PROBLEMS OF MAJORIZATION OF FEYNMAN DIAGRAMS

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An interesting method of majorization of diagrams has been suggested in papers of Nambu and Symanzik. My report is devoted to the extension of the majorization method [1)].

Let us consider strongly connected graphs with l_N external nucleon and l_M external π-meson lines, in each vertex of which three and only three lines enter: an even number (2 or 0) of baryon lines and an odd number (1 or 3) of meson lines. The set of all such graphs is designated by R. We suppose that the external momenta are Euclidean vectors and their scalar products are real. Let us denote by $G(D)$ the domain of the momentum values in which matrix elements of the diagram D has no singularities.

If it is known for two graphs, $D \in R$ and $D' \in R$, that $G(D) \subseteq G(D')$, then for finding the intersection of the regions $G_R = \cap G(D)$ among the two graphs, D and D', it is sufficient to take into account only the graph D. In this case we will say that the graph D majorizes the graph D'. Let us introduce the concept of sub-diagram. If, after removing from a diagram $D \in R$ some internal lines and internal vertices, a graph $D' \in R$ is obtained, then we shall call the graph D' a sub-diagram of the graph D (with respect to R). It is not difficult to prove the following theorems:

Theorem I: Each diagram can be majorized by any of its sub-diagrams.

Theorem II: Let the graph D contain a closed loop with $n+1$ vertices, to n sides of which the mass M corresponds, and to one side the mass $m \leq M$. Changing the masses on these sides in the following way: $M \to m$, $m \to M$, yields a new graph D; which majorizes the graph D.

We will call the diagram D of R primitive in R, if it is impossible to find in R with the aid of theorems I and II a diagram majorizing the diagram D. Denote by R_0 the set of all diagrams primitive in R. We have found R_0 for the diagrams satisfying the following condition: $l_N + l_M \leq 4$. Due to the lack of time, I shall not discuss the method of search for the primitive diagrams. I would only like to note that searching for the primitive diagrams is essentially a problem of finding in a diagram some minimal

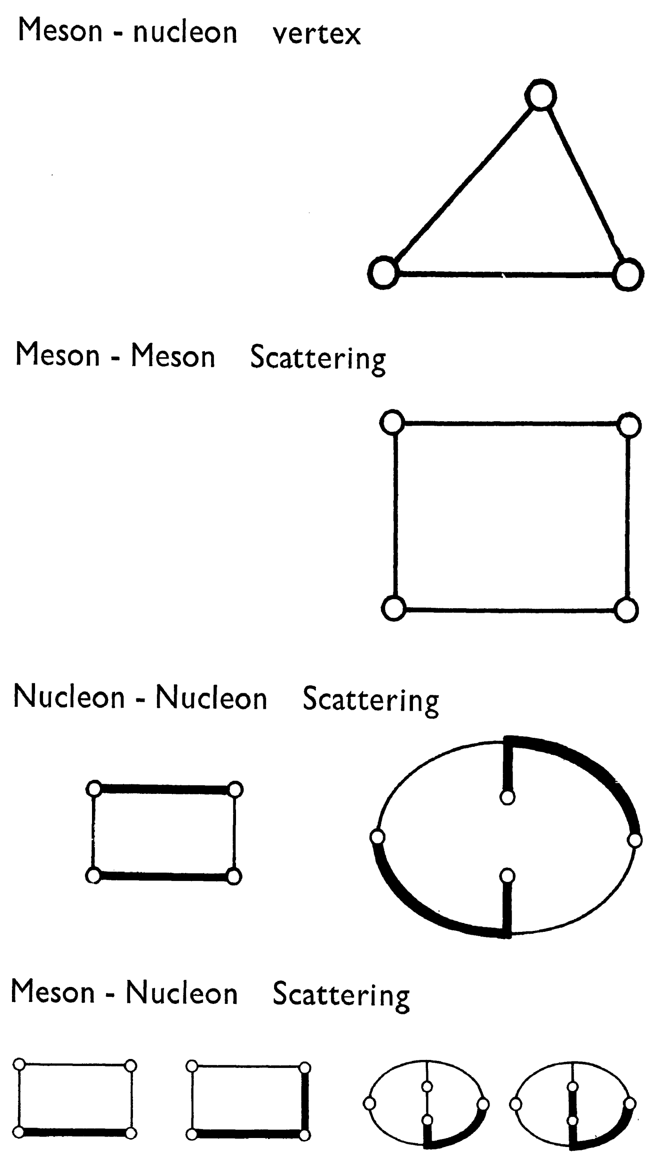

Fig. 1. Majorizing diagrams for some processes.

system of internal lines which are strongly connected [2] and which do not violate nucleonic charge conservation. To compare the primitive diagrams with each other, more detailed information about the quadratic forms is required. Symanzik's theorem and theorem III (see below) are effective means for this purpose. To begin with, note that the function:

$$\bar{f}_D(\rho) = \max_{\alpha} \sqrt{\frac{A_D(\alpha,\rho)}{M_D^2(\alpha)}}\,, \tag{1}$$

possesses all properties of the norm, where $A_D(\alpha, \rho)$ is a quadratic form of diagram D, and $M_D^2(\alpha)$ is a mass term.

Theorem III: Let there be given $K+1$ diagrams $D_\sigma(\sigma = 0,1,...K)$ of the same process. Let $\bar{f}_{D_\sigma}(\rho)$ be the norms corresponding to the graphs D_σ. In order that for all Euclidean ρ the inequality

$$\bar{f}_{D_0}(\rho) \leq \max_{\sigma=1,\dots,k} \bar{f}_{D_\sigma}(\rho) = \bar{f}_{1,\dots,k}(\rho) \tag{2}$$

holds, it is necessary and sufficient that for all Euclidean z the conjugated norms would satisfy the inequality: [3]

$$f_{1,\dots,k}(z) \leq f_{D_0}(z) = \min_{z_{\text{int}}} \sum_{\nu=1}^{l} m_\nu \left| \sum_{i=1}^{n-1} l_{i\nu} z_i \right|. \tag{3}$$

$f_{1,\dots,k}(z)$ is the largest norm, satisfying the condition

$$f_{1,\dots,k}(z) \leq \min_{\sigma=1,\dots,k} f_{D_\sigma}(z) \tag{4}$$

This theorem gives a necessary and sufficient condition for diagrams D_σ to majorize the diagram D_0. A sufficient condition for the fulfillment of condition (2) is:

$$\min_{\sigma=1,\dots,k} f_{D_\sigma}(z) \leq f_{D_0}(z) \tag{5}$$

Using Symanzik's theorem and theorem III, we can reduce the number of majorizing diagrams. In this way we obtain the majorizing diagrams for some processes shown in Fig. 1.

LIST OF REFERENCES AND NOTES

1. For a detailed presentation, see Chernikov, N. A., Logunov, A. A. and Todorov, I. T.; preprint JINR (1960).
2. A diagram is called strongly connected when it still remains connected after withdrawing any line.
3. The incidence matrix E $= (l_{i\nu})$, $i = 1,...n$; $\nu = 1,...l$; of such a graph is determined in the following manner:

$$l_{i\nu} = \begin{cases} 1, & \text{if the line } \nu \text{ leaves the vertex } i, \\ -1, & \text{if the line } \nu \text{ enters the vertex } i, \\ 0, & \text{if the vertex } i \text{ does not belong to the line } \nu. \end{cases}$$

DISCUSSION

EDEN: I just want to comment that the theorems of majorization are also contained in the discriminant formalism which I was using earlier. If we differentiate the discriminant $D(\alpha,s,t)$ with respect to α, we find

$$\frac{\partial D(\alpha,s,t)}{\partial \alpha_i} = D(\alpha_i\,,\, s,\, t,\, \alpha_i^{-1}) - m_i^2 C(\alpha) - \alpha_i m_i^2 C(\alpha_i\,,\, \alpha_i^{-1})$$

where the first term on the right means the disciminant of the diagram from which the line i has been removed. The other two terms are negative. Since at a singularity we have $\dfrac{\partial D}{\partial \alpha_i} = 0$, we cannot have a singularity of the more complicated diagram in the region where the discriminant of the simpler diagram is negative. In particular, if one goes to the Euclidean region, the simpler diagrams have their discriminants negative and, therefore, the more complicated diagrams with lines added cannot have singularities. One can also deduce that they are negative by the formula $D = \dfrac{1}{n}\sum_i \alpha_i \dfrac{\partial D}{\partial \alpha_i}$. This is just Euler's formula, which follows from the fact that the discriminant D is homogeneous in the alpha variables. This also can be applied to vertex parts. I think this formalism is not so general as that described by Logunov in that Logunov has taken account of conservation laws in forming reduced diagrams, but I believe that this could be extended also to take account of conservation laws.