BRANE PHENOMENOLOGY

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We review motivations and properties of low-scale string models and phenomenological applications to brane world scenarios with large extra dimensions.

1 Introduction

String theory is probably the best candidate for a fundamental quantum theory of all interactions, including the Einstein gravity. The theory contains only one free parameter, the string scale M_s. The four-dimensional (4d) gauge group and the matter content depend on geometric properties of the compact space. The Standard Model is supposed to correspond to a particular compact space or vacuum configuration. There is therefore in principle the hope to understand the empirically observed pattern of the parameters in the Standard Model.

It was conjectured [3] that in Type I strings the string scale can be lowered all the way down to the TeV range. Similar ideas for lowering the fundamental Planck scale in theories with (sub)millimeter gravitational dimensions [4] appeared, as an alternative solution to the gauge hierarchy problem, and, simultaneously, a new way for lowering the GUT scale in theories with large (TeV) dimensions[5]. The new emerging picture found a simple realization in a perturbative Type I setting[6] with low (in the TeV range) string scale and became subject of an intense activity, (mostly) on the phenomenological side and on the theoretical side. Recently, a toy-model for localizing gravity was also proposed [13], in which extra dimensions can be infinitely large !

Our goal is to review some of the ideas which emerged from this new picture: milimeter and TeV extra dimensions, gauge coupling unification, neutrino masses and localized gravity.

2 Millimeter and TeV^{-1} large extra dimensions

The presence of branes [1] in Type I, Type II strings and M-theory open new perspectives for particle physics phenomenology. Indeed, in Type I strings the string scale is not necessarily tied to the Planck scale. In view of the new D-brane picture which emerged, let us take a closer look to the simplest example of compactified Type I string with only D9 branes present. Let us split the compact volume into two parts, $V = V^{(1)}V^{(2)}$, where $V^{(1)}$, of dimension $6 - n$, is of order one in string units and $V^{(2)}$, of dimension n, is very small. The Kaluza-Klein states of the brane fields along $V^{(2)}$ are much heavier than the string scale and therefore difficult to excite. The physics is then better captured in this case by performing T-dualities along $V^{(2)}$, which read $\lambda_I' = \lambda_I / V^{(2)} M_I^n$, $V_\perp = 1/V^{(2)} M_I^{2n}$. In the T-dual picture we get, neglecting numerical factors

$$M_P^2 \sim \frac{1}{\alpha_{GUT} \lambda_I'} V_\perp M_I^{2+n} \quad , \quad \frac{1}{\alpha_{GUT}} \sim \frac{V_{||} M_I^{6-n}}{\lambda_I'} \ , \tag{1}$$

where we redefined for simplicity of notation $V^{(1)} \equiv V_{||}$. After the n T-dualities, the D9 brane becomes a D(9-n) brane, since the T-dual winding modes of the bulk (orthogonal) compact space are very heavy and therefore the brane fields cannot propagate in the bulk. As seen from (1), for a very large value of the bulk volume the string scale can be very low $M_I << M_P$. The geometric picture here is that we have a D-brane with some compact radii parallel to it, of the order M_I^{-1}, and some very large, orthogonal compact radii. In particular, if the full compact space is orthogonal to the brane ($n = 6$), then from (1) the T-dual string coupling is fixed by the unified coupling $\lambda_I' \sim \alpha_{GUT}$ and therefore we find [6]

$$M_P^2 \sim V_\perp M_I^{2+n} \ , \tag{2}$$

a relation similar to that proposed in the field-theoretical scenario of [4].

Le us now imagine a "world-brane" picture in which the Standard Model gauge group and charged fields are confined to the D-brane under consideration. We can then ask a very important question: what are the present experimental limits on parallel $R_{||}$ and perpendicular $R_\perp$ type radii ? The Standard Model fields have light KK states in the parallel directions $R_{||}$. Their possible effects in accelerators were studied in detail [9] and the present limits are $R_{||}^{-1} \geq 4 - 5$ TeV. On the other hand, Standard Model excitations with respect to $R_\perp$ are very heavy and are basically irrelevant. The main constraints on $R_\perp$ come from the presence of very light winding (KK after T-dualities) gravitational excitations, which can therefore generate deviations of the gravitational attraction from the Newton law. The actual experimental limits on such deviations are in the cm range and experiments in the near future are planned to improve them [10]. For $M_I \sim$ TeV in (2), the case of only one extra dimension is clearly excluded, since it asks for $R_\perp^{-1} \sim 10^8$ Km. However, already for two extra dimensions, we find $R_\perp^{-1} \sim 1$mm, which is not yet excluded by the present experimental data.

There are clearly a lot of other challanging questions that such a scenario must answer in order to be seriously considered as an alternative to the conventional "desert picture" of supersymmetric unification at energies of the order of 10^{16} GeV. Serious questions concern gauge coupling unification, which in this case, if exists, must be completely different from the conventional MSSM one and also supersymmetry breaking [2]. Also, there is more and more convincing evidence for neutrino masses and mixings, and the conventional picture provides an elegant explanation via the seesaw mechanism with a mass scale of the order of the $10^{12} - 10^{15}$ GeV, surprisingly close to the usual GUT scale. The new scenario described above must therefore provide at least a qualitative picture for neutrino masses and mixings. There are also cosmological, astrophysical and accelerator physics tests [4] which puts strong constraints, too, on the low-scale string scenario.

3 Gauge coupling unification

At the same time as brane-world models with low-string scale as an alternative to supersymmetry to the gauge hierarchy problem, models with gauge-coupling unification at low energy triggered by Kaluza-Klein states were independently proposed [5]. It was soon realized that low-scale string models were the natural framework for this fast-driven unification. We separate here the discussion into two steps: the field-theoretic picture originally proposed in [5] and then the Type I string approach developped in [7,8] which brings some new, interesting features.

The essential ingredient in the field theory approach are the KK excitations of the Standard Model gauge bosons and matter multiplets and their contribution to the energy evolution of the physical gauge couplings. The KK excitations give power-law corrections to be interpreted at low energy as threshold corrections. If the energy is higher than the KK compactification scale $1/R$, these corrections are really to be interpreted as a power-law accelerated evolution of gauge couplings which, under some reasonable assumptions, can bring the couplings together at low energies.

Let us start, for reasons to be explained later on, with the MSSM in 4d and try to extend it in 5d, where the fifth dimension is a circle of radius $R_{//}$, with the notations introduced in the previous section. Consider for concreteness gauge couplings of a D9 brane and consider δ large compact dimensions $R_{||}M_I >> 1$ parallel to D9 and orthogonal to D5. Then 99 states will have associated KK states, but 95 states do not. By evaluating the gauge couplings at one-loop, we find [5]

$$
\begin{aligned}
\frac{1}{\alpha_a(\mu)} &= \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi}\ln\frac{\mu}{M_Z} - \frac{\tilde{b}_a}{2\pi}\int_{1/\mu^2}^{1/\mu_0^2}\frac{dt}{t}\theta_3^\delta\left(\frac{it}{\pi R_{||}^2}\right) \\
&\simeq \frac{1}{\alpha_a(M_Z)} - \frac{b_a}{2\pi}\ln\frac{\mu}{M_Z} + \frac{\tilde{b}_a}{2\pi}\ln(\mu R_{||}) - \frac{\tilde{b}_a}{2\pi}[(\mu R_{||})^\delta - 1]\ . \qquad (3)
\end{aligned}
$$

The coefficients b_a in (3) denote usual MSSM beta function coefficients and $\tilde{b}_a$ denote one-loop beta-function coefficients of massive KK modes, to be computed in a specific model. The important term contained in (3) is the power-like term $(\mu R_{||})^\delta >> 1$, which takes over the logarithmic terms in higher-dimensional regime and govern the eventual unification pattern. Notice that compactifying on a circle a supersymmetric theory in 5d gives a 4d theory with at least $\mathcal{N}=2$ supersymmetries. The simplest way to avoid this is to compactify on an *orbifold*. We consider as example the case of a Z_2 orbifold which breaks supersymmetry down to $\mathcal{N}=1$. Interestingly enough, in the simplest extension of MSSM in higher dimensions, the gauge couplings unify with a surprisingly good precision, for any compact radius 10^3 GeV $\leq R_{||}^{-1} \leq 10^{15}$ GeV, at a energy scale roughly a factor of 20 above the compactification scale $R_{||}^{-1}$. This fast unification with KK states is another numerical miracle, similar to the MSSM unification and up to now is the only hint pointing into the possible relevance of extra dimensions in our world.

In a superstring model, the one-loop threshold corrections have contributions from $\mathcal{N}=4$, $\mathcal{N}=2$ and $\mathcal{N}=1$ sectors, respectively. The $\mathcal{N}=4$ sectors, containing the full compactification lattice, have a 10d origin and give no contribution to threshold corrections. The $\mathcal{N}=2$ sectors contain the lattice of one compact torus. In these sectors only BPS KK states contribute to threshold corrections and string oscillators decouple [7]. Their contribution to the evolution of gauge couplings does not stop therefore at the string scale M_I, but rather, as we will see, at a heavy KK scale. The $\mathcal{N}=1$ sectors have no KK excitations and give a moduli-independent contribution to threshold-corrections, interpreted as the $\mathcal{N}=1$ contribution to gauge couplings, running up to M_I.

The string one-loop threshold corrections coming from $\mathcal{N}=2$ sectors were computed in a generic model in [8]. The complete one-loop gauge couplings reads

$$\frac{4\pi^2}{g_a^2(\mu)} = \frac{1}{l} + \sum_k s_{ak} m_k + \frac{1}{4} b_a^{(\mathcal{N}=1)} \ln \frac{M_I^2}{\mu^2} - \frac{1}{4} \sum_{i=1}^{3} b_{ai}^{(\mathcal{N}=2)} \ln(\sqrt{G_i}\mu^2 |\eta(U_i)|^4 \mathrm{Im} U_i) , \qquad (4)$$

where for a rectangular torus of radii R_1, R_2, we have $\sqrt{G_i} = R_1 R_2$ and $\mathrm{Im} U = R_1/R_2$. In (4) $b_{ai}^{(\mathcal{N}=2)}$ denote beta function coefficients from $\mathcal{N} = 2$ sectors having KK excitations in the compact torus T^i and m_k are twisted closed string moduli.

Let us consider now the field-theory limit of the corrections given by an $\mathcal{N} = 2$ sector, depending on a torus of radii $R_{1,2}$. In the limit $R_1 \to \infty$ and R_2 fixed, the corrections are linearly divergent $\Lambda_2 \sim R_1/R_2$. These power-law corrections can be used for the purpose of lowering the unification scale [5] in models with a low value of the string scale M_I. Notice that in all the above computations, μ denoted an infrared energy scale, smaller than any KK mass scales. Actually, for energies $\mu >> R_1^{-1}$, relevant for the $R_1 \to \infty$ limit, it can be seen that the previous factor R_1/R_2 really becomes $R_1\mu$, reproducing therefore the field-theory derivation (3) with $\delta = 1$. In this case, to get unification we need 10^3 GeV $\leq R_1^{-1} \leq 10^{15}$ GeV.

On the other hand, in the limit $R_1, R_2 \to \infty$ with R_1/R_2 fixed, $\Lambda_2 \sim \ln(R_1 R_2 \mu^2)$, instead of the quadratic divergence ($\delta = 2$ in (3)) expected in the field theory approach. The same result holds in the $R_1, R_2 \to 0$ limit. This result can be understood by the following argument [7]. After T-duality, the two directions are very large and perpendicular to the brane under consideration. One-loop threshold corrections can also be understood as tree-level coupling of gauge fields to closed sector fields, which have a bulk variation reproducing the threshold dependence on the compact space. The bulk variation can be computed in a supergravity approximation, solving classical field equations for closed fields coupled to various sources subject to global neutrality (or global tadpole cancellation) in the compact space. As the Green function in two-dimensions has a logarithmic behaviour, this explain the logarithmic term $\ln(R_1 R_2 \mu^2)$. The same argument in one compact dimension explains also the linearly divergent term previously discussed.

4 Bulk physics: Neutrino masses with large extra dimensions

The most elegant mechanism for explaining the smallness of neutrino masses postulate the existence of right-handed neutrinos with very large associated Majorana masses $10^{11}GeV \leq M \leq 10^{15}GeV$. Via the seesaw mechanism very small neutrino masses, of the order of $m_\nu \sim v^2/M$, are generated, where $v \simeq 246GeV$ is the vev of the Higgs field. This suggests the presence of a large (intermediate or GUT) scale in the theory, presumbly related to new physics. On the other hand, low-scale string models do not have such a large scale and superficially have therefore problems to accomodate neutrino masses. It will be argued here that actually there is a natural way to find very small neutrino masses. The scenario is based on the observation that right-handed neutrinos can be put in the bulk space of a very large (mm size) compact space [12], perpendicular to the brane where we live. We consider for simplicity the case of one family of neutrinos. The model consists of our brane with the left-handed neutrino ν_L and Higgs field stuck on it and one bulk Dirac neutrino $\Psi = (\psi_1, \bar{\psi}_2)^T$ in Weyl notations in one (again for simplicity) compact perpendicular direction y. The compact direction is taken here to be an orbifold S^1/Z_2, since as is well known circle compactifications are not phenomenologically realistic. The Z_2 orbifold acts on the spinors as $Z_2\Psi(y) = \pm\gamma_5\Psi(-y)$, such that one of the two-component Weyl spinors, e.g. ψ_1, will be even under the Z_2 action $y \to -y$, while the other spinor ψ_2 will be odd. If the left-handed neutrino ν_L is restricted to a brane located at the orbifold fixed point $y = 0$, then ψ_2 vanishes at this point and so ν_L couples only to ψ_1. This then results in a Lagrangian of the form

$$\mathcal{L} \;=\; \frac{1}{2}\int d^4x\, dy\, M_s \left\{ \bar{\psi} i \bar{\gamma}^M \partial_M \psi - \partial_M \bar{\psi} i \bar{\gamma}^M \psi \right\}$$

$$+ \int d^4x \left\{ \bar{\nu}_L i\bar{\sigma}^\mu D_\mu \nu_L \;+\; (\hat{m}\nu_L \psi_1|_{y=0} + \text{h.c.}) \right\} . \tag{5}$$

Here M_s is the mass scale of the higher-dimensional fundamental theory (e.g., a reduced Type I string scale) and the M spacetime index runs over all five dimensions: $x^M \equiv (x^\mu, y)$. The first line represents the kinetic-energy term for the 5d Ψ field and the second line the kinetic energy of the 4d two-component neutrino field ν_L as well as the coupling between ν_L and ψ_1. Note that in 5d, a bare Dirac mass term for Ψ would not have been invariant under the action of the Z_2 orbifold, since $\bar{\Psi}\Psi \sim \psi_1\psi_2+$ h.c.

Next, we compactify the Lagrangian (5) down to 4d by expanding the 5d Ψ field in Kaluza-Klein modes. Imposing the orbifold relations $\psi_{1,2}(-y) = \pm\psi_{1,2}(y)$ implies that our Kaluza-Klein decomposition takes the form

$$\psi_1(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \psi_1^{(n)}(x)\, \cos(ny/R) \;,\;\; \psi_2(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \psi_2^{(n)}(x)\, \sin(ny/R) \;. \tag{6}$$

However, a more general possibility emerges naturally from the Scherk-Schwarz compactification. Let us consider performing a local rotation in (ψ_1, ψ_2) space of the form

$$\begin{pmatrix} \hat{\psi}_1 \\ \hat{\psi}_2 \end{pmatrix} \equiv \mathcal{R} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \qquad \text{where} \;\; \mathcal{R} \equiv \begin{pmatrix} \cos(\omega y/R) & -\sin(\omega y/R) \\ \sin(\omega y/R) & \cos(\omega y/R) \end{pmatrix} \;, \tag{7}$$

with ω a real (for the moment) number. The effect of the matrix $\mathcal{R}$ in (7) is to twist the fermions after a $2\pi R$ rotation on y. Such twisted boundary conditions are allowed in field and string theory if the higher-dimensional (5d in this case) theory has an appropriate $U(1)$ symmetry. The 4d Lagrangian of the component fields coming from the 5d Lagrangian reads from (5) by replacing everywhere $\psi_i \to \hat{\psi}_i$. For convenience, we shall define in the following the linear combinations $N^{(n)} \equiv (\psi_1^{(n)} + \psi_2^{(n)})/\sqrt{2}$ and $M^{(n)} \equiv (\psi_1^{(n)} - \psi_2^{(n)})/\sqrt{2}$ for all $n > 0$.

Inserting (7) into (5) and integrating over the compactified dimension then yields

$$\begin{aligned}
\mathcal{L} \;=\; \int d^4x \, \Bigg\{ & \bar{\nu}_L i\bar{\sigma}^\mu D_\mu \nu_L + \bar{\psi}_1^{(0)} i\bar{\sigma}^\mu \partial_\mu \psi_1^{(0)} + \sum_{n=1}^{\infty} \left(\bar{N}^{(n)} i\bar{\sigma}^\mu \partial_\mu N^{(n)} + \bar{M}^{(n)} i\bar{\sigma}^\mu \partial_\mu M^{(n)} \right) \\
& + \Bigg\{ \frac{1}{2} M_0\, \psi_1^{(0)}\psi_1^{(0)} \;+\; \frac{1}{2}\sum_{n=1}^{\infty} \left[\left(M_0 + \frac{n}{R} \right) N^{(n)}N^{(n)} + \left(M_0 - \frac{n}{R} \right) M^{(n)}M^{(n)} \right] \\
& + \; m \left[\nu_L \psi_1^{(0)} + \nu_L \sum_{n=1}^{\infty} \left(N^{(n)} + M^{(n)} \right) \right] \;+\; \text{h.c.} \Bigg\} \Bigg\} .
\end{aligned} \tag{8}$$

where the Majorana mass is $M_0 = \omega/R$. Here the first line gives the four-dimensional kinetic-energy terms, while the second line gives the Kaluza-Klein and Majorana mass terms. The third line of (8) describes the coupling between the 4d neutrino ν_L and the 5d field Ψ. Note that in obtaining this Lagrangian, it is necessary to rescale the individual $\psi_1^{(0)}$, $N^{(n)}$, and $M^{(n)}$ Kaluza-Klein modes so that their 4d kinetic-energy terms are canonically normalized. This then results in a suppression of the Dirac neutrino mass $\hat{m}$ by the factor $(2\pi M_s R)^{1/2}$. In the third line, we have therefore simply defined the effective Dirac neutrino mass couplings $m \equiv \hat{m}/\sqrt{2}\sqrt{\pi M_s R}$. Given the Lagrangian (8), we see that the Standard-Model neutrino ν_L will mix with the entire tower of Kaluza-Klein states of the higher-dimensional Ψ field. Indeed, if we restrict our attention to the case of only one extra dimension for simplicity and define $\mathcal{N}^T \equiv (\nu_L, \psi_1^{(0)}, N^{(1)}, M^{(1)}, N^{(2)}, M^{(2)}, ...)$ we see that the mass terms in the Lagrangian (8) take the form $(1/2)(\mathcal{N}^T\mathcal{M}\mathcal{N}+\text{h.c.})$, where the mass matrix is symmetric and takes the form

$$\mathcal{M} = \begin{pmatrix} 0 & m & m & m & m & m & \dots \\ m & M_0 & 0 & 0 & 0 & 0 & \dots \\ m & 0 & M_0+1/R & 0 & 0 & 0 & \dots \\ m & 0 & 0 & M_0-1/R & 0 & 0 & \dots \\ m & 0 & 0 & 0 & M_0+2/R & 0 & \dots \\ m & 0 & 0 & 0 & 0 & M_0-2/R & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{9}$$

Let us start for simplicity by disregarding the possible bare Majorana mass term, setting $M_0 = 0$. In this case, the characteristic polynomial which determines the eigenvalues λ of the mass matrix (9) can be exactly worked out and takes the form

$$\lambda R = \pi(mR)^2 \cot(\pi\lambda R) \ . \tag{10}$$

All the eigenvalues can be determined from this equation, as functions of the product mR. This equation can be analyzed graphically [12]. In the limit $mR \to 0$ (corresponding to $m \to 0$), the eigenvalues are k/R, $k \in Z$, with a double eigenvalue at $k = 0$. Conversely, in the limit $mR \to \infty$, the eigenvalues with $k > 0$ smoothly shift to $(k+1/2)/R$, while those with $k < 0$ shift to $(k-1/2)/R$ and the double zero eigenvalue splits towards the values $\pm 1/(2R)$. In order to derive general analytical expressions valid in the limit $mR \ll 1$, we can solve (10) iteratively by power-expanding the cotangent function. To order $\mathcal{O}(m^5R^5)$, this gives the solutions

$$\lambda_\pm = \pm m\left(1-\frac{\pi^2}{6}m^2R^2+\ldots\right) , \qquad \lambda_{\pm k} = \pm\frac{k}{R}\left(1+\frac{m^2R^2}{k^2}-\frac{m^4R^4}{k^4}+\ldots\right) , \tag{11}$$

where $\lambda_{\pm k}$ are the two eigenvalues at each Kaluza-Klein level k and $\lambda_\pm$ are the "light" eigenvalues at $k = 0$.

Let us now come to the more general case of $M_0 \neq 0$. It turns out to be useful to define $k_0 \equiv [M_0R]$, $\epsilon \equiv M_0 - \frac{k_0}{R}$, where $[x]$ denotes the integer nearest to x. Thus, ϵ is the smallest diagonal entry in the mass matrix (9), corresponding to the excited Kaluza-Klein state $M^{(k_0)}$. In other words, we have $\epsilon \equiv M_0$ (modulo R^{-1}), satisfying $-1/(2R) < \epsilon \leq 1/(2R)$. The remaining diagonal entries in the mass matrix can then be expressed as $\epsilon \pm k'/R$ where $k' \in Z^+$. Unlike M_0, we see that $|\epsilon| \sim \mathcal{O}(R^{-1})$. Thus, the heavy Majorana mass scale M_0 completely *decouples* from the physics. Indeed, the value of M_0 enters the results only through its determinations of k_0 and the precise value of ϵ. Therefore, interestingly enough, the presence of the infinite tower of regularly-spaced Kaluza-Klein states ensures that only the value of M_0 modulo R^{-1} plays a role.

The easiest way to solve for the eigenvalues $\lambda_\pm$ in this case is to integrate out the Kaluza-Klein modes. It turns out that there are two cases to consider, depending on the value of ϵ. If $|\epsilon| \gg m$ (which can arise when $mR \ll 1$), then *all* of the Kaluza-Klein modes are extremely massive relative to m, and we can integrate them out to obtain an effective $\nu_L\nu_L$ mass term of size

$$\begin{aligned} |\epsilon| \gg m : \qquad m_\nu &= m^2/\epsilon + m^2\sum_{k'=1}^{\infty}\left(\frac{1}{\epsilon+k'/R}+\frac{1}{\epsilon-k'/R}\right) \\ &= \pi m^2 R\cot(\pi R\epsilon) \ . \end{aligned} \tag{12}$$

We shall discuss the special case $\epsilon = 1/2R$ later on. Alternatively, if $|\epsilon| \not\gg m$, then the lightest Kaluza-Klein mode $M^{(k_0)}$ should not be integrated out, and we obtain an effective $\nu_L\nu_L$ mass

Diagonalizing the final resulting 2×2 mass matrix between ν_L and $M^{(k_0)}$ in the presence of this mass term then yields the result

$$|\epsilon| \not\gg m: \qquad \lambda_\pm = \frac{1}{2}\left[(\mu+\epsilon) \pm \sqrt{(\mu-\epsilon)^2+4m^2}\right] \quad . \tag{13}$$

Thus, as $M_0 \to 0$ (or as $M_0 \to n/R$ where $n \in Z$), we see that $\epsilon, \mu \to 0$, and we recover the eigenvalues given in (11). We reiterate that our effective seesaw scale is $M_{\rm eff} \sim \mathcal{O}(R^{-1})$.

In string theory, however, there are additional topological constraints (coming from the preservation of the form of the worldsheet supercurrent) that permit only *discrete* values of ω. In particular, in a compactification from five to four dimensions, this restriction limits us to the only non-trivial possibility $\omega = 1/2$. Taking $\omega = 1/2$ then implies $\psi_{1,2}(2\pi R) = -\psi_{1,2}(0)$, which shows that lepton number is broken globally (although not locally) as the spinor is taken around the compactified space. In order to obtain the corresponding neutrino mass, we note that for $\epsilon = 1/2R$, the assumption $mR \ll 1$ translates into $\epsilon >> m$, whereupon the result (12) is valid. Thus, for $\epsilon = 1/2R$, we find the remarkable result that $m_\nu = 0$! In obtaining this result, one might worry that (12) is only approximate because it relies on the procedure of integrating out the Kaluza-Klein states rather than a full diagonalization of the corresponding mass matrix. However, it is straightforward to show that when $\epsilon = 1/2R$, the characteristic eigenvalue equation $\det(\mathcal{M} - \lambda I) = 0$ has an exact trivial solution $\lambda = 0$, corresponding to an exactly massless neutrino. Thus, we conclude that $m_\nu = 0$ for $\epsilon = 1/2R$, regardless of the relative sizes of m and R.

Note that the massless neutrino eigenstate is *primarily* composed of the neutrino gauge eigenstate ν_L (since $mR \ll 1$), as required phenomenologically. It should be stressed that this combined neutrino mass eigenstate is exactly massless in the limit that the full, infinite tower of Kaluza-Klein states participates in the mixing[a]. This result is valid *regardless* of the value of neutrino Yukawa coupling m or of the scale R^{-1} of the Kaluza-Klein states.

It is also interesting to notice that exactly the desired value of the Majorana mass $M_0 = 1/2R$ emerges naturally from a Scherk-Schwarz decomposition, for reasons that are *topological* and hence do not require any fine-tuning.

The scenario(s) presented have also other interesting consequences. The neutrino eigenstate can now oscillate into an infinite tower of right-handed KK neutrinos with a probability that can be reliably estimated and experimentally tested. Moreover, even if in the last scenario presented the physical neutrino is massless, its probability of oscillation into the tower of KK states is nonvanishing. In particular, the neutrino mass difference $\Delta m \sim 10^{-2} eV$, which fit the experimental data could well be explained by an oscillation of the massless neutrino into the first KK state, for a radius $R^{-1} \sim 10^{-2} eV$, which is precisely in the mm region we are interested in !

5 Conclusions

The last years changed a lot our current understanding of string physics and its possible implications for low energy physics. In particular, there is a real hope to experimentally test scenarios with a low string scale, large compactification (TeV) radii and eventual (sub)millimeter gravitational dimensions. Some of the relevant issues (gauge coupling unification, supersymmetry breaking, gauge hierarchy problem) were already analyzed at string level by using quasirealistic

[a]Actually, our field theory approach breaks down for KK masses of the order of the fundamental string scale M_s. If we cut our summation at $k_{max} = RM_s$, the physical neutrino is not exactly massless anymore, but aquires a small mass $m_\nu \sim m^2/M_s$. For phenomenologically interesting values $m \sim R^{-1} \simeq 10^{-2} eV$ and $M_s \sim$ TeV, this mass is however negligibly small $m_\nu \sim 10^{-15} eV$.

string models, other issues (flavor physics, for example) were mainly studied at field theory level and more detailed string studies would be very useful.

It is however important to keep in mind that, despite the beautiful new ideas dealing with large (or infinite) extra dimensions which appeared recently, the good old picture of the "desert" between the weak scale and a large (of the order of 10^{16} GeV) unification scale is still a viable possibility. Only new experimental results can provide a hint for the real value of the string scale or, more generally, for the real picture of the physics beyond the Standard Model.

Acknowledgments

It is a pleasure to thank the organizers of the Electroweak Session of Moriond for a very enjoyable conference.

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