



α clustering in the $^{18,16}\text{O} + ^{13,12}\text{C}$ fusion-evaporation reactions

Bing Wang^a, Zhongzhou Ren^{b,c,*}, Dong Bai^a

^a School of Physics, Nanjing University, Nanjing 210093, China

^b School of Physics Science and Engineering, Tongji University, Shanghai 200092, China

^c Key Laboratory of Advanced Micro-Structure Materials, Ministry of Education, Shanghai 200092, China



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ABSTRACT

α clustering plays an important role in studying not only nuclear structures but also nuclear reactions of light nuclei. In this work, we study the $^{18,16}\text{O} + ^{13,12}\text{C}$ fusion-evaporation reactions at the energy near the Coulomb barrier using a modified statistical model with the α -cluster structures taken into consideration explicitly in the nuclear level densities. This modified statistical model reproduces successfully the experimentally measured α -emission cross sections, and thus provides a possible way to resolve the underestimation discrepancy of the α emission in the original statistical model. Moreover, the modified statistical model could also describe well the angular distributions of the evaporation residues and the energy spectra of the emitted α particles. Therefore, our work shows that α -cluster structures could play an important role in the fusion-evaporation reactions of light ions, and the modified statistical model is a reliable theoretical model for these physical processes.

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1. Introduction

α -cluster structures are ubiquitous across the nuclide chart and play a crucial role in our understanding of nuclear structures and nuclear reactions [1–8]. In various nuclear many-body systems, protons and neutrons tend to gather together and form α clusters in order to lower down the total energy or increase the stability of the system. For light nuclei, the existence of α clustering is well-established. Famous examples include the ground state of ^8Be and the Hoyle state of ^{12}C , which are widely believed to be a gas-like condensate state with two and three α clusters [9–11]. Such α -condensate states are conjectured to exist in other heavier self-conjugate nuclei as well, which have been studied intensively these years from both the theoretical and experimental viewpoints [12–17]. Recently, inspired by the studies of α condensates, non-localized clustering is proposed by one of the authors (ZR) and his collaborators as a new concept in nuclear cluster physics [18,19] and has been applied to study α -cluster structures in various light nuclei [20–24], as well as the α -knockout reactions [25]. α clusters could also exist in medium-mass and heavy nuclei. Thanks to the recent progress on theoretical descriptions of α decay and α clus-

tering in the medium-mass and heavy-mass region, the landscape of the α -cluster formation probabilities could be obtained by exploiting the rich experimental data on α decays and nuclear radii, and various major-shell and sub-shell effects have been identified [26–35].

α clusters could also be important in the theoretical descriptions of the fusion-evaporation reactions. The standard framework for the fusion-evaporation reaction is the statistical model [36], which assumes that the compound nucleus produced by the entrance channel reaches the thermal equilibrium and all the physical states available for the many-body system are occupied with equal probabilities. For light nuclei, besides protons and neutrons, to certain extent α clusters also play as effective degrees of freedom, and it is interesting and important to see whether these α -cluster structures could be survived through the fusion-evaporation reactions. For the carbon + oxygen fusion-evaporation reactions, it is found recently that the measured α -emission cross sections are typically much larger than the theoretical predictions given by the original statistical model which does not consider the impacts of α clustering [37–40]. Moreover, these disagreements could also be found in other light-ion fusion-evaporation reactions [41–43].

In this work, we study the $^{18,16}\text{O} + ^{13,12}\text{C}$ fusion-evaporation reactions, where the oxygen projectile with the beam energies $E_{\text{c.m.}} = 6$ MeV to 15 MeV interacts with the carbon target. In order to resolve the underestimation discrepancy of the α emission in the original statistical model, we introduce explicitly the effect

* Corresponding author at: School of Physics Science and Engineering, Tongji University, Shanghai 200092, China.

E-mail addresses: dg1722033@smail.nju.edu.cn (B. Wang), zren@tongji.edu.cn (Z. Ren), dbai@itp.ac.cn (D. Bai).

of α clustering into the model and propose the modified statistical model. This model is then used to describe the de-excitation of the compound nucleus. The rest part of this article is organized as follows. In Section 2, the physical picture and the theoretical formalism of our work are described in detail. In Section 3, the numerical results of the modified statistical model are presented and compared with the experimental data. In Section 4, the conclusions are given.

2. Theoretical framework

In this work, we attempt to study the $^{18,16}\text{O} + ^{13,12}\text{C}$ fusion-evaporation reactions using the modified statistical model. Before going to the theoretical formalism, we would like to introduce first the physical picture that underlines our analysis. Take the $^{16}\text{O} + ^{12}\text{C}$ reaction as an example. In the original statistical model, ^{16}O and ^{12}C are treated as assemblies of protons and neutrons and no α -cluster structures are considered explicitly. After the collision, these protons and neutrons interact rapidly with each other, go through all the possible excited states of the compound nucleus $^{28}\text{Si}^*$ that are consistent with the conservation laws, and reach the thermal equilibrium, during which the compound nucleus $^{28}\text{Si}^*$ loses the memory of the entrance channel (i.e., the Bohr independence hypothesis [44]). The light-particle emission processes then take place as the de-excitation of the compound nucleus. This physical picture looks reasonable and works well in many fusion-evaporation reactions. However, as mentioned before, in the $^{16}\text{O} + ^{12}\text{C}$ reaction, it is found experimentally that the α -emission cross section predicted by the original statistical model is much smaller than the realistic experimental data. This discrepancy might be related to the fact that there are rich α -cluster structures in ^{12}C and ^{16}O . Indeed, in some extreme α -cluster models, ^{12}C (^{16}O) is viewed to be made by three (four) point-like α particles. Within this picture, the α -cluster structures are then inherited naturally by the compound nucleus $^{28}\text{Si}^*$ produced by the fusion-evaporation reaction and the thermal equilibrium is achieved between different α -cluster states. For the later convenience, the statistical model based on such a picture is called the extreme α -cluster statistical model. An immediate consequence of the extreme α -cluster statistical model is the dictatorship of the α emission in the final state, which explains qualitatively the dominance of α emission observed in the experiments. The realistic situation of the $^{16}\text{O} + ^{12}\text{C}$ reaction seems to lie somewhere between the original statistical model and the extreme α -cluster statistical model, where the α clusters in the entrance channel, to some extent, seems to be survived through the fusion-evaporation reaction, and the thermal equilibrium in the compound and residual nuclei might be achieved in an “ α -philic” subset of all the possible excited states.

With the above physical picture in mind, the basic formalism of this work could be described as follows. For the fusion-evaporation reaction $\underbrace{a(\text{projectile}) + A(\text{target})}_{\alpha} \rightarrow C^*(\text{compound nucleus}) \rightarrow$

$\underbrace{b(\text{light emitter}) + B(\text{evaporation residue})}_{\beta}$, the cross section from the entrance channel α to the exit channel β could be given by the Hauser-Feshbach formalism [45] as

$$\sigma_{\alpha\beta}(J, S') = \sigma_{\alpha}(J) \frac{\sum_{\ell', j'} T_{\ell'}(\varepsilon_{\beta}) \rho_B(E_B^*, S')}{\sum_{\gamma, \ell'', j''} T_{\gamma, \ell''} \rho_G(E_G^*)}, \quad (1)$$

which is the master formula of our numerical calculations. Here, J and S' are the spin of the compound and residual nucleus respectively. ε_{β} is the kinetic energy of the channel β . E_B^* is the

excitation energy of the residual nucleus. ℓ' and j' are the orbital angular momentum and the total spin for the exit channel β . $\sigma_{\alpha}(J)$ is the partial cross section for the fusion reaction from the entrance channel α , and T_{ℓ} is the transmission coefficient.

$\sum_{\gamma, \ell'', j''} T_{\gamma, \ell''} \rho_G(E_G^*)$ means summing over all the possible exit channels, with G being the residual nucleus in the channel γ . Because of the α clustering, the level density ρ in Eq. (1) should be different from the original statistical model. However, at present little is known for sure about the details of these level densities in the $^{18,16}\text{O} + ^{13,12}\text{C}$ fusion-evaporation reaction and reasonable guesses have to be made to make progress. After a large number of trials and errors, we find that the level densities in Eq. (1) could take a modified Gilbert-Cameron form [46]

$$\rho(E^*, J) = f(J) \rho(E^*), \quad (2)$$

$$f(J) = \frac{2J+1}{2\sigma^2} \exp\left[-\frac{J(J+1)}{2\sigma^2}\right], \quad (3)$$

$$\rho(E^*) = \frac{c}{T} \exp\left[\frac{E^* - E_0}{T}\right], \quad E^* < E_x, \quad (4)$$

$$= \frac{c}{12(2\sigma^2)^{\frac{1}{2}} a^{\frac{1}{4}} (E^* - \Delta - k\delta(\alpha))^{\frac{5}{4}}} \times \exp\left\{2[a(E^* - \Delta - k\delta(\alpha))]^{\frac{1}{2}}\right\}, \quad E^* \geq E_x, \quad (5)$$

where E_x is the connection point between the constant temperature formula ($E^* < E_x$) and the Fermi-gas-like formula ($E^* \geq E_x$), T and E_0 are determined from the smooth condition, and σ^2 is the spin cutoff parameter. The level density parameter a and pairing correction Δ are given by

$$a = \{0.00917[S(N) + S(Z)] + 0.142\}A, \quad (6)$$

$$\Delta = P(N) + P(Z), \quad (7)$$

where $P(N)$ ($P(Z)$) and $S(N)$ ($S(Z)$) are the pairing and shell corrections for neutrons (protons). Occasionally, the level density parameter a is also taken to be energy dependent to achieve a better phenomenological agreement [47],

$$a(E^*, A) = \tilde{a}(A) \left\{1 + \frac{S}{E^*} \left[1 - \exp\left(-\frac{E^*}{E_D}\right)\right]\right\}, \quad (8)$$

where $\tilde{a}(A)$ is the asymptotic level density parameter at the high excitation energies, S is the shell correction energy, and E_D is the damping energy. Compared with the original Gilbert-Cameron formula, the level density given by Eq. (4) and (5) is characterized by the additional term $k\delta(\alpha)$ in Eq. (5) and the overall scaling constant c , which quantify the effects of the entrance-channel α clustering on the level density of the residual nucleus. $\delta(\alpha)$ is taken from Ref. [48]

$$\delta(\alpha) = (-)^{Z+N+1} \frac{1}{2} [S_n(Z-1, N) - 2S_n(Z, N) + S_n(Z+1, N)], \quad (9)$$

$$S_n(Z, N) = B(Z, N) - B(Z, N-1), \quad (10)$$

where $B(Z, N)$ is the binding energy for the nucleus with Z protons and N neutrons.

The Fermi gas model [49] is commonly used to describe the nuclear level density. Despite of its simplicity, the Fermi gas model reproduces the exponential increase of the level density with the excitation energy and is the starting point to construct more realistic models. In Ref. [46], the Fermi gas model is improved by introducing further the pairing and shell corrections (given by Δ

Table 1

The input values of the parameter k in the level density formula for different entrance channels (rows) and exit channels (columns). Here, ER stands for the evaporation residue, N stands for the proton or neutron, and α stands for the α particle.

Channel	ER + N	ER + 2 N or α	ER + N + α	ER + 2 α
$^{16}\text{O} + ^{13}\text{C}$	3.5	0	−3	−6
$^{16}\text{O} + ^{12}\text{C}$	3	0	−3	−5
$^{18}\text{O} + ^{12}\text{C}$	3	0	−3	−6

and a in Eq. (5)), which turns out to be successful and influential. Besides the pairing correlations between two like nucleons, nuclear many-body systems have also α -like correlations which could play an important role in many situations. With this in mind, the corrections $k\delta(\alpha)$ are introduced naturally in Eq. (5) to capture the α -like correlations. The quantity $\delta(\alpha)$ is proposed in Ref. [48] to study α -clustering in the ground states of heavy nuclei. An empirical parameter k is introduced to help extrapolate this measure of α clustering from ground states to excited states populated in fusion-evaporation reactions and from heavy nuclei to light nuclei involved in the fusion-evaporation reactions studied in this work. Three sets of optimal values of the parameter k are found by seeking for better agreement with the experimental data, and are given in Table 1. As we can see, for three different $^{18,16}\text{O} + ^{13,12}\text{C}$ reactions the k values are almost same with each other for each category, which indicates that the k values obtained here have some kind of universality. Further understanding of this universality lies beyond the scope of this work and will be sought for in future publications. As shown in Section 3, the numerical results with the k values from Table 1 agree well with the experimental data, which provides concrete evidence for introducing the empirical parameter k .

In Table 1, the k values for ER + N + α and ER + 2 α turn out to be negative, which can be understood as follows. It is well-known that, the pairing energies in the mass formula of the liquid-drop model can be positive, zero, and negative [50]. The appearance of negative pairing energies is related to the choice of the energy reference. Typically, the pairing energy of the odd- A nuclei is calibrated to be zero, as a result of which the odd-odd nuclei have negative pairing energies and the even-even nuclei have positive pairing energies. Similarly, in this work, the appearance of the negative k values is also related to the choice of the energy reference and could be eliminated by choosing a different energy reference. With the k values in Table 1, the theoretical results of the α -emission cross sections agree well with the experimental data, which shows the usefulness of the present value sets of the parameter k . It is also interesting to note that the k values of the one-nucleon emission channel and the two-nucleon emission channel are different, which is likely to be the same if the two-nucleon emission process takes place sequentially. The reason for this anomaly might be related to the strong correlations between the two nucleons, which makes the realistic two-nucleon emission different from the idealized two-nucleon sequential emission. In recent years, the non-sequential two-nucleon emission has been observed experimentally and is a hot topic in theoretical studies of nuclear structures. See Ref. [51] for a comprehensive review. Another reason might be related to the differences between the compound nuclei in the first-nucleon and second-nucleon emissions, which generally have different average excitation energies. The discussions here could also be applied to the one- α and two- α emissions.

The overall constant c is introduced to scale the modified level density universally. As mentioned before, it might be the case that the thermal equilibrium in the compound and residual nuclei is achieved only among a subset of all the possible excited states due to the impacts of α clustering in the entrance channel. As a

result, the modified level density should not exceed the original Gilbert-Cameron level density, which could be viewed as a measure of the total level density. By introducing the overall constant c , this could always be achieved. The introduction of the factor c is also supported by a microscopic study of the relation between the total level density and the α -cluster level density in Ref. [52], which shows explicitly that the level densities of the α -cluster states could be much smaller than the total ones. In this work, the absolute value of c plays no role and it is introduced for the sake of self-consistency, as in the master formula Eq. (1) what really matters is the relative size of the level densities and all the dependence on c cancels out neatly. This has been verified by explicit numerical calculations.

3. Numerical results

The numerical calculations are carried out based on the code EVAPOR [53] with α clustering taken into account in the way outlined in Section 2. The partial cross section for the formation of the compound nucleus with the total angular momentum J is given by

$$\sigma_{\alpha}(J) = \frac{\pi}{k^2} \frac{2J+1}{1 + e^{(J-J_{\text{cr}})/\Delta J}}. \quad (11)$$

Here, $\Delta J = 0.3\hbar$ is the diffuseness parameter. J_{cr} is determined by matching the theoretical value of the total fusion cross section $\sigma_{\alpha} = \sum_{J=J_0}^{\infty} \sigma_{\alpha}(J)$ with the experimental data [38,39,54,55].

For the $^{16}\text{O} + ^{12}\text{C}$ reaction, the level density parameter a is given by Eq. (6), while for the $^{18}\text{O} + ^{12}\text{C}$ and $^{16}\text{O} + ^{13}\text{C}$ reactions, the energy-dependent level parameter in Eq. (8) is used instead, with $E_D = 18.5$ MeV, $\tilde{a}(A) = \frac{A}{14.6} \left(1 + \frac{3.114}{A^{1/3}} + \frac{5.626}{A^{2/3}} \right)$ [59], and the shell correction energy $S(Z, N) = M_{\text{exp}}(Z, N) - M_{\text{LD}}(Z, N)$. Here, $M_{\text{exp}}(Z, N)$ is the experimental value for the mass, and $M_{\text{LD}}(Z, N)$ is the liquid-drop component of the mass formula. The transmission coefficients for neutrons, protons, and α particles are obtained by the optical-model calculations with the potential parameters taken from Refs. [56–58]. The values for the parameter k for each reaction could be found in Table 1.

The main results are presented in the following. First, we study the α -emission cross section and its relative cross section for the three $^{18,16}\text{O} + ^{13,12}\text{C}$ fusion-evaporation reactions, and the numerical results could be found in Fig. 1, 2, and 3. For the $^{16}\text{O} + ^{13}\text{C}$ fusion-evaporation reaction, it is shown in Fig. 1 that, unlike the original EVAPOR which underestimates significantly the absolute and relative α -emission cross section, our modified EVAPOR gives theoretical predictions that agrees well with the experimental data. Similar situations could also be found in the $^{16}\text{O} + ^{12}\text{C}$ fusion-evaporation reactions as shown in Fig. 2, where the theoretical results given by the modified EVAPOR agree better with the experimental data than the original EVAPOR. The agreement between theoretical results given by the modified EVAPOR and experimental data is somehow less satisfactory for the $^{18}\text{O} + ^{12}\text{C}$ reaction as shown in Fig. 3. This might be related to the unresolved impacts of the extra neutrons in ^{18}O and the ignorance of the three-particles emission channel in the present study. All these results show that the effect of α clustering plays an important role in the fusion-evaporation reactions of light nuclei, and our implementation of this effect in the modified statistical model does capture the main feature of the underlying physics and could provide a possible resolution to the underestimation discrepancy of the original statistical model.

We study further the angular distributions of the evaporation residues in the laboratory frame for the $^{18}\text{O} + ^{12}\text{C}$ reaction at the different incident energies E_{lab} . The numerical results could be

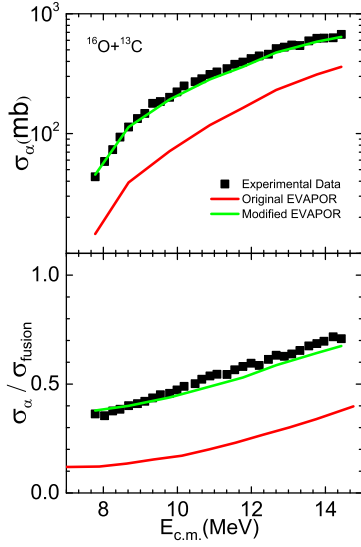


Fig. 1. The α -emission cross section (top panel) and its relative value to the fusion cross section (bottom panel) versus the center-of-mass incident energy $E_{c.m.}$ for the $^{16}\text{O} + ^{13}\text{C}$ reaction. Solid points are the experimental data taken from Ref. [38]. Solid lines are theoretical results given by the original EVAPOR (the red line) [39] and the modified EVAPOR (the green line). As can be seen, there is a significant discrepancy between the experimental data and the original EVAPOR which doesn't consider the effect of α clustering. However the modified EVAPOR agrees well with the experimental data after introducing the effect of α clustering.

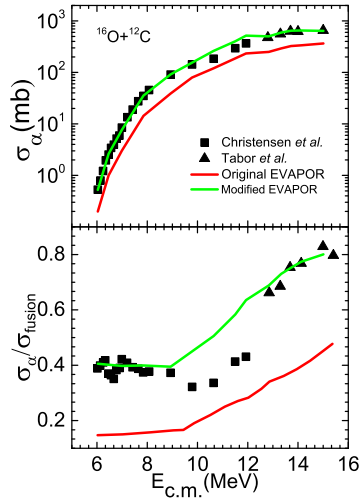


Fig. 2. The α -emission cross section (top panel) and its relative value to the fusion cross section (bottom panel) versus the center-of-mass incident energy $E_{c.m.}$ for the $^{16}\text{O} + ^{12}\text{C}$ reaction. Solid points are the experimental data taken from Refs. [54,55]. Solid lines are the theoretical results given by the original EVAPOR (the red line) [39] and the modified EVAPOR (the green line). As can be seen, there is a significant discrepancy between the experimental data and the original EVAPOR which doesn't consider the effect of α clustering. However the modified EVAPOR agrees well with the experimental data after introducing the effect of α clustering.

found in Fig. 4, as well as the corresponding experimental data. The experimental data show two bumps, with the first one centered at the small angles and the second one centered at the large angles. It is easy to see that, the first bump corresponds to the nucleon emissions from the compound nucleus, while the second bump corresponds to the α emissions. The theoretical results given by the original EVAPOR are plotted in the red line, which reproduce successfully the angular distribution of the residual nuclei at the small angles but typically underestimate the results at the large angles. By including the effect of α clustering, the modified EVAPOR gives satisfactory results that reproduce both the small-

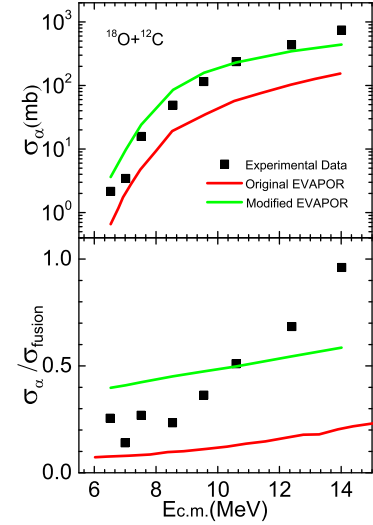


Fig. 3. The α -emission cross section (top panel) and its relative value to the fusion cross section (bottom panel) versus the center-of-mass incident energy $E_{c.m.}$ for the $^{18}\text{O} + ^{12}\text{C}$ reaction. Solid points correspond to the experimental data from Ref. [39]. Solid lines are theoretical results given by the original EVAPOR (the red line) [39] and the modified EVAPOR (the green line). As can be seen, there is a significant discrepancy between the experimental data and the original EVAPOR which doesn't consider the effect of α clustering. However the modified EVAPOR agrees well with the experimental data after introducing the effect of α clustering.

angle and the large-angle behaviors of the angular distribution. The energy spectra of the emitted α particles at the different bombarding energies are also studied and the results are plotted in Fig. 5. The energy distributions given by the modified EVAPOR are slightly better than those given by the original EVAPOR, and are consistent very well with the experimental data. These give extra supports to the effectiveness of the modified statistical model in describing the fusion-evaporation reactions of light ions.

4. Conclusions

In this work, we study various aspects of α emissions from the $^{16}\text{O} + ^{13}\text{C}$, $^{16}\text{O} + ^{12}\text{C}$, and $^{18}\text{O} + ^{12}\text{C}$ fusion-evaporation reactions using a modified statistical model, in which the effects of α clustering are considered explicitly. Compared with the original statistical model implemented in the program EVAPOR, the modified statistical model generally gives theoretical results that agree better with the experimental data. Explicitly, for the three fusion-evaporation reactions, we study the variations of the α -emission cross sections from the low bombarding energy to the high bombarding energy, as well as their relative values to the fusion cross section. It is found that, the modified statistical model gives the theoretical results that agree better with the experimental data than the original one, thus providing a possible way to resolve the underestimation discrepancy of the α emission. For the $^{18}\text{O} + ^{12}\text{C}$ reaction, we also analyze the angular distributions of the evaporation residues, which consist of two bumps with the decreasing trend. Our results describe well the experimental data both at the small angles and at the large angles. Furthermore, the energy spectra of the emitted α particles for the $^{18}\text{O} + ^{12}\text{C}$ reaction could also be reproduced by the modified statistical model. Therefore, all these results show that α clustering could play an important role in the fusion-evaporation reactions of light ions, and the modified statistical model with the effects of α clustering taken into consideration could provide good theoretical descriptions of these physical processes, which may help deepen our understanding of α clustering in nuclear reactions and be a useful reference for future experimental studies. It is an important task to seek for the

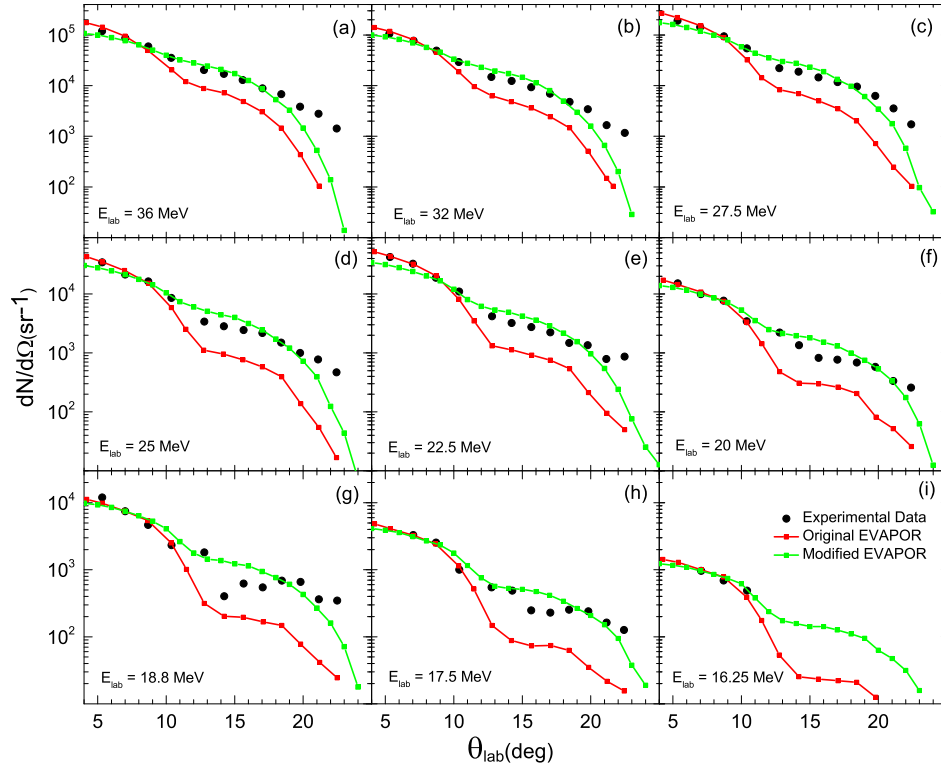


Fig. 4. The angular distributions of the evaporation residues in the laboratory frame for the $^{18}\text{O} + ^{12}\text{C}$ reaction at different bombarding energies E_{lab} . Experimental data (solid symbols) are taken from Ref. [39]. The red and green lines are the theoretical results given by the original EVAPOR [39] and the modified EVAPOR, respectively. When the effect of α clustering is introduced, the large discrepancies between the experimental data and the original EVAPOR disappear and our results given by the modified EVAPOR well reproduce the experimental data.

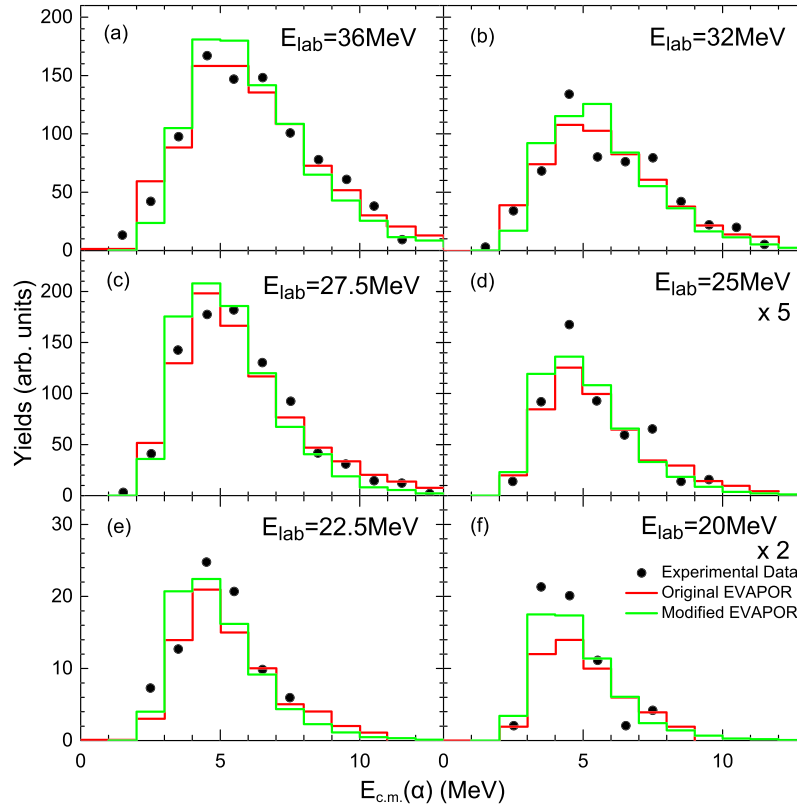


Fig. 5. The energy distributions of α particles in the center-of-mass frame for different bombarding energies E_{lab} . Solid points are experimental data taken from Ref. [39]. The solid lines depict the theoretical results given by the original EVAPOR (the red line) [39] and the modified EVAPOR (the green line). By introducing the effect of α clustering, the modified EVAPOR are slightly better than the original EVAPOR and consistent very well with the experimental data.

microscopic understanding of the modified Gilbert-Cameron level density and the fusion-evaporation reactions. Generally, this is not an easy task. It might be possible to calculate the level density by using microscopic methods such as the antisymmetrized molecular dynamics [60,61], which describes the mean-field and cluster configurations simultaneously. See also Ref. [62] for the state of the art of the mean-field and shell-model approaches to the level density. We are still working on the project of the microscopic understanding of the modified Gilbert-Cameron level density and might report some progress in future publications.

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