s}$ the asymmetry is near $+7 \cdot 10^{-2}$ while $Br(b \to ds\bar{s}) \simeq 7 \cdot 10^{-5}$.

The general conclusion we infer from our analysis supports the idea that in general even for Penguin processes and even at the somewhat academic assumptions that $m_b^2 \gg m_c^2 \gg \Lambda_{QCD}^2$ the FSI phases do not arise from small distances and are thus non-local. In principle they may be even not too small. For example it is only for the inclusive decays that the complexity of the Penguin amplitude necessary contains the charm mass suppression factor. For a general exclusive decay there is of course no reason for the phase to vanish, say in the limit $m_c > m_b/2$, so the contributions of other rescattering processes could easily compete with the small δ_P. Moreover, we feel that the terminology itself about the FSI phases of amplitudes is somewhat misleading for common heavy flavour decays, and it is probably more adequate to speak directly about various on-mass-shell rescattering contributions. Nevertheless for inclusive observables the QCD expansion parameter for the effective FSI complexities is $\alpha_s(m_b^2)$ and at least parametrically the inclusive CP odd asymmetries can be treated in the perturbative framework.

At present the measurement of the inclusive CP odd asymmetries in beauty particles seems to be an extremely difficult experimental problem. Indeed, the main decay channel $b \to s + q\bar{q}$ has probability only about $2.5 \cdot 10^{-3}$ whereas the allowed decay chain $b \to c + \bar{u}d \to su\bar{d} + \bar{u}d$ occurs in a half of the b decays and leads to the same quark flavour content. So here one would probably need an extremly fine secondary vertex resolution to suppress the background by two orders of magnitude. Nevertheless the QCD calculations we have made are not a kind of useless business. In fact these corrections enter any quark diagram that proceeds via Penguins. Our scepticism about calculability of the FSI phases for exclusive decays is in fact not more but the statement that in this case there could be other and even larger sources for these phases.

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