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On the spin correlations of muons and tau leptons produced in the high-energy annihilation processes $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$

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The electromagnetic processes of annihilation of (e^+e^-) pairs, generated in high-energy nucleus-nucleus and hadron-nucleus collisions, into heavy flavor lepton pairs are theoretically studied in the one-photon approximation, using the technique of helicity amplitudes. For the process $e^+e^- \to \mu^+\mu^-$, it is shown that – in the case of the unpolarized electron and positron – the final muons are also unpolarized but their spins prove to be strongly correlated. For the final $(\mu^+\mu^-)$ system, the structure of triplet states is analyzed and explicit expressions for the components of the spin density matrix and correlation tensor are derived; besides, the formula for angular correlation at the decays of final muons μ^+ and μ^- is obtained.

It is demonstrated that the spin correlations of muons in the process $e^+e^- \to \mu^+\mu^-$ have the purely quantum character, since one of the Bell-type incoherence inequalities for the correlation tensor components is always violated (i.e. there is always at least one case when the modulus of sum of two diagonal components exceeds unity). Besides, the additional contribution of the weak interaction of lepton neutral currents through the virtual Z^0 boson is considered in detail, and it is established that, when involving the weak interaction contribution, the qualitative character of the muon spin correlations does not change.

Analogous analysis can be wholly applied as well to the final tau leptons formed in the annihilation process $e^+e^- \to \tau^+\tau^-$, which becomes possible at considerably higher energies.

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1. Helicity amplitudes for the annihilation process $e^+e^- \to \mu^+\mu^-$

In the first non-vanishing approximation over the electromagnetic constant $e^{\,2}/\hbar c$, the process of conversion of the electron-positron pair into the muon pair is described by the well-known one-photon Feynman diagram [1].

Due to the electromagnetic current conservation, the virtual photon with a time-like momentum transfers the angular momentum $J = 1$ and negative parity. Taking into account that the internal parities of muons μ^+ and μ^- are opposite, the $(\mu^+\mu^-)$ pair is generated in the triplet states (the total spin $S = 1$) with the total angular momentum $J = 1$ and with the orbital angular momenta $L = 0$ and $L = 2$, being the superpositions of the states ${}^3S_1$ and ${}^3D_1$ with the negative space parity.

The respective helicity amplitudes have the following structure:

$$f_{\Lambda'\Lambda}(\theta,\phi) = R_{\Lambda'\Lambda}(E)\, d^{(1)}_{\Lambda'\Lambda}(\theta)\, \exp(i\Lambda\phi), \tag{1.1}$$

where θ and ϕ are the polar and azimuthal angles of the flight direction of the positively charged muon (μ^+) in the center-of-mass (c.m.) frame of the considered reaction with respect to the initial positron momentum;
$d^{(1)}_{\Lambda'\Lambda}(\theta)$ are the Wigner functions (elements of the finite rotation matrix) for the angular momentum $J = 1$;
Λ is the difference of helicities of the positron and electron, coinciding with the projection of total spin and with the projection of total angular momentum of the (e^+e^-) pair onto the direction of positron momentum in the c.m. frame (the projection of orbital angular momentum onto the momentum direction equals zero);
Λ' is the difference of helicities of the muons μ^+ and μ^-, coinciding with the projection of total angular momentum of the $(\mu^+\mu^-)$ pair onto the direction of momentum of the positively charged muon μ^+ in the c.m. frame (see, for example, [1, 2]).

Due to the factorizability of the Born amplitude, we can write:

$$R_{\Lambda'\Lambda}(E) = r^{(\mu)}_{\Lambda'}(E)\, r^{(e)}_{\Lambda}(E). \tag{1.2}$$

Here Λ' and Λ take the values $+1, 0, -1$; in doing so, the parameters $r^{(\mu)}_{\Lambda'}$, $r^{(e)}_{\Lambda}$ depend upon the initial energy E of the positron (electron) in the c.m. frame of the pair e^+e^-, but do not depend upon the angles θ and ϕ.

On account of the space parity conservation in the electromagnetic interactions, we have:

$$r^{(\mu)}_{+1} = r^{(\mu)}_{-1} = r^{(\mu)}_{1}, \quad r^{(e)}_{+1} = r^{(e)}_{-1} = r^{(e)}_{1}. \tag{1.3}$$

In accordance with the structure of electromagnetic current for the pairs e^+e^- and $\mu^+\mu^-$ in the c.m. frame [1], the following relations are valid:

$$r^{(\mu)}_{0} = \frac{m_\mu}{E} r^{(\mu)}_{1} = \sqrt{1-\beta_\mu^2}\, r^{(\mu)}_{1}, \quad r^{(e)}_{0} = \frac{m_e}{E} r^{(e)}_{1}, \tag{1.4}$$

where m_μ and m_e are the masses of the muon and electron, respectively, β_μ is the muon velocity in the c.m. frame. Since for the process $e^+e^- \to \mu^+\mu^-$ the inequality $E \geq m_\mu \gg m_e$ is always satisfied, the contribution of electron-positron states with antiparallel spins (equal helicities) can be neglected. In doing so, $R_{\Lambda 0}(E) \approx 0$.

The calculation of the one-photon diagram gives:

$$r_1^{(\mu)}(E) = r_1^{(e)}(E) = \frac{|e|}{\sqrt{2E}}, \tag{1.5}$$

where e is the electron charge. If the relativistic invariant

$$s = (p_{e^+} + p_{e^-})^2 = (p_{\mu^+} + p_{\mu^-})^2 = 4E^2$$

is introduced, the expression for the cross section of the process $e^+e^- \to \mu^+\mu^-$ takes the following form [1]:

$$\sigma = \frac{4\pi}{3}\frac{e^2}{s}\left(1 + \frac{2m_\mu^2}{s}\right)\sqrt{1 - \frac{4m_\mu^2}{s}}. \tag{1.6}$$

Taking into account the explicit formulas for d-functions corresponding to the angular momentum $J = 1$ [1, 2], we find the angular distribution of muon emission, normalized by unity, in the c.m. frame:

$$dW_{\mu^+\mu^-} = \frac{3}{16\pi}\frac{1 + \cos^2\theta + (m_\mu^2/E^2)\sin^2\theta}{1 + (m_\mu^2/2E^2)}d\Omega = \frac{3}{8\pi}\frac{2 - \beta_\mu^2\sin^2\theta}{3 - \beta_\mu^2}d\Omega, \tag{1.7}$$

where $d\Omega$ is the element of solid angle.

2. Structure of the triplet states of the $(\mu^+\mu^-)$ system formed in the process $e^+e^- \to \mu^+\mu^-$

Taking into account the relations (1.1)-(1.4) for the helicity amplitudes, it is clear that if the positron and electron are totally polarized along the positron momentum in the c.m. frame, then the $(\mu^+\mu^-)$ system is produced in the triplet state of the following form:

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2 - \beta_\mu^2\sin^2\theta}}\left(\frac{1+\cos\theta}{2}|+1\rangle - \sqrt{1-\beta_\mu^2}\frac{\sin\theta}{\sqrt{2}}|0\rangle + \frac{1-\cos\theta}{2}|-1\rangle\right). \tag{2.1}$$

Here $\beta_\mu = \sqrt{1 - (m_\mu^2/E^2)}$ is the velocity of each of the muons, as before;

$$|+1\rangle = |+1/2\rangle^{(\mu^+)} \otimes |+1/2\rangle^{(\mu^-)}, \quad |-1\rangle = |-1/2\rangle^{(\mu^+)} \otimes |-1/2\rangle^{(\mu^-)},$$

$$|0\rangle = \frac{1}{\sqrt{2}}\left(|+1/2\rangle^{(\mu^+)} \otimes |-1/2\rangle^{(\mu^-)} + |-1/2\rangle^{(\mu^+)} \otimes |+1/2\rangle^{(\mu^-)}\right)$$

are the states with the projection of total spin of the $(\mu^+\mu^-)$ pair onto the direction of momentum of the muon μ^+ in the c.m. frame of the reaction $e^+e^- \to \mu^+\mu^-$, equaling $+1$, -1 and 0, respectively.

Let us note that the real values of the coefficients of superposition of the triplet states $|+1\rangle$, $|0\rangle$ and $|-1\rangle$ in the state $|\Psi\rangle^{(+1)}$ (2.1) correspond to the choice of the quantization axes z' and z along the positron momentum and μ^+ momentum, respectively, in the c.m. frame of the reaction $e^+e^- \to \mu^+\mu^-$, and the axis y - along the normal to the plane of this reaction.

If the positron and electron are totally polarized in the direction being antiparallel to the positron momentum, then the $(\mu^+\mu^-)$ pair is generated in the following triplet state:

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2\sin^2\theta}}\left(\frac{1-\cos\theta}{2}|+1\rangle + \sqrt{1-\beta_\mu^2}\,\frac{\sin\theta}{\sqrt{2}}|0\rangle + \frac{1+\cos\theta}{2}|-1\rangle\right). \tag{2.2}$$

3. Spin density matrix and correlation tensor of the $(\mu^+\mu^-)$ pair

If the positron and electron are not polarized, then, since $r^{(e)} \approx 0$, the final state of the $(\mu^+\mu^-)$ pair represents a noncoherent mixture of spin states $|\Psi\rangle^{(+1)}$ and $|\Psi\rangle^{(-1)}$, each of them being realized with the relative probability of $1/2$. Taking into account Eqs. (2.1) and (2.2), we can find the elements of the spin density matrix of the $(\mu^+\mu^-)$ system in the representation of triplet states $|+1\rangle$, $|0\rangle$ and $|-1\rangle$ [3].

The spin states of two particles with spin 1/2 are characterized by the polarization vectors $\vec{\zeta}_1 = \langle\hat{\vec{\sigma}}^{(1)}\rangle$, $\vec{\zeta}_2 = \langle\hat{\vec{\sigma}}^{(2)}\rangle$ and the components of the correlation tensor $T_{ik} = \langle\hat{\sigma}_i^{(1)}\otimes\hat{\sigma}_k^{(2)}\rangle$. Here $\hat{\vec{\sigma}} = \{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ is the vector Pauli operator, $\hat{\sigma}_i$, $\hat{\sigma}_k$ are the Pauli matrices, $i,k \to \{1,2,3\} \to \{x,y,z\}$; the axis z is directed along the momentum of the positively charged muon μ^+ in the c.m. frame of the considered reaction, and the axis y is directed along the normal to the reaction plane; the symbol $\langle...\rangle$ denotes the averaging over the quantum ensemble. If both the particles are not polarized and the correlations are absent, then $T_{ik} = 0$. For two independent particles with the polarization vectors $\vec{\zeta}_1$ and $\vec{\zeta}_2$ the correlation tensor is factorized: $T_{ik} = \zeta_i\zeta_k$.

It is easy to see that, at the annihilation $e^+e^- \to \mu^+\mu^-$ of the unpolarized positron and electron, the produced muons μ^+ and μ^- are unpolarized ($\vec{\zeta}_{\mu^+} = \vec{\zeta}_{\mu^-} = 0$), but their spins are correlated: the correlation tensor components have the following form (see also [5]):

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2-\beta_\mu^2)\sin^2\theta}{2-\beta_\mu^2\sin^2\theta}, \quad T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2\sin^2\theta}{2-\beta_\mu^2\sin^2\theta}, \quad T_{zz}^{(\mu^+\mu^-)} = \frac{2\cos^2\theta+\beta_\mu^2\sin^2\theta}{2-\beta_\mu^2\sin^2\theta},$$

$$T_{xz}^{(\mu^+\mu^-)} = -\frac{(1-\beta_\mu^2)^{1/2}\sin 2\theta}{2-\beta_\mu^2\sin^2\theta}, \quad T_{xy}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = 0. \tag{3.1}$$

The "trace" of the correlation tensor of the $(\mu^+\mu^-)$ pair is:

$$T^{(\mu^+\mu^-)} = \langle\hat{\vec{\sigma}}_{\mu^+}\otimes\hat{\vec{\sigma}}_{\mu^-}\rangle = T_{xx}^{(\mu^+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1, \tag{3.2}$$

just as it should hold for any triplet state [1)].

4. Angular correlations at the joint registration of decays of the final muons μ^+ and μ^-

The "trace" of the correlation tensor T determines the angular correlation between flight directions for the products of decay of two unstable particles with spin $1/2$ in the case when space parity is not conserved [4-8].

[1)] For the singlet state $T = -3$; in the general case, $T = \rho_t - 3\rho_s$, where ρ_t and ρ_s are the fractions of the triplet and singlet state, respectively [4-6].

Actually, the angular distribution at the decay of any polarized unstable particle with spin $1/2$ under space parity nonconservation, normalized by unity, has the form (see, for example, [9]):

$$dW = \frac{1}{4\pi}(1+\alpha\vec{\zeta}\,\mathbf{n})d\Omega_{\mathbf{n}},$$

where $\vec{\zeta}$ is the polarization vector of the unstable particle, α is the angular asymmetry coefficient, $\mathbf{n}$ is the unit vector along the momentum of the particle, formed in the decay, in the rest frame of the decaying unstable particle.

Then the double distribution for the flight directions of the decay products of two unstable particles under space parity nonconservation, normalized by unity, is as follows [4–5]:

$$d^2W = \frac{1}{16\pi^2}(1+\alpha_1\vec{\zeta_1}\,\mathbf{n_1}+\alpha_2\vec{\zeta_2}\,\mathbf{n_2}+\alpha_1\alpha_2\sum_{i=1}^{3}\sum_{k=1}^{3}T_{ik}\,n_{1,i}\,n_{2,k})d\Omega_{\mathbf{n}_1}\,d\Omega_{\mathbf{n}_2}. \tag{4.1}$$

Here $\vec{\zeta_1}$ and $\vec{\zeta_2}$ are the polarization vectors of the first and second unstable particle, α_1 and α_2 are the coefficients of angular asymmetry for the decays of the first and second particle; $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors defined in the rest frames of the first and second unstable particle, respectively, and specified with respect to a unified system of spatial coordinate axes [7, 8]; just as before, $i,k \to \{1,2,3\} \to \{x,y,z\}$.

Using the method of moments, the components of the polarization vectors and correlation tensor can be found as a result of averaging the corresponding combinations of trigonometric functions of angles over the double distribution of decay directions [4–5].

The integration of the double distribution of flight directions over all angles, except the angle δ between the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, leads to the following formula for the angular correlation [4–5]:

$$dW = \frac{1}{2}\left(1+\frac{\alpha_1\alpha_2 T}{3}\cos\delta\right)d(-\cos\delta); \quad \cos\delta = \mathbf{n}_1\mathbf{n}_2. \tag{4.2}$$

Let us apply Eq. (4.2) to the decays of the muons μ^+ and μ^- produced in the process of electron-positron pair annihilation $e^+e^- \to \mu^+\mu^-$. According to Eq. (3.2), in this case the "trace" of the correlation tensor of the muon pair is equal to unity ($T=1$). It is known that the asymmetry coefficient in the angular distribution of electrons at the decay of the polarized negatively charged muon $\mu^- \to e^-\nu_\mu\bar{\nu}_e$, integrated over the electron energy spectrum, equals $-1/3$ $(\alpha_1 = -1/3)$ [9]. Due to the CP-invariance, the asymmetry coefficient in the angular distribution of positrons at the decay of the polarized positively charged muon $\mu^+ \to e^+\bar{\nu}_\mu\nu_e$, integrated over the positron energy spectrum, amounts to $+1/3$ $(\alpha_2 = +1/3)$. As a result, we obtain the following formula for the angular correlation at the decays $\mu^- \to e^-\nu_\mu\bar{\nu}_e$ and $\mu^+ \to e^+\bar{\nu}_\mu\nu_e$:

$$dW^{(\mu^+\mu^-)} = \frac{1}{2}\left(1-\frac{1}{27}\cos\delta\right)d(-\cos\delta). \tag{4.3}$$

5. Coherent properties of the correlation tensor and violation of "classical" incoherence inequalities in the process $e^+e^- \to \mu^+\mu^-$

Previously it was shown in the papers [4–5] that in the case of incoherent mixtures of factorizable states of two particles with spin $1/2$ the modulus of the sum of any two (and three) diagonal components of the correlation tensor cannot exceed unity, i.e. the following inequalities hold:

$$|T_{xx}+T_{yy}| \leq 1, \quad |T_{xx}+T_{zz}| \leq 1, \quad |T_{yy}+T_{zz}| \leq 1, \quad |T| = |T_{xx}+T_{yy}+T_{zz}| \leq 1.$$

However, for nonfactorizable (entangled) states some of these inequalities may be violated.

In the process of annihilation of the unpolarized positron and electron $e^+e^- \to \mu^+\mu^-$, the muon pair is produced in the nonfactorizable two-particle quantum states $|\Psi\rangle^{(+1)}$ and $|\Psi\rangle^{(-1)}$ (see Eqs. (2.1) and (2.2)). In so doing, one of the incoherence inequalities is violated: indeed, using Eqs. (3.1), we obtain at the angle $\theta \neq 0$:

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1 - T_{yy}^{(\mu^+\mu^-)} = \frac{2}{2-\beta_\mu^2 \sin^2\theta} > 1. \tag{5.1}$$

Thus, we see that the spin correlations of muons in the process $e^+e^- \to \mu^+\mu^-$ have the strongly pronounced quantum character.

Certainly, the above consideration can be wholly applied also to the annihilation process $e^+e^- \to \tau^+\tau^-$ with the generation of a tau-lepton pair (which becomes possible at substantially higher energies) – following the replacements $m_\mu \to m_\tau$, $\beta_\mu \to \beta_\tau$.

Finally, let us remark that at ultrarelativistic energies $E \gg m_\mu$ (m_τ), when $\beta_\mu, \beta_\tau \to 1$, the nonzero components of the correlation tensor for the final lepton pair take – in accordance with Eqs. (3.1) – the following values:

$$T_{xx} = \frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{yy} = -\frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{zz} = 1, \tag{5.2}$$

and we see that one of the incoherence inequalities is still violated : $T_{xx} + T_{zz} \geq 1$.

6. Incorporation of the weak interaction of lepton neutral currents through the virtual Z^0 boson

At very high energies the annihilation processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ are conditioned not only by the electromagnetic interaction through the virtual photon, but also by the weak interaction of neutral currents through the Z^0 boson [9].

The interference of amplitudes of the purely electromagnetic and weak interaction leads to the charge asymmetry in lepton emission and to the effects of space parity violation. In the framework of the standard model of electroweak interaction, at the electron-positron pair annihilation the pairs $\mu^+\mu^-$, $\tau^+\tau^-$ are produced in the states ${}^3S_1$, ${}^3D_1$ with the negative space parity and, due to the weak interaction, also in the state ${}^3P_1$ with the positive space parity. In doing so, the total angular momentum is $J=1$ and CP-parity of the pairs is positive.

If the weak interaction contribution is neglected, then the lepton pairs, generated at the annihilation of the unpolarized positron and electron, are correlated but unpolarized. Analysis shows that, due to the weak interaction through the exchange by the virtual Z^0 boson with the nonconservation of space parity, the final leptons acquire the longitudinal polarization. Since the lepton pairs are produced in the triplet states, the polarization vectors of the positively and negatively charged leptons are the same, and their average helicities $\lambda_+ = -\lambda_-$ have different signs in consequence of the opposite directions of momenta in the c.m. frame [3].

The structure of the correlation tensor of the final leptons is, on the whole, similar to that for the case of purely electromagnetic annihilation at very high energies ($\beta_\mu \to 1, \beta_\tau \to 1$). In doing so, the nonzero components of the correlation tensor are: $T_{zz} = 1$, $T_{xx} = -T_{yy}$, as before. Again one of the incoherence inequalities for the correlation tensor components is violated: $T_{xx} + T_{zz} > 1$ (see [3]).

Thus, the consequences of the quantum-mechanical coherence for two-particle quantum systems with nonfactorizable internal states manifest themselves very distinctly in spin correlations of lepton pairs produced in the annihilation processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$, and they can be verified experimentally (see also our papers [3], [10 – 12]) .

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