



# Holographic inflation

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## ABSTRACT

We apply the holographic principle at the early universe, obtaining an inflation realization of holographic origin. Such a consideration has equal footing with its well-studied late-time application, and moreover the decrease of the horizons at early times naturally increases holographic energy density at inflationary scales. Taking as Infrared cutoff the particle or future event horizons, and adding a simple correction due to the Ultraviolet cutoff, whose role is non-negligible at the high energy scales of inflation, we result in a holographic inflation scenario that is very efficient in incorporating inflationary requirements and predictions. We first extract analytically the solution of the Hubble function in an implicit form, which gives a scale factor evolution of the desired e-foldings. Furthermore, we analytically calculate the Hubble slow-roll parameters and then the inflation-related observables, such as the scalar spectral index and its running, the tensor-to-scalar ratio, and the tensor spectral index. Confronting the predictions with Planck 2018 observations we show that the agreement is perfect and in particular deep inside the  $1\sigma$  region.

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## 1. Introduction

The holographic principle originates from black hole thermodynamics and string theory [1–4], and establishes a connection of the Ultraviolet cutoff of a quantum field theory, which is related to the vacuum energy, with the largest distance of this theory, which is related to causality and the quantum field theory applicability at large distances [5]. This consideration has been applied extensively at a cosmological framework at late times, in which case the obtained vacuum energy constitutes a dark energy sector of holographic origin, called holographic dark energy [6] (for a review see [7]). In particular, the holographic energy density is proportional to the inverse squared Infrared cutoff  $L_{\text{IR}}$ , which since is related to causality it must be a form of horizon, namely

$$\rho = \frac{3c^2}{\kappa^2 L_{\text{IR}}^2}, \quad (1)$$

with  $\kappa^2$  the gravitational constant and  $c$  a parameter. Holographic dark energy proves to have interesting phenomenology, both in its basic [6–14] as well as in its various extensions [15–22], and it can fit observations [23–29].

Despite the extended research on the application of holographic principle in late-time cosmology and dark-energy epoch, there has not been any attempt in applying it at early universe, namely to obtain an inflationary realization of holographic origin. Nevertheless, such consideration has equal footing with its late-time application, and moreover, observing the form of (1), we deduce that since at early times the largest distance is small, the holographic energy density is naturally suitably large in order to lie in the inflationary scale.

In the present Letter we are interested in investigating holographic inflation, namely to acquire a successful inflation triggered by the energy density of holographic origin. Since the involved energy scales at this epoch are high, we should additionally consider a correction coming from the Ultraviolet cutoff. As we show, although the basic scenario is very simple and natural, it can be very efficient and results to inflationary observables in perfect agreement with observations. Furthermore, as we show, one can extend the basic scenario to more subtle constructions.

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## 2. Holographic inflation

In this section we will construct the basic model of holographic inflation. We consider a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) geometry with metric

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right), \quad (2)$$

where  $a(t)$  is the scale factor and with  $k = 0, +1, -1$  corresponding respectively to flat, close and open spatial geometry. In this work for convenience we focus on the flat case, nevertheless the generalization to non-flat geometry is straightforward.

In a general inflation scenario the first Friedmann equations is written as

$$H^2 = \frac{\kappa^2}{3} \rho_{\text{inf}}, \quad (3)$$

where  $\rho_{\text{inf}}$  is the energy density of the (effective) fluid that drives inflation, which can originate from a scalar field, from modified-gravity, or from other sources and mechanisms. Note that as usual we have neglected the contributions from other components, such as the matter and radiations sectors.

In this work we consider that inflation has a holographic origin, namely that its source is the holographic energy density. Hence, imposing that  $\rho_{\text{inf}}$  is  $\rho$  of (1), the Friedmann equation (3) for an expanding universe becomes simply

$$H = \frac{c}{L_{\text{IR}}}. \quad (4)$$

As it is well-known,  $H^{-1}$  corresponds to the radius of the cosmological horizon which gives the limit of the causal relations from the viewpoint of the classical geometry, namely any information beyond the horizon cannot affect us at least at the time when we measure  $H$ . The original idea of the holographic energy paradigm comes by identifying the classical horizon radius with the infrared cutoff in the quantum field theory, since we neglect any contribution coming from energy scales smaller than the cutoff scale. This implies that we may relate, as we will see later, the horizon radius  $H^{-1}$  with the Infrared cutoff, since the information in energy scales larger than the Ultraviolet cutoff scale is irrelevant.

The simplest choice is that  $L_{\text{IR}}$  should be exactly the Hubble radius, which however cannot be used at late-times application since it cannot lead to an accelerating universe [30], and this is the case for the next guess, namely the particle horizon. Hence, one can use the future event horizon [6], the age of the universe or the conformal time [31,32], the inverse square root of the Ricci curvature [33], a combination of Ricci, Gauss-Bonnet [34] or other curvature invariants, or even consider a general Infrared cutoff as an arbitrary function of the above and their derivatives [35].

Contrary to the case of late-time application of holographic principle, namely the holographic dark energy, in the present inflationary application almost all the above choices can be successful in driving inflation, due to the absence of matter sector, apart from the simplest case of the Hubble radius which leads to a trivial result. In particular, we may consider the particle horizon  $L_p$  or the future event horizon  $L_f$ , which are given as

$$L_p \equiv a \int_0^t \frac{dt}{a}, \quad L_f \equiv a \int_t^\infty \frac{dt}{a}. \quad (5)$$

Inserting these into (4) we obtain

$$\frac{d}{dt} \left( \frac{c}{aH} \right) = \frac{m}{a}, \quad (6)$$

where  $m = 1$  corresponds to the particle horizon and  $m = -1$  to the future event horizon. In the second case, and for  $c = 1$ , we immediately extract the de Sitter solution

$$a = a_0 e^{H_0 t}, \quad (7)$$

with  $a_0, H_0$  the two integration constants. Hence, as we observe, the basic inflationary feature can be straightforwardly obtained.

Since we investigate the application of holographic principle at early times, and thus at high energy scales, apart from the Infrared cutoff we should consider the effects of the Ultraviolet cutoff  $\Lambda_{\text{UV}}$  too. In particular, at this regime the quantum effects become important, and thus the Infrared cutoff acquires a correction by the Ultraviolet one, which as was shown in [36] takes the simple form

$$L \equiv \sqrt{L_{\text{IR}}^2 + \frac{1}{\Lambda_{\text{UV}}^2}}. \quad (8)$$

Therefore, inserting this corrected expression into (4), with  $L_{\text{IR}}$  being either the particle horizon  $L_p$  or the future event horizon  $L_f$ , we obtain

$$\frac{m}{a} = \frac{d}{dt} \left( \frac{1}{a} \sqrt{\frac{c^2}{H^2} - \frac{1}{\Lambda_{\text{UV}}^2}} \right), \quad (9)$$

or equivalently

$$\dot{H} = -\frac{H^3}{c^2} \left\{ m \sqrt{\frac{c^2}{H^2} - \frac{1}{\Lambda_{\text{UV}}^2}} + H \left( \frac{c^2}{H^2} - \frac{1}{\Lambda_{\text{UV}}^2} \right) \right\}. \quad (10)$$

As we will soon see, the above simple modification caused by  $\Lambda_{\text{UV}}$  has a crucial effect in the inflation realization, namely it can cause a successful exit from the de-Sitter solution (7) obtained for  $\Lambda_{\text{UV}} \rightarrow \infty$ , and most importantly it can lead to inflationary predictions in perfect agreement with observations.

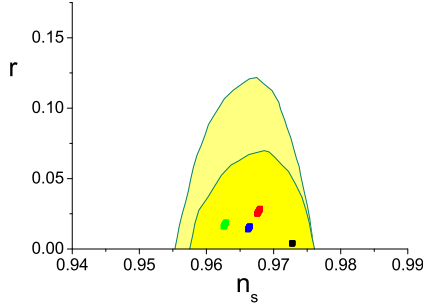
The general solution of (10), for  $\Lambda_{\text{UV}}$  not being equal to 0 or infinity, can be written in an implicit form as

$$\begin{aligned} & \frac{\sqrt{c^2 - \frac{H^2}{\Lambda_{\text{UV}}^2}}}{(c^2 - 1)^{\frac{3}{2}} \Lambda_{\text{UV}}^2 \sqrt{H^2 - c^2 \Lambda_{\text{UV}}^2}} \tan^{-1} \left[ \frac{mH}{\sqrt{c^2 - 1} \sqrt{H^2 - c^2 \Lambda_{\text{UV}}^2}} \right] \\ & + \frac{m \sqrt{c^2 - \frac{H^2}{\Lambda_{\text{UV}}^2}} - c^2}{c^2 (c^2 - 1) H \Lambda_{\text{UV}}^2} + \frac{\tanh^{-1} \left[ \frac{H}{\sqrt{c^2 - 1} \Lambda_{\text{UV}}} \right]}{(c^2 - 1)^{\frac{3}{2}} \Lambda_{\text{UV}}^3} = -\frac{t}{c^2 \Lambda_{\text{UV}}^2} + C_0, \end{aligned} \quad (11)$$

whose integration provides the scale factor evolution. Note that the above solution is real and well behaved for all  $c > 0$ . Hence, since the evolution of  $H(t)$  is known, we can straightforwardly obtain the Hubble slow-roll parameters  $\epsilon_n$  (with  $n$  positive integer), defined as [37–40]

$$\epsilon_{n+1} \equiv \frac{d \ln |\epsilon_n|}{dN}, \quad (12)$$

with  $\epsilon_0 \equiv H_{\text{ini}}/H$  and  $N \equiv \ln(a/a_{\text{ini}})$  the e-folding number, and where  $a_{\text{ini}}$  is the scale factor at the beginning of inflation and  $H_{\text{ini}}$  the corresponding Hubble parameter (inflation ends when  $\epsilon_1 = 1$ ). Thus, we can calculate the values of the inflationary observables, namely the scalar spectral index of the curvature perturbations  $n_s$ , its running  $\alpha_s \equiv dn_s/d \ln k$  with  $k$  the absolute value of the wave number  $\vec{k}$ , the tensor spectral index  $n_T$  and the tensor-to-scalar ratio  $r$ , as [39]



**Fig. 1.**  $1\sigma$  (yellow) and  $2\sigma$  (light yellow) contours for Planck 2018 results (Planck +TT +lowP) [42], on  $n_s - r$  plane. Additionally, we present the predictions of holographic inflation for  $m = -1$  (i.e. the future event horizon is used), with  $c = 1.007$ ,  $\Lambda_{UV} = 20$  (black points),  $c = 1.009$ ,  $\Lambda_{UV} = 19$  (red points),  $c = 1.009$ ,  $\Lambda_{UV} = 18$  (blue points), and  $c = 1.01$ ,  $\Lambda_{UV} = 18$  (green points), in units where  $\kappa^2 = 1$ , for e-folding number between  $N = 50$  and  $N = 60$  (for some of the cases the results for  $N$  varying between 50 and 60 cannot be distinguished at the resolution scale of the figure).

$$r \approx 16\epsilon_1, \quad (13)$$

$$n_s \approx 1 - 2\epsilon_1 - 2\epsilon_2, \quad (14)$$

$$\alpha_s \approx -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3, \quad (15)$$

$$n_T \approx -2\epsilon_1, \quad (16)$$

where the first three  $\epsilon_n$  are straightforwardly extracted from (12) to be

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}, \quad (17)$$

$$\epsilon_2 \equiv \frac{\ddot{H}}{H\dot{H}} - \frac{2\dot{H}}{H^2}, \quad (18)$$

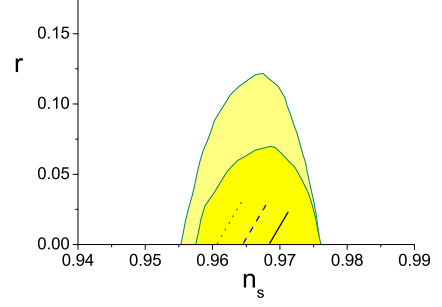
$$\epsilon_3 \equiv \left(\ddot{H}H - 2\dot{H}^2\right)^{-1} \cdot \left[ \frac{H\dot{H}\ddot{H} - \ddot{H}(\dot{H}^2 + H\ddot{H})}{H\dot{H}} - \frac{2\dot{H}}{H^2}(H\ddot{H} - 2\dot{H}^2) \right]. \quad (19)$$

We mention here that in principle in order to extract the exact expressions for the inflationary observables we should perform a full perturbation analysis for this specific holographic model, based on the detailed perturbation analysis of holographic dark energy performed in [41]. Nevertheless, for the purpose of the present work the approximate relations (13)-(16), that hold for every scenario as long as  $H(t)$  is known [39], are adequate in order to provide the approximate values for the inflationary observables.

Relations (13)-(16) are very useful, since they allow for a comparison of the predictions of holographic inflation with observations. In Fig. 1 we present the estimated tensor-to-scalar ratio of the specific scenario for four parameter choices and for e-folding numbers varying between  $N = 50$  and  $N = 60$ , on top of the  $1\sigma$  and  $2\sigma$  contours of the Planck 2018 results [42]. As we observe, the agreement with observations is very efficient, and in particular well inside the  $1\sigma$  region.

In the scenario of holographic inflation examined above, the differential equation (10) allows to eliminate the time-derivatives of  $H$  in terms of  $H$  in (17)-(19), and therefore extract analytical and quite simple expressions for the inflationary observables. In particular, doing so we find that

$$r = 16 \left( 1 - \frac{H^2}{c^2 \Lambda_{UV}^2} + \frac{m \sqrt{c^2 - \frac{H^2}{\Lambda_{UV}^2}}}{c^2} \right), \quad (20)$$



**Fig. 2.**  $1\sigma$  (yellow) and  $2\sigma$  (light yellow) contours for Planck 2018 results (Planck +TT +lowP) [42], on  $n_s - r$  plane. Additionally, we present the analytical predictions of holographic inflation, namely the parametric curve (24), for  $m = -1$  and  $q = -1$ , and for  $c = 1.008$  (black-solid curve), for  $c = 1.009$  (blue-dashed curve), and for  $c = 1.010$  (red-dotted curve).  $\Lambda_{UV}$  does not explicitly determine the form of the curves, however since it affects the duration of inflation and the e-folding number it implicitly determines their bounds (we have considered  $\Lambda_{UV}$  to vary between 1 and 100 in units where  $\kappa^2 = 1$  and the e-folding number to vary between  $N = 50$  and  $N = 60$ ).

$$n_s = -1 - \frac{2m}{\sqrt{c^2 - \frac{H^2}{\Lambda_{UV}^2}}} - \frac{2H^2}{c^2 \Lambda_{UV}^2}, \quad (21)$$

$$\alpha_s = \frac{mH^4 \left( m + \sqrt{c^2 - \frac{H^2}{\Lambda_{UV}^2}} \right)}{c^4 \Lambda_{UV}^2 (c^2 \Lambda_{UV}^2 - H^2)}, \quad (22)$$

$$n_T = -\frac{r}{8}. \quad (23)$$

Hence, we can now eliminate  $\frac{H^2}{\Lambda_{UV}^2}$  between (20), (21) and between (20), (22), and obtain the relation of  $n_s$  and  $\alpha_s$  in terms of  $r$ , namely

$$n_s = -3 + \frac{r}{8} + \frac{2 - l\sqrt{c^2 r + 4}}{2c^2} - \frac{8m}{\sqrt{c^2 r + 4} - 4l\sqrt{c^2 r + 4}}, \quad (24)$$

and

$$\alpha_s = \frac{m \left[ 8 + c^2(r - 16) - 4q\sqrt{c^2 r + 4} \right]^2}{64c^4 \left( 8 + c^2 r - 4q\sqrt{c^2 r + 4} \right)} \cdot \left( 4m + \sqrt{8 + c^2 r - 4q\sqrt{c^2 r + 4}} \right), \quad (25)$$

where  $l = \pm 1$  corresponds to two solutions branches. Interestingly enough, we observe that as long as  $\Lambda_{UV}$  is finite and non-zero it does not appear in the above relations (24) and (25), although its effect was crucial in generating a new solution branch that did not exist in the case where  $\Lambda_{UV}$  in (8) was absent (namely when  $\Lambda_{UV} \rightarrow \infty$ , i.e. when there is no Ultraviolet cutoff). Of course it does appear in the solution for  $H$  (relation (11)), and hence along with  $c$  it determines the duration of inflation and the e-folding number, and thus the bounds of the above parametric curves.

However, the most significant feature is that relation (24) leads to  $r$  and  $n_s$  values in perfect agreement with observations, as long as one chooses suitably the holographic parameter  $c$ . In particular, one can deduce that the most interesting case is when  $m = -1$  (i.e. the future event horizon is used) and when  $q = -1$ . In this case, as one can see from Fig. 2, with  $c$  slightly larger than 1 we can obtain a  $n_s$  inside its observational bounds and  $r$  adequately small. Finally, calculations of  $\alpha_s$  using (25) result to typical values of the order of  $10^{-7}$ , and thus well inside the observational bounds [42]. These are the main result of the present work and reveal the capabilities

of holographic inflation, since it is well-known that the majority of inflationary models cannot result to adequately small  $r$ .

We close this analysis by making a comment on reheating, which is a necessary phase that should follow the inflationary one. In our model, the reheating can be obtained in a similar way to Starobinsky  $R^2$  inflation, namely driven by scalaron (composite) degree of freedom. Hence, with the addition of this mechanism, we can obtain the transition to the post-inflationary thermal history of the universe.

### 3. Generalized scenarios

In this section we apply the holographic principle at early times, however considering extended Infrared cutoffs. Such extensions have been applied in the late-time universe, resulting in generalized holographic dark energy [35]. In particular, in these holographic constructions one considers a general Infrared cut-off  $L_{\text{IR}}$ , which could be a function of both  $L_p$  and  $L_f$  [14] and their derivatives, or additionally of the Hubble horizon and its derivatives as well as of the scale factor [35], namely

$$L_{\text{IR}} = L_{\text{IR}}(L_p, \dot{L}_p, \ddot{L}_p, \dots, L_f, \dot{L}_f, \ddot{L}_f, \dots, a, H, \dot{H}, \ddot{H}, \dots). \quad (26)$$

Hence, applying the above general Infrared cutoff at the early universe gives as enhanced freedom to obtain a successful inflationary realization.

Without loss of generality we consider the following specific example. We start by considering an Infrared cutoff of the form

$$L_{\text{IR}} = -\frac{1}{6\alpha\dot{H}^2 a^6} \int^t dt a^6 \dot{H}, \quad (27)$$

with  $\alpha$  the model parameter. Inserting this expression into the inflationary Friedmann equation (4) and taking  $c = 1$  for simplicity, leads to the differential equation

$$3H^2 = \alpha \left( -108H^2\dot{H} + 18\dot{H}^2 - 36H\ddot{H} \right). \quad (28)$$

Having in mind that the Ricci scalar in FRW geometry is just  $R = 6(2H^2 + \dot{H})$ , the above differential equation can be re-written in the form

$$\frac{F(R)}{2} = 3 \left( H^2 + \dot{H} \right) F'(R) - 18 \left( 4H^2\dot{H} + H\ddot{H} \right) F''(R), \quad (29)$$

with  $F(R) = R + \alpha R^2$ . Equation (29) is just the first Friedmann equation of the  $R^2$ -gravity [43–46] (see [47,48] for reviews in  $F(R)$  gravity) in the absence of matter. Therefore, the scenario of holographic inflation, under the generalized Infrared cutoff (27) can reproduce Starobinsky  $R^2$  inflation [43], which is known to lead to inflationary observables in a very good agreement with observations [42].

In similar lines, one can consider other generalized Infrared cutoffs in order to obtain a correspondence with other geometrical inflationary models, such as Gauss-Bonnet and  $f(G)$  inflation [49],  $f(T)$  inflation, etc. These capabilities act as an additional advantage in favour of generalized holographic inflation (see also [50]).

### 4. Conclusions

In this work we applied the holographic principle at the early universe, obtaining an inflation realization of holographic origin. Although holographic energy density has been well studied at late times, giving rise to holographic dark energy, up to now it had not been incorporated at early times, although such consideration has equal footing, and moreover despite the fact that the decrease of

the horizons at early times naturally increases holographic energy density at inflationary scales.

Taking as Infrared cutoff the particle or future event horizons, and adding a simple correction due to the Ultraviolet cutoff, whose role is non-negligible at the high energy scales of inflation, we resulted in a holographic inflation scenario that is very efficient in incorporating inflationary requirements and predictions. In particular, we first extracted analytically the solution of the Hubble function in an implicit form, which can give a scale factor evolution of the desired e-foldings.

Furthermore, we analytically calculated the Hubble slow-roll parameters and then the inflation-related observables, such as the scalar spectral index and its running, the tensor-to-scalar ratio, and the tensor spectral index, which were found to follow simple expressions. Confronting the predictions with Planck 2018 observations, we showed that the agreement is perfect, and in particular deep inside the  $1\sigma$  region. Additionally, we found that with  $n_s$  being inside its observational bounds  $r$  can be adequately small. These are the main result of the present work and reveal the capabilities of holographic inflation, since it is well-known that the majority of inflationary models cannot lead to adequately small  $r$ .

Finally, we constructed generalizations of holographic inflation which are based on extended Infrared cutoffs. Under these considerations we showed that we can reconstruct Starobinsky inflation, which is also known to be in a very good agreement with the data, as well as obtain a correspondence with other inflationary scenarios of geometrical origin. In summary, holographic inflation proves to have a very interesting phenomenology, and thus it is a good candidate for the description of the early universe.

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