# DYNAMICS OF STRONG INTERACTIONS

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| Chairman | G.F. CHEW |
| Rapporteur | W.R. FRAZER |
| Discussion Leader | M. TOLLER |
| Secretaries | R. JENGO<br>W. RÜHL |

# STRONG INTERACTION DYNAMICS

**W.R. Frazer**

University of California, San Diego, La Jolla, Calif.

## 1. FINITE-ENERGY SUM RULE BOOTSTRAPS

A very exciting "new bootstrap" scheme is provided by the finite-energy sum rules (FESR), which have already shown their utility in another session of this Conference, where they were used to determine parameters for high-energy Regge-pole fits from low-energy data. This type of work was pioneered by Igi[1] in 1962, but the current wave of activity came about through an unusually tortuous route. Motivated by current algebra (which turned out to be irrelevant) de Alfaro, Fubini, Furlan and Rossetti[2] formulated superconvergence relations. They observed that if an amplitude $A(\nu,t)$, where $\nu = (s-u)/4$, satisfies a dispersion relation

$$\frac{1}{\pi}\int_{-\infty}^{\infty} d\nu' \,\frac{\mathrm{Im}\, A(\nu', t)}{\nu' - \nu} = A(\nu,t) \tag{1}$$

and if, moreover, $A(\nu,t)$ falls off faster than $1/\nu$ as $\nu \to \infty$, then it must be true that

$$\int_{-\infty}^{\infty} d\nu \;\mathrm{Im}\, A(\nu,t) = 0 \tag{2}$$

This is a superconvergence relation. At the time of the Berkeley Conference two years ago, attempts were being made to extract dynamics from superconvergence relations by saturating the integrals with resonance contributions. Some success was achieved, but the results were uneven.

The improvement over superconvergence relations which made possible the work I am going to describe was made independently by several groups[3-6]. They followed Igi's technique of subtracting from $A(\nu,t)$ a function $R(\nu,t)$ which is asymptotically equal to $A(\nu,t)$, and which is usually taken to be a sum of Regge-pole terms. By including enough Regge poles in R it is then possible to write a superconvergence relation for any scattering amplitude of the form

$$\int d\nu \;\mathrm{Im}\, \left[A(\nu,t) - R(\nu,t)\right] = 0\,. \tag{3}$$

If one then divides the region of integration into the parts $\nu < N$ and $\nu > N$ such that the amplitude is well-represented by its Regge expansion for $\nu > N$, one obtains the famous FESR:

$$\int_0^N d\nu\, \nu^n \;\mathrm{Im}\, A(\nu,t) = \sum_r \beta_r(t)\, \frac{N^{\alpha_r(t)+n+1}}{\alpha_r(t)+n+1}\,. \tag{4}$$

Here we have taken the high-energy form to be

$$\mathrm{Im}\, A(\nu,t) \sim \sum_r \beta_r(t)\, \nu^{\alpha_r(t)}\,, \tag{5}$$

although in principle branch points as well as poles can be included. We have also generalized the sum rule by taking the $n^{\text{th}}$ moment and by using a crossing-symmetric amplitude.

Now a bootstrap equation results if the left-hand side of Eq. (4) can be saturated by resonance contributions, and if the same resonances give rise to the Regge terms on the right-hand side. Although the pioneering works already cited contain the suggestion of bootstrap possibilities, the first elaborating of these possibilities was made by Mandelstam[7], followed closely by several authors[8-10]. An example, which we shall consider in detail in a moment, is the reaction $\pi\pi \to \pi\omega$, where a reasonable hypothesis is to saturate both sides by the $\rho$ contribution.

A delicate point arises here: where should one choose the cut-off N? If it is very large, then the Regge-pole approximation should be valid and the use of the FESR would provoke no controversy. But like most ideas that provoke no controversy, it would not be very interesting. Saturation of the integral by a small number of resonances would not be likely to produce a reasonable approximation. Therefore the cut-off is usually chosen quite low -- 1 to 2 GeV.

Such a low cut-off means that we are using the Regge representation in an energy region where no one would have had the temerity to apply it a few years ago. In fact, it means using the Regge representation in a region where there are still resonances. Now our FESR [Eq. (4)] tells us that in fact the extrapolated high-energy fit should indeed reproduce the average behaviour of the amplitude in the resonance region, where the average is taken from threshold to N. Dolen, Horn and Schmid[3]) verified this phenomenon in pion-nucleon scattering, but went on to make the more striking hypothesis now known as the Dolen-Horn-Schmid duality: the Regge-pole fit to the high-energy data is equal to a local average of the scattering amplitude in the low-energy region. The term "local average" is intentionally vague, but qualitatively it means that the resonating low-energy amplitude oscillates about the tail of the Regge term, as in the example shown in Fig. 1.

We have heard in other sessions about the repercussions of this idea on data fitting in the intermediate energy region and even on the concept of a

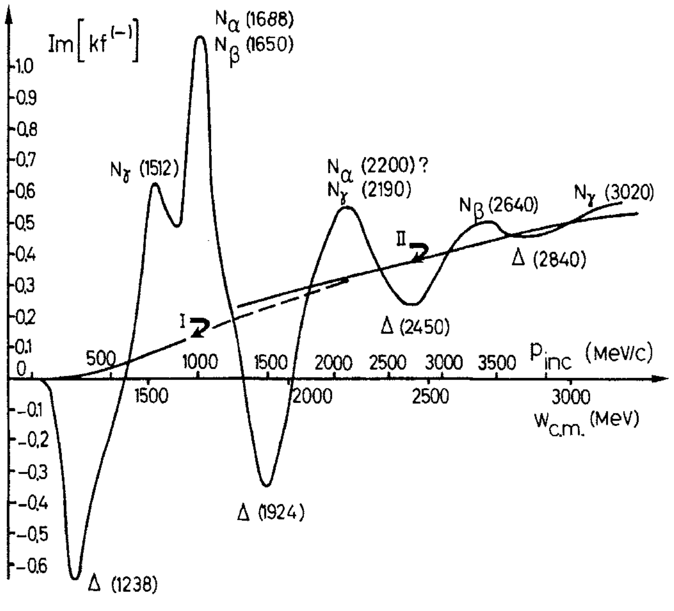


Fig. 1 Comparison of extrapolated Regge pole fit to high-energy region with actual low-energy amplitude for forward pion-nucleon scattering [from C. Chiu and A. Stirling, Phys. Letters 26B, 236 (1968)].

resonance. If it is true that a whole sequence of s-channel resonances can be approximately described by a t-channel Regge pole, an exciting advance has been made in our understanding of the S-matrix.

It is most important to find out to what extent the duality is true. I am unable to present you a careful compilation of evidence made by an objective, dispassionate investigator, because I have not yet located the said investigator. The evidence I have seen looks encouraging, but I stress the importance of establishing the limits of validity of the duality hypothesis.

Why do we expect the duality on theoretical grounds? Schmid argues that the validity of a set of several higher-moment FESR's [Eq. (4)] requires some sort of local average equality. But the fact that the FESR is only an approximate equality makes this argument difficult to quantify.

Now let us get back to our subject of bootstrap dynamics by looking in some detail at the $\pi\pi \to \pi\omega$ system, which Ademollo, Rubinstein, Veneziano and Virasoro[11]) have analysed so successfully. This reaction is a particularly felicitous choice because it is described by a single invariant amplitude A, symmetric in s, t, and u. The only possible resonances have I = 1, G = +, normal parity, and odd J. Among reasonably well-known particles there are only the $\rho$ and the G(1650), which is probably $3^-$, and lies on the $\rho$ trajectory. So we assume, for the moment, a single Regge pole on the right-hand side of the FESR, which takes the form

$$\int_0^N \nu^n \, \mathrm{Im}\, A(\nu,t)\, d\nu = \frac{\beta(t)}{\alpha(t)+N}\left(\frac{N}{\nu_1}\right)^{\alpha(t)+n} \nu_1^{\,n+1} , \quad (6)$$

where $\nu_1$ is the usual arbitrary scale factor, and $\nu = (s-u)/4$. The authors take

$$\beta(t) = \frac{\bar{\beta}(t)}{\Gamma[\alpha(t)]} , \quad (7)$$

in agreement with usual Reggeization procedure, but then make the arbitrary simplifying assumption that $\bar{\beta}(t)$ is a constant. Moreover, the $\rho$ trajectory is assumed linear, $\alpha(t) = \alpha_0 + \alpha' t$. The left-hand side is assumed to be saturated by the $\rho$ (at this stage N is chosen to be below the G mass), whose contribution is calculated in the narrow-width approximation. It turns out, reassuringly, that the optimum choice for N is half-way between the last resonance included, the ρ, and the first one left out, the G. This turns out to be quite general, but I will not go into detail. Then evaluation of Eq. (6) yields

$$2m_\rho^2 - 3m_\pi^2 - m_\omega^2 + t = \frac{\alpha}{\alpha'}\,\Phi_1(\alpha)\left(1/2\nu_1\alpha'\right)^{\alpha-1}, \quad (8)$$

where

$$\Phi_1(\alpha) \equiv \left(\frac{\alpha+2}{2}\right)^{\alpha+1}\Gamma^{-1}(\alpha+2)\,. \quad (9)$$

The left-hand side has a zero at $t = m_\omega^2 - 2m_\rho^2 + 3m_\pi^2 = -0.53\ \mathrm{GeV}^2$, which implies a zero of α at this point, in good agreement with the value determined from the dip in πN charge exchange scattering! Imposing this condition on α reduces Eq. (8) to the form

$$1 = \Phi_1(\alpha)/(2\nu_1\alpha')^{\alpha-1}\,. \quad (10)$$

This equation is well satisfied for a wide range of t with the choice

$$2\nu_1\alpha' = 1\,. \quad (11)$$

With this choice the two sides of Eq. (8) agree, as shown in Fig. 2.

Now ARVV go on to check the stability of the bootstrap scheme under displacement of the cut-off N. They find that if one more resonance is included in the integral, the G, then the dip prediction is stable, and the equation is satisfied for an even wider range of t (Fig. 3). But then when they repeated this process to include still more resonances,

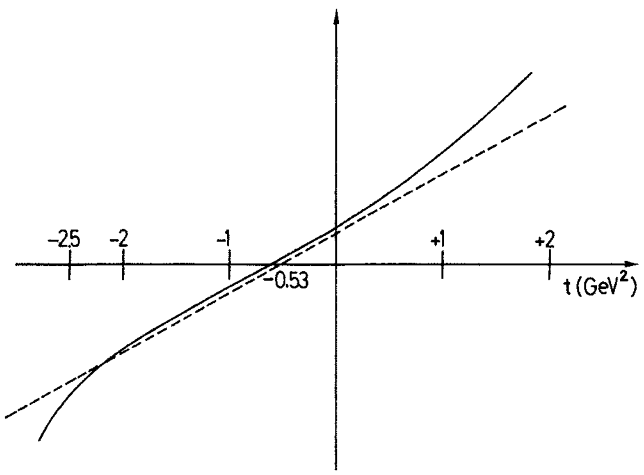


Fig. 2 Comparison of left- and right-hand sides of Eq. (8) (from Ref. 11).

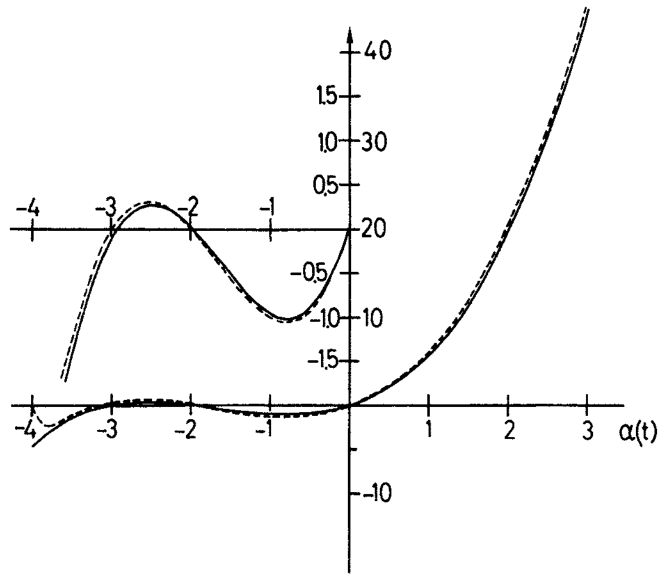


Fig. 3 Saturation of the same sum rule as in Fig. 2 with the ρ and its first Regge recurrence ($3^-$). In the upper left side the most important region is shown in larger scale (from Ref. 11).

they found the resonance side becoming smaller and smaller with respect to the Regge side. They could not achieve a bootstrap model in which one trajectory sustains itself.

The resolution of this dilemma was suggested by the Schmid[12] partial-wave projection of the Regge exchange. Every partial wave continues to circle as the energy increases, suggesting further resonances. Moreover, the masses of these resonances are such that they lie on parallel daughter trajectories with $\Delta J = 2$ spacing. These daughters were then included in their bootstrap by ARVV, and good results were obtained only if the daughter trajectories were taken parallel to the parent. If r resonances are included, the dip remains stable and the bootstrap condition takes the form

$$\Phi_r(\alpha) = 1\,, \quad (12)$$

where

$$\Phi_r(\alpha) \equiv \left(\frac{\alpha+4r-2}{2}\right)^{\alpha+1}\frac{\Gamma(2r-1)}{\Gamma(2r+\alpha)}\,. \quad (13)$$

This is a very amusing function, as you will see from Fig. 4, where Φ is shown. Furthermore, it has the property

$$\Phi_r(\alpha) \underset{r\to\infty}{\to} 1 \text{ for } \alpha \text{ fixed}\,, \quad (14)$$

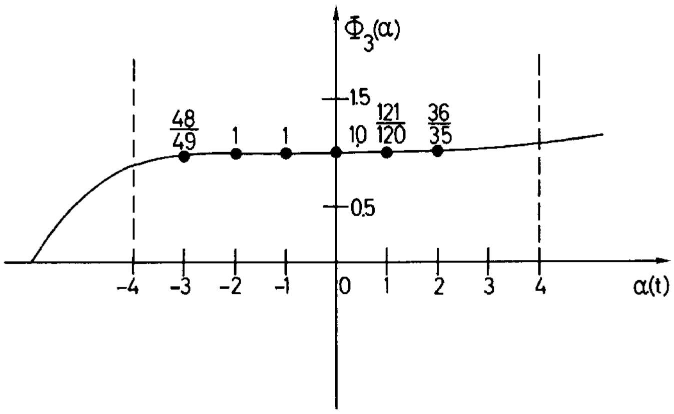


Fig. 4 The function $\Phi_3$ (from Ref. 11).

so a consistent bootstrap can be achieved as the cut-off is made large.

After all this work was done, Veneziano[13] invented a simple formula which summarizes almost all of it, and which answers many questions which sceptics of the duality question have been asking. He undertook to write down a representation of a scattering amplitude which has Regge asymptotic behaviour in all channels, and resonances in all channels. For $\pi\pi \to \pi\omega$ his representation is

$$A(s,t,u) = \frac{\bar{\beta}}{\pi}\left[\frac{\Gamma[1-\alpha(s)]\Gamma[1-\alpha(t)]}{\Gamma[2-\alpha(s)-\alpha(t)]} + (s \to u) + (t \to u)\right]. \tag{15}$$

Look at the first term. The s-channel resonances, which have zero width, are given by the poles of $\Gamma(1-\alpha_s)$. The $\Gamma$ function in the denominator is just right to make the residues of these poles polynomials in t. The polynomials are not Legendre polynomials, so daughters appear, but with unit spacing $\Delta J = 1$. To avoid these odd daughters Veneziano imposes the condition

$$\alpha(s) + \alpha(t) + \alpha(u) = 2 . \tag{16}$$

Assuming again a linear trajectory with $\alpha(m_\rho^2) = 1$, one finds that Eq. (16) is just the dip condition again, that $\alpha(m_\omega^2 - 2m_\rho^2 + 3m_\pi^2) = 0$!

From the relation

$$\frac{\Gamma(z+a)}{\Gamma(z+b)} \underset{z\to\infty}{\to} z^{a-b} , \tag{17}$$

the asymptotic behaviour of the first term in Eq. (15) can be found to be

$$\frac{\bar{\beta}}{\Gamma(\alpha_t)\sin\pi\alpha_t}\left[-\alpha(s)\right]^{\alpha(t)-1} . \tag{18}$$

Since $\alpha(s) = \alpha_0 + \alpha' s$ this is Regge behaviour $(\nu/\nu_1)^{\alpha_t - 1}$, with the scale factor $\nu_1 = 1/2\alpha'$ just as ARVV[11] found. Moreover, the residue contains the required factor $1/\Gamma(\alpha_t)$ and no further t-dependence.

Of course, the zero-width approximation is a shortcoming of the Veneziano representation. In fact, the asymptotic formulae I have used are not valid for $s \to \infty$ along the line of poles, and Regge behaviour holds asymptotically only for $\arg s \neq 0$. In order to obtain Regge behaviour on the real axis, the poles must be displaced; for example, by giving $\alpha(s)$ an imaginary part. If $\mathrm{Im}\,\alpha \to \infty$ one finds that Regge behaviour is approached after low-energy oscillations, similar to those in Fig. 1. Such modification of the Veneziano representation requires further study, however.

As a second example, Veneziano[13] applies his representation to the reaction $\pi\eta \to \pi\rho$, where it yields the result $\alpha'_\rho(0) = \alpha'_{A_2}(0)$, as well as the remarkable mass relation (implicit in Ref. 13)

$$2m_{A_2}^2 = 9m_\rho^2 - 3m_\omega^2 - 7m_\pi^2 + m_\eta^2 . \tag{19}$$

Using the right-hand side to predict the mass of the $A_2$, one finds $m_{A_2} = 1340$ MeV, versus 1315 experimentally.

I have concentrated on a single FESR bootstrap calculation in the hope of making the method clear to you, at the expense of being manifestly unfair to the work of other authors[14-24]. I have only enough time left to sketch the direction in which other work is going. One extension is to drop some of the simplifying assumptions, such as zero width, linear trajectories, etc. The paper by Chu, Epstein, Kaus, Slansky and Zachariasen[14] does this by using a modified Cheng-Sharp representation for the trajectory, which leads to a bootstrap scheme which they solve numerically. One qualitative conclusion is that the straight-line approximation is remarkably good.

Another extension of the FESR bootstrap is to include more channels, usually with some degeneracy or symmetry assumption. One then obtains coupling constant relations of the form

$$g_i = \Phi \sum_j C_{ij}\, g_j\ , \tag{20}$$

where $\Phi$ is the function defined in Eq. (13), or some similar function, and where $C_{ij}$ is a crossing matrix. Assuming the consistency of the bootstrap, that $\Phi \approx 1$, one is left with coupling constant relations of the type studied by Capps, Cutkosky, and others several years ago, and pursued further by Mandelstam[15] in a paper presented here. All the FESR bootstrap has done in this connection is to present a cleaner derivation of the relations, in the context of a scheme which seems to be making quantitatively accurate predictions.

The multichannel FESR bootstrap goes on to give mass relations, such as the ones obtained by ARVV and other Weizmann Institute collaborators[16,17],

$$\begin{aligned} m_{A_2}^{\,2} &= 3(m_\rho^{\,2} - m_\pi^2), \\ m_B &= m_{A_2} - m_\pi\ , \end{aligned} \tag{21}$$

and others. They are well satisfied ($\approx$ 10%).

To summarize, the FESR bootstrap is a tremendous advance over older schemes, both in the palatability of its approximations and in its predictive power. Its claims are still, however, relatively modest. There is no prospect of its leading to a complete dynamical theory.

## 2. MULTIPERIPHERAL BOOTSTRAP

Another new bootstrap scheme has been presented to this Conference; in fact, so new that no written report of it exists at this time. I hope you will forgive the inaccuracies in my reporting of it.

Chew and Pignotti have submitted to this Conference a paper entitled "A Multiperipheral Bootstrap Model"[25]. Subsequent to the writing of that paper, the collaboration of Goldberger and Low has refined the mathematics into the scheme which Chew described in the parallel session. It is based on approximating production amplitudes by multi-Regge exchange diagrams, such as the one shown in Fig. 5. The amplitude for such a diagram is of the form

$$\prod s_i^{\alpha_i(q_i^2)}\ \beta_{0i}\left(q_0^{\,2}, q_i^{\,2}, \omega\right)\ , \tag{22}$$

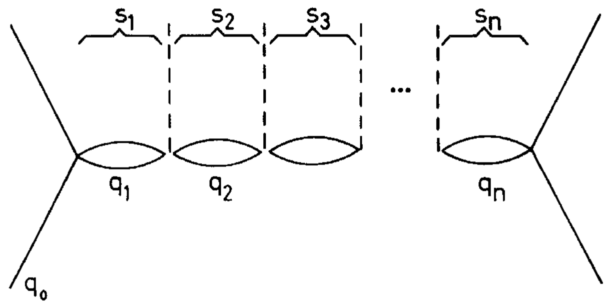


Fig. 5 Multi-Regge exchange graph. Double lines indicate Regge poles.

where $s_i$ are the sub-energies, as shown in the figure, and the $\beta$'s are form factors at the vertices. I will not attempt to trace the tortuous history of the model, but only give a few recent references from which this history can be traced[25-28]. The comparison of this model with production data has been discussed extensively in a previous session by Dr. Czyzewski. The results are very encouraging.

An important assumption made by Chew and Pignotti[28] is the validity of the Dolen-Horn-Schmid duality, which justifies ignoring explicit resonance production diagrams. That is, in Fig. 5 only stable particles are included as outgoing lines. (In fact, Chew and Pignotti simplify by considering only pions as the dotted lines.) The resonances are supposed to be taken care of approximately by the Regge-pole exchanges, as we have already discussed in connection with FESR bootstraps. Without this assumption it would be very difficult to avoid double counting; for example, by including both two-pion production and $\rho$ production.

Now the contribution of $M_n$ to the total cross-section is given by $A_n$, where

$$A_n = \int d\Phi_n\, |M_n|^2\ , \tag{23}$$

and where $\int d\Phi_n$ indicates n-body phase space. The quantity $A_n$ can be represented graphically as in Fig. 6 *). The total cross-section is then given by $\sigma_T \propto A = \Sigma_n A_n$.

*) Interference terms (crossed rungs) are provisionally neglected because, in so far as the $\beta$'s favour small t, they contribute in different regions of phase space and are negligible. This argument is weakened by the small values of the sub-energies involved.

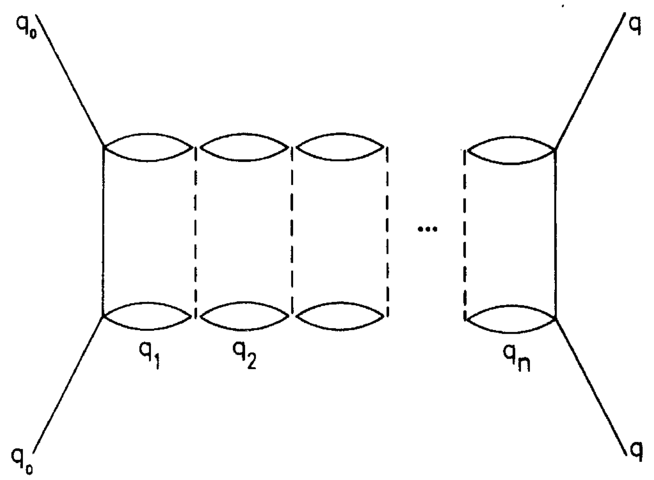


Fig. 6 Multi-Regge exchange contribution to absorptive part.

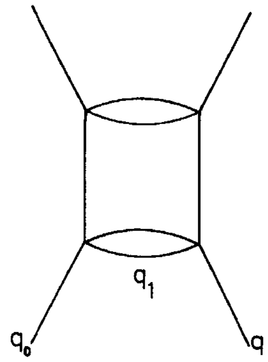


Fig. 7 Inhomogeneous term in the integral equation for B.

The familiar ladder graph in Fig. 6 reminds one of the work of Amati, Stanghellini and Fubini, and Bertocchi and Tonin[29] in which such graphs are summed by means of a Bethe-Salpeter type integral equation. Such a technique cannot immediately be applied here, however, because of the Regge factors $s_i^{\alpha_i(q_i^2)}$. So the following trick is employed: a new amplitude $B_n$ is defined as follows:

$$A_n(q,q_0) = \int d^4q_1\,\delta\left[(q_0-q_1)^2 - m^2\right] g^2\beta^2(q_0^2,q_1^2) \times \times B_n(q_0,q_1,q)\ . \quad (24)$$

The final integration is left undone in B. At the cost of having B depend on an extra variable, one is now able to introduce correlations such as factors of $s_i^{\alpha}$. Then if we define $B = \Sigma B_n$, we find the following integral equation for B:

$$B(q_0,q_1,q) = g^2 B_2(q_0,q_1,q) + + g^2\int d^4q_2\, B_2(q_0,q_1,q_2)\, B(q_1,q_2,q)\ , \quad (25)$$

where $B_2$, which is derived from the amplitude shown in Fig. 7, is given by*)

$$B_2(q_0,q_1,q) = g^2\beta^2(q_0^2,q_1^2)\, s_1^{2\alpha_1(q_1^2)}\,\delta\left[(q_1-q)^2 - m^2\right]\ . \quad (26)$$

This equation can be simplified by a sort of partial-wave projection. Since this is being done at fixed momentum transfer, and the momentum transfer is a space-like vector, the appropriate little group is O(2,1), not the rotation group O(3), so the machinery developed by Toller is applicable (see Ref. 40). In fact, at t = 0, the relevant point for calculating total cross-sections as we are doing here, the little group is enlarged to O(3,1), and the projection reduces Eq. (25) to an integral equation in a single variable, but we cannot go into detail here.

The resulting equation has not yet been investigated in any detail, but Chew presented a simple model which results from assuming a factorizable kernel. If b(J) is the projection of B, one finds an algebraic equation of the form

$$b(J) = q^2 b_2(J) + q^2 b_2(J) b(J)\ . \quad (27)$$

The conversion from B to A via Eq. (24) introduces nothing new, so we can just as well write Eq. (27) for a(J), the projection of A. Solving this equation gives the result

$$a(J) = \frac{g^2 a_2(J)}{1 - g^2 a_2(J)}\ . \quad (28)$$

Now this equation, admittedly derived under some drastic assumptions which will undoubtedly be relaxed later, already has some very interesting properties. Note that $a_2(J)$ carries the Amati-Fubini-Stanghellini[29] branch point in the J-plane, as shown in Fig. 8 *). The strength of this cut is

*) I oversimplify -- end vertices may differ from internal ones, but I ignore this and other details to keep things simple.

*) The reader who remembers that Mandelstam[30] proved that the AFS cut really was not there may be confused at this point. His proof was for genuine Feynman graphs, not for the multi-Regge graphs we are using here.

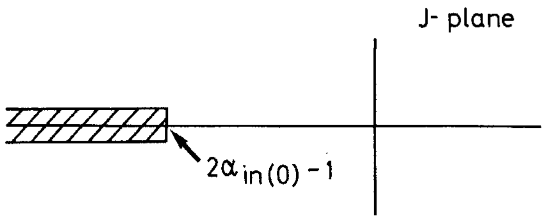


Fig. 8 Singularities of $a_2(J)$: the AFS cut.

proportional to $g^2$. But Eq. (28) carries a denominator which damps the cut for large $g^2$, unless the two terms in the denominator almost cancel -- that is, unless there is a nearby pole in the J-plane. We have the same situation as we have found to be the rule in the energy plane: a branch cut usually makes an important contribution only if there is a nearby pole (resonance). It seems likely that cuts in the J-plane will turn out to be important only if they are associated with a nearby pole, in which case they can be approximately replaced by a pole in doing phenomenology! Speaking more conservatively, these methods will at any rate lead to a relation between cuts and poles.

Now to get a preview of the sort of information one can obtain from this multi-Regge model, let us make another rough approximation, replacing the cut in $a_2(J)$ by an effective pole,

$$a_2(J) \approx \frac{1}{J-\bar{\alpha}} \quad , \tag{29}$$

where we choose the effective pole position at the branch point,

$$\bar{\alpha} \approx 2\alpha_{in} - 1 \, . \tag{30}$$

We use the subscript "in" to show that this Regge pole was put into the inhomogeneous term $a_2(J)$. Then Eq. (28) gives

$$\alpha(J) = \frac{g^2}{J-\bar{\alpha}-g^2} \quad . \tag{31}$$

Note that the pole representing the branch point has disappeared, and instead a pole in the J-plane has been generated at $J = \alpha$, where

$$\alpha = 2\alpha_{in} - 1 + g^2 \, . \tag{32}$$

(All $\alpha$'s are evaluated at $t = 0$.)

Suppose that the input pole is the Pomeranchuk pole, with $\alpha_{in} = 1$. Then we have

$$\alpha_P = 1 + g_P^2 \ , \tag{33}$$

which implies a total cross-section violating the Froissart bound. This difficulty has already been noted by Finkelstein and Kajantie[31], and by Gribov and Migdal[32]. It implies that either $g_P^2 = 0$ or $\alpha_P \neq 1$. The latter choice is the one of CGLP. One can then look for a self-consistent solution of Eq. (32), with $\alpha_P = \alpha_{in}$. The result is

$$\alpha_P = 1 - g_P^2 \quad . \tag{34}$$

This result shows that the model we have considered, involving only the Pomeranchuk, is incomplete. It is known that $\alpha_P \gtrsim 0.9$. Moreover, it is possible to calculate the average multiplicity from the model, with the result

$$\bar{n} \simeq g^2 \ln\left(\frac{E_L}{M}\right) \ . \tag{35}$$

Fits by Chew and Pignotti find $g^2 \approx 1.5$, so clearly this is not the $g_P^2$ of Eq. (34). There must be another trajectory which is coupled much more strongly. Chew and Pignotti introduce a "meson" trajectory, which is supposed to summarize the average effect of the meson exchanges. Letting $g_M$ represent the coupling at a vertex such as that in Fig. 9, and neglecting the Pomeranchuk contribution which we have found to be small, we find from Eq. (32)

$$\alpha_P = 2\alpha_{in}^M - 1 + g_M^2 \ . \tag{36}$$

Using the observed fact that $\alpha_P \approx 1$, we find

$$g_M^2 \approx 2(1-\alpha_M) \ . \tag{37}$$

Identifying $g_M^2$ with the dominant coupling in Eq. (35) we find $\alpha_M \approx 0.25$, which is not unreasonable.

The details of these crude models will undoubtedly be refined, but the following interesting features

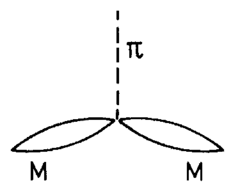


Fig. 9 The coupling $g_M$.

should persist: 1) The emergence of Regge asymptotic behaviour from Eq. (25) plus the Toller projection. This is much more general than the specific multi-Regge form used. It depends on the recursive nature of the multiperipheral diagrams, which permits one to formulate an integral equation. 2) The cut-pole relationship, given in simplified form by Eq. (28). 3) The weak coupling of the Pomeranchuk pole, leading to the dominance of $\sigma_{\text{inel}}$ by meson exchange. 4) The emergence of a new bootstrap scheme in which no two-particle truncation of the unitarity condition is used.

## 3. GRIBOV-MIGDAL DIAGRAMS FOR REGGE POLES AND CUTS

In the parallel session, Dr. Ter-Martirosyan summarized recent work by V.N. Gribov and A.A. Migdal on extracting Regge poles and cuts from Feynman diagrams. It is sufficiently intricate that I can only attempt to give you some idea of the method and some important results. Earlier work on this problem by Gribov was discussed in the report by Omnès at the Heidelberg Conference[33].

At large values of s and small values of t, the Regge-pole exchange graph in Fig. 10, which is obtained from ladder graphs, contributes an amplitude

$$M^{(1)} = \eta\, g^2\, e^{-k^2\xi}, \qquad \xi = \ln(s/s_0), \tag{38}$$

where

$$\eta \equiv i + \frac{\pi}{2}\frac{\partial}{\partial\xi} \tag{39}$$

gives the usual signature factor if $k^2 \ll 1$. Here $k^2 = \alpha'\kappa^2$, where $\kappa$ is the transverse momentum ($t = -\kappa^2$). It is assumed that the Pomeranchuk trajectory has the form $\alpha_P(t) = 1 - k^2$.

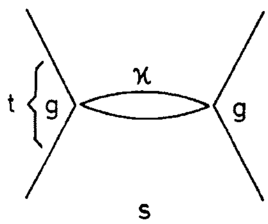


Fig. 10 Single Reggeon exchange.

Now the graph in Fig. 11 gives a cut in the angular momentum plane, whose contribution Gribov[32]

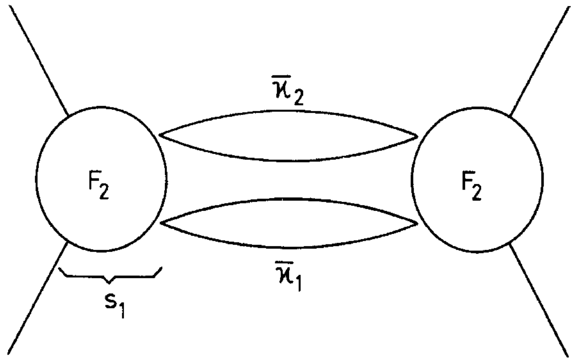


Fig. 11 Double Reggeon exchange, leading to cuts.

has found to be

$$M^{(2)}(k,s) = \frac{1}{2!}\int n_2^2(\bar{k}_1,\bar{k}_2)\, i\,\eta^2\, e^{-(k_1^2+k_2^2)\xi}\,\frac{d^2k_1}{2(2\pi)^3\alpha'}$$

$$\approx -\frac{n_2^2(0,0)}{32\pi\alpha'}\,\eta\,\frac{1}{\xi}\,e^{-k^2\xi/2}, \tag{40}$$

where

$$n_2(\bar{k}_1,\bar{k}_2) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} F_2(\bar{k}_1,\bar{k}_2,s_1)\,ds\;. \tag{41}$$

Gribov and Migdal have been able to show that $n_2 = g^2 + \Delta n_2$, with $\Delta n_2 > 0$. This leads, after an impressive amount of diagram-summing, to the following interesting result: the sum of all these graphs gives an amplitude which can be written

$$M = \eta\, g^2\left[e^{-k^2\xi} - \frac{1}{\xi}B_{12}(\xi,k^2)\right], \tag{42}$$

where, at $t = 0$,

$$B_{12}^0 = \frac{n_2^2}{32\pi\alpha'} \geq \frac{\sigma_{\text{tot}}}{32\pi\alpha'}\;. \tag{43}$$

This says that total cross-sections will approach a constant asymptotic limit from below, with sizeable deviations ($\sim$ 10%) dying out only logarithmically.

## 4. THE t = 0 PROBLEM: CONSPIRACY, LORENTZ SYMMETRY, ETC.

This subject is technically one of the most intricate of the subjects I am trying to present today, so I will have to limit my report to brief, largely qualitative remarks. The birth of this subject occurred in 1959, when MacDowell[34] found that pion-nucleon partial-wave amplitudes of the same J but opposite parity are equal at t = 0, and when Goldberger, Grisaru, MacDowell and Wong[35] found that the

NN partial waves were related by analyticity at t = 0 (these are t-channel partial waves). Then in 1962 Volkov and Gribov[36] recognized the implications of this relation for Regge poles: either each term vanishes separately at t = 0 (evasion) or several trajectories must coincide and have related residues (conspiracy). This result was ignored until some features of the experimental data, namely the pion exchange peak in backward np scattering, encouraged Gell-Mann and Leader to speculate that conspiring trajectories might exist[37]. The first candidate was the pion[38] and that suggestion is still with us, neither confirmed nor ruled out.

In the meantime the theoretical activity in this subject has been intense[39]. Toller[40] followed by Freedman and Wang[41] showed that the conspiracy relations reflected an additional symmetry of the scattering amplitude at q = 0, where q is the four-momentum transfer. The group is isomorphic to the homogeneous Lorentz group, so the symmetry is often called Lorentz symmetry. I will not go into detail, because the subject has been reviewed at previous conferences[39]. Recall, however, that Toller's approach led to the introduction of a new quantum number M at t = 0. All the well-known trajectories with intercepts $\alpha(0) > 0$ have been shown to have M = 0. The pion may be an example of an M = 1 trajectory, which would involve a parity doublet trajectory (called the pion's conspirator, or $\pi_c$) degenerate with the pion trajectory at t = 0.

The quantum number M has recently been given a more physical interpretation by a number of authors working both from the group-theoretic approach and from the analyticity-factorization approach[42-49]: the asymptotic form of the s-channel helicity amplitudes behaves at t = 0 as follows

$$f^{s}_{\lambda_3\lambda_4,\lambda_1\lambda_2} \sim \sqrt{t}^{\,|(|M|-\mu)|}\;\sqrt{t}^{\,|(|M|-\lambda)|} \quad , \qquad (44)$$

where

$$\lambda = |\lambda_1 - \lambda_3| \, , \qquad \mu = |\lambda_2 - \lambda_4| \; .$$

This elegant formula is consistent with the earlier work of Sawyer[50].

At the time of the last Berkeley Conference the group-theory approach was in its infancy, and was inadequate in the following respects: it applied only to EE reactions (equal masses going to equal masses, such as $NN \to \pi\pi$) and only at t = 0. Formalisms which overcame some of these limitations were proposed by Sawyer[50], Delbourgo, Salam and Strathdee[51], and by Domokos and Tindle[52]. In particular, Domokos[53] remarked that in the Bethe-Salpeter equation, the particle spectrum at t = 0 can be classified according to O(4) even in the unequal-mass case where O(4) is not a symmetry of the scattering amplitude.

A very elegant approach to the problem of extending the O(3,1) symmetry to unequal masses and $t \neq 0$, which has been carried out by Cosenza, Sciarrino and Toller[47], is to enlarge the group to SL(2,C). One apparent disadvantage of this method is that daughter trajectories remain parallel to the parent at $t \neq 0$, in contradiction to the results of Bethe-Salpeter models[54-56]. Perhaps, however, the Veneziano[13] representation is teaching us that nature really chooses parallel daughters.

Another approach, carried out by Domokos, Kövesi-Domokos and Surányi[57], and by Halpern, Lipinski, Snider and myself[48], combines a perturbation expansion of the Bethe-Salpeter equation in the momentum transfer $q_\mu$ with an expansion in O(4) basis states. This permits the derivation of trajectory formulae as well as general properties of residue functions.

The other approach to the t = 0 problem, via analyticity and factorization, has made impressive progress. Practically all the results of the group theorists have been recovered by these means, and some recent work goes further. Independently, a large number of authors[45,58-60] have found that the constraints of analyticity applied to UU and UE amplitudes, when coupled with the factorization requirement, imply that daughter Regge poles constitute a single Toller pole. Previously there had been the worrisome possibility of an "anticonspiracy" of Toller poles.

In conclusion, let us examine the present status of the question of the Toller quantum number of the pion. Dr. Chan Hong-Mo has summarized the phenomenological situation: some reactions, especially charged pion photoproduction, seem to call for an

M = 1 pion, whereas an M = 1 pion cannot be the dominant contribution in $\pi N \to \rho\Delta$. Arbab and Brower[61] have got around this with a fit relying heavily on the $A_1$, but the situation is still confusing. It is clear from photoproduction that some important J-plane singularity has M = 1, but it need not be the pion.

The theoretical situation has also been confusing. Mandelstam[62] attempted to make a connection between M = 1 and PCAC, but Sawyer[63] recognized a fundamental difficulty, which we can see from Eq. (44): if the pion has M = 1, then at t = 0 (that is, $m_\pi^2 = 0$) it will decouple from all "sense" amplitudes. This is much stronger than the Adler self-consistency condition, which only requires that soft-pion amplitudes vanish. A second difficulty is that an M = 1 trajectory is nonsense-choosing at J = 0 (implying no physical pion!) as long as the continuation from t = 0 to J = 0 is smooth. A loophole is the possibility of the M = 1 trajectory crossing an M = 0 trajectory, as seen in the models constructed by Blankenbecler and Sugar[64] and by Lipinski, Snider and myself[65]. But it is not clear that this differs significantly from the model proposed by Sawyer[63], in which the pion lives on an M = 0 trajectory, with an M = 1 trajectory lying very nearby. I hope we will soon see some clarification of this situation.

## 5. PADE APPROXIMANT METHOD

I will conclude by briefly reviewing progress with the Padé Approximant method of calculating scattering amplitudes. The work of Bessis and Pusterla[66] was reviewed by Omnès[33] at the Heidelberg Conference, so I refer you to those proceedings for a discussion of the basic method. Two extensions have been presented to this Conference: Remiddi, Pusterla and Mignaco[67] have used the Padé method to calculate low-energy pion-nucleon phase shifts. Basdevant, Bessis and Zinn-Justin[68] have considered the coupled $\pi\pi$ and $K\bar{K}$ system to calculate the resonance positions and widths. The results show some impressive numerical agreement, but are marred by the prediction of degeneracies not observed in nature. The latter authors also propose an alternative viewpoint on the Padé method which does not tie it so closely to field theory. They regard it as the summation of a formal power series in "coupling constants", which are defined as values of scattering amplitudes at a certain point. Unitarity and crossing symmetry enable one to determine the first few terms in the series, whereupon the Padé method is used to approximate the sum of the series. In this view, the Padé Approximant is used as a tool for constructing scattering amplitudes which are approximately unitary and crossing-symmetric.

## Acknowledgements

This report was originally scheduled to be given by Dr. G. Domokos who unfortunately was unable to be present. In preparing this report I have benefitted enormously from the draft which he had prepared.

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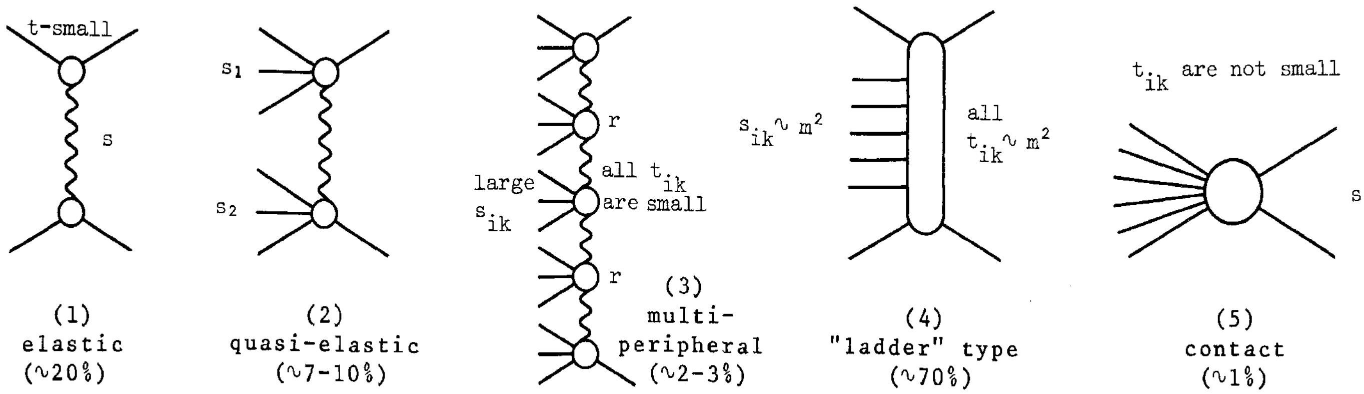
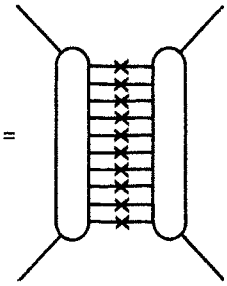
## DISCUSSION

TER-MARTIROSYAN: I have some small comments connected with the work by Chew, Low and Goldberger. As I understood they consider elastic scattering as a result of inelastic multiperipheral collisions. At high energy all collisions can be grouped into five classes illustrated by the following diagrams:



They are of elastic (1), quasi-elastic (2), multiperipheral (3), ladder (4), and contact types (5). They are all familiar to physicists today. Perhaps the two last ones have to be commented on. The most important is the collision of the ladder type. Its contribution to the elastic scattering amplitude has the form of a ladder

$$\mathrm{Im}\, M(0,s) = \sigma^{(tot)}(s) =$$ 

corresponding to a Regge pole. It gives the main contribution to the amplitude M(0,s). In collisions of the contact type, all momentum transfers are large, the cross-section is very small and decreases rapidly with energy. The percentages given at the diagrams are taken from experiment. They indicate the order of magnitude of the contribution of each diagram in the region around 20 GeV.

All this corresponds very well to the complex angular momentum picture, where at high energy the main contributions are Regge poles (i.e. inelasticity of the ladder type), and the corrections, which are of the order of $1/\ln(s/m^2)$, are due mainly to elastic and quasi-elastic scattering.

I want to stress that, as experiment shows, the multiperipheral contribution is always small (2-3% only). Indeed, the same follows from theory if the vertex corresponding to the emission of a particle or a group of particles by a Reggeon vanishes at small Reggeon momentum.

CHEW: I believe that the general answer to Ter-Martirosyan is that the use of either duality or something like the Chan, Loskiewicz, Allison model, described in the review by Czyzewski, will produce the essential results of the multiperipheral equation described by Frazer.

NAUENBERG: I would like to ask a question concerning the finite energy sum rules. You told us that a representation due to Veneziano was an example which satisfies the sum rule, but the form you wrote down is purely real for zero width resonances. Furthermore, asymptotically it misses the signature factor of a Regge pole. Assuming this point is clarified, how does Veneziano's representation explain that finite-energy sum rules work: that is, why there exist discrete values of the cut-off N for which the equation holds for different values of t?

VENEZIANO: 1) Concerning the first question, the representation I wrote down for real trajectories has only poles and it is, for the remainder, real. Im A as obtained from the iε-prescription is a sum of δ-functions. Im $A_{\mathrm{Regge}}$ is defined as the average discontinuity and can be also obtained in the limit $s \to \infty$ in the case $\mathrm{Im}\,\alpha\,(s)_{s\to\infty}^{\to}\infty$. In this last case, one gets nice Breit-Wigner shapes at low energy, which, as the energy increases, are dumped and overlap until the smooth asymptotic Regge form is obtained.

2) To have a value of N which gives good results in a large region of t could seem quite a mystery. On the other hand, if one accepts the Dolen-Horn-Schmid duality, the finite-energy sum rules require that a local average is taking place and consequently there should be a value of N for which the sum rules are satisfied.

SCHMID: I should like to make a comment about the finite-energy sum rule bootstrap. It is addressed to those who wonder whether we have here just another scheme, as we had, for example, the N/D bootstrap before. The new important element is that we can check the approximations and estimate the errors, which are typically of the order of 20%. These are not *a priori* estimates, rather we use experiments as a guide. Nature tells us which terms we can drop, for example, how important the non-resonating background is or how good an approximation the leading Regge poles are at a rather low energy. Note that in the N/D bootstrap scheme there was no way to estimate the errors on the left-hand cut, and nobody has ever measured the left-hand cut.

RATTI: The idea of the multiperipheral model in the Reggeized version is that what happens to each vertex depends only on the nearly exchanged trajectories. If this is true, there is a way to check in some detail the plausibility of the extrapolation of the Regge model to high multiplicities. In fact, in a quasi-two-body reaction, the decay properties of a given resonance (for instance the t-dependence of the spin density matrix elements) are experimentally known. My question is whether the same properties have to be shown by the same resonance produced at an external vertex of a multiperipheral process, provided the exchanged trajectory at the vertex is the same.

CHEW: The external vertices in a peripheral chain are independent of how many legs are in the chain. There should correspondingly be common features for the end of the chain, independently of the rest of the chain.

SULLIVAN: I have a question concerning the relation of the cuts obtained in the absorption (Glauber) model and the cuts obtained in the multi-Regge model. Is it correct that the absorption model cuts are the input cuts to the latter model?

CHEW: Yes.