



# Beyond self-acceleration: Force- and fluid-acceleration

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## ABSTRACT

The notion of self acceleration has been introduced as a convenient way to theoretically distinguish cosmological models in which acceleration is due to modified gravity from those in which it is due to the properties of matter or fields. In this paper we review the concept of self acceleration as given, for example, by [1], and highlight two problems. First, that it applies only to universal couplings, and second, that it is too narrow, i.e. it excludes models in which the acceleration can be shown to be induced by a genuine modification of gravity, for instance coupled dark energy with a universal coupling, the Hu-Sawicki  $f(R)$  model or, in the context of inflation, the Starobinski model. We then propose two new, more general, concepts in its place: force-acceleration and field-acceleration, which are also applicable in presence of non universal cosmologies. We illustrate their concrete application with two examples, among the modified gravity classes which are still in agreement with current data, i.e.  $f(R)$  models and coupled dark energy.

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## 1. Introduction

The origin of cosmic acceleration is one of the most puzzling and debated questions in contemporary cosmology: the biggest challenge relies in understanding whether acceleration is caused by a cosmological constant or not. Alternatives to a cosmological constant roughly divide into two main approaches: the first considers an additional dynamical new fluid that adds to the usual cosmologically relevant matter fields, i.e. dark matter, baryons, radiation, neutrinos; the second, appealing path, tries to attribute acceleration to a change in gravity and General Relativity itself (which typically also requires new degrees of freedom associated with new fields). If a new field is present, this could couple minimally (i.e. purely gravitationally, as in the standard General Relativity) or non-minimally to the other matter fields or to gravity or both. Moreover, the coupling could be either universal or depend on the matter field, thereby violating the equivalence principle. The notion of self acceleration was introduced after the discovery of cosmic acceleration to denote accelerated models without a cosmological constant (see e.g. [2–6]). This classification was then generalized and made more precise in [1], and it is to this definition that we refer specifically in the following. Since then, it has often been used to highlight those cosmologies in which cosmic acceleration is due to a ‘pure’ modification of gravity (see e.g. [7–13]), and also employed to motivate (or not) further investigation of certain classes of modified gravity models.

In this paper we review what this concept actually means and discuss more precisely its limitations. We then propose a new classification that addresses such limitations, with the same original motivation in mind as for self acceleration, but applicable also to non universal couplings to matter species. This is particularly interesting in view of the fact that non universal coupled dark energy are among the classes of modified gravity models which are still compatible with all current observations: if baryons are minimally coupled to gravity, in fact, local gravitational constraints are automatically satisfied, as well as the recent constraints on the speed of gravitational waves.

It is to be noted that the concept of self-acceleration is purely theoretical. Since all we can measure from cosmological observations in a homogeneous and isotropic universe is the expansion rate  $H(z)$  and the space curvature, it is impossible to determine from cosmological observations alone whether our universe is self-accelerated or not. We will not discuss the issue of observability further in this paper and

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rather concentrate on the concept of self-acceleration as presented in literature, i.e. as a mean to determine whether, within a given theory, the predicted (theoretical) cosmic acceleration is to be attributed to a modification of gravity or not.

## 2. Reference frames

In order to discuss self-acceleration, its shortcomings and possible extensions, it is necessary to review the concept of reference frames in some detail.

Observational data are in general obtained under the reasonable assumption that our reference laboratory system (units of time, distance, mass) does not vary in time and space. This means that the *observed* reference system is the one in which baryons are coupled to that metric for which this is true: we define then the *observational metric* the one in which baryons follow geodesics. We refer to  $S_{obs}$ , as to the action in which this condition holds.

Let's consider a model containing Einstein gravity, matter, and a scalar field. If  $T_{\mu\nu}^{(b)}$  is the baryon Energy-Momentum Tensor (EMT), then the observational metric (or frame) is the one in which the EMT for baryons is conserved:

$$\nabla_{\mu} T_{\nu}^{(b)\mu} = 0 \quad (\text{observational frame}) \quad (1)$$

If there is only one matter component, or if all matter components are coupled to the same metric, i.e. in the case of universal couplings, the frame in which baryons follow geodesics (with metric  $g_{\mu\nu}$ ) is also called Jordan frame: in other words, for a universal coupling, the observational frame happens to be coincident with the Jordan frame, in which matter is conserved in a covariant way. For a universal coupling, it is often convenient to consider also a second relevant reference frame: the one in which any additional scalar degree of freedom is only minimally coupled to gravity through a factor  $\sqrt{-g}$ , i.e. the Einstein frame. Of course, one could define also mixed reference frames in which both baryons and field are not minimally coupled.

In any other metric non-trivially related to  $g_{\mu\nu}$ , the EMT will, in general, not be conserved. For example, for a metric related to  $g_{\mu\nu}$  via a conformal coupling, such that  $\tilde{g}_{\mu\nu} = f(\phi)g_{\mu\nu}$ , with  $f$  a generic function of a scalar field  $\phi$ , the right hand side in Eq. (1) does not vanish anymore and is a function of  $\phi$ , so that the EMT of each species is no longer separately conserved<sup>1</sup>:

$$\tilde{\nabla}_{\mu} T_{\nu}^{(b)\mu} = -\frac{f_{,\phi}}{2f} T^{(b)}_{\phi,\nu} . \quad (2)$$

Let's consider now the possibility that the coupling is not universal, i.e. such that different matter species may couple to gravity with different strengths, mediated by a scalar field. For example, baryons may still be coupled only minimally to gravity, while a non-minimal coupling may affect dark matter or neutrinos. In this case, there are (at least) three relevant frames: one in which only baryons are conserved, one in which only dark matter (or neutrinos) is conserved, and one in which the scalar field is conserved, i.e. is minimally coupled to gravity. The first one is the observational frame, the third one can be called the Einstein frame; the second is, in general, neither Jordan nor Einstein, and we can refer to it as being the "dark matter (or neutrinos) frame". Here again, one could also equivalently define an infinity of other frames in which all three components are coupled. Let's explicit now the actions corresponding to these three relevant frames.

For an Action that contains gravity, a scalar field, baryons and dark matter one has the three main cases which can be obtained one from another through a conformal transformation:

$$S_E = \int d^4x \sqrt{-g} [R + L_{\phi}(g_{\mu\nu}) + L_b(f_b(\phi)g_{\mu\nu}) + L_{dm}(f_{dm}(\phi)g_{\mu\nu})], \quad (3)$$

$$S_{dm} = \int d^4x \sqrt{-\tilde{g}} [f_g(\phi)\tilde{R} + \tilde{L}_{\phi}(\tilde{g}_{\mu\nu}) + \tilde{L}_b(\tilde{f}_b(\phi)\tilde{g}_{\mu\nu}) + \tilde{L}_{dm}(\tilde{g}_{\mu\nu})], \quad (4)$$

$$S_{obs} = \int d^4x \sqrt{-\hat{g}} [\hat{f}_g(\phi)\hat{R} + \hat{L}_{\phi}(\hat{g}_{\mu\nu}) + \hat{L}_b(\hat{g}_{\mu\nu}) + \hat{L}_{dm}(\hat{f}_{dm}(\phi)\hat{g}_{\mu\nu})] \quad (5)$$

where  $R$ ,  $\tilde{R}$ ,  $\hat{R}$  are the curvature scalars written in terms of  $g$ ,  $\tilde{g}$ ,  $\hat{g}$ , respectively, defined as

$$\tilde{g}_{\mu\nu} = f_{dm}(\phi)g_{\mu\nu}, \quad (6)$$

$$\hat{g}_{\mu\nu} = \tilde{f}_b(\phi)\tilde{g}_{\mu\nu} \quad (7)$$

and the following relations hold,

$$f_g = f_{dm}^{-1}, \quad (8)$$

$$\tilde{f}_b = f_b f_{dm}^{-1}, \quad (9)$$

$$\hat{f}_{dm} = \tilde{f}_b^{-1} = f_b^{-1} f_{dm}, \quad (10)$$

$$\hat{f}_g = f_g \tilde{f}_b^{-1} = f_g f_b^{-1} f_{dm} = f_b^{-1} \quad (11)$$

The metric  $g$  is always defined as the one from which the uncoupled Ricci scalar of General Relativity is built. If  $f_{dm} = f_b$  (universal coupling), then the observational frame coincides with the  $dm$  frame, which in this case is also the standard Jordan frame. If  $f_b = 1$ ,

<sup>1</sup> In the following, we will provide examples that are related to Einstein metric  $g_{\mu\nu}$  via a conformal coupling; we note however that similar considerations can be extended to a general coupling metric  $\hat{g}_{\mu\nu} = f(\phi, g_{\mu\nu})$ , including the so-called disformal metric  $\hat{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\phi_{\mu}\phi_{\nu}$ .

then the observational frame coincides with the Einstein frame. For any other choice, the three frames are different choices of reference frames, although they are physically equivalent and one can always pick the most convenient one for calculation purposes. We stress that  $S_E = S_{dm} = S_{obs}$ : they are just the same Action written with different variables. In Appendix A we show this explicitly.

### 3. The concept of self-acceleration and its shortcomings

Although in agreement with observations, a cosmological constant is affected by fine tuning and coincidence problems. One needs to tune the initial conditions with very high precision in order to justify the observed, very small non-zero value of the cosmological constant  $\Lambda$ ; in addition, if the whole contribution of dark energy is entirely due to a cosmological constant, then its energy density happens to be of the same order of magnitude of matter density just at the present time, while it has been completely negligible in the past and it will dominate in the future, over all other species. These choices impact the naturalness of the model and lack a more fundamental prediction of the value assumed by  $\Lambda$ . Most dark energy models have then been introduced in order to address these issues, trying to replace the cosmological constant with a fluid or a modification of gravity. Although a combination of cosmological constant and modified gravity would still be interesting to detect, an important question and a desirable property for dark energy models is to understand whether they are able to explain acceleration without the introduction of any cosmological constant in the Lagrangian. More in general, it would be interesting to explain acceleration without the action of matter field, i.e. without fluid negative pressure, be it vacuum energy or a non-standard kinetic term. This property is often embodied in the condition of so-called *self-acceleration* formalized by Wang et al. in [1] (we refer to this paper as WHK in the following): a model is self-accelerated if the model does not produce acceleration in the frame where gravity is uncoupled (Einstein frame) while it is accelerated in the observational frame. Or, as WHK say, “by self-acceleration, we mean accelerated expansion in the Jordan frame, while the Einstein-frame expansion rate is not accelerating. This is a sensible definition, for the lack of acceleration in the Einstein frame - where the Einstein, and therefore the standard Friedmann, equations hold - is equivalent to the lack of dark energy. In self-accelerating theories, the observed (Jordan-frame) cosmic acceleration stems entirely from the conformal transformation [...], i.e., a genuine modified gravity effect.”

Note that the WHK condition of self-acceleration given above does not really prevent the existence of a potential or even of a cosmological constant: it just requires this term not to be sufficient for the observed present acceleration. The idea is that if the model is not accelerated in the Einstein frame, where gravity and matter are clearly separated (minimally coupled), while it is so in the observational frame, then the origin of the acceleration cannot be entirely due to vacuum energy but has to come from the transformation of the metric that brings the Einstein frame into the observational frame: in other words, it has to be due to a modification of gravity rather than to the matter sector. Whether one can really distinguish observationally between these two cases is another question that will not be discussed here except to notice that, obviously, if one allows for a completely arbitrary matter sector (including arbitrary deviation from a perfect fluid tensor) then no observation based purely on gravitational effects can distinguish between an effect due to the right-hand side of Einstein equations to an effect due to the left-hand side.

Although we agree that the declaration of principle that led to such definition is worth being pursued, we feel there are two main limitations with the current definition of self-acceleration. First, this definition can only be literally applied to the case of universal coupling (as indeed carefully specified in Ref. [1]) since in the non-universal coupling case the observational frame can coincide with the Einstein frame and the self-acceleration condition would become contradictory. Second, even if we restrict ourselves to a universal coupling, this definition excludes from “self-accelerated cosmologies” even some models in which the acceleration in the Einstein frame is not due to vacuum energy (or negative pressure) but rather to the effect of a coupling between scalar field and matter, as we will show below. That is, it might exclude also models in which acceleration is actually driven by a modification of gravity, contrary to the intended scope.

Goal of this short note is to elaborate a more general definition of self-acceleration that, while keeping the original motivation, addresses these two shortcomings. Our new proposal is valid also when the coupling is non universal, and we will show that in literature there are indeed already examples of non-universal self-accelerated models that fulfil our new definition, i.e. in which acceleration is only a consequence of a modification of gravity. Moreover, we will show that there are also cases with a universal coupling in which the acceleration is indeed driven by a modification of gravity and yet would be classified as non-self-accelerated according to WHK definition, but not according to ours.

### 4. Force-acceleration and field-acceleration

Let's then go back to the original motivation for introducing the concept of self-acceleration. The reason why this distinction proves useful is because most proposals for modifying gravity are motivated by the observation of acceleration, so it is important to characterize them according to whether they really induce acceleration or just modify the acceleration that is already driven by the uncoupled matter sector. In other words, whether modifying gravity is really needed, at least for what concerns acceleration. Let's then assume that we would like to distinguish between models that explain acceleration with vacuum energy or negative pressure and models that explain it with a *modification of gravity*, i.e. with the addition of a propagating degree of freedom that mediates an extra force (assumed to be a scalar field for definiteness). A first attempt to generalise the definition of self-acceleration that extends also to non-universal couplings might be: *a model is self-accelerated if the universe expansion is accelerated when the couplings of the scalar field (to matter or to gravity, universal or not) are non-zero but is not accelerated when they are taken to vanish*. Here, “coupling to gravity” means specifically the existence of a covariant term in the Lagrangian that includes the scalar field and second derivative tensors of the metric.<sup>2</sup> Consider then a cosmological model with matter and a scalar field: we want to check whether this is self-accelerated according to this definition. First, we would switch off the couplings: this means looking at the limit in which we are in a cosmology with standard gravity, plus a scalar field, plus matter. If

<sup>2</sup> This applies to models with single-metric gravity and a scalar field. In case of multi-metric theories, see e.g. [14,15], the direct coupling between metrics also acts as a modification of gravity. Our classification in force- and fluid-accelerated models remains valid.

acceleration is present, then this cannot be due to a modification of gravity, but rather to the field or matter, having for example strongly negative pressure, and there would therefore be no self-acceleration. If instead there is no acceleration when the couplings are set to zero, while there is acceleration when one or more couplings are switched on, this must be due to the new interactions (forces) rather than to the matter or field content. This definition has the advantage that it makes no use of the Jordan frame vs. Einstein frame transformation, which is ambiguous for non-universal couplings.

Once one recognises that the divide between vacuum-energy-driven and modified-gravity-driven acceleration is actually a divide between uncoupled models and coupled ones, the expression “self-acceleration” appears not sufficiently clear. We propose then that a more useful distinction is between *force-acceleration* and *fluid-acceleration*. The distinction is an operational one. A model is

- *force-accelerated* when the acceleration in the observational frame becomes deceleration when all the couplings between matter and scalar field or between scalar field and gravity are switched off, i.e.

$$f_g = f_{dm} = f_b = 1 ; \quad (12)$$

- *fluid-accelerated* when the acceleration in the observational frame persists in absence of such couplings.

In a sufficiently complicated model one can have periods of force-acceleration and periods of fluid-acceleration. Here, for simplicity, we will refer to the present-time cosmic acceleration (but also show one possible example that applies to inflation in Sect. 5). A model which is neither force- nor fluid-accelerated, has no acceleration and is therefore uninteresting to describe present cosmology.

Let's apply the definitions above to some quick examples of different theories, to clarify their scope. Consider an Action without any interaction term between the scalar field and matter or gravity. If any of the matter fluids accelerates the expansion (e.g. some form of dark matter with negative pressure), then we are in a fluid-accelerated case. The same if the scalar field induces acceleration because of its potential or a non-standard kinetic term. If instead there is no acceleration in absence of couplings, but there is when they are switched on, then we are in a force-accelerated case.

Taking the Horndeski Action [16,17] as another example, we see that the two functions  $\alpha_M, \alpha_T$  (defined in [18]) parameterize the coupling of the Horndeski field to gravity, i.e. to the second-derivative tensors  $R$  and  $R_{\mu\nu}$ . When  $\alpha_{M,T} = 0$  the scalar field is minimally coupled to gravity but it possesses, in general, a non-canonical kinetic sector. If in this case the expansion is accelerated, then it is a case of fluid-acceleration. If the acceleration occurs only when either  $\alpha_M$  or  $\alpha_T$  differ from zero, then we are in presence of force-acceleration.

In the next section we highlight more in detail how our new definition applies to two of the classes of theories for dark energy and modified gravity that are still interesting at present, as both survive present constraints from cosmology and from gravitational waves.

We also remark that in this paper we only consider actions that include a conformally coupled scalar field, beside gravity and matter. In this case, the concept of “coupling” is quite simple, and refers exclusively to the  $f$  functions introduced in Section 2. This choice reflects the same choice of the WHK paper, and is of course a great simplification, while still capturing a large class of models. In more general cases, the straightforward extension of our classification would be to consider a “coupling” any term in the Lagrangian that includes the metric tensor or the matter fields and one or more additional field of any kind (scalar, vector, tensor), beside the usual minimal gravitational coupling. For instance, in bimetric massive gravity [19,20], the terms that depend on combinations of the two metric tensors should be switched off to see whether the model is force- or fluid-accelerated.

## 5. Example 1: $f(R)$ models

The first class we consider here is based on a  $f(R)$  cosmology. This class is a subclass of more general scalar tensor theories, and can be redefined in terms of a scalar field that couples to gravity in a non-minimal way. Such cosmologies have been studied for a long time and constrained in many ways (e.g. [21–23] among many others; see also [24,25] for a review). After many years they still offer a range of values that fits current data; their tensor equation, describing gravitational wave propagation, is also in agreement with a speed of  $c_T = 1$ , and therefore survives gravitational wave detection [26]. As they are equivalent to having a universal coupling to gravity for all matter fields, including baryons, they would typically require some screening mechanism active in high density regions, to guarantee that solar system constraints are also satisfied. Also, since the coupling to gravity is universal, the Jordan frame coincides with the observational frame. In this frame, this class of models can be described with the following Action (we choose units such that  $16\pi G = 1$ ):

$$S = \int d^4x \sqrt{-g} [f(R) + L_{dm}(g_{\mu\nu}) + L_b(g_{\mu\nu})] . \quad (13)$$

In order to apply our definition, we want to express this Action into the scalar tensor form of Eq. (5). As well known, under certain conditions, this can be done via a Legendre transformation [27–29]. We define

$$\phi \equiv df/dR , \quad (14)$$

and we write the Legendre transform as  $\hat{f}(\phi) \equiv f - \phi R$ . By differentiation, since  $df = \phi dR$ , and  $d\hat{f} = df - R d\phi - \phi dR = -R d\phi$ , one finds that  $d\hat{f}/d\phi = -R$ , and therefore  $\hat{f}$  depends only on  $\phi$  (and not on derivatives of  $\phi$ ). We can then always redefine  $f(R)$  in terms of a potential  $V(\phi)$  such that:

$$f = \phi R + \hat{f}(\phi) \equiv \phi R - V(\phi) , \quad (15)$$

with

$$V(\phi) = -f(R) + \phi R , \quad (16)$$

which becomes a function of  $\phi$  alone when expressing  $R$  in terms of  $\phi$  by inverting  $\phi = df/dR$ . Comparing Eq. (15) with Eq. (5), we then have that any action of the kind of Eq. (13) can be rewritten as

$$S = d^4x \sqrt{-g} [\phi R - V(\phi) + L_{dm}(g_{\mu\nu}) + L_b(g_{\mu\nu})], \quad (17)$$

where now  $f_g = \phi$  and where the potential is given by Eq. (16).

Suppose now, for instance, that the model of interest within the class in Eq. (13) is

$$f(R) \equiv R + \alpha R^2 - \Lambda; \quad (18)$$

then  $\phi = 1 + 2\alpha R$ , Eq. (17) holds and

$$V(\phi) = -R - \alpha R^2 + \Lambda + (1 + 2\alpha R)R \quad (19)$$

$$= \Lambda + \alpha R^2 = \Lambda + \frac{(\phi - 1)^2}{4\alpha}. \quad (20)$$

According to our definition above, the model Eq. (18) is fluid-accelerated if it is accelerated when the couplings (in this case the conformal factors coupling non-minimally to gravity) are switched off, i.e. if we rewrite the Action (17) with  $f_g = 1$ , that is as

$$S = d^4x \sqrt{-g} [R - V(\phi) + L_{dm}(g_{\mu\nu}) + L_b(g_{\mu\nu})]. \quad (21)$$

When varying with respect to  $\phi$ , one gets  $\phi = 1$ , so  $V = \Lambda$  and finally this Action is equivalent to a standard  $\Lambda$ CDM action. The model Eq. (18) is therefore indeed fluid-accelerated, provided of course  $\Lambda > 0$  and sufficiently large today. This simple example helps clarifying the content of our definition: the presence of  $\Lambda$  in the observed Action Eq. (18) means obviously that there is no further need of modified gravity to explain acceleration.

If instead we start from

$$f(R) = R + \alpha R^2, \quad (22)$$

(also known as Starobinski's inflation [30]), switching off the coupling term reduces the model to a pure matter model without acceleration, because  $V(\phi = 1) = 0$ . Since Starobinski's model is accelerated (not today, but we are using it only as illustration), then we should classify it as force-accelerated model, i.e. past inflationary acceleration in this model is really a pure gravitational effect.

For more complicated forms of  $f(R)$ , (and in general for scalar-tensor theories or extended quintessence [31] with canonical kinetic term) the criterion for fluid-acceleration vs. force-acceleration reduces (as expected) to simply checking whether  $V(\phi)$  alone is capable of supporting acceleration, in absence of couplings as in Eq. (21). The general condition on  $f(R)$  for this to occur can be derived as follows. In the form (21), Einstein equations are obtained varying with respect to  $\phi$ . This implies  $\partial V / \partial \phi = 0$ , and therefore, using Eq. (16) into Eq. (21) and considering  $\phi$  and  $R$  as independent degrees of freedom,  $R(\phi) = 0$ . Inserting this condition in  $V(\phi)$  (Eq. (16)) we have immediately  $V = -f(0)$ . Then, if  $f(0)$  is a non-zero constant, the uncoupled version of the model is  $\Lambda$ CDM and therefore fluid-accelerated (again, provided  $\Lambda$  is large enough to overcome matter's deceleration), otherwise either is not accelerated at all or is force-accelerated. So the Hu-Sawicki model [32], for instance, is such that  $f(R=0) = 0$ , i.e. it is decelerated in absence of couplings, but accelerated with coupling: it is then force-accelerated, according to our definition above. Indeed, one of the main motivations behind the Hu-Sawicki model was to satisfy the condition  $f(R \rightarrow 0) \rightarrow 0$ .

If now we apply WHK's classification, we find that neither model Eq. (18) nor Eq. (22) is self-accelerated, since they both are accelerated also in the Einstein frame. Starobinski's model is however a clear case in which acceleration is driven entirely by a modification of gravity, and we believe it should be classified as such. More in general, any evolution such that  $\phi = \text{const}$  (as in a typical de Sitter state) is not self-accelerated under WHK condition, since then the Jordan and Einstein frames coincide. For instance, as we show in App. B, depending on the value of the parameters, the Hu-Sawicki model turns out to be WHK self-accelerated only near the present time, but not in the asymptotic future when it approaches a de Sitter solution.

The force-acceleration classification applies also when one considers a non-universal coupling. In fact, one can generalize the  $f(R)$  models so that baryons are decoupled, thereby escaping all local gravity constraints. For this, it is sufficient to write in Eq. (13)  $L_b(f_b g_{\mu\nu})$  with  $f_b = f_g = \phi$ . In this way, when we remove the coupling  $f_g$  from the gravity sector via a conformal transformation, we remove also the coupling  $f_b$  from the baryonic sector. The frame becomes the observational one, since in this frame the baryons are uncoupled, and local gravity constraints are automatically bypassed. Switching off the coupling in the observational frame brings us back to Eq. (17), therefore the conclusions for the universally coupled  $f(R)$  model apply in exactly the same way to the non-universal case. In this way, the reach of our classification is extended with respect to the WHK one.

## 6. Example II: coupled quintessence with universal coupling

In this second example we discuss another model that would be classified as non self-accelerated by the WHK criterion, but as force-accelerated according to ours. Since this model is actually accelerated due to the coupling of matter to the scalar field, we believe the acceleration is "a genuine modified gravity effect" and fulfils the rationale for a self-acceleration. The model in the Einstein frame is

$$S_E = \int d^4x \sqrt{-g} [R + L_\phi(g_{\mu\nu}) + L_b(f_b(\phi)g_{\mu\nu}) + L_{dm}(f_{dm}(\phi)g_{\mu\nu})], \quad (23)$$

with

$$L_\phi(g_{\mu\nu}) = -\frac{1}{2} \phi_{,\mu} \phi^{,\mu} - V_0 e^{-\lambda \phi}, \quad (24)$$

where  $V_0$  and  $\lambda$  are constants. The coupling can be either universal or non universal, but here we consider only a universal one. Let us take

$$f_b = f_{dm} \equiv f \equiv e^{2\beta\phi} \quad (25)$$

where for simplicity we assume  $\beta$  (also constant) and  $\lambda$  to be positive (but the results are completely general), and from now on we use a subscript  $m$  to denote both baryons and dark matter. This class of models has been studied many times in the past, beginning with [33].

The conservation equations are (see also App. C)

$$\nabla_{\mu}^{(g)} T_{(\phi)\nu}^{\mu} = \beta\phi_{,\nu} T_{(m)} \quad (26)$$

$$\nabla_{\mu}^{(g)} T_{(m)\nu}^{\mu} = -\beta\phi_{,\nu} T_{(m)} \quad (27)$$

where  $T_{(m)\nu}^{\mu} = T_{(m)\nu}^{\mu(0)}$  (see also [28,34] and references in [35]). The upshot [34] is that the model leads to asymptotic acceleration (i.e., to a global stable attractor) either for a)  $\lambda^2 < 2$  when  $\lambda(\beta + \lambda) < 3$  or for b)  $\beta > \lambda/2$  when  $\lambda(\beta + \lambda) > 3$ , where  $\lambda$  is the slope of the potential in Eq. (24). Neither case is an exact de Sitter Universe.

To apply now the WHK classification and ours, we need to derive the Jordan frame, which in this case of universal coupling coincides with the observational one. Then the Action becomes

$$S_{obs} = \int d^4x \sqrt{-\hat{g}} [F(\phi)\hat{R} + L_{\phi}(\hat{g}_{\mu\nu}) + L_m(\hat{g}_{\mu\nu})] \quad (28)$$

where

$$F(\phi) = e^{-2\beta\phi} \quad (29)$$

and

$$L_{\phi}(\hat{g}_{\mu\nu}) = -\frac{1}{2}(1 - 6\beta^2)F(\phi)\phi_{,\mu}\phi^{,\mu} - V_0 e^{-(\lambda+4\beta)\phi} \quad (30)$$

In this frame, one finds that the conditions for asymptotic acceleration are just the same as in the Einstein frame (see for instance [36], Chap. 9.2, where  $Q = \beta$  and  $\lambda$  is our  $\lambda + 4\beta$ ). This immediately shows that WHK's criterion leads to the puzzling result that such coupled models are either not accelerated at all, or not self-accelerated, notwithstanding the fact that gravity is indeed modified with an arbitrarily high strength.

Let us apply now our classification. We begin with case a). If  $\lambda^2 < 2$ , we have acceleration in the observational frame, and since we have acceleration also for  $\beta = 0$ , we would classify the model as fluid-accelerated. If instead  $\lambda^2 > 2$ , we would classify the model as not accelerated at all. So for case a) our definition agrees with WHK.

In case b),  $\beta > \lambda/2$ . If  $\lambda^2 > 2$ , while in absence of coupling we would again fall back in a decelerated case, there is instead acceleration in the observational frame when (and only when) the coupling to gravity  $\beta$  is switched on. This case is therefore for us a force-accelerated model. Acceleration is here induced by a large universal coupling  $\beta > \sqrt{2}/2$  and is not present when  $V(\phi)$  only is active, and therefore represents a clear case of modified gravity acceleration (with a universal coupling) that would have been missed if using the WHK definition for self-acceleration.

In Appendix B we express this model in the Horndeski form and further discuss our result in view of Ref. [8], which employs the WHK definition to exclude a vast class of models.

## 7. Conclusions

In this paper we have rediscussed the condition of self-acceleration, often used in literature to theoretically distinguish cosmological models in which acceleration is due to a pure modification of gravity from those where acceleration is the effect of the energy associated to a fluid. Such classification has been employed to motivate further (or not) investigation of classes of models. While keeping the same original motivation, as presented in [1], we highlighted, via examples, two limitations of the self-acceleration definition, as currently adopted in literature: first, self-acceleration cannot be directly applied to non-universal couplings; second, even in presence of a universal coupling, some models in which acceleration is clearly an effect of modified gravity would be considered as not self-accelerated.

We then proposed in Sect. 4 to reformulate the self-acceleration concept in two new, more general, definitions that address such limitations – fluid-acceleration and field-acceleration – and further discussed in a few examples how these definitions are applied in practice, with respect to self-acceleration.

In particular, in Sect. 5 we discussed  $f(R)$  (in various forms, including Hu-Sawicky) and the condition for this class to be fluid-accelerated or force-accelerated; we highlighted how this condition reduces, as expected, to testing whether the potential  $V(\phi)$  is able to produce acceleration, once the theory has been expressed in the form Eq. (21) with non-minimal couplings switched off. Specifically, we find that while Starobinsky's  $f(R)$  inflation is not self-accelerated according to WHK, it should be classified as force-accelerated according to our definitions. Moreover, we find the Hu-Sawicky model to be always force-accelerated at the present time and in the future, while the WHK criterion classifies it as self-accelerated only near the present time and for  $n > 0.75$ . In a second example (Sect. 6), we showed how the self-acceleration and our definitions lead again to different results: in particular, applying WHK's definition would exclude a range of models in presence of a universal coupling (class b) in Eq. (28), with  $\beta > \lambda/2 > \sqrt{2}/2$ , even when the acceleration is clearly due to a modification of gravity.

As noted already for example in [37,38], we further remark that at present non-universal couplings are among the (few) classes of models which survive gravitational wave detection and local constraints (see [12] for a review on models surviving with a universal

coupling). This is because, by construction, baryonic interactions are standard and satisfy solar system constraints; furthermore the speed of gravitational waves in these models is  $c_T = 1$  and therefore in agreement with gravitational wave detection. It has also been noted (see for example [39–41] and the update in [35]) that models in which a non-universal coupling between dark matter particles is considered would also solve the tension in the measurement of the Hubble parameter [42] due to the degeneracy  $\beta - H_0$  first noted in Ref. [43].

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## Appendix A

Here we explicitly write some relations holding for a conformal coupling transformation, which are used in the text and are convenient to keep at hand. We also explicitly recall and show that  $S_E = S_{dm} = S_{obs}$ : they are just the same Action written with different variables.

If  $R$  is the curvature written in terms of the Einstein metric  $g_{\mu\nu}$ , a conformal transformation of the type

$$\tilde{g}_{\mu\nu} = f(\phi)g_{\mu\nu}, \quad (31)$$

where  $f$  is a generic function of  $\phi$ , leads to the following relations:

$$\sqrt{-g} = f^{-2}\sqrt{-\tilde{g}}, \quad (32)$$

$$R = f(\tilde{R} + 6\tilde{\square}\omega - 6\tilde{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu}), \quad (33)$$

$$\tilde{\square} = f^{-1}[2\omega_{,\mu}g^{\mu\nu}\partial_\nu + \square], \quad (34)$$

where  $\omega_{,\mu} \equiv f_{,\mu}/2f = f'\phi_{\mu}/(2f)$  and  $f' \equiv df/d\phi$ , and  $\tilde{R}$  is the curvature scalar written in terms of  $\tilde{g}$ . It follows that

$$f^{-1}(R - 6f\tilde{\square}\omega_{,\mu} + 6f\tilde{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu}) = \tilde{R}, \quad (35)$$

$$f^{-1}(R - 6\omega_{,\mu}g^{\mu\nu}\omega_{,\nu} - 6\square\omega) = \tilde{R}. \quad (36)$$

Here we further give an explicit example of the conformal transformation procedure from  $S_E$  to  $S_{dm}$ :

$$S_E = \int d^4x \sqrt{-g} [R + L_\phi(g_{\mu\nu}) + L_b(f_b g_{\mu\nu}) + L_{dm}(f_{dm} g_{\mu\nu})] \quad (37)$$

$$= \int d^4x \sqrt{-\tilde{g}} f_{dm}^{-2} [f_{dm}(\tilde{R} + 6\tilde{\square}\omega - 6\tilde{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu}) + L_\phi(g_{\mu\nu}) + L_b(f_b g_{\mu\nu}) + L_{dm}(\tilde{g}_{\mu\nu})] \quad (38)$$

$$= \int d^4x \sqrt{-\tilde{g}} \{ f_{dm}^{-1} \tilde{R} + [f_{dm}^{-2} L_\phi(f_{dm}^{-1} \tilde{g}_{\mu\nu}) + f_{dm}^{-1} (6\tilde{\square}\omega - 6\tilde{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu})] + f_{dm}^{-2} L_b(f_b f_{dm}^{-1} \tilde{g}_{\mu\nu}) + f_{dm}^{-2} L_{dm}(\tilde{g}_{\mu\nu}) \} \quad (39)$$

$$= \int d^4x \sqrt{-\tilde{g}} \{ f_{dm}^{-1} \tilde{R} + \tilde{L}_\phi(\tilde{g}_{\mu\nu}) + \tilde{L}_b(f_b f_{dm}^{-1} \tilde{g}_{\mu\nu}) + \tilde{L}_{dm}(\tilde{g}_{\mu\nu}) \} = S_{dm} \quad (40)$$

where  $\tilde{L}_{b,dm} = f_{dm}^{-2} L_{b,dm}$ , while the scalar Lagrangian is

$$\tilde{L}_\phi(\tilde{g}_{\mu\nu}) = f_{dm}^{-2} L_\phi(f_{dm}^{-1} \tilde{g}_{\mu\nu}) + f_{dm}^{-1} (6\tilde{\square}\omega - 6\tilde{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu}) \quad (41)$$

If

$$L_\phi = -\frac{1}{2}\phi_{,\mu}g^{\mu\nu}\phi_{,\nu} - V(\phi) \quad (42)$$

then (here  $f \equiv f_{dm}$ )

$$\tilde{L}_\phi(\tilde{g}_{\mu\nu}) = f^{-2} L_\phi(f^{-1} \tilde{g}_{\mu\nu}) + f^{-1} (6\tilde{\square}\omega - 6\tilde{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu}) \quad (43)$$

$$= -\frac{1}{2}\phi_{,\mu}f^{-1}\tilde{g}^{\mu\nu}\phi_{,\nu} - f^{-2}V(\phi) + f^{-1}(6\tilde{\square}\omega - 6\tilde{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu}) \quad (44)$$

$$= -\frac{1}{2}\phi_{,\mu}f^{-1}\tilde{g}^{\mu\nu}\phi_{,\nu} - f^{-2}V(\phi) + 6f^{-2}f'\phi_{,\mu}\tilde{g}^{\mu\nu}\omega_{,\nu} - 6f^{-1}\tilde{g}^{\mu\nu}\omega_{,\mu}\omega_{,\nu} \quad (45)$$

$$= f^{-1}\tilde{g}^{\mu\nu}(-\frac{1}{2}\phi_{,\mu}\phi_{,\nu} + 6f^{-1}f'\phi_{,\mu}\omega_{,\nu} - 6\omega_{,\mu}\omega_{,\nu}) - f^{-2}V(\phi) \quad (46)$$

$$= f^{-1}\tilde{g}^{\mu\nu}(-\frac{1}{2}\phi_{,\mu}\phi_{,\nu} + 3\left(\frac{f'}{f}\right)^2\phi_{,\mu}\phi_{,\nu} - 6\left(\frac{f'}{2f}\right)^2\phi_{,\mu}\phi_{,\nu}) - f^{-2}V(\phi) \quad (47)$$

$$= -\frac{1}{2}f^{-1}\tilde{g}^{\mu\nu}[1 - 3\left(\frac{f'}{f}\right)^2]\phi_{,\mu}\phi_{,\nu} - f^{-2}V(\phi) \quad (48)$$

(in the third line we used integration by parts) which can be written in fully canonical form defining a new field and a new potential

$$\psi_{,\mu} = f^{-1/2} \left[ 1 - 3 \left( \frac{f'}{f} \right)^2 \right]^{1/2} \phi_{,\mu} , \quad (49)$$

$$U(\psi) = \frac{V(\phi)}{f^2(\phi)} \Big|_{\phi(\psi)} . \quad (50)$$

Similarly, one can show that  $S_{dm} = S_{obs}$  as follows.

$$S_{dm} = \int d^4x \sqrt{-\tilde{g}} [f_g(\phi) \tilde{R} + \tilde{L}_\phi(\tilde{g}_{\mu\nu}) + \tilde{L}_b(\tilde{f}_b(\phi) \tilde{g}_{\mu\nu}) + \tilde{L}_{dm}(\tilde{g}_{\mu\nu})] \quad (51)$$

$$= \int d^4x \sqrt{-\tilde{g}} \tilde{f}_b^{-2} [f_g \tilde{f}_b (\hat{R} + 6\hat{\square}\omega - 6\hat{g}^{\mu\nu} \omega_{,\mu} \omega_{,\nu}) + \tilde{L}_\phi(\tilde{f}_b^{-1} \hat{g}_{\mu\nu}) + \tilde{L}_b(\hat{g}_{\mu\nu}) + \tilde{L}_{dm}(\tilde{f}_b^{-1} \hat{g}_{\mu\nu})] \quad (52)$$

$$= \int d^4x \sqrt{-\tilde{g}} \{ \tilde{f}_b^{-1} f_g (\hat{R} + 6\hat{\square}\omega - 6\hat{g}^{\mu\nu} \omega_{,\mu} \omega_{,\nu}) + \tilde{f}_b^{-2} \tilde{L}_\phi(\tilde{f}_b^{-1} \hat{g}_{\mu\nu}) + \tilde{f}_b^{-2} \tilde{L}_b(\hat{g}_{\mu\nu}) + \tilde{f}_b^{-2} \tilde{L}_{dm}(\tilde{f}_b^{-1} \hat{g}_{\mu\nu}) \} \quad (53)$$

$$= \int d^4x \sqrt{-\tilde{g}} \{ \tilde{f}_b^{-1} f_g (\hat{R}) + [\tilde{f}_b^{-2} \tilde{L}_\phi(\tilde{f}_b^{-1} \hat{g}_{\mu\nu}) + \tilde{f}_b^{-1} f_g (6\hat{\square}\omega - 6\hat{g}^{\mu\nu} \omega_{,\mu} \omega_{,\nu})] + \tilde{f}_b^{-2} \tilde{L}_b(\hat{g}_{\mu\nu}) + \tilde{f}_b^{-2} \tilde{L}_{dm}(\tilde{f}_b^{-1} \hat{g}_{\mu\nu}) \} \quad (54)$$

$$= \int d^4x \sqrt{-\tilde{g}} \{ \hat{f}_g \hat{R} + [\tilde{f}_b^{-2} \tilde{L}_\phi(\tilde{f}_b^{-1} \hat{g}_{\mu\nu}) + \tilde{f}_b^{-1} f_g (6\hat{\square}\omega - 6\hat{g}^{\mu\nu} \omega_{,\mu} \omega_{,\nu})] + \hat{L}_b(\hat{g}_{\mu\nu}) + \hat{L}_{dm}(\hat{f}_{dm} \hat{g}_{\mu\nu}) \} \quad (55)$$

$$= \int d^4x \sqrt{-\tilde{g}} \{ \hat{f}_g \hat{R} + \hat{L}_\phi(\hat{g}_{\mu\nu}) + \hat{L}_b(\hat{g}_{\mu\nu}) + \hat{L}_{dm}(\hat{f}_{dm} \hat{g}_{\mu\nu}) \} = S_{obs} \quad (56)$$

where  $\hat{L}_{b,dm} = \tilde{f}_b^{-2} \tilde{L}_{b,dm}$ , while the scalar Lagrangian is

$$\hat{L}_\phi(\hat{g}_{\mu\nu}) = \tilde{f}_b^{-2} \tilde{L}_\phi(\tilde{f}_b^{-1} \hat{g}_{\mu\nu}) + \tilde{f}_b^{-1} f_g (6\hat{\square}\omega - 6\hat{g}^{\mu\nu} \omega_{,\mu} \omega_{,\nu}) \quad (57)$$

If now we take  $\tilde{L}_\phi$  in Eq. (48), written in the canonical form for the rescaled field  $\psi$ ,

$$\tilde{L}_\phi = -\frac{1}{2} \psi_{,\mu} \tilde{g}^{\mu\nu} \psi_{,\nu} - U(\psi) = -\frac{1}{2} \tilde{f}_b \psi_{,\mu} \hat{g}^{\mu\nu} \psi_{,\nu} - U(\psi) \quad (58)$$

then

$$\hat{L}_\phi(\hat{g}_{\mu\nu}) = \tilde{f}_b^{-2} \tilde{L}_\phi(\tilde{f}_b^{-1} \hat{g}_{\mu\nu}) + \tilde{f}_b^{-1} f_g (6\hat{\square}\omega - 6\hat{g}^{\mu\nu} \omega_{,\mu} \omega_{,\nu}) \quad (59)$$

$$= -\tilde{f}_b^{-1} \frac{1}{2} \psi_{,\mu} \hat{g}^{\mu\nu} \psi_{,\nu} - \tilde{f}_b^{-2} U(\psi) + \tilde{f}_b^{-1} f_g (6\hat{\square}\omega - 6\hat{g}^{\mu\nu} \omega_{,\mu} \omega_{,\nu}) \quad (60)$$

$$= -\frac{1}{2} \psi_{,\mu} \tilde{f}_b^{-1} \hat{g}^{\mu\nu} \psi_{,\nu} - \tilde{f}_b^{-2} U(\psi) - 6(\tilde{f}_b^{-1} f_g)' \psi_{,\mu} \hat{g}^{\mu\nu} \omega_{,\nu} - 6\tilde{f}_b^{-1} f_g \hat{g}^{\mu\nu} \omega_{,\mu} \omega_{,\nu} \quad (61)$$

$$= -\frac{1}{2} \psi_{,\mu} \tilde{f}_b^{-1} \hat{g}^{\mu\nu} \psi_{,\nu} - \tilde{f}_b^{-2} U(\psi) - 6(\tilde{f}_b^{-1} f_g)' \psi_{,\mu} \hat{g}^{\mu\nu} \frac{\tilde{f}_b'}{2\tilde{f}_b} \psi_{,\nu} - 6\tilde{f}_b^{-1} f_g \hat{g}^{\mu\nu} \left( \frac{\tilde{f}_b'}{2\tilde{f}_b} \right)^2 \psi_{,\mu} \psi_{,\nu} \quad (62)$$

$$= \psi_{,\mu} \psi_{,\nu} \left[ -\frac{1}{2} \tilde{f}_b^{-1} \hat{g}^{\mu\nu} - 6(\tilde{f}_b^{-1} f_g)' \hat{g}^{\mu\nu} \frac{\tilde{f}_b'}{2\tilde{f}_b} - 6\tilde{f}_b^{-1} f_g \hat{g}^{\mu\nu} \left( \frac{\tilde{f}_b'}{2\tilde{f}_b} \right)^2 \right] - \tilde{f}_b^{-2} U(\psi) \quad (63)$$

$$= -\frac{1}{2} \tilde{f}_b^{-1} \psi_{,\mu} \psi_{,\nu} \hat{g}^{\mu\nu} \left[ 1 - 3(f_g - 2\tilde{f}_b \frac{f_g'}{f_b}) \left( \frac{\tilde{f}_b'}{f_b} \right)^2 \right] - \tilde{f}_b^{-2} U(\psi) \quad (64)$$

where the prime here is derivative with respect to  $\psi$ . As earlier, this can be written in fully canonical form defining a new field and a new potential

$$\chi_{,\mu} = \tilde{f}_b^{-1/2} \left[ 1 - 3(f_g - 2\tilde{f}_b \frac{f_g'}{f_b}) \left( \frac{\tilde{f}_b'}{f_b} \right)^2 \right]^{1/2} \psi_{,\mu} , \quad (65)$$

$$W(\chi) = \frac{U(\psi)}{\tilde{f}_b^2} \Big|_{\psi(\chi)} . \quad (66)$$

## Appendix B

In Ref. [8] it was shown that WHK's self-acceleration is equivalent to the following condition for Horndeski models with  $\alpha_T = 0$ :

$$\left( 1 + \frac{H'}{H} \right) \left( 1 + \frac{\alpha_M}{2} \right) + \frac{\alpha'_M}{2} \leq 0 \quad (67)$$

( $H$  is the Hubble function in the Jordan frame and prime is  $d/d \ln a$ ) while at the same time  $1 + \frac{H'}{H} > 0$ . In coupled quintessence we have  $\alpha_T = 0$  and

$$\alpha_M = \frac{F'}{F} = -2\beta\phi' \quad (68)$$

where  $F(\phi)$  is given in Eq. (29) (notice that in Horndeski notation,  $F = G_4$ , see e.g. [18]). Now, on any asymptotic acceleration solution, or more in general on any critical point of the coupled quintessence phase-space system (see [36]),  $\phi' = \text{const}$ , so  $\alpha_M = \text{const}$ , and therefore condition Eq. (67) reduces to

$$1 + \frac{\alpha_M}{2} = 1 - \beta\phi' \leq 0 \quad (69)$$

For case a),  $\phi' = \frac{\lambda}{1+\beta\lambda}$ , and one sees immediately that condition Eq. (69) is never satisfied (we are assuming  $\beta, \lambda \geq 0$ ). For case b), one has instead

$$\phi' = \frac{3}{\lambda + 4\beta} \quad (70)$$

and the self-acceleration condition becomes

$$1 - \frac{3\beta}{\lambda + 4\beta} \leq 0 \quad (71)$$

It is not difficult to see that this condition cannot be realized since it contradicts the existence of case b), namely  $\lambda(\lambda + \beta) > 3$ . Therefore, also case b) is always not self-accelerated, according to WHK and [8], notwithstanding the fact that acceleration only arises because  $\beta$  is large. More in general, while according to Ref. [8] values  $\alpha_M > -2$  (assuming constant  $\alpha_M$ ) are to be excluded because non self-accelerating, they might be acceptable according to our definition of force-acceleration (for  $\beta \gg \lambda$ , for instance, we have  $\alpha_M = -3/2$ ).

One can use condition Eq. (67) also to confirm that Starobinski's  $f(R)$  model, in the context of inflation, is not self-accelerated if one applies WHK condition. In fact, one can see from Eq. (17) that the Horndeski  $\alpha_M$  parameter for  $f(R)$  models is simply

$$\alpha_M = \frac{\phi'}{\phi} = \frac{f_{,RR} R'}{f_{,R}} \quad (72)$$

where the comma denotes the derivative with respect to  $R$  and  $\phi = f_{,R}$ . In Starobinski's model during inflation, i.e. for  $R \approx 12H^2 \gg 1/\alpha$ , one has  $H \approx H_i - (t - t_i)/(36\alpha)$  (see e.g. [25]). It follows

$$\alpha_M = \frac{\phi'}{\phi} \approx \frac{R'}{R} \approx \frac{2H'}{H} \approx -\frac{1}{18\alpha H^2} \quad (73)$$

which is much less than unity during inflation, while  $\alpha'_M \sim \alpha_M^2$  is even smaller, and therefore Eq. (67) is not satisfied.

The same analysis can be extended to the Hu-Sawicki model, for which<sup>3</sup>

$$f(R) = R - m^2 \frac{c_1(R/m^2)^n}{c_2(R/m^2)^n + 1} \quad (74)$$

where  $n, m, c_1, c_2$  are positive constants. According to [32], near the present epoch and in the future one can approximate the model as

$$f \approx R - A + BR^{-n} \quad (75)$$

with  $A, B$  positive constants formed out of  $m, c_1, c_2$ . In this regime one has [32]

$$R \approx 3m^2(a^{-3} + 4\frac{1 - \tilde{\Omega}_m}{\tilde{\Omega}_m}) \quad (76)$$

where  $a$  is the cosmic scale factor normalized to unity today and  $\tilde{\Omega}_m$  approximates the true present matter density fraction. Then we have

$$\alpha_M \approx \frac{3(n+1)}{1 + 4a^3 \frac{1 - \tilde{\Omega}_m}{\tilde{\Omega}_m}} \quad (77)$$

Now the condition Eq. (67) can be written as

$$S(a) \equiv -\left(\frac{1}{2} + \frac{3}{2}w_{DE}\Omega_{DE}\right)\left(1 + \frac{\alpha_M}{2}\right) + \frac{\alpha'_M}{2} \leq 0 \quad (78)$$

where  $w_{DE}, \Omega_{DE}$  are the equation of state and matter density parameter, respectively. For  $\tilde{\Omega}_m = 0.3$  and  $w_{DE} = -1$  (since the Hu-Sawicki model approximates a  $\Lambda$ CDM expansion), one has today  $S_0 \approx 0.24 - 0.31n$ , which is indeed negative for any  $n > 0.75$ . However,  $S(a)$  becomes (and remains) positive at some point in the future: for instance, at  $a = 1.23$  for  $n = 2$  and at  $a = 1.5$  for  $n = 4$ . In other words, applying the WHK definition, the Hu-Sawicki models turns out to be self-accelerated only near the present time. For an overview on constraints by current data see for example [44–46].

<sup>3</sup> Notice that our  $f(R)$  is  $R + f(R)$  in the notation of [32].

### Appendix C

In the main text we discussed only the case of a pure conformal coupling. Here we write down the explicit form of the coupling for a more general scalar-scalar interaction (but still without derivative couplings). Consider two scalar fields  $\phi$  and  $\chi$  with Lagrangian density

$$\frac{\mathcal{L}}{\sqrt{-g}} = R + L_\phi + L_\chi, \quad (79)$$

where

$$L_\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi), \quad (80)$$

$$L_\chi = -\frac{1}{2}K(\phi)\partial^\mu\chi\partial_\mu\chi - I(\phi, \chi), \quad (81)$$

with  $V(\phi)$  a potential and  $I(\phi, \chi)$  an interaction term. We obtain the following equations of motion

$$-\nabla^\nu(\partial_\nu\phi) + V_{,\phi} = -\frac{\partial I}{\partial\phi} - \frac{1}{2}K_{,\phi}\partial^\mu\chi\partial_\mu\chi, \quad -K^{-1}\nabla^\nu(K\partial_\nu\chi) = -K^{-1}\frac{\partial I}{\partial\chi}. \quad (82)$$

We define now the energy-momentum tensors

$$T_{\mu\nu}^{\phi,\chi} = -\frac{2}{\sqrt{-g}}\frac{\partial\sqrt{-g}L(\phi,\chi)}{\partial g^{\mu\nu}}. \quad (83)$$

While the total energy momentum tensor  $T_{\mu\nu} = T_{\mu\nu}^\phi + T_{\mu\nu}^\chi$  is conserved, the individual components  $T_{\mu\nu}^\phi$  and  $T_{\mu\nu}^\chi$  are not.

For timelike  $\nabla^\mu\phi$  and  $\nabla^\mu\chi$  ( $\nabla^\mu\phi\nabla_\mu\phi < 0$ ,  $\nabla^\mu\chi\nabla_\mu\chi < 0$ ), we can write

$$T_{\mu\nu} \equiv (p_\phi + \rho_\phi)u_\mu u_\nu + p_\phi g_{\mu\nu}, \quad T_{\mu\nu}^\phi \equiv (p_\phi + \rho_\phi)u_\mu u_\nu + p_\phi g_{\mu\nu}, \quad T_{\mu\nu}^\chi \equiv (p_\chi + \rho_\chi)u_\mu u_\nu + p_\chi g_{\mu\nu}, \quad (84)$$

where  $u_\mu$  are 4-velocities and

$$\rho = \rho_\phi + \rho_\chi, \quad p = p_\phi + p_\chi, \quad u_\mu u^\mu = -1, \quad (85)$$

and

$$\rho_\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + V(\phi), \quad p_\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi), \quad (86)$$

$$\rho_\chi = -\frac{1}{2}K(\phi)\partial^\mu\chi\partial_\mu\chi + I(\phi, \chi), \quad p_\chi = -\frac{1}{2}K(\phi)\partial^\mu\chi\partial_\mu\chi - I(\phi, \chi). \quad (87)$$

For an homogeneous and isotropic Friedmann-Robertson-Walker cosmology, the (background) field equations read

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\frac{\partial I}{\partial\phi} + \frac{1}{2}K_{,\phi}\dot{\chi}^2, \quad \ddot{\chi} + (3H + (\ln K)_{,\phi}\dot{\phi})\dot{\chi} = -K^{-1}\frac{\partial I}{\partial\chi}. \quad (88)$$

Taking into account Eqs. (86) and (87), we can rewrite the equations of motion Eq. (88) as

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = +\beta(\rho_\chi - p_\chi)\dot{\phi} - \gamma(\rho_\chi + p_\chi)\dot{\phi}, \quad (89)$$

$$\dot{\rho}_\chi + 3H(\rho_\chi + p_\chi) = -\beta(\rho_\chi - p_\chi)\dot{\phi} + \gamma(\rho_\chi + p_\chi)\dot{\phi}, \quad (90)$$

with

$$\beta \equiv -\frac{1}{2}\frac{\partial \ln I(\phi, \chi)}{\partial\phi}, \quad \gamma \equiv -\frac{1}{2}\frac{\partial \ln K(\phi)}{\partial\phi}. \quad (91)$$

When  $K(\phi) = 1$  and for a separable interaction term  $I(\phi, \chi) = I_1(\phi)I_2(\chi)$ , the effective couplings Eq. (91) become  $\chi$ -independent. In particular, for  $I = m^2(\phi)\chi^2/2$ , we have

$$\beta(\phi) = -\frac{\partial \ln m(\phi)}{\partial\phi}. \quad (92)$$

If the Action Eq. (79) has been obtained through a conformal transformation of an Action with two non-interacting bosons  $\phi, \psi$  in which  $\phi$  couples conformally to gravity, then  $I$  is separable and  $\gamma = 2\beta$  (because the kinetic term has an extra  $g^{\mu\nu}$  term with respect to the potential term) and only the couplings  $\pm\beta(\rho_\chi - 3p_\chi)\phi_{,\nu}$  are left on the right-hand-side of the conservation equations (89)-(90). The covariant form of the conservation equations becomes then as in Eqs. (26)-(27). A similar procedure for a fermion-boson coupling leads to the same result.

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