DOUBLE STRATONOVICH - HUBBARD TRICK AND NOVEL PATH INTEGRAL FOR A SYSTEM OF INTERACTING FERMIONS

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1. Introduction

The calculation techniques that come under the path integral approach have proven to be a powerful and effective tools in a quantum mechanical applications. Thirty years ago Stratonovich [1] and Hubbard [2] laid the basis for a widely used method of calculating the partition function $Z(\beta)$ of a many-particle system with two-body interactions, by expressing $Z(\beta)$ as a Gaussian average over partition functions for systems interacting with "time-dependent" external fluctuating classical fields. These fluctuating fields are fictitious and therefore the Stratonovich-Hubbard (S-H) functional integrals should be distinguished from Feynman path integrals. Unfortunately, the structure of the partition function in the S-H representation for realistic Hamiltonians is very complicated: one must carry out averaging of the exponents of non-commuting time-dependent operators. A general and universal method is not known for elimination of the operators, in contrast to the Feynman path integral method. While "Gaussian functional average" methods for statistical [3], solid state [4,5] and nuclear [6,7] physics, are of considerable importance, but the methods are at present incomplete.

In this paper we give a new functional averaging procedure for a many-fermion system. It is based on a Grassmanian version of the S-H transformation (plus the usual one) and on a disentangling of the evolution operator utilizing properties of the relevant super-Weyl group SW(N), and fermionic coherent state methods for computation of the trace.

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2. Fermionic Hamltonians and Double S-H Trick

Consider the Hamiltonian H for a system of fermions interacting via a two-body potential V

$$H = \sum_i \varepsilon_i \hat{c}_i^+ \hat{c}_i + \frac{1}{2} \sum_{ijkl} \langle i, j | V | k, l \rangle \hat{c}_i^+ \hat{c}_j^+ \hat{c}_k \hat{c}_l ,$$

Where $\{\hat{c}_k^{\#}\}$ are the usual fermionic operators; the index k includes both momentum and spin indices $k \equiv (\vec{k}, \sigma)$; $\varepsilon_k = (\vec{k}^2/2m_o - \mu)$ is the one-particle energy, m_o is the mass of the fermion and μ - the chemical potential.

Usually, the methods of determination of some values connected with such Hamiltonians involve the linearization scheme based on a mean-field approximation:

$\hat{A}\hat{B} \approx \hat{A}\langle B \rangle + \hat{B}\langle A \rangle$,

where $\hat{A}$ and $\hat{B}$ are an appropriate quadratic in $\hat{c}_i$, $\hat{c}_j^+$ operators, and $\langle A \rangle$, $\langle B \rangle$ are the expectation values in some ground state. The resulting approximate (mean-field) Hamiltonian H_{m-f} is diagonalizable because it becomes linear in the generators of any compact Lie algebra (Dynamical Algebra of the mean-field Hamiltonian). Moreover, the partition function and the many-fermion Green functions of the reduced problem can be built up from the factors completely determined by a Dynamical Algebra [8]. A convenient approach suggestion for the calculation of the mean-field theory corrections is the main purpose of this paper.

For simplicity we now restrict attention to a generic model with H an operator-valued function of the generators $\hat{X}_{ab}(\vec{k})$ of a Lie algebra

$$H = \sum_k \varepsilon_a(\vec{k}) \hat{X}_{ab}(\vec{k}) + \frac{1}{2} \sum_{\vec{k},\vec{k}'} V_{ab\,a'b'}(\vec{k},\vec{k}') \hat{X}_{ab}(\vec{k}) \hat{X}_{ab}(\vec{k}) \quad (1)$$

(sum over all repeated indices)

Having applications in solid state physics in mind, we make explicit use of a quasimomentum label $\vec{k}$ and write $\hat{X}_{ab}(\vec{k}) = \hat{f}_a^+(\vec{k})\hat{f}_b(\vec{k})$. The notation $\{\hat{f}_a^{\#}\}$ permits us to distinguish various quasimomenta ($\pm\vec{k}$, $\vec{k}\pm\vec{Q}$, etc.) and, e.g. in the BCS case permits a correspondence with familiar creation/annihilation operators $(\hat{f}_1(\vec{k}), \hat{f}_2(\vec{k})) \equiv (\hat{c}_{\vec{k}\uparrow}, \hat{c}^{+}_{-\vec{k}\downarrow})$etc. An SU(8) model along these lines unifying, superconductivity and charge and spin density waves discussed elsewhere [9].

Here the operators $\hat{X}_{ab}(\vec{k})$ obey the commutation relations

$$[\hat{X}_{ab}(\vec{k}), \hat{X}_{a'b'}(\vec{k}')] = \delta_{\vec{k}\vec{k}'} \cdot (\delta_{ba'} \cdot \hat{X}_{ab'}(\vec{k}) - \delta_{ab'} \cdot \hat{X}_{a'b}(\vec{k})) \qquad (2)$$

of the Lie algebra gl(N,R), if a, b, ...= $\overline{1,\ N}$ (the number N depends upon the model considered) in formula (1), $\varepsilon(\vec{k})$ is the excitation energy of the fermionic oscillation of the kind "a" which may differ in sign from the corresponding one-particle energy.

The matrix element of the interaction operator V can be expanded in a series

$$\frac{1}{2} V_{aba'b'}(\vec{k}, \vec{k}') = \sum_{e} v^{(e)}_{ab}(\vec{k})\, \bar{v}^{(e)}_{a'b'}(\vec{k}') \qquad (3)$$

Consider the simplest case of a separable potential ($e \equiv 1$) and represent the right side of eqn. (3) as $v_{ab}(\vec{k})\bar{v}_{a'b'}(\vec{k}')$. (This restriction is not required for the following, but in the general case the calculations are more unwieldy).

Then, our Hamiltonian has the structure

$$\hat{H} = \hat{H}_{m-f} + \hat{V}, \qquad (4)$$

where $\hat{H}_{m-f} = \sum_{\vec{k}} \varepsilon_a(\vec{k}) \hat{X}_{aa}(\vec{k})$

and $\hat{V} = \left(\sum_{\vec{k}} v_{ab}(\vec{k}) \hat{X}_{ab}(\vec{k})\right) \left(\sum_{\vec{k}'} \bar{v}_{a'b'}(\vec{k}') \hat{X}_{a'b'}(\vec{k}')\right) \equiv \hat{A}\hat{B}$.

(Here we consider a case when the mean-field Hamiltonian has a diagonal form in the Dynamical Algebra generators; we have to combine our approach with a Bogoliubov-like diagonalization procedure in more general cases).

The Grand Partition Function for the Hamiltonian of (4) can be written

$$Z(\beta) = \mathrm{Tr}\, \exp(-\beta \hat{H}) = \mathrm{Tr}\left[\exp(-\beta \hat{H}_{m-f})\, \hat{T}_d \exp(-\int_0 \hat{V}(\tau) d\tau)\right], \qquad (5)$$

where $\hat{T}_d$ - the time ordering operator, and $\hat{V}(\tau) \equiv \exp(\tau \hat{H}_{m-f})\hat{V} \exp(-\tau \hat{H}_{m-f})$ is $\hat{V}$ in the interaction representation.

We now follow a familiar procedure : in eqn. (5) divide the interval $[0, \beta]$ into m equal segments as $[0, \beta_1] \ldots [\beta_{m-1}, \beta_m = \beta]$ $\beta_l = (\beta l/m)$; use the S-H transformation for each factor in the product.

$$\exp(-(\beta/m)\hat{V}(\tau_l)) \equiv \exp(-(\beta/m)\hat{A}(\tau_l)\hat{B}(\tau_l)) =$$

$$\int d\,\mathrm{Re}\, z(\tau_l)\, d\,\mathrm{Im}\, z(\tau_l)\, (\pi m/\beta)^{-1} \exp(-(\beta/m)\bar{z}(\tau_l) z(\tau_l)) \times$$

$$\times \exp\left[-(\beta/m)\, z(\tau_l)\, \hat{A}(\tau_l) + (\beta/m)\, \bar{z}(\tau_l)\, \hat{B}(\tau_l) \right] \qquad (6)$$

(valid to terms of order $(\beta/m)^2$); then using (6) and recalling that operators labelled by different $\vec{k}$ commute, we obtain

$$Z(\beta) = \prod_{\vec{k}} \lim_{m\to\infty} \mathrm{Tr}\left[\hat{T}_d \int \ldots \int \prod_{l=1}^{m} d\,\mathrm{Re}\, z(\tau_l) d\,\mathrm{Im}\, z(\tau_l) (\pi m/\beta)^{-1} \times \right.$$

$$\times \exp(-(\beta/m) z(\tau_l)\bar{z}(\tau_l)) \exp(-(\beta/m)\varepsilon_a(\vec{k})\, \hat{X}_{aa}(\vec{k})) \times$$

$$\left. \times \exp\left[(\beta/m)(\bar{z}(\tau_l)\bar{v}_{ab}(\vec{k}) - z(\tau_l) v_{ab}(\vec{k}))\, \hat{X}_{ab}(\vec{k}, \tau_l)\right] \right]. \qquad (7)$$

Note in the product (7) the "time dependent" operators $\hat{X}_{ab}(\vec{k}, \tau_l)$ are some linear combinations of the "time-independent" operators $\hat{X}_{ab}(\vec{k})$ which preserve the Lie algebraic structure (2) for the simple case of (4) $\hat{X}_{ab}(\vec{k}, \tau_l) = \hat{X}_{ab}(\vec{k}) \exp\left[\tau_l(\varepsilon_a(\vec{k}) - \varepsilon_b(\vec{k}))\right]$.

Below, we shall omit the explicit $\vec{k}$ dependence where it can cause no confusion. Taking the formal limits $m \to \infty$, and interchanging integration and trace, we get

$$Z(\beta) = \prod_{\vec{k}} \int D(\bar{z}, z) \exp(-\int \bar{z}(\tau) z(\tau)\, d\tau)\, \mathrm{Tr}\left[\exp(-\beta\varepsilon_a \hat{X}_{aa}) \times \right.$$

$$\left. T_d \exp\left[-\int (z(\tau) v_{ab} - \bar{z}(\tau)\bar{v}_{ab}) \exp(\tau(\varepsilon_a - \varepsilon_b))\, \hat{X}_{ab}\right]\right] \equiv$$

$$\equiv \prod_{\vec{k}} \langle \chi^G(\vec{k}, \beta) \rangle_{GAUSS} , \tag{8}$$

Where $D(\bar{z}, z) = \lim_{m\to\infty} \prod_{l=1}^{m} d\,\mathrm{Re}\, z(\tau_l)\, d\,\mathrm{Im}\, z(\tau_l)\, (\pi m/\beta)^{-1}$,

and χ^G is the character of the (reducible) representation of Lie group $G \equiv GL(N,R)$, or of some particular subgroup depending on specific form of $\hat{H}$. These methods are well known [3-7]. Unfortunately we cannot obtain the characters χ^G explicitly since the $\bar{z}(\tau)$ and $z(\tau)$ are general fluctuating functions of τ. Often a "static" path approximation is used: $z(\tau) \equiv$ const, which is equivalent to mean-field theory.

To overcome this limitation we return to eqn. (7) and introduce the Grassmanian version of the S-H transformation. Namely, if $\hat{F}_1$ and $\hat{F}_2$ are anticommuting operators $\{\hat{F}_1, \hat{F}_2\} = 0$, then

$$\exp(\hat{F}_1 \hat{F}_2) = \int d\xi^* d\xi \exp(-\xi^*\xi + \hat{F}_1 \xi + \xi^* \hat{F}_2).$$

Here ξ and ξ^* are Grassmanian anticommiting variables:
$\{\xi, \xi^*\} = 0,\ \xi^2 = \xi^{*2} = 0,\ \{\xi, \hat{F}_{1,2}\} = \{\xi^*, \hat{F}_{1,2}\} = 0.$
To apply this transformation in our case, we use an assymmetric definition of operators $\hat{F}_1$ and $\hat{F}_2$:

$$\hat{F}_1 = \sqrt{\beta/m}\ (\bar{z}(\tau_l)\bar{v}_{ac} - z(\tau_l)\, v_{ac})\exp(\tau\varepsilon_a)\, \hat{f}_a^+ ,$$

$$\hat{F}_2 = \sqrt{\beta/m}\ \exp(-\tau\varepsilon_b)\, \hat{f}_b .$$

In this case the anticommutator $\{\hat{F}_1, \hat{F}_2\} \sim (\beta/m)$ and can be neglected in the limit $m \to \infty$.

Consider the eqn. (7) and represent the last exponent, as a product, and for each factor, use the Grassmanian S-H trick to obtain:

$$Z(\beta) = \prod_{\vec{k}} \lim_{m\to\infty} \mathrm{Tr}\Big[\exp(-\beta\varepsilon_a \hat{f}_a^+ \hat{f}_a) \hat{T}_d \int \dots \int \Big[d\,\mathrm{Re} z(\tau_l) d\,\mathrm{Im} z(\tau_l) \times$$
$$\times (\pi m/\beta)^{-1} \prod_{ab} d\xi^*_{ab}(\tau_l) d\xi_{ab}(\tau_l) (\beta/m)^{-1} \exp(-(\beta/m)\bar{z}(\tau_l) z(\tau_l)) \times$$
$$\exp(-(\beta/m)\xi^*_{ab}(\tau_l)\xi_{ab}(\tau_l)) \exp(-(\beta/m)(\hat{f}_a^+ \zeta_a(\tau_l) - \bar{\zeta}_a(\tau_l)\hat{f}_a))\Big]\Big] \tag{9}$$

(Under the time-ordering operation T_d all operators commute.) In eqn. (9) we introduced notation

$$\zeta_a(\tau_1) = \exp(\tau_1 \varepsilon_a) \sum_b (z(\tau_1)v_{ab} - \bar{z}(\tau_1)\bar{v}_{ab})\xi_{ab}(\tau_1),$$
$$\tilde{\zeta}_a(\tau_1) = \exp(-\tau_1 \varepsilon_a) \sum_b \xi^*_{ba}(\tau_1).$$

Hence, in the limit $m \to \infty$, $Z(\beta)$ has been represented as a double Gaussian average over the fluctations af the complex fields $\bar{z}(\tau)$, $z(\tau)$ and the Grassmanian fields $\xi^*_{ab}(\tau)$ and $\xi_{ab}(\tau)$:

$$Z(\beta) = \prod_{\vec{k}} \mathrm{Tr}\left[\exp(-\beta\varepsilon_a \hat{f}^+_a \hat{f}_a) \int D(\bar{z}, z; \xi^*, \xi) \exp\left[-\int_0^\beta (\bar{z}z + \sum_{ab} \xi^*_{ab}\xi_{ab})\, d\tau\right] \hat{T}_d \exp\left[\sum_a \int_0^\beta (\hat{f}^+_a \zeta_a - \tilde{\zeta}_a \hat{f}_a) d\tau\right]\right], \quad (10)$$

where

$$D(\bar{z}, z; \xi^*, \xi) = D(\bar{z}, z) \lim_{m \to \infty} \prod_{l=1}^m \prod_{ab} d\xi^*_{ab}(\tau_l) d\xi_{ab}(\tau_l)(\beta/m)^{-1}.$$

3. Super-Weyl Group and Novel Path Integral Structure

The Dyson-Exponent in the right side of (10) satisfies the Schrodinger-like equation

$$-\frac{d}{d\tau}\hat{T}_d \exp[\ldots] = \mathcal{H}(\tau)\hat{T}_d \exp[\ldots] \quad (11)$$

here the Hamiltonian $\mathcal{H}(\tau)$ is a linear combination of the operators $\hat{f}^+_a$, $\hat{f}_a$ with time-dependent Grassmanian coefficients. (This Dyson-exponent violates hermicity, but it is restored after integration over both Grassmanian and complex variables.) In order to present a factorized form of exponent in eqn.(11) we seek a "disentangled" solution of eqn.(11) as:

$$\hat{T}_d \exp\left[\ldots\right] = \exp(\lambda(\tau)\hat{I}) \prod_a \exp(\hat{f}^+_a \eta_a(\tau)) \prod_{a'} \exp(\tilde{\eta}_{a'}(\tau)\hat{f}_{a'}) \quad (12)$$

Observe that the set $\hat{f}^+_a$, $\hat{f}_a(a=1,\ldots,N)$ plus the unit operator $\hat{I}$

generate the super-Weyl Group SW(N) which is the dynamical group of the intermediate Hamiltonian $\mathcal{H}(\tau)$ of eqn.(11). Now substituting (12) into (11) we get a system of equations for the coefficients λ, η_a and $\tilde{\eta}_a$. These can be solved to give

$$\eta_a(\tau) = \int_0^{\tau} \zeta_a(\tau')\, d\tau' , \quad \tilde{\eta}_a(\tau) = -\int_0^{\tau} \tilde{\zeta}_a(\tau')\, d\tau' ,$$

$$\lambda(\tau) = \sum_a \int_0^{\tau}\int_0^{\tau} \theta(\tau'-\tau'')\, \zeta_a(\tau')\, \tilde{\zeta}_a(\tau'')\, d\tau'\, d\tau'' ,$$

where $\theta(\tau)$ is the usual step-function with the boundary condition $\theta(0) = 0$.

In order to simplify the expression for $Z(\beta)$, let us introduce the system of fermionic coherent states (FCS) [10] :

$$|\,\theta_1,\ldots,\theta_N\rangle = \exp(-\theta_1 \hat{f}_1^+)\ldots\exp(-\theta_N \hat{f}_N^+)\,|\,0,\ldots,0\rangle$$

(here $|\,0,\ldots,0\rangle$ is the Fock-vacuum vector) with the scalar product

$$\langle\theta_1,\ldots,\theta_N\,|\,\theta_1',\ldots,\theta_N'\rangle = \exp(\theta_1^*\theta_1' + \ldots + \theta_N^*\theta_N'). \quad (13)$$

Note that our definition of the FCS is slightly different from Ref.10 . For the trace of any operator $\hat{R}(\hat{f}^+, \hat{f})$ we have :

$$\mathrm{Tr}\,\hat{R} = \int \langle\theta_1,\ldots,\theta_N|\,\hat{R}\,|\theta_1,\ldots,\theta_N\rangle \exp(\theta_1^*\theta_1 + \ldots + \theta_N\theta_N^*)\, d\theta_1 d\theta_1^* \ldots d\theta_N d\theta_N^* \quad (14)$$

Moreover, as in the case of Glauber coherent states it is easy to prove that

$$\prod_{a=1}^{N} \exp(s_a \hat{f}_a^+ \hat{f}_a)\,|\,\theta_1,\ldots,\theta_N\rangle = |\,e^{s_1}\theta_1,\ldots,e^{s_N}\theta_N\rangle . \quad (15)$$

Using the trace formula (14), computing the matrix element of the "evolution" operator between FCS, and taking into account the formulas (13) and (15), we can calculate the trace exactly, because the integrals over the external Grassmanian variables θ_1, $\theta_1^*,\ldots$, θ_N, θ_N^* are Gaussian, and get the following path integral representation for the partition function:

$$Z(\beta) = \prod_{\vec{k}} Z_0(\beta, \vec{k}) \int D(\bar{z}, z; \xi^*, \xi) \exp\left[-\int_0^\beta (\bar{z}z + \sum_{ab} \xi^*_{ab}\xi_{ab})\, d\tau\right] \times$$

$$\times \exp\left[\sum_a \int_0^\beta\!\!\int_0^\beta (\theta(\tau-\tau')-n_a)\, \zeta_a(\tau)\, \tilde{\zeta}_a(\tau')\, d\tau\, d\tau'\right], \quad (16)$$

where $Z_0(\beta, \vec{k}) = \prod_a [1 + \exp(-\beta\varepsilon_a(\vec{k}))]$ is the partition function of the system of noninteracting fermionic oscillators; and $n_a = [1+\exp(-\beta\varepsilon_a(\vec{k}))]^{-1}$ is the average number of noninteracting fermions of kind "a". Examining eqn.(16) we may first integrate over all the complex fields $\bar{z}(\tau)$, $z(\tau)$. This integrals have the structure

$$\int D(\bar{z}, z) \exp[-\int_0^\beta\!\!\int_0^\beta \bar{z}(\tau)\,\delta(\tau-\tau')\,z(\tau')\,d\tau\, d\tau' + \int_0^\beta (Jz+\tilde{J}\bar{z})\,d\tau],$$

where "the currents" $J(\vec{k}, \tau)$ and $\tilde{J}(\vec{k}, \tau)$ are bilinear combinations of ξ^*_{ab} and ξ_{ab}

$$J(\vec{k}, \tau) = \sum_{abc} v_{ab} \int_0^\beta [\theta(\tau-\tau_1)-n_a(\vec{k})]\, e^{\varepsilon_a(\vec{k})(\tau-\tau_1)}\, \xi_{ab}(\tau)\, \xi^*_{ca}(\tau_1)\, d\tau_1,$$

$$\tilde{J}(\vec{k}, \tau) = -\sum_{abc} \bar{v}_{ab} \int_0^\beta [\theta(\tau-\tau_1)-n_a(\vec{k})]\, e^{\varepsilon_a(\vec{k})(\tau-\tau_1)}\, \xi_{ab}(\tau)\, \xi^*_{ca}(\tau_1)\, d\tau_1.$$

Calculating those integral, we obtain a path integral over Grassmanian variables only, namely:

$$Z(\beta) = \prod_{\vec{k}} Z_0(\beta, \vec{k}) \int D(\xi^*, \xi) \exp[-\int_0^\beta\!\!\int_0^\beta \xi^*_{ab}(\tau)\,\delta(\tau-\tau')\,\xi_{ab}(\tau')\,d\tau\, d\tau']$$

$$\exp[\int_0^\beta\!\!\int_0^\beta \tilde{J}(\vec{k}, \tau)\,\delta(\tau-\tau')\,J(\vec{k}, \tau')\, d\tau\, d\tau'] \quad (17)$$

It is essential to stress that in the derivation of this formula we do not make any approximation, once we were given $\hat{H}$. Since the path integral (17) is not Gaussian we cannot compute it exactly, but it is useful for perturbation treatments. For this purpose we can utilize a power series expansion of the last exponent in (17) and, permute the path and "time " integrations, and thus obtain these integrals (for each term in series) using the generating function method. It is natural to formulate this calculations using diagrammatic techniques, and these calculations will be reported elsewhere.

Alternatively consider integration over the Grassmanian fields in the functional integral (16). This is a more difficult problem in the general case. The integral which involves only the Grassman field integrations we may write as

$$\int D(\xi^*, \xi)\exp[-\int_0^\beta\int_0^\beta (\vec{\xi}^*(\tau))^T K(\tau,\tau')\vec{\xi}(\tau')\, d\tau\, d\tau'] \qquad (18)$$

where $\vec{\xi}$ is(in general) the N^2- dimensional column with the compo- components $\xi_{ab}$and K is the $N^2 \times N^2$ functional matrix whose matrix elements depends on $\bar{z}$and z. It is easy to show that the K has a structure $K = I + M$, where I is unity matrix, the matrix elements of which are equal δ-functions $\delta(\tau - \tau')$ on the main diagonal, and M is a matrix with zero trace. Calculation of the integral (18) leads us to the following formal relation

$$Z(\beta) = \prod_{\vec{k}}\int D(\bar{z},z)Z_0(\beta,\vec{k})\det K(z,\bar{z};\vec{k})\exp[-\int_0^\beta\int_0^\beta \bar{z}\,\delta\, z\, d\tau\, d\tau'] \qquad (19)$$

Comparing expression (19) with eqn. (8), we observe that the determinant of the matrix K multiplied by Z_0 is equal to the character of the group G. In the simple cases (small number N) $\det K$ can be computed exactly because the members of the power series $\det K$ = $\Sigma_n[(-1)^n/n]\,\mathrm{tr}\, M^n$ may vanish for $n > n_{max}$ (some finite number). It is convenient to use the partition function representation in the last form (19) for the derivation of the 1/N expansion for the study of the thermodynamic behaviour of the system in the limit $N\to\infty$ (the saddle-point calculations).

4. Simple example and conclusions

As a brief illustration of an applications of our method, consider a simple (but non-trivial) "Lipkin-like" model

$$\hat{H} = 2\varepsilon\ \hat{J}_z - 4\lambda\hat{J}_x^2 \tag{20}$$

where $\hat{J}_x$, $\hat{J}_y = i[\hat{J}_x, \hat{J}_y]$, $\hat{J}_z$ generate SU(2) group. This model can describe a system of N types of fermions occupying two different states [6]. If N >> 1 there are two significant regimes in parameter space: 1) $\lambda < \varepsilon/2N$, and 2) $\lambda > \varepsilon/2N$, as described in ref. [6]. The mean-field Hamiltonian in 1) is: $H_{m-f} = -2\varepsilon J_z$, while in 2) $H''_{m-f} = -2\varepsilon \hat{J}_z + 8\lambda \langle J_x\rangle \hat{J}_x$. Here $\langle J_x\rangle \neq 0$ is the order parameter which can be calculated via a variational procedure. To use our approach, introduce two types of Grassmanian variables ξ^*_a, ξ_a (a=1,2) and applying eqn.(17), we find in region 1):

$$Z(\beta) = [2(1+\mathrm{ch}\,\varepsilon\beta)]^N[1+\lambda\beta/(1+\mathrm{ch}\,\varepsilon\beta) + \ldots]^N \tag{21}$$

and for the full system energy $E = -(d/d\beta)\ \ln Z(\beta)$, $E = -N\lambda/2 - N\varepsilon^2\beta/2 + O(\beta^2)$ ($\beta \to 0$; $N \to \infty$). This coincides with the usual pertubative result.

The application of our approach to computing the corrections to the mean-field theory for this model and for several model Hamiltonians - BCS SU(2) model, the SU(8) unified superconductivity and density wave model, and the Anderson model will be discussed elsewhere [11].

Summarizing: using the double S-H trick and disentangling method for the dynamical super-Weyl group SW(N) developed here we obtained a new factorized path integral form of the Grand Partition Function Z of many-fermion systems which offers the possibility of alternate and systematic corrections to the mean-field theory. We believe such approach is effective at least for certain model four-fermion Hamiltonians.

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References:

1. R.L. Stratonovich: Dokl. Akad. Nauk. SSSR 115 (1957) ; Translated Sov. Phys. Dokl. 2 416 (1958).
2. J Hubbard: Phys. Rev. Lett. 3 77 (1959).
3. F.W. Wiegel: Phys. Reports 16C 57 (1975).
4. B. Muhlschlegel: J. Math. Phys. 3 522 (1962); and in Path Integrals and their Applications in Quantum, Statistical, and Solid State Physics, ed. by G.J. Papadopoulos and J.T. Devreese (Plenum Press, New York and London, 1978).
5. Dai, Xian-xi and Chin-Sin Ting: Phys. Rev. 2B 5243 (1984).
6. P. Arve, G. Bertsch, B. Lauritzen an G. Puddu: Ann. Phys. (USA) 183 309 (1988).
7. R. Cenni and P. Saracco: Nuovo Cimento 11D 303 (1989).
8. J.L. Birman and A.I. Solomon: Prog. Theor. Phys. (Kyoto), Suppl. 80 62 (1984).
9. A.I. Solomon and J.L. Birman: J. Math. Phys. 28 1526 (1987).
10. J.W.F. Valle: J. Math. Phys. 22 1521 (1981).
11. A.V. Gorokhov and J.L. Birman (in preparation).