Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

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To cite this article: Paolo Gambino *et al* 2019 *J. Phys.: Conf. Ser.* **1137** 012005

Masses, decay constants and HQE matrix elements of pseudoscalar and vector heavy-light mesons in LQCD

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Abstract. We present a precise lattice computation of masses and decay constants of pseudoscalar and vector heavy-light mesons with $m_\ell = m_{u/d}$, m_s and m_h in the range $(m_c, \sim 3m_c)$. We employ the ETMC gauge configurations with both $N_f = 2$ and $N_f = 2+1+1$ dynamical quarks and the ETMC ratio method to reach the b-quark mass. In the case of the vector decay constants an unusual quenching effect of the strange quark is observed. Specific masses combinations are then analyzed in terms of the Heavy Quark Expansion (HQE) to extract matrix elements up to dimension-6, including $\overline{\Lambda}$, μ_π^2 and μ_G^2 with a good precision. These parameters play a crucial role in the inclusive determination of the V_{ub} and V_{cb} matrix elements.

1. Introduction

An important role in heavy flavor physics is played by the vector (V) and pseudoscalar (P) heavy-light mesons $H_\ell^{(*)}$: $D_{(s)}^{(*)}$ and $B_{(s)}^{(*)}$. They are characterized by their masses M_H and decay constants f_H, the latter parametrize the matrix elements of the vector current $V_\mu = \overline{h}\gamma_\mu\ell$ and the pseudoscalar density $P = \overline{h}\gamma_5\ell$:

$$f_{H_\ell^*} M_{H_\ell^*}\, \varepsilon_\mu^\lambda = \langle 0|V_\mu|H_\ell^*(\vec{p},\lambda)\rangle \quad \text{and} \quad f_{H_\ell} M_{H_\ell}^2 = (m_h + m_\ell)\langle 0|P|H_\ell(\vec{p})\rangle\,, \tag{1}$$

where m_h, m_ℓ are the heavy- and light-quark masses and ε_μ^λ is the vector meson polarization. In lattice QCD (LQCD) ground-state masses and decay constants can be straightforwardly determined by studying the two-point correlation functions at large time distances, viz.

$$C_V(t) = \frac{1}{3}\langle \sum_{i,\vec{x}} V_i(\vec{x},t)V_i^\dagger(0,0)\rangle \xrightarrow[t\geq t_{\min}]{} \sum_i |\langle 0|V_i(0)|H_\ell^*(\lambda)\rangle|^2\, \frac{\cosh[M_{H_\ell^*}(T/2-t)]}{3M_{H_\ell^*}} e^{-M_{H_\ell^*}\frac{T}{2}}\,, \tag{2}$$

$$C_P(t) = \langle \sum_{\vec{x}} P(\vec{x},t)P^\dagger(0,0)\rangle \xrightarrow[t\geq t_{\min}]{} |\langle 0|P(0)|H_\ell\rangle|^2\, \frac{\cosh[M_{H_\ell}(T/2-t)]}{M_{H_\ell}} e^{-M_{H_\ell}\frac{T}{2}}\,, \tag{3}$$

where $t_{\min}$ is the minimum time step at which the ground state can be considered isolated. In particular, being the decay modes of the vector mesons dominated by the strong and electromagnetic decays, it is unlikely that their decay constants can be experimentally measured. Thus, a non perturbative approach based on first principles, like LQCD simulations, is crucial.

[3] Acknowledges financial support from *La Caixa-Severo Ochoa* scholarship.

The few lattice calculations of the vector-meson decay constants with either $N_f = 2$ [1, 2] or $N_f = 2+1(+1)$[3, 4, 5] dynamical quarks display a non-negligible difference between $N_f = 2$ results and those including the strange quark. We discuss the results of our analysis of Ref. [3] using the two-point correlation functions computed over the gauge ensembles generated by the European Twisted Mass Collaboration (ETMC) with $N_f = 2+1+1$ [6, 7] dynamical quarks and in addition we perform the same analysis with the ETMC $N_f = 2$ configurations [8, 9]. In both cases the fermions are regularized in the maximally twisted-mass (Mtm) Wilson formulation. The valence strange and charm quark masses are close to their physical values, while, in order to extrapolate up to the b-quark sector, we have considered seven values of the valence heavy-quark mass, m_h, up to $\sim 3\, m_c \simeq 0.75\, m_b$.

As shown in Ref. [10], our precise results for the heavy-light meson masses allow to extract the b-quark mass and the dimension four, five and six matrix elements entering in the Operator Product Expansion (OPE) of the inclusive semileptonic B decay rate, viz.

$$\Gamma_{B\to X_c e\nu} = |V_{cb}|^2 \frac{G_F\, m_b^5}{192\pi^3} a^{(0)} A_{ew} \left[1 + a^{(1)}\frac{\alpha_s}{\pi} + a^{(2)}\left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\mu_\pi^2|_B}{m_b^2}\left(-\frac{1}{2} + b^{(1)}\frac{\alpha_s}{\pi}\right) + \right. \quad (4)$$

$$\left. \frac{\mu_G^3|_B}{m_b^3}\left(c^{(0)} + c^{(1)}\frac{\alpha_s}{\pi}\right) + \frac{\rho_D^3|_B}{m_b^3}\left(d^{(0)} + d^{(1)}\frac{\alpha_s}{\pi}\right) + \frac{\rho_{LS}^3|_B}{m_b^3}\left(e^{(0)} + e^{(1)}\frac{\alpha_s}{\pi}\right) + \mathcal{O}\left(\frac{1}{m_b^4}\right)\right],$$

In this expression a - e are functions of m_c^2/m_b^2, and the main ingredients are the quark masses m_c, m_b and the Heavy Quark Expansion (HQE) matrix elements: μ_π^2 , μ_G^2 , ρ_D^3 , ρ_{LS}^3 computed at the B-meson point. In Ref. [11] the Cabibbo?Kobayashi?Maskawa (CKM) matrix element $|V_{cb}|$ is extracted together with m_c , m_b , μ_π^2 , ρ_D^3 from a global fit of the experimental semileptonic moments, where usually μ_G^2 is fixed by the $B^* - B$ splitting and ρ_{LS}^3 from Heavy Quark Sum Rules. We present the first unquenched lattice determination, obtained in [10], of the parameters appearing in the OPE analysis of the inclusive B-meson decays. The aim is to improve the precision of the global fit of the inclusive determination of $|V_{cb}|$ that is crucial in searches for new physics effects especially in view of the anomalies in $B \to D^{(*)}\tau\nu$. In fact, the same parameters appear also as coefficients of the HQE for the spin-averaged M_{av} and the hyperfine splitting ΔM of the V and P heavy-light meson masses, namely

$$M_{av} \equiv \frac{M_P + 3M_V}{4} \quad : \quad \frac{M_{av}}{\widetilde{m}_h} = 1 + \frac{\overline{\Lambda}}{\widetilde{m}_h} + \frac{\mu_\pi^2}{2\widetilde{m}_h^2} + \frac{\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3}{4\widetilde{m}_h^3} + \frac{\sigma^4}{\widetilde{m}_h^4}\,, \quad (5)$$

$$\Delta M \equiv M_V - M_P \quad : \quad \widetilde{m}_h\, \Delta M = \frac{2}{3} c_G(\widetilde{m}_h, \widetilde{m}_b)\mu_G^2(\widetilde{m}_b) + \frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3}{3\widetilde{m}_h} + \frac{\Delta\sigma^4}{\widetilde{m}_h^2}\,, \quad (6)$$

where μ_i^2, ρ_i^2 refer to matrix elements of asymptotically heavy mesons, which are related to $\mu_i^2|_B$, $\rho_i^2|_B$ for the physical B-meson (see Ref. [10]). They can be determined having meson masses with the heavy-quark mass between the physical c- and b-quark masses, m_c and m_b [12], or even above m_b [10]. In this contribution we adopt the ETMC ratio method [13] to employ lattice QCD as a virtual laboratory and to compute these fictitious meson masses with good accuracy. To use the Eq. (5,6) we need to define the mass scheme, i.e. the meaning of $\widetilde{m}_h$. A natural choice would be the pole mass $m_h^{\rm pole}$, however in perturbation theory this is well known to suffer from infrared (IR) renormalons of $\mathcal{O}(\Lambda_{\rm QCD})$. A viable option are the so called subtraction masses. In our analysis we use the kinetic mass scheme, defined subtracting to $m_h^{\rm pole}$ the perturbative contributions of the HQE paramenters at a separation scale $\mu_{\rm soft} = 1$GeV:

$$\widetilde{m}_h \equiv m_h^{\rm pole} - \delta m_h(\mu_{\rm soft})_{\rm IR} = m_h^{\rm pole} - \left[\overline{\Lambda}(\mu_{\rm soft})\right]_{\rm pert} - \frac{\left[\mu_\pi^2(\mu_{\rm soft})\right]_{\rm pert}}{2\widetilde{m}_h} - \frac{\left[\rho_D^3(\mu_{\rm soft})\right]_{\rm pert}}{4\widetilde{m}_h^2} \cdots . \quad (7)$$

The full expression can be found in Ref. [10]. Our choice is coherent with the scheme adopted for the semileptonic analysis of Ref. [11].

2. Masses and decay constants of $D^*_{(s)}$ and $B^*_{(s)}$ mesons

For a better control over statistical and systematic, we have analyzed the following V to P ratios

$$R^M_\ell(m_h) = M_{H^*_\ell}/M_{H_\ell} \quad \text{and} \quad R^f_\ell(m_h) = f_{H^*_\ell}/f_{H_\ell} \quad \text{with} \quad \ell = u/d, s, \tag{8}$$

for either $N_f = 2+1+1$ and $N_f = 2$ lattice data. These ratios are expected to go to 1 in the static limit, i.e. $\lim_{m_h\to\infty} R^M = 1$ and $\lim_{m_h\to\infty} R^f/c_R(m_h) = 1$ (where c_R is the appropriate Wilson coefficient computed in [14] allowing for the matching between QCD and HQET).

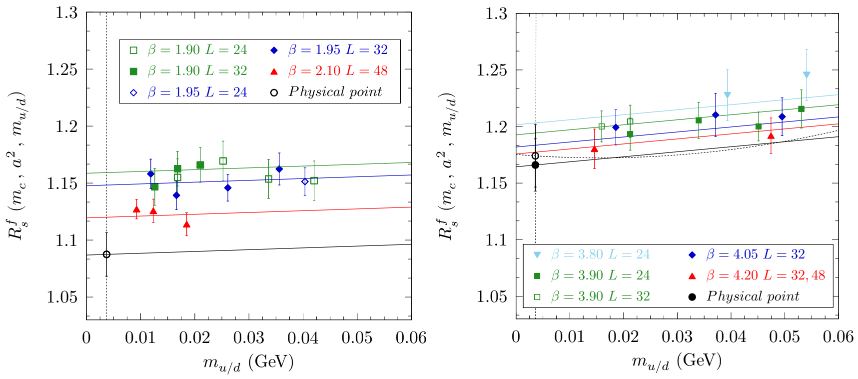

Figure 1. Chiral and continuum extrapolations of $f_{D^*_s}/f_{D_s}$ based on the polynomial fit (9) for $N_f = 2+1+1$ (Left panel) and $N_f = 2$ (Right panel). The black points represent the values at the physical pion point and in the continuum limit.

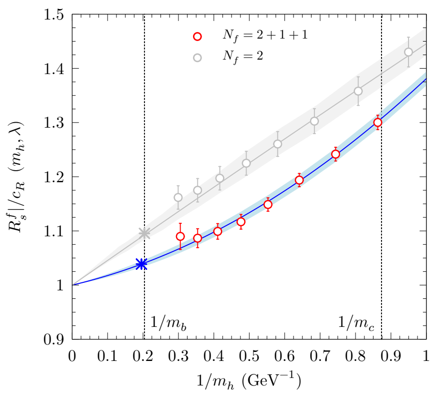

Figure 2. The dependence of R^f_s on the inverse heavy-quark mass.

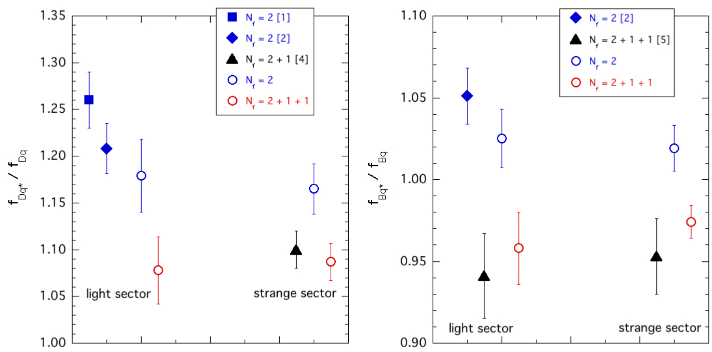

Figure 3. Comparison with previous estimates. The empty dots correspond to our values for the cases $N_f = 2$ (blue) and $N_f = 2+1+1$ (red).

We can study the dependence of R^M_ℓ and R^f_ℓ on the renormalized up/down quark mass $m_{u/d}$ and the lattice spacing a through a combined chiral and continuum extrapolation, based on a polynomial expansion of the form

$$R^{fit}(a^2, m_{u/d}) = P_0 + P_1\, m_{u/d} + P_2 a^2 (+ P_3\, m^2_{u/d} + P_4\, a^4)\,, \tag{9}$$

where for our Mtm setup the discretization effects involve only even powers of the lattice spacing. The quadratic $m^2_{u/d}$ and quartic a^4 terms have been considered to estimate the uncertainty related to the chiral and continuum extrapolation, respectively. In Fig. 1 the extrapolations for the case $h = c$, $\ell = s$ are shown for both the analysis $N_f = 2+1+1$ and $N_f = 2$. The ratios are interpolated to the charm point through a smooth interpolation, while, in order to get to the b-quark point, we perform a correlated polynomial fit in $1/m_h$ imposing the static limit constraint, namely

$$\overline{R}^{fit}(m_h) \quad = \quad 1 + \overline{D}_1/m_h + \overline{D}_2/m_h^2 + \overline{D}_3/m_h^3$$

where the linear term is absent in the case of the mass ratio (i.e. $\overline{D}_1 = 0$). In Fig. 2 an example of the inverse heavy-quark mass interpolation is shown for the case $\ell = s$ together with the results corresponding to the physical c- and b-quark masses. The latter ones, summarized also in Fig. 3, read

$$
\begin{array}{rrcll rcl}
N_f = 2+1+1: & M_{D^*}/M_D & = & 1.0769\,(79) & , & M_{B^*}/M_B & = & 1.0078\,(15)\ , \\
 & M_{D_s^*}/M_{D_s} & = & 1.0751\,(56) & , & M_{B_s^*}/M_{B_s} & = & 1.0083\,(10)\,, \\
 & f_{D^*}/f_D & = & 1.078\,(36) & , & f_{B^*}/f_B & = & 0.958\,(22)\,, \\
 & f_{D_s^*}/f_{D_s} & = & 1.087\,(20) & , & f_{B_s^*}/f_{B_s} & = & 0.974\,(10)\ , \\
N_f = 2: & M_{D^*}/M_D & = & 1.087(15) & , & M_{B^*}/M_B & = & 1.0078\,(15)\,, \\
 & M_{D_s^*}/M_{D_s} & = & 1.076(15) & , & M_{B_s^*}/M_{B_s} & = & 1.0074\,(15)\,, \\
 & f_{D^*}/f_D & = & 1.187\,(33) & , & f_{B^*}/f_B & = & 1.025\,(18)\,, \\
 & f_{D_s^*}/f_{D_s} & = & 1.172\,(23) & , & f_{B_s^*}/f_{B_s} & = & 1.019\,(14)\,.
\end{array}
$$

These results can be combined with the corresponding pseudoscalar values from Ref. [16] to get the absolute quantities. Quite remarkably in all cases meson masses are fully compatible with experimental values, while we can confirm a tension between the $N_f = 2$ and $2+1+1$ estimates although reduced from $\sim 8\%$ to $\sim 4\%$ (typically expected $\leq 1\%$).

3. HQE matrix elements

In order to apply the ETMC ratio method [13] to the quantities M_{av} and ΔM in Eq. (5,6), we construct a sequence of heavy-quark masses $\{\widetilde{m}_h^{(n)}\}$ with a common fixed ratio λ: $\widetilde{m}_h^{(n)} = \lambda\,\widetilde{m}_h^{(n-1)}$. The series starts at the physical charm quark mass $\widetilde{m}_h^{(1)} = \widetilde{m}_c = 1.219\,(57)$ GeV corresponding to $\overline{m}_c(2\text{ GeV}) = 1.176(36)$ GeV. For each gauge ensemble the quantities $M_{av}(\widetilde{m}_c)$, $\Delta M(\widetilde{m}_c)$ can be computed by a smooth interpolation of the results in the charm region and a subsequent extrapolation to the physical pion mass and to the continuum limit using a linear fit analogous to Eq. (9). We get $M_{av}^{\rm phys}(\widetilde{m}_c) = 1.967(25)$ GeV and $\Delta M^{\rm phys}(\widetilde{m}_c) = 140(11)$ MeV, which agree with the experimental values from PDG: $(M_D + 3M_{D^*})/4 = 1.973$ GeV and $M_{D^*} - M_D = 141.4$ MeV, as well as with the result of [3]. We now consider the following ratios

$$
y_M(\widetilde{m}_h^{(n)}) = \lambda^{-1}\frac{M_{av}(\widetilde{m}_h^{(n)})}{M_{av}(\widetilde{m}_h^{(n-1)})} \quad , \quad y_{\Delta M}(\widetilde{m}_h^{(n)}) = \lambda\frac{\Delta M(\widetilde{m}_h^{(n)})}{\Delta M(\widetilde{m}_h^{(n-1)})}\,\frac{c_G(\widetilde{m}_h^{(n-1)},\widetilde{m}_b)}{c_G(\widetilde{m}_h^{(n)},\widetilde{m}_b)}\ , \tag{10}
$$

with $n = 2, 3, \ldots$. They are built to have a well defined static limit $\lim_{\widetilde{m}_h\to\infty} y_{(\Delta)M} = 1$. Each ratio is extrapolated to the physical point obtaining values denoted by $\overline{y}_{(\Delta)M}$. The $\widetilde{m}_h$-dependence of $\overline{y}_{(\Delta)M}$ can be described as a series expansion in terms of $1/\widetilde{m}_h$ analogous to Eq. (10), we show the quality of these fits in Fig. 4. The following chain equations

$$
\frac{M_{av}(\widetilde{m}_h^{(n)})}{\widetilde{m}_h^{(n)}} = \frac{M_{av}(\widetilde{m}_c)}{\widetilde{m}_c}\prod_{i=2}^{n}\overline{y}_M(\widetilde{m}_h^{(i)})\ ,\ \ \widetilde{m}_h^{(n)}\frac{\Delta M(\widetilde{m}_h^{(n)})}{c_G(\widetilde{m}_h^{(n)},\widetilde{m}_b)} = \widetilde{m}_c\frac{\Delta M(\widetilde{m}_c)}{c_G(\widetilde{m}_c,\widetilde{m}_b)}\prod_{i=2}^{n}\overline{y}_{\Delta M}(\widetilde{m}_h^{(i)}) \tag{11}
$$

allow to reach the b-quark point and determine the b-quark mass $\widetilde{m}_b$ in an iterative way, requiring that, tuning the parameter λ, after K steps the quantity $M_{av}(\widetilde{m}_b)$ matches the experimental value $(M_B + 3M_{B^*})/4 = 5.314$ GeV . Then the b-quark mass $\widetilde{m}_b$ is directly given by $\widetilde{m}_b = \widetilde{m}_h^{(K+1)} = \lambda^K\ \widetilde{m}_c$. Adopting $K = 10$ we find $\lambda = 1.1422(10)$, which yields

$$
\widetilde{m}_b = 4.605\ (120)_{\rm stat}\ (57)_{\rm syst}\ \text{GeV}\ . \tag{12}
$$

In the $\overline{\rm MS}$ scheme the result (12) corresponds to $\overline{m}_b(\overline{m}_b) = 4.257\ (120)$ GeV, which is well compatible with the ETMC determination $\overline{m}_b(\overline{m}_b) = 4.26\ (10)$ GeV in Ref. [15] and consistent with other lattice determinations within one standard deviation (see, e.g., the FLAG review [17]). Eq. (11) allows also to go beyond the b-quark point and we have considered $n \lesssim 20$ ($\widetilde{m}_h \lesssim 4\,\widetilde{m}_b$). Taking into account the correlations between lattice data, we have performed the HQE fit ansatze (5,6), as shown in Fig. 5. For the heavy-light and the heavy-strange cases we obtain

$$\begin{array}{llcll}
\text{light:} & \overline{\Lambda} = 0.552\,(26)\ \text{GeV} & , & \mu_G^2 = 0.253\,(25)\ \text{GeV}^2\,, \\
 & \mu_\pi^2 = 0.321\,(32)\ \text{GeV}^2 & , & \rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = \text{-}0.158\,(84)\ \text{GeV}^3\,, \\
 & \rho_D^3 + \rho_{\pi\pi}^3 - \rho_S^3 = 0.153\,(34)\ \text{GeV}^3 & , & \Delta\sigma^4 = 0.0092\,(60)\ \text{GeV}^4\,, \\
 & \sigma^4 = 0.0071\,(56)\ \text{GeV}^4 & , & \\
\text{strange:} & \overline{\Lambda} = 0.636\,(16)\text{GeV} & , & \mu_G^2 = 0.303\,(16)\ \text{GeV}^2\,, \\
 & \mu_\pi^2 = 0.431\,(23)\ \text{GeV}^2 & , & \rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = \text{-}0.359\,(60)\ \text{GeV}^3\,, \\
 & \rho_D^3 + \rho_{\pi\pi}^3 - \rho_S^3 = 0.204\,(27)\ \text{GeV}^3 & , & \Delta\sigma^4 = 0.0272(56)\ \text{GeV}^4\,, \\
 & \sigma^4 = 0.0128\,(42)\ \text{GeV}^4 & . &
\end{array}$$

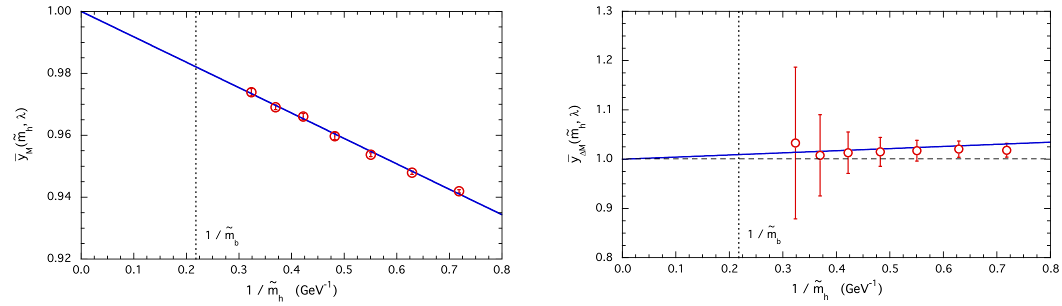

Figure 4. Linear fit for the ratios $\overline{y}_M(\widetilde{m}_h, \lambda)$ (Left panel) and $\overline{y}_{\Delta M}(\widetilde{m}_h, \lambda)$ (Right panel) versus the inverse heavy-quark mass $\widetilde{m}_h$ taking into account the correlations among the lattice points.

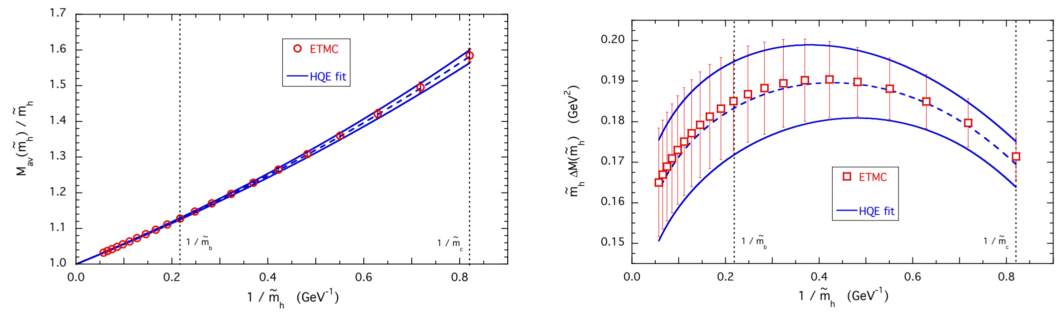

Figure 5. HQE fit for the quantity $M_{av}(\widetilde{m}_h)/\widetilde{m}_h$ (Left panel) and $\widetilde{m}_h \Delta M(\widetilde{m}_h)$ (Right panel) versus the inverse heavy-quark mass $\widetilde{m}_h$ taking into account the correlations among the lattice points.

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