THE ASTROPHYSICAL JOURNAL, 223:314–323, 1978 July 1

NEUTRON STAR MATTER AT HIGH TEMPERATURES AND DENSITIES. I. BULK PROPERTIES OF NUCLEAR MATTER

JAMES M. LATTIMER
Department of Astronomy, University of Illinois at Urbana–Champaign

AND

D. G. RAVENHALL
Department of Physics, University of Illinois at Urbana–Champaign
Received 1977 October 24; accepted 1978 January 18

ABSTRACT

We perform calculations of the thermodynamic properties of uniform dense matter at finite temperature using the Skyrme nuclear interaction. The calculations are valid for arbitrary proton concentrations and temperatures. The equation of state is compared with earlier investigations done for a few restricted cases. In order to understand the conditions under which one might expect to find nuclei immersed in a sea of nucleons, we explore the coexistence of two fluid phases with different proton concentrations. At zero temperature, nuclei with proton fractions in the range 0.044–0.335 may coexist with a pure neutron fluid, but at lower proton fractions protons "drip" as well. At finite temperatures, nuclear droplets with proton fractions up to 0.5 may coexist with fluid also containing up to 50% protons. A phase diagram for nuclear matter is presented. The critical temperature, as a function of proton concentration, above which no coexistence is possible and nuclei evaporate, is established. Its maximum value is $k_B T = 20$ MeV, which occurs in the case of symmetric nuclear matter.

Subject headings: dense matter — equation of state — stars: collapsed

I. INTRODUCTION

One of the most important current problems in astrophysics is the understanding of supernovae. In supernova models, a high-temperature equation of state for a system of interacting nucleons is crucial. Necessary information includes not only the relationship between the pressure, density, and temperature, but also the abundance and composition of the nuclei present, in order correctly to estimate the neutrino opacities and electron capture rates of hot, dense matter. A finite-temperature nuclear model is also needed in calculations of the decompression of neutron star matter (Lattimer *et al.* 1977) where nuclear fission processes are expected to raise the temperature to greater than 10^{10} K. Hot dense matter may also be ejected in the aftermath of a supernova explosion and may eventually nucleosynthesize the so-called *r*-process nuclei (Schramm and Norman 1976).

Much of the recent work on nuclear matter equations of state starting from a microscopic viewpoint has been confined to zero-temperature models (see Baym and Pethick 1975 for a review), or to finite-temperature homogeneous interacting nucleon gases (Buchler and Coon 1977; El Eid and Hilf 1977). It is known, however, that at zero temperature dense matter in the range 10^{11} g cm^{-3} to 10^{14} g cm^{-3} consists of nuclei immersed in a pure neutron fluid (Baym and Pethick 1975). The problem of the coexistence of two nuclear phases with different proton concentrations (i.e., the nucleus and a "dripped" neutron fluid) has been examined at zero temperature by Baym, Bethe, and Pethick (1971), hereafter referred to as BBP. Their treatment has been revised by Ravenhall, Bennett, and Pethick (1972), hereafter referred to as RBP. Coexistence at finite temperature has theretofore been investigated only for the case of symmetric nuclear matter (Küpper, Wegmann, and Hilf 1974), which, however, is physically far removed from neutron star matter. As a first step toward calculating a realistic finite-temperature equation of state, we shall investigate the coexistence problem for arbitrary proton concentrations and temperatures. We address ourselves to these problems by studying a system of interacting nucleons, employing a Skyrme contact pseudopotential (Vautherin and Brink 1970). A major advantage of the Skyrme model is that the thermodynamic properties of the matter are described by relatively simple equations: only the appearance of the well-known Fermi integrals renders them nonanalytic. The Skyrme interaction is discussed in § II, and the bulk properties of dense matter are determined in § III. In § IV the coexistence problem is investigated by determining the equilibrium phases possible in this matter. In § V we evaluate the equation of state of uniform dense matter in special limits in order to compare results with other recent investigations. In another paper (Ravenhall and Lattimer 1978) the surface energy of finite temperature nuclei will be calculated in the Thomas-Fermi approximation. The full equation of state of hot neutron star matter may then be determined, following the compressible liquid drop nuclear model of BBP and RBP.

II. THE NUCLEAR HAMILTONIAN

The approximate representation of the nucleon-nucleon interaction by a contact pseudopotential, as suggested by Skyrme (1956, 1959), produces the great simplification that the Hamiltonian density, as well as the single-particle potential and effective mass, are local functions of the proton and neutron particle densities $\rho_t(x)$ and the kinetic-energy densities $\tau_t(x)$,

$$\rho_t(x) = \sum_a n_{a,t}|\phi_{a,t}(x)|^2\,, \qquad \tau_t(x) = \sum_a n_a|\nabla\phi_{a,t}(x)|^2\,. \tag{1}$$

Here $n_{a,t}$ are the occupation numbers of the single-particle orbitals $\phi_{a,t}(x)$, and $t = n$ or p. The Hamiltonian density we use, very similar to that obtained by Vautherin and Brink (1970), has the form

$$\begin{aligned} H[\rho_n, \rho_p, \tau_n, \tau_p] = {}& \frac{\hbar^2}{2m_n}\tau_n + \frac{\hbar^2}{2m_p}\tau_p + \tfrac{1}{2}t_0[(1 + \tfrac{1}{2}x_0)(\rho_n + \rho_p)^2 - (\tfrac{1}{2} + x_0)(\rho_n{}^2 + \rho_p{}^2)] \\ & + \tfrac{1}{4}(t_1 + t_2)(\rho_n + \rho_p)(\tau_n + \tau_p) + \tfrac{1}{8}(t_2 - t_1)(\rho_n\tau_n + \rho_p\tau_p) \\ & + \tfrac{1}{4}t_3[\rho_n\rho_p + \lambda(\tfrac{1}{4}(\rho_n + \rho_p)^2 - \rho_p\rho_n)](\rho_n + \rho_p) \\ & + \tfrac{1}{16}(3t_1 - t_2)(\nabla\rho_n + \nabla\rho_p)^2 - \tfrac{1}{32}(3t_1 + t_2)[(\nabla\rho_n)^2 + (\nabla\rho_p)^2]\,. \end{aligned} \tag{2}$$

The adjustable parameters t_0, x_0, t_1, t_2, and t_3 (with an extra parameter for a spin-orbit term which we do not use) were determined, by Vautherin and Brink (1970), by fitting properties of a range of physical nuclei, including rms radii and shapes of charge distributions and total binding energies. We have added the parameter λ in the three-body interaction, and modified the two-body parameter x_0, so as to also obtain agreement with the properties of pure neutron matter as calculated by Siemens and Pandharipande (1971). These modifications have been designed to have zero effect both when $x = \frac{1}{2}$ and when $\rho = 0.154\ \mathrm{fm}^{-3}$, the saturation density at $x = \frac{1}{2}$. Thus the modification has little effect in the interior of normal nuclei, and in the surface regions the Hamiltonian is dominated by the gradient two-body terms. It is not expected to disturb the Brink and Vautherin fits to normal nuclei. The adopted values of these parameters are $t_0 = -1057.3$ MeV fm^3, $t_1 = 235.9$ MeV fm^5, $t_2 = -100$ MeV fm^5, $t_3 = 14{,}463.5$ MeV fm^6, $x_0 = 0.2885$, and $\lambda = 0.5162$.

A tacit assumption we have made is that the only temperature dependence possessed by the model is that contained in the occupation numbers $n_{a,t}$ in ρ_t and τ_t. This is not necessarily true since the effective interaction itself may be temperature dependent. The various terms associated with the adjustable parameters are a phenomenological representation of an s-wave finite range interaction (t_0, x_0, t_1), a p-wave two-body interaction (t_2), and an effect of three-body correlations (t_3, λ). The evaluation of those constants may be approached alternatively from the viewpoint of a microscopic two-nucleon finite-range interaction, as has been done by Negele and Vautherin (1973). But even there the ultimate choice of parameters comes from an empirical fit to some properties of physical nuclei. An alternative procedure to the one we propose has been used by Buchler and Coon (1977), who start with a realistic potential, the Reid potential, and calculate a finite-temperature two-body K-matrix using the method of Bloch and De Dominicis (1958). It is to be observed, however, that more accurate calculations of uniform nuclear matter using the Reid potential fail to give proper saturation properties (Day 1978; Wiringa and Pandharipande 1978). Thus while the calculations of Buchler and Coon (1977) could contain finite-temperature effects additional to those our method includes, their results may not be accurate, even at zero temperature, for matter at nuclear densities. We do not know of any way to include such additional finite-temperature contributions into our Skyrme scheme, since the interaction must, as we have noted, be determined by comparison with properties of physical nuclei at zero temperature. To the extent that the Skyrme model fits also the excited states of closed shell nuclei (see, for example, Liu and Brown 1976), these additional finite-temperature contributions may not be very important.

We note here an approach similar to ours by Küpper, Wegmann, and Hilf (1974) and El Eid and Hilf (1977), who also employ an effective interaction based on fitting properties of physical nuclei. However, our interaction has more adjustable parameters and also contains a density dependence, which theirs lacks. Our objectives also differ, in that we wish to examine the coexistence problem for arbitrary proton concentrations.

The Hartree-Fock equations for the single-particle orbitals $\phi_{a,t}(x)$ are

$$\{-\nabla[\hbar^2/2m_t{}^*(x)]\nabla + U_t(x)\}\phi_{a,t}(x) = \epsilon_{a,t}\phi_{a,t}(x)\,. \tag{3}$$

They contain effective masses $m_t{}^*$ and single-particle potentials U_t for both protons and neutrons of the form

$$\frac{\hbar^2}{2m_t{}^*} = \frac{\hbar^2}{2m_t} + \tfrac{1}{4}(t_1 + t_2)(\rho_n + \rho_p) + \tfrac{1}{8}(t_2 - t_1)\rho_t\,, \tag{4}$$

$$\begin{aligned} U_{n,p} = {}& t_0[(\rho_n + \rho_p)(1 + \tfrac{1}{2}x_0) - \rho_{n,p}(\tfrac{1}{2} + x_0)] + \tfrac{1}{4}(t_1 + t_2)(\tau_n + \tau_p) \\ & + \tfrac{1}{8}(t_2 - t_1)\tau_{n,p} + \tfrac{1}{4}t_3[\tfrac{3}{4}\lambda(\rho_n + \rho_p)^2 + \rho_{p,n}(\rho_n + \rho_p + \rho_{n,p})(1 - \lambda) \\ & - \tfrac{1}{8}(3t_1 - t_2)\nabla^2(\rho_n + \rho_p) + \tfrac{1}{16}(3t_1 + t_2)\nabla^2\rho_{n,p}\,. \end{aligned} \tag{5}$$

We now assume the matter to be uniform, with the intention of using the compressible liquid drop model to obtain the equation of state. In this approximation, the single-particle energies $\epsilon_{a,t}$ associated with the plane-wave orbital $\phi_{a,t}(\boldsymbol{x}) \propto \exp(i\boldsymbol{k}\cdot\boldsymbol{x})$ become

$$\epsilon_{k,t} = (\hbar^2/2m_t^*)k^2 + V_t\,, \tag{6}$$

where V_t is given by equation (5) for ρ_n and ρ_p constant (i.e., not dependent on $\boldsymbol{x}$), so that $\nabla\rho_t = 0$.

III. THERMODYNAMIC PROPERTIES OF BULK NUCLEAR MATTER

The densities ρ_t, τ_t of equation (1) become in the uniform-density case

$$\rho_t = \frac{1}{\pi^2}\int_0^\infty k^2 f_t(k)dk\,, \tag{7}$$

$$\tau_t = \frac{1}{\pi^2}\int_0^\infty k^4 f_t(k)dk\,, \tag{8}$$

where

$$f_t(k) = \{\exp[\beta(\epsilon_{k,t} - \mu_t] + 1\}^{-1} \tag{9}$$

is the Fermi distribution function, μ_t are the chemical potentials, and $\beta = (k_B T)^{-1}$. The internal energy density E of the uniform nuclear matter is given by equation (2) with $\nabla\rho_t = 0$. To the same approximation as equations (1) and (2) the entropy densities S_t are given by

$$S_t = -k_B \sum_a [n_{a,t}\log n_{a,t} + (1 - n_{a,t})\log(1 - n_{a,t})]\,, \tag{10}$$

which for uniform matter can be manipulated upon integration by parts into the particularly simple form

$$S_t = T^{-1}[\tfrac{5}{3}(\hbar^2/2m_t^*)\tau_t + \rho_t(V_t - \mu_t)]\,. \tag{11}$$

The pressure P of the matter is found from the thermodynamic relationship

$$P = T(S_n + S_p) + \mu_n\rho_n + \mu_p\rho_p - E\,. \tag{12}$$

We note that E and P when expressed in terms of ρ_t and τ_t contain no explicit temperature dependence, a conceptually simplifying property.

The expression (7), (8) for ρ_t and τ_t each depend not only on the chemical potentials μ_t, as yet undetermined, but also on ρ_t and τ_t themselves through the single-particle energies $\epsilon_{k,t}$. For given values of ρ_n, ρ_p, and β, the consistent values of τ_t and μ_t may be obtained as follows. Equations (7) and (8) are rewritten in terms of Fermi integrals

$$F_i(y) = \int_0^\infty u^i du/(e^{u+y} + 1):$$

$$\rho_t = \frac{1}{2\pi^2}(\hbar^2\beta/2m_t^*)^{-3/2}F_{1/2}(y_t)\,, \tag{13}$$

$$\tau_t = \frac{1}{2\pi^2}(\hbar^2\beta/2m_t^*)^{-5/2}F_{3/2}(y_t)\,, \tag{14}$$

where

$$y_t = \beta(V_t - \mu_t)\,. \tag{15}$$

The first of these equations can be inverted numerically to obtain y_n and y_p because m_t^* depends only on ρ_t and not on τ_t. Equation (14) is then used to find τ_t, after which V_t can be determined from equation (5), allowing μ_t to be found from equation (15). Equations (2), (11), and (12) may then be used to calculate the nucleon contributions to the equation of state.

IV. DELINEATION OF PHASES

At zero temperature and at densities between $\sim 3 \times 10^{11}$ g cm^{-3} and $\sim 2 \times 10^{14}$ g cm^{-3}, neutron-rich nuclei coexist with a sea of surrounding neutrons. That range of densities corresponds to a proton fraction $x = \rho_p/(\rho_p + \rho_n)$ inside the nuclei ranging from ~ 0.34 down to ~ 0.05. As is demonstrated in BBP, the most important criterion for coexistence of the nuclei and the neutron sea is the equilibrium of the two kinds of bulk matter. The quantities

ρ and x inside the nuclei, and the density ρ_d of the dripped neutrons, must be adjusted until the pressure and neutron chemical potentials are equal for the two fluids.

At finite but small temperatures the same possibilities for equilibrium must occur. Because of the tail on the Fermi distribution at finite T, protons in the nuclei may also cross the boundary into the sea, so that there are dripped protons also. Thus equilibrium at finite temperatures must involve ρ and x for the nuclei and ρ_d and x_d for the sea. It requires that the three quantities P, μ_n, and μ_p be made equal across the boundary. Compared to the zero-temperature problem, this adds one dimension of difficulty into the process. As a first step toward extending the compressible liquid-drop model of BBP, we have explored computationally the bulk equilibrium described above.

A convenient way to display the equilibrium graphically is to plot isobars on a (μ, x)-diagram. This is done, for uniform nuclear matter of the kind detailed in § II, at temperatures $k_BT = 0$, 3, 10, and 20 MeV in Figures 1, 2, 3, and 4 respectively. The dashed portion of each curve has negative compressibility, and is therefore unstable. At $k_BT = 20$ MeV the curves resemble qualitatively those of a noninteracting fluid. The effects of the nucleon-nucleon interaction are seen in the convolutions of Figures 1, 2, and 3. The decreasing importance of these interactions, compared to the thermal motion, is seen in the progressive simplification in the structure of the curves as the temperature increases. This progression is best understood by representing the proton curves for $0 < x < 0.5$ as neutron curves for $0.5 < x < 1$, in which case they are continuous and smooth. For example, the curve j of Figure 1 then has one lobe projecting from $x = 0$ and another lobe projecting from $x = 1$. As T increases from zero, these lobes touch, and afterwards the j curve has two branches, a central closed part resembling a figure eight and another monotonically decreasing branch. Both branches are used to achieve equilibrium. We note that as the temperature becomes large, the chemical potentials μ_t approach negative infinity as P tends to zero, which is their classical limit. At $T = 0$ and a given pressure P, coexistence is possible if a point $(\mu_n, 0)$ (the neutron sea) on the isobar P can be connected to a point (μ_n, x) on the same isobar by a horizontal line. (They then have the same P and μ_n.) Coexistence at $T = 0$ of the kind we have just described is indicated by the dotted line (n) in Figure 1. The proton chemical potential μ_p is given also in Figure 1, the isobars for μ_n and μ_p being continuous at $x = 0.5$. At $T = 0$, the value of μ_p on the coexistence curve (dotted line p) for a given P and for x values greater than ~ 0.04 is always less than the corresponding value of μ_p at $x = 0$ on the isobar P, implying that proton drip does not occur over this range of x. It may be concluded from Figure 1 that coexistence between nuclei and a dripped sea is possible only in the domain $x < 0.335$, at zero temperature. To find the density range to which this corresponds requires evaluation of the remaining contributions to the total energy and pressure—for example, the surface, Coulomb, and electron contributions. This has been done at $T = 0$ by Ravenhall, Bennett, and Pethick (1973) in the spirit of BBP.

We now consider coexistence in the presence of dripped protons. Since x_d is no longer zero, coexistence is possible at a given P only if the pair of points (μ_p, x) and (μ_p, x_d), as well as the pair (μ_n, x) and (μ_n, x_d), along the isobar P, can be connected by horizontal lines. In other words, only if the set (μ_p, x), (μ_p, x_d), (μ_n, x), and (μ_n, x_d) form a rectangle will the two phases for a given P, with proton fractions x and x_d, coexist. An example is shown in Figure 3, at a temperature of $k_BT = 10$ MeV.

In Figures 1–4 the coexistence curves for neutrons and protons are indicated by the dotted lines n and p for the heavier matter, and by n_d and p_d for the dripped matter. The vertical slash in each coexistence curve indicates the point at which $\rho = \rho_d$ and thus separates the heavier and dripped portions of the curve. As in the $T = 0$ results of BBP, the coexistence curves in Figure 1 go to decreasing μ_n and increasing μ_p as x increases. This feature remains at finite T, but the coexistence curves shift to larger values of x as T is increased. In fact, at finite temperatures, coexistence extends all the way to $x = x_d = 0.5$. At temperatures below about 1 MeV, x_d is nearly zero along most of the coexistence curve. At higher temperatures (Figs. 2, 3, and 4) the lower limit of x_d, as well as that of x, on the coexistence curve tends to larger x values as the temperature is increased. A surprising result of this investigation is that protons are able to "drip" at zero temperature if $x \leqslant 0.04$. This is in contrast to the results of BBP, who in their consideration of bulk equilibrium at zero temperature did not find any indication of proton drip.

These features have been summarized in Figure 5, which shows, as a function of temperature, the range of x and x_d for which coexistence with a dripped medium is possible. Figure 5 may be interpreted as a phase diagram. For a given temperature, nuclear pressure P, and average proton fraction Y_e, Figure 5 shows whether or not coexistence is possible, and if it is, what the relative mass fractions of nuclei and dripped matter are. This can be seen as follows. Choose T, P, and Y_e. If the point (Y_e, T) lies to the left of the dashed (drip) portion of the curve for that P, i.e., if $Y_e < x_d(P, T)$, no coexistence is possible, and the state of the matter is that of a uniform medium. If this point lies between the dashed and solid (nuclei) portions of the P curves, i.e., if $x_d(P, T) < Y_e < x(P, T)$, then coexistence is possible, and the mass fraction of heavy nuclei is, from mass and charge conservation, simply

$$f_H = \frac{Y_e - x_d(P, T)}{x(P, T) - x_d(P, T)} . \tag{16}$$

Finally, if the point (Y_e, T) lies to the right of the solid portion of the P curve, i.e., if $Y_e > x(P, T)$, then coexistence is again no longer possible, and the matter again consists of a uniform medium.

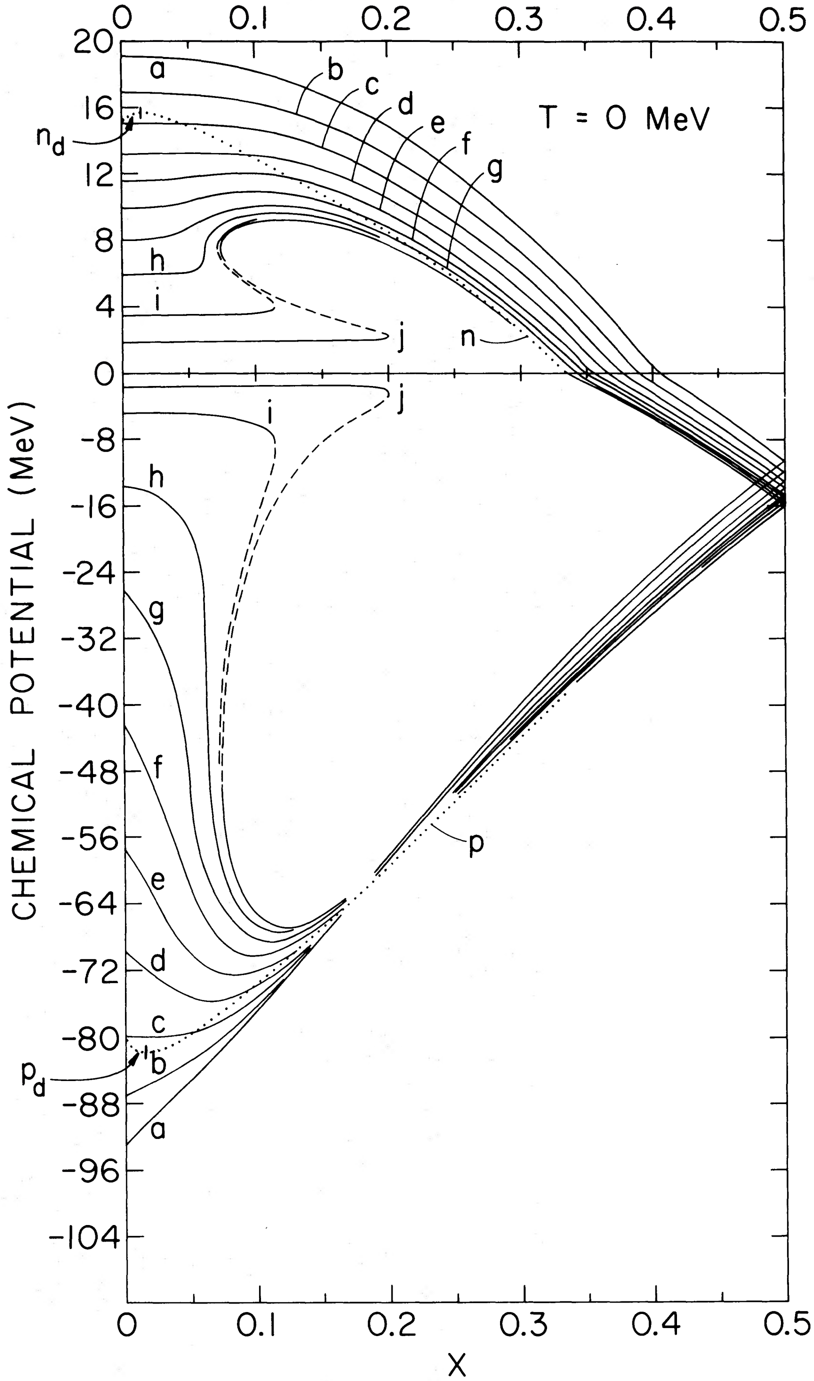

FIG. 1.—Isobars showing the neutron and proton chemical potentials, in units of MeV, of bulk matter at zero temperature plotted as a function of the proton concentration x. The upper curves represent μ_n and have pressures of (*a*) 0.88, (*b*) 0.646, (*c*) 0.458, (*d*) 0.31, (*e*) 0.198, (*f*), 0.115, (*g*) 0.058, (*h*), 0.023, (*i*) 0.005, and (*j*) 0.001 MeV fm^{-3}, respectively. The lower curves represent μ_p and have the same pressures, in order, from the lower to upper left. The dashed portion of each curve has negative compressibility and is unstable. Note the change in scale of the ordinate at 0.0 MeV. The dotted line n intersecting the μ_n curves is the coexistence curve; for a given value of μ_n this curve gives the proton concentration x of bulk nuclear matter that has the same pressure and μ_n as, and is thus in equilibrium with, dripped matter with zero proton concentration. The absolute value of $\mu_p(P, x)$ for $x > 0.04$ (dotted line p) is greater than $\mu_p(P, 0)$, so proton drip does not occur. For $x < 0.04$, proton drip occurs, and the dotted lines (n and p) show the proton fraction that has the same P, μ_n, and μ_p as, and hence is in equilibrium with, dripped matter with proton fraction x_d (dotted lines n_d and p_d). The vertical slash in each coexistence curve show where the densities of the two phases are equal.

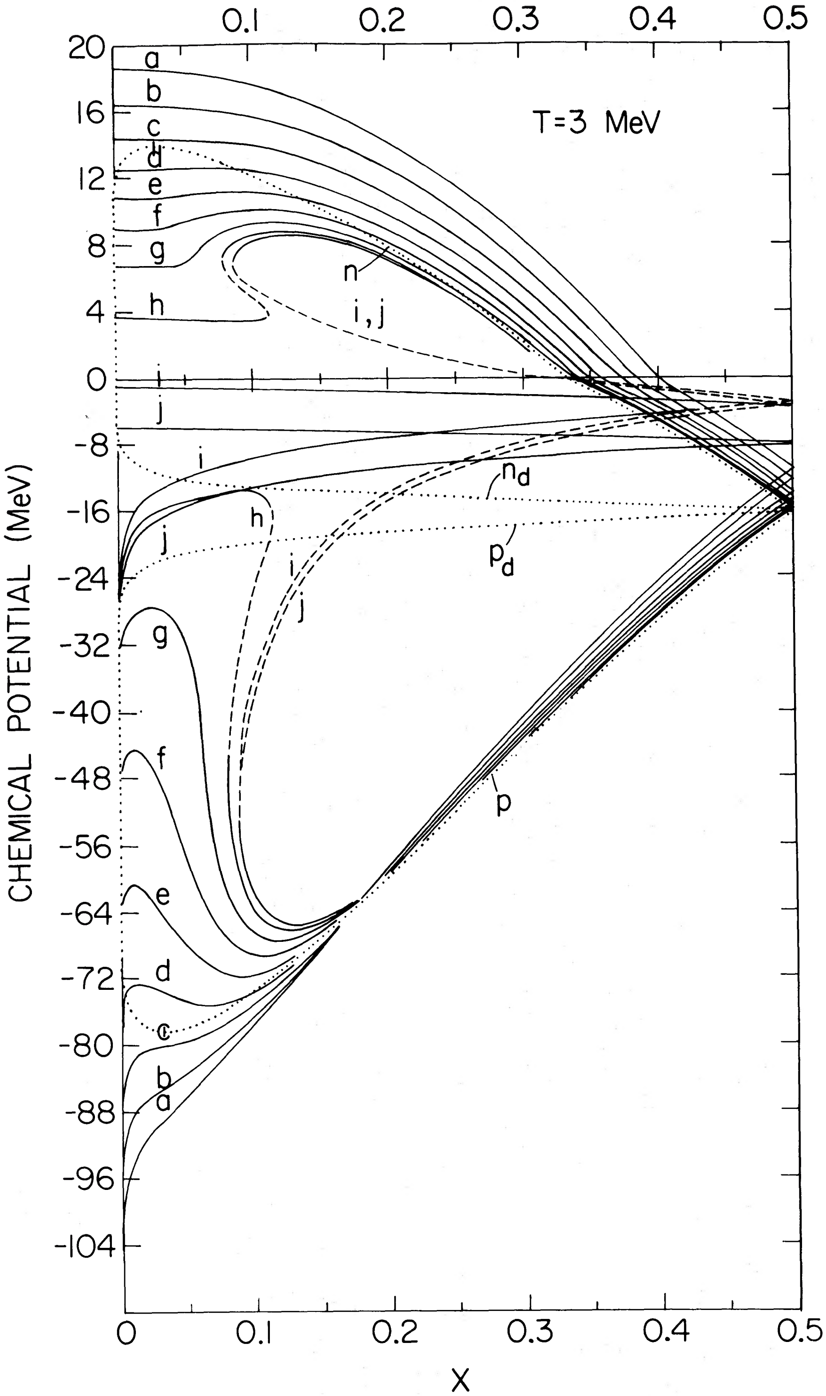

FIG. 2.—Same as Fig. 1 except for a temperature of $T = 3$ MeV

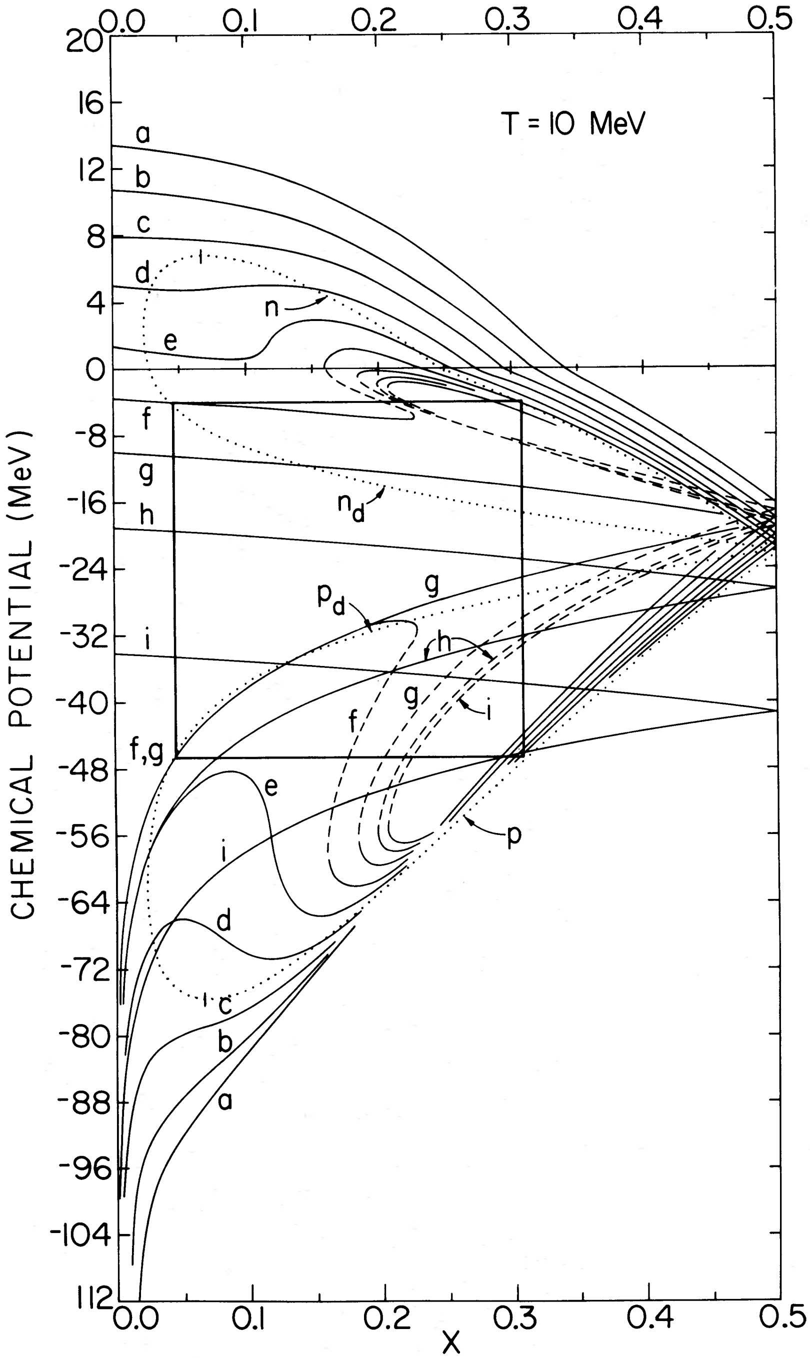

FIG. 3.—Same as Fig. 1 except for $T = 10$ MeV and the isobar (*j*), $P = 0.001$ MeV fm^{-3}, is not displayed. The superimposed reactangle shows graphically an example of coexistence between nuclear droplets with $x = 0.308$ and a dripped medium with $x_d = 0.044$. All four points of the rectangle lie on $P = 0.115$ MeV fm^{-3}(*f*) isobars: the upper pair have the same μ_n, while the lower pair have the same μ_p.

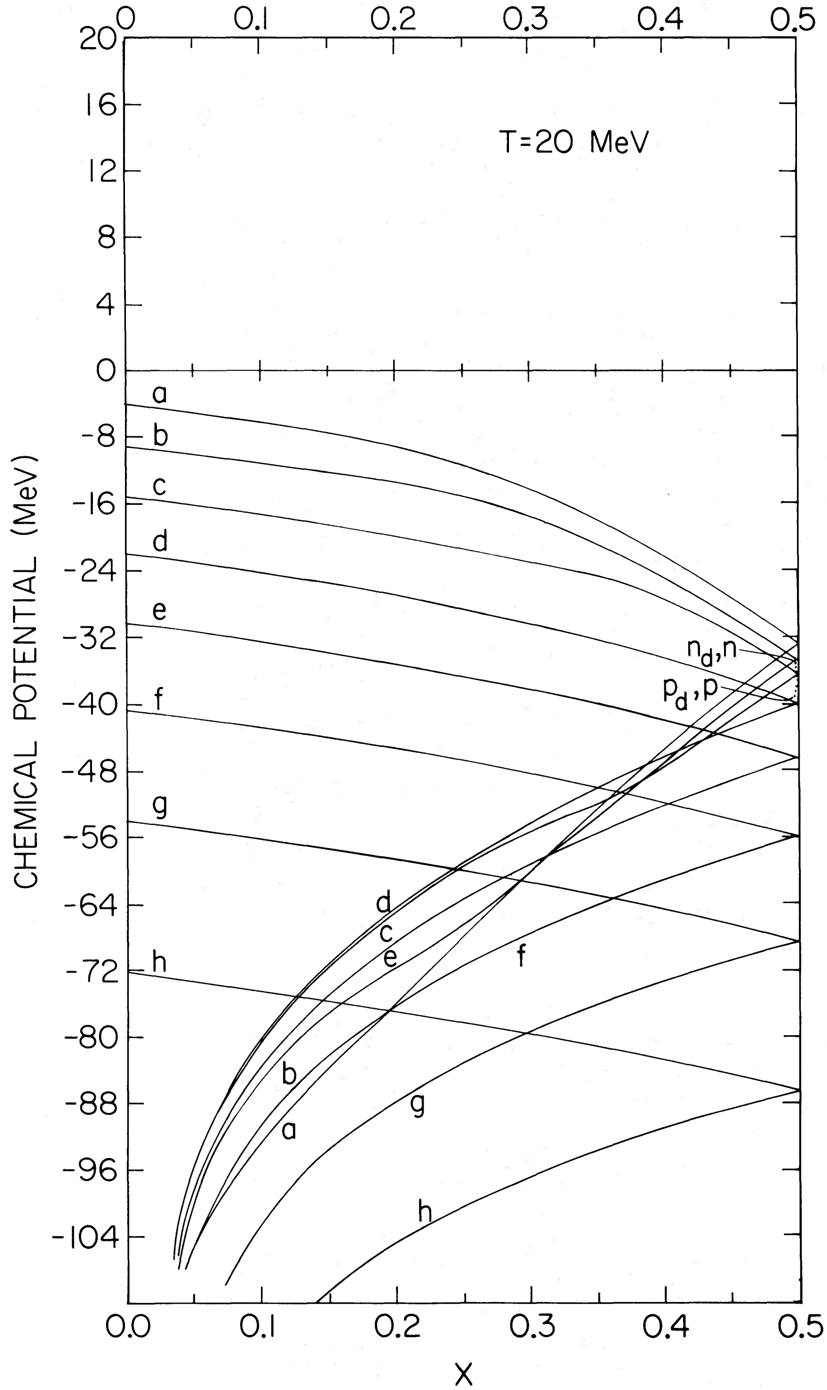

FIG. 4.—Same as Fig. 1 except for T = 20 MeV and the isobars (*i*) and (*j*) are not displayed

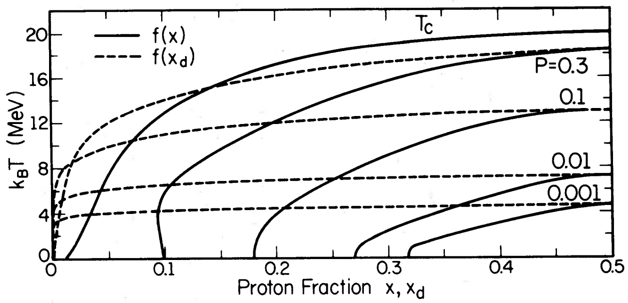

FIG. 5.—Phase diagram for nuclei and a dripped medium. Isobars, with pressures of 0.3, 0.1, 0.01, and 0.001 MeV fm^{-3}, show the proton fractions in nuclei (*solid curves*) that are in equilibrium with a dripped sea of proton fraction x_d (*dashed curves*). The critical temperature is given by the solid line labeled T_c: coexistence is impossible at any pressure if $T > T_c(x)$.

Also displayed in Figure 5 is the critical temperature, T_c, which is the maximum temperature, at a given nuclear x, for which equilibrium with a dripped sea is possible. One way to understand its significance is as follows: Choose a P, T, and Y_e such that the point (Y_e, T) lies on the T_c curve, i.e., $T = T_c(Y_e)$, but that the pressure is low enough for coexistence to be possible [i.e., $x_d(P, T) < Y_e < x(P, T)$]. If the pressure is gradually increased, one can show that the volume fraction occupied by the nuclei increases. When the pressure is such that $x_d(P, T) = x(P, T) = Y_e$, this fraction is unity. A further pressure increase results in a phase transition to a uniform medium. Thus the maximum pressure for which coexistence is possible is also given by the T_c curve. One sees that for symmetric nuclear matter, $x = 0.5$, the critical temperature is $k_B T_c = 20.19$ MeV. This compares with the result of Küpper, Wegmann, and Hilf (1974), using the same nuclear interaction as El Eid and Hilf (1977), who find $k_B T_c = 17.35$ MeV.

The special situation of charge-symmetric matter ($x = 0.5$) can be examined analytically in the limit that $T \to 0$. The matter in nuclear droplets (subscript nuc) is degenerate, thus approximately temperature-independent, with approximately the values pertaining at $T = 0$: $2\rho_{t,\mathrm{nuc}} = \rho_{\mathrm{nuc}} = 0.155\ \mathrm{fm}^{-3}$, $\mu_t = -16$ MeV. The dripped matter (subscript d) has very low density, and has the properties of a nondegenerate gas. Thus

$$\rho_{t,d} = \frac{1}{2}\left(\frac{\hbar^2\beta\pi}{2m_t}\right)^{-3/2} \exp(\beta\mu_t)\,, \tag{17}$$

and $P = 2\rho_{t,d}/\beta$. When $\beta = 10\ \mathrm{MeV}^{-1}$, the density of the dripped gas is only $10^{-74}\ \mathrm{fm}^{-3}$, small enough that it has no effect on the nuclear-matter properties. Such a treatment can be extended to values of x somewhat less than 0.5, although for $x \leqslant 0.34$ a regime occurs in which $\rho_{n,d} \neq 0$ as $T \to 0$.

V. COMPARISON WITH PREVIOUS CALCULATIONS

Figures 6 and 7 compare the results of the work with those of previous investigators for uniform matter in two special limits. The equation of state for pure neutron matter ($x = 0$) is displayed in Figure 6. First, comparing the results of the present work (*solid line*) with a noninteracting neutron gas (*dashed line*), one sees the influence of the effective interaction: the pressure is lowered by the attractive nuclear potential until nuclear densities ($\rho_b \sim 0.12\ \mathrm{fm}^{-3}$) are reached, at which point the potential becomes repulsive and the equation of state is stiffer than that of the noninteracting gas. (Stiffness is used in this context to mean the slope on this logarithmic plot.) Buchler and Coon's (1977) neutron matter is softer at all densities than the present work, and does not become stiff at nuclear densities. This may be due partly to their use of the soft-core Reid potential, and partly to their neglect of three-body correlations. The El Eid and Hilf (1977) equation of state is initially softer than ours, at least for $k_B T < 10$ MeV, until nuclear densities are reached. Above nuclear densities, their pressure is considerably greater than ours.

Figure 7 displays the equation of state of a neutron-proton-electron mixture in beta-equilibrium with zero charge density. The results of the present work, compared with those of El Eid and Hilf (1977), show the same tendencies as in the pure neutron matter case, Figure 6. Differences between them are a reflection of the different nuclear interactions assumed. Both nuclear Hamiltonians are based on a fit to the properties of physical nuclei, although the one used by us (Vautherin and Brink 1970) contains more adjustable parameters, including one that allows for a dependence on density. For physical nuclei the difference is probably not important. The nuclear matter involved in the comparison, however, has no protons (Fig. 6) or few (Fig. 7), and is thus far removed from physical nuclei. Our Hamiltonian has been adjusted, in addition, to theoretical properties of pure neutron matter (Siemens and Pandharipande 1971). We therefore believe that in principle its results may be superior to those of El Eid and Hilf.

The forms of matter explored in Figures 6 and 7 are not, however, of primary astrophysical interest. Matter at subnuclear densities and below the critical temperature will be the finite-temperature extension of that explored

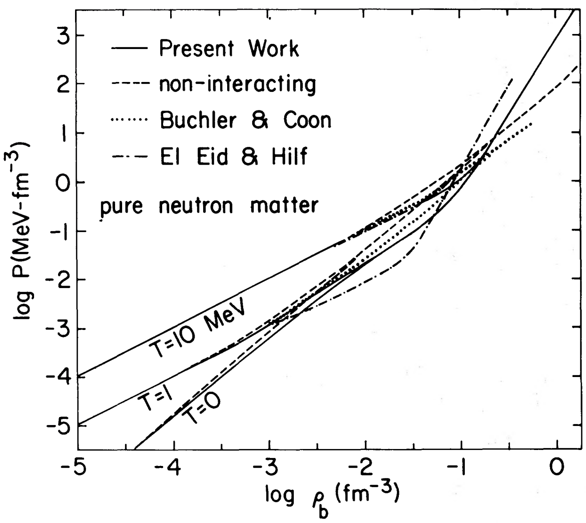

FIG. 6

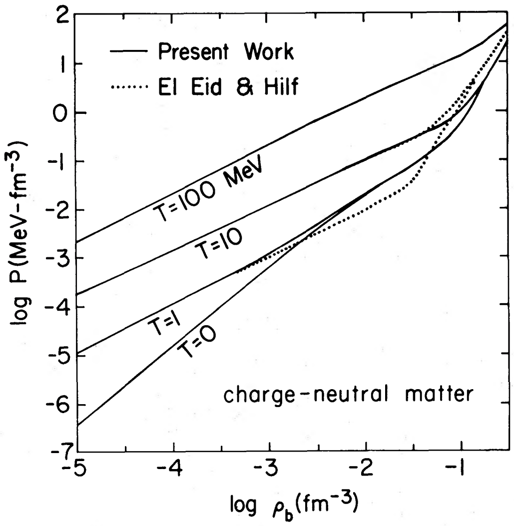

FIG. 7

FIG. 6.—The equation of state of a pure neutron gas ($x = 0$) at various temperatures. Solid lines show the results of this work; dashed lines are for a noninteracting neutron gas, dot-dashed lines indicate the results of El Eid and Hilf (1977), and the dots are taken from Buchler and Coon (1977).

FIG. 7.—The equation of state of a neutron-proton-electron mixture in beta-equilibrium with zero charge density at various temperatures. Solid lines show the results of this work, and dotted lines indicate the results of El Eid and Hilf (1977).

in BBP and RBP: dense nuclear droplets surrounded by a dripped medium of lower density, the bulk properties of which have been explored in this paper. Of the additional ingredients required, the most important unknown quantity is the nuclear surface energy at finite temperature. This will be discussed in a forthcoming paper.

In summary, we have investigated the thermodynamic properties of hot, dense, uniform nuclear matter employing a Skyrme nuclear interaction. The equation of state of uniform matter is harder at subnuclear densities and softer at supernuclear densities than that of El Eid and Hilf (1977), and harder than that of Buchler and Coon (1977) at all densities. The coexistence of two nuclear phases with different proton concentrations has been studied, and the critical temperature above which nuclei evaporate has been determined. The critical temperature of symmetric nuclear matter is found to be 20.19 MeV, about 2.8 MeV higher than that found by Küpper, Wegmann, and Hilf (1974).

We thank Professors C. Pethick, D. Q. Lamb, and G. Baym for enlightening discussions regarding this work. This research was supported in part by National Science Foundation grants AST 73-05117, AST 76-22673 and PHY 75-21590.

REFERENCES

Baym, G., Bethe, H. A., and Pethick, C. J. 1971, *Nucl. Phys.*, **A175**, 225 (BBP).
Baym, G., and Pethick, C. J. 1975, *Ann. Rev. Nucl. Sci.*, **25**, 82.
Bloch, C., and De Domincis, C. 1958, *Nucl. Phys.*, **2**, 459.
Buchler, J. R., and Coon, S. A. 1977, *Ap. J.*, **212**, 807.
Day, B. D. 1978, *Rev. Mod. Phys.*, in press.
El Eid, M. F., and Hilf, E. R. 1977, *Astr. Ap.*, **57**, 243.
Küpper, W. A., Wegmann, G., and Hilf, E. R. 1974, *Ann. Phys.*, **88**, 454.
Lattimer, J. M., Mackie, F., Ravenhall, D. G., and Schramm, D. N. 1977, *Ap. J.*, **213**, 225.
Liu, K. F., and Brown, G. E. 1976, *Nucl. Phys.*, **A265**, 385.
Negele, J. W., and Vautherin, D. 1973, *Nucl. Phys.*, **A207**, 298.
Ravenhall, D. G., Bennett, C. D., and Pethick, C. J. 1972, *Phys. Rev. Letters*, **28**, 978 (RBP).
Ravenhall, D. G., and Lattimer, J. M. 1978, in preparation.
Schramm, D. N., and Norman, E. J. 1976, Proc. 3d International Conference on the Properties of Nuclei Far from the Region of Beta Stability (CERN Report).
Siemens, P. J., and Pandharipande, V. R. 1971, *Nucl. Phys.*, **A173**, 561.
Skyrme, T. H. R. 1956, *Phil. Mag.*, **1**, 1043.
———. 1959, *Nucl. Phys.*, **9**, 615.
Vautherin, D., and Brink, D. M. 1970, *Phys. Letters*, **B32**, 149.
Wiringa, R. B., and Pandharipande, V. R. 1978, *Nucl. Phys.*, in press.

JAMES M. LATTIMER: Department of Astronomy, University of Illinois at Urbana–Champaign, Urbana, IL 61801

D. G. RAVENHALL: Department of Physics, University of Illinois at Urbana–Champaign, Urbana, IL 61801