DETERMINATION OF α_s AND $\sin^2\vartheta_W$ FROM THE R MEASUREMENTS AT PEP AND PETRA

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The analysis of all the available data on the total multihadron cross section in the e^+e^- annihilation above 14 GeV allows a good determination of the strong coupling constant and the electroweak mixing angle. Using a procedure which takes into account the correlation between measurements we obtain from the global fit $\alpha_s\,(34^2\ \mathrm{GeV}^2) + 0.165 \pm 0.030$ and $\sin^2\vartheta_W = 0.236 \pm 0.020$. Fixing $\sin^2\vartheta_W$ at the world average value of 0.23 yields $\alpha_s\,(34^2\ \mathrm{GeV}^2) = 0.169 \pm 0.025$.

1. Introduction

In the e^+e^- physics, R is defined as the ratio between the total multihadron cross section in the annihilation channel and the lowest order muon cross section ($4\pi\alpha^2/3s$). The simple prediction of the Quark Parton Model, $R = 3\Sigma_f Q^2_f$, is expected to be modified by the gluon radiation in the final state, as described by QCD, and by the presence of the Z° propagator at the highest PETRA energies. The complete prediction for R can be calculated in the Standard $SU(3)_c \otimes SU(2)_L \otimes U(1)$ Model and depends on the free parameters of the theory, i.e. the mixing angle of the electroweak interactions, given by $\sin^2\vartheta_W$, and the strong coupling constant α_s. The precise measurement of R at different c.m. energies is a good tool to test the standard model and measure the two quantities, since:

- the increase in R from Z° exchange is sizeable at the highest PETRA energies and the measurement of $\sin^2\vartheta_W$ is an independent test of the SM in completely different channels and kinematics region respect to other measurements;
- a direct measurements of α_s from R has clear advantages with respect to the other methods [1]:
  - it is an inclusive measurement, not depending on a certain class of events and it is then insensitive of the fragmentation models or of the details of the multijet cross section;
  - the calculation up to second order is not controversial;
  - the second order term is already small (the QCD correction is of the form $1+\alpha_s/\pi + O(1)\,(\alpha_s/\pi)^2$, thus we can safely neglet higher order.

Experimentally R is determined by

$$R = \frac{N_H - N_{BG}}{L\,\varepsilon_H\,(1+\delta_H)\,\frac{4\,\pi\alpha^2}{3\,s}}$$

where N_H and N_{GB} are the number of the observed multihadrons and of the background events. L is the integrated luminosity, determined by the central Bhabha's and $\varepsilon_H(1 + \delta_H)$ is the multihadron efficiency multiplied by the radiative correction. Typical systematic errors on the luminosity and the efficiency are 2% each. Table 1 shows the different contributions to the systematic error in the case of the CELLO data [2].

Beside the experimental errors, we would like to remind what are the theoretical uncertainties on the determination of R:

- as far as the hadronization of the partons is concerned, the model dependence of the fragmentation scheme as well as the parameter dependence of the model itself is negligeable, if the model parameters have been previously tuned to reproduce the observed topology of the multihadrons. It has to be noticed instead how crucial is the assumption that the multihadrons are produced at high energy via the primary production of $q\bar{q}$ pair. This hypothesis has been checked observing the angular distribution of the event axis.

• The main uncertainties on the radiative corrections come from the hard initial state bremsstrahlung and the higher order (> α^3) corrections to the Bhabha and the multihadron cross section. The first one comes from the pour knowledge of the hadron cross section at low energy. In fact the probability for a photon to be emitted in the initial state depends on the hadron cross section at the remaining $\sqrt{s'}$ after radiation. However the small efficiency of the apparatus to this kind of events makes the final uncertainty on R smaller than 1%. The uncertainties on the O(> α^3) corrections is what presently limits more the analysis of the R values. No full calculation is available. Tsai [3] gives an estimate of all higher order corrections by summing all leading logarithms. Since these calculations are not complete and give correction factors at the percent level with errors of the order of the correction itself, we have not applied these corrections to the data. We will show by how much the results change if a hypothetical value for the corrections is assumed.

2. Analysis method

The combination of data coming from different experiments is always a delicate procedure. In fact this requires that:

• all the data are homogeneous, i.e. they have been treated in the same way, and that their errors have similar meanings;

• the analysis method must take into account for the correlation between the data points, eventually also between different experiments.

In order to do this properly, some of the data points have been modified in the following way. Radiative corrections involving Z° exchange (up to 2% for PETRA) had been made only by CELLO, MAC [4] and Mark [5], and then had to be applied to all other experiments. As said before, we decided not to put the uncertainty on the O(> α^3) radiative corrections as an error in the data. MAC had applied these corrections, but also quotes on R-value without it. This second value has been used in our analysis. TASSO [6] includes in the common normalization error an estimate of the uncertainty due to the O(> α^3) corrections of 2%, which has been taken out. Table 2 shows all the data points used in the analysis after our modifications [2, 4, 5, 6, 7, 8, 9]. In some case updated versions of the data, on respect to published ones, have been used.

We have used the following procedure to take into account the correlation between data points:

one define a N×N error matrix V_{ij} for the N measurements. The diagonal elements are given, for each experiment by the quadratic sum of the statistical, point-to-point and normalization errors. The correlation between data points is contained in the offdiagonal element V_{ij}, given by the square of the error common to i and j. Since all the errors are quoted in percent of the R values, in practice the element V_{ij} has been estimated by the product of the values calculated for i and j. In this way, we could also take into account for different normalization errors in different running periods of the same experiments. In the covariance matrix, no common errors between different experiments have been included. In fact, a common error could come from the theoretical uncertainties:

• MC generator for the hadronization. We said already before that this uncertainty is small. Moreover it is known that each collaboration uses slightly different turned version of the programs.

• Hard bremsstrahlung. It depends essentially on the individual detector efficiency for peculiar events containing a hard photon and a low invariant mass hadronic system.

• Higher order radiative corrections. This gives the real systematic uncertainty to the result, but we cannot put it as a probabilistic uncertainty in the covariance matrix. Its influence will be quoted later.

To conclude, we assign no common error between different experiments, but we will discuss the effect of a hidden common error of the percent level.

Once the covariance matrix has been built (Fig. 1 gives a graphic representation of it), we proceed as follows. The expression to be minimized is

$$\chi^2 = \Delta^T V^{-1} \Delta$$

where Δ is the vector of the residuals $R_{exp} - R_{fit}$. In case the off diagonal elements V_{ij} are all zero, χ^2 reduces to the usual expression for independent measurements. Such a procedure avoids fitting the normalization factors (eleven or more in our case) as free parameters in addition to the physical ones, with enormous advantages as far as the convergence of the fit, the interpretation of the results and the confidence on the same is concerned.

The statistical methods require that the errors appearing in the χ^2 expressions are gaussian. We believe that in our analysis this assumption is well founded. The normalization errors summarize in fact the total effect of many little uncertainties of probabilistic nature depending on the single experiment (trigger, efficiency, calibrations, detector simulation, background subtraction, etc.). The result is then guaranteed to be gaussian by the central limit theorem also if the individual contributions are not.

3. Results

We have compared the R measurements with the expectations of the Standard Model. The QCD effects have been included up to the second order in the perturbation theory, both in the definition of R and in the running of α_s. The $\overline{MS}$ renormalization scheme has been used and the reference point for α_s has been chosen at $s = 34^2$ GeV^2 where the PETRA luminosity has been maximum. For the E-W effect, we have used Mz = 93.3 GeV/c^2 and Γz = 2.5 GeV/c^2 and left $\sin^2\vartheta_W$ as free parameter. The radiative corrections on $\sin^2\vartheta_W$ are absorbed in the experimental definition of the Fermi coupling constant, that we have used to parametrize the strength of the E-W interactions [7]. Mass effect terms, important for the low energy data, have been used at the best of our knowledge [11]. The complete expression of R used in the fit can be found in Ref. [2].

From the fit to the CELLO data alone we obtain:

$$\alpha_s\,(34^2\ \mathrm{GeV}^2) = 0.19 \pm 0.05 \text{ and } \sin^2\vartheta_W = 0.20 \pm 0.03 \quad (X^2/\text{d.o.f.} = 3/7)$$

If we impose the world average of $\sin^2\vartheta_W = 0.23$ in the fit, we obtain for the strong coupling constant, $\alpha_s\,(34^2\ \mathrm{GeV}^2) = 0.16 \pm 0.05$, consistent with the previous CELLO measurements [12].

From the simultaneous fit to the combined data of PEP and PETRA one finds

$$\alpha_s\,(34^2\ \mathrm{GeV}^2) = 0.165 \pm 0.030 \text{ and } \sin^2\vartheta_W = 0.236 \pm 0.020 \quad (X^2/\text{d.o.f.} = 51/61)$$

The errors are given by $\Delta\chi^2 = 1$ around $\chi_{minimun}$ and include the statistical and detector effects. The degree of correlation between the two quantities is - 0.49. Fig. 2 shows the agreement between experimental data and fit. If $\sin^2\vartheta_W$ is kept fixed at 0.23 we obtain for the strong coupling constant

$$\alpha_s\,(34^2\ \mathrm{GeV}^2) = 0.169 \pm 0.025$$

This value corresponds to $\Lambda_{\overline{MS}} = 610^{+470}_{-340}$ MeV

As discussed before, the quoted errors do not take into account the uncertainty of the $O(> \alpha^3)$ radiative correction. A hypothetical correction of +1% would reduce all R values by 1% and yield $\alpha_s = 0.145 \pm 0.024$, while the influence on $\sin^2\vartheta_W$ is negligeable.

Several checks have been performed in order to test the basic assumptions of the analysis and the stability of the results against possible arbitrariness in the analysis procedures:

- The results of the global fit describes very well the single experiment. Moreover we have fitted the normalization factors of each experiment in order to get the best agreement with the combined fit. These factors spread symmetrically around 1 and the difference from 1 is always compatible with the quoted normalization error.
- The separation of the systematic errors into point-to point and common can be subject to some uncertainty, but we have checked that also a dramatic change of the off diagonal terms by ± 50% (keeping constant the total error) has a very small effect on α_s and $\sin^2\vartheta_W$ and on their error, only changing their correlation degree.
- We have assumed no correlation between the experiments. The introduction of a hypotetical common error of 1% would change α_s by -0.007 and $\sin^2\vartheta_W$ by -0.005, and increase their errors by 0.005 and 0.001 respectively.
- The numerical value of α_s depends clearly on the normalization scheme and on its running. To give an experimental result independent of these assumptions we have parametrized the "non-E-W" contribution to R by a linear expression

$$R = R_{EW}\,(a + b\,(E - 34\ \mathrm{GeV}))$$

and fitted it keeping $\sin^2\vartheta_W$ constant at 0.23. We find a = 1.062 ± 0.011 and b = (-0.75 ± 0.73) x 10^{-3} GeV^{-1} and very small correlation between a and b. Interpreting the "non-EW" factor as completely due to QCD, we recover the value of α_s previous found, while b = $(1/\pi)(\partial\alpha_s/\partial E)$ is a direct measurement of the running of the strong coupling constant. This result imply an 85% probability for α_s to run in the "right direction", i.e. b < 0, with a slope compatible with the QCD expectation (b ~ -1.3 10^{-3} GeV^{-1}).

• We have preferred to restrict our fit to a region free of resonances, although QCD is expected to describe the continuum below open $b\bar{b}$ production too (apart from resonance effects). If we include lower energy data (Table 3) from CESR [13] and DORIS [14] above 7.3 GeV (excluding the Y resonances and the data above the Y(4S)) and keep $\sin^2\vartheta_W$ fixed at 0.23, we find $\alpha_s(34^2\ GeV^2)$ = 0.166 ± 0.023, in good agreement with the high energy data (see Fig. 3).

The measurement of $\sin^2\vartheta_W$ is in good agreement with the value determined by completely different processes: neutrino scattering masses of the weak bosons, polarized electron-deuton scattering and asymmetries in lepton pair production in e^+e^-. Fig. 2 and Fig. 3 show a clear rise of R due to the Z° exchange. The statistical significance of the effect has been evaluated comparing the "high energy" data with the QCD extrapolation of the "low energy" data. The separation between the two samples has been put very safely at 37 GeV. A rigorous statistical calculation using the propagation of the covariance matrix of the measurements yield a probability of ~ 10^{-3} for the "high energy" rise to be just a statistical fluctuation.

4. Conclusions

The $\sin^2\vartheta_W$ determination from the combined data of the PEP and PETRA experiments has reached a precision of about 10%, which is similar to that from the asymmetry measurements of purely leptonic final state in e^+e^- annihilation [15]. Of course, neutrino scattering [16] and the determination of the Z° and $W^\pm$ masses [17] have provided more accurate measurements of $\sin^2\vartheta_W$, but its determination from R is an independent test of the Standard Model.

The importance of the determination of α_s from R stems from the fact that R is an inclusive quantity, which do not depend on the event topology and is therefore insensitive to the parton hadronization. Using a method, which correctly treats normalization errors, and combining the results from various experiments, we find $\alpha_s(34^2\ GeV^2)$ to be 0.169 with a total error less than 15%, if we exclude the theoretical uncertainty from higher order QED radiative corrections, which could lower α_s by a similar amount.

5. References

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Table 1 : Systematic errors on R from CELLO: common normalization error (σ_{norm}) and systematic point to point errors (σ_{ptp}). The errors do not include uncertainties from higher order QED radiative corrections (see text).

Source	$\sigma_{\rm norm}$(%)	$\sigma_{\rm ptp}$(%)
Luminosity		
trigger	0.5	
selection	0.7	
acollinearity cut	0.6	
energy cut	0.4	
acceptance	0.7	0.0 – 1.3
calibration		0.5
tracking efficiency		0.9
	1.3	**1.0 – 1.7**
Multihadrons		
trigger	0.4	
data reduction	0.7	
MC generator	0.5	
rad. corrections		
(hard bremsstrahlung)	0.5	0.0 – 0.7
selection cuts		0.6 – 1.2
	1.1	**0.6 – 1.4**
Total	1.7	**1.2 – 2.2**

Table 2: R-values used in the fit. If the point to point error σ_{ptp} is not given, it is included in the statistical error. If a second normalization error σ_{norm2} is given, it has to be added in quadrature to the first one. Note that for some experiments small corrections to published values have been made (see text).

Experiment	$\sqrt{s}$	R	$\sigma_{stat}(\%)$	$\sigma_{ptp}(\%)$	$\sigma_{norm1}(\%)$	$\sigma_{norm2}(\%)$
HRS	29.00	4.20	0.8		7.0	
MAC	29.00	4.00	0.8		2.1	
CELLO	14.04	4.10	2.6	2.2	1.7	
	22.00	3.86	3.0	2.1	"	
	33.80	3.74	2.6	1.9	"	
	38.28	3.89	2.6	1.7	"	
	41.50	4.03	4.1	1.8	"	
	43.60	3.97	2.0	1.4	"	
	44.20	4.01	2.5	1.2	"	
	46.00	4.09	5.1	1.9	"	
	46.60	4.20	8.5	1.7	"	
JADE	14.04	3.94	3.6		2.4	
	22.00	4.11	3.2		"	
	25.01	4.24	6.8		"	
	27.66	3.85	12.5		"	
	29.93	3.55	11.3		"	
	30.38	3.85	4.9		"	
	31.29	3.84	7.3		"	
	33.89	4.17	2.4		"	
	34.50	3.94	5.1		"	
	35.01	3.94	2.5		"	
	35.45	3.94	4.6		"	
	36.38	3.72	5.7		"	
	40.32	4.07	4.7		"	0.9
	41.18	4.24	5.2		"	"
	42.55	4.24	5.2		"	"
	43.53	4.05	5.0		"	"
	44.41	4.04	5.0		"	"
	45.59	4.47	5.0		"	"
	46.47	4.11	5.9		"	"

Continuation of Table 2.

Experiment	$\sqrt{s}$	R	$\sigma_{stat}(\%)$	$\sigma_{ptp}(\%)$	$\sigma_{norm1}(\%)$	$\sigma_{norm2}(\%)$
MARK J	22.00	3.66	2.2	3.0	2.1	
	25.00	3.89	5.4	"	"	
	30.60	4.09	3.4	"	"	
	33.82	3.71	1.6	"	"	
	34.63	3.74	0.8	"	"	
	35.11	3.85	1.6	"	"	
	36.36	3.78	4.0	"	"	
	37.40	3.97	9.3	"	"	
	38.30	4.16	2.2	"	"	
	40.36	3.75	4.0	"	"	
	41.50	4.32	4.6	"	"	
	42.50	3.85	5.2	"	"	
	43.58	3.91	1.5	"	"	
	44.23	4.14	1.9	"	"	
	45.48	4.17	4.8	"	"	
	46.47	4.35	3.9	"	"	
PLUTO	27.60	4.07	7.1		6.0	
	30.80	4.11	3.2		"	
TASSO	14.00	4.14	7.3		3.5	2.0
	22.00	3.89	4.4		"	"
	25.00	3.72	10.2		"	"
	33.00	3.74	7.2		"	"
	34.00	4.14	3.1		"	"
	35.00	4.23	2.1		"	"
	27.50	3.91	8.2		"	"
	30.10	3.94	4.6		"	"
	31.10	3.67	4.9		"	2.0
	33.20	4.49	6.3		"	"
	34.00	4.10	4.9		"	"
	35.00	4.04	4.2		"	"
	36.10	3.94	4.3		"	"
	41.50	4.11	2.9		"	3.0
	44.20	4.28	3.8		"	"

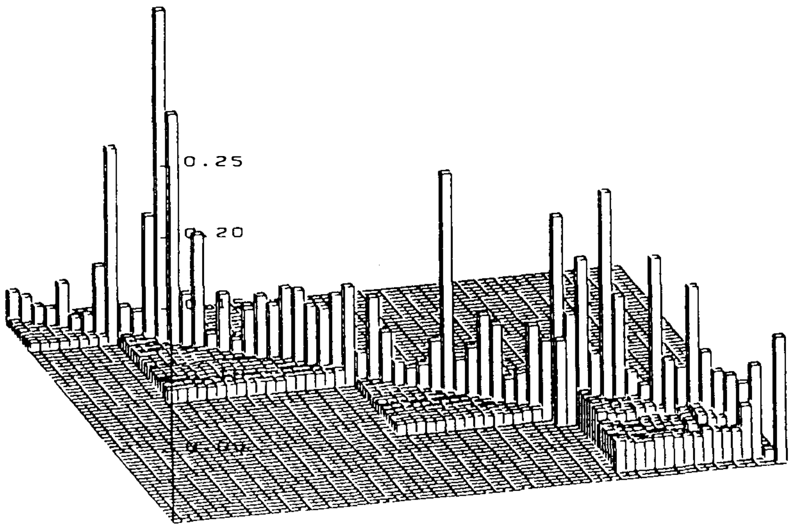

Figure 1 : The error correlation matrix for all PEP and PETRA experiments. The vertical axis is proportional to V_{ij}.

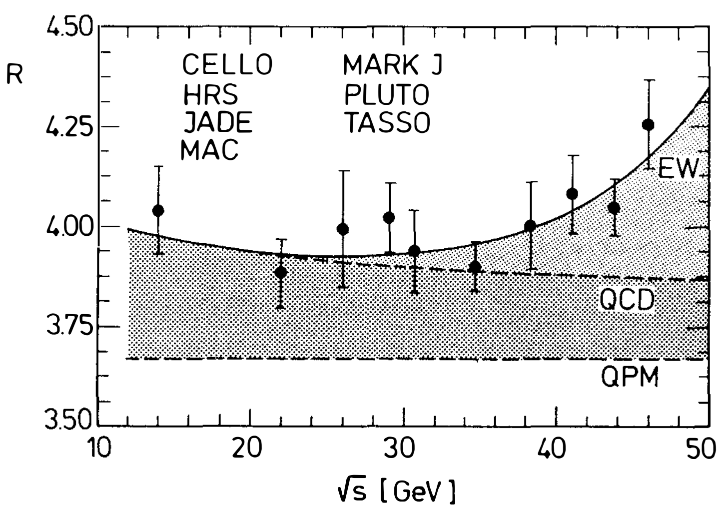

Figure 2 : **Averaged R values as function of $\sqrt{s}$.**
The errors include statistical and correlated normalization errors.

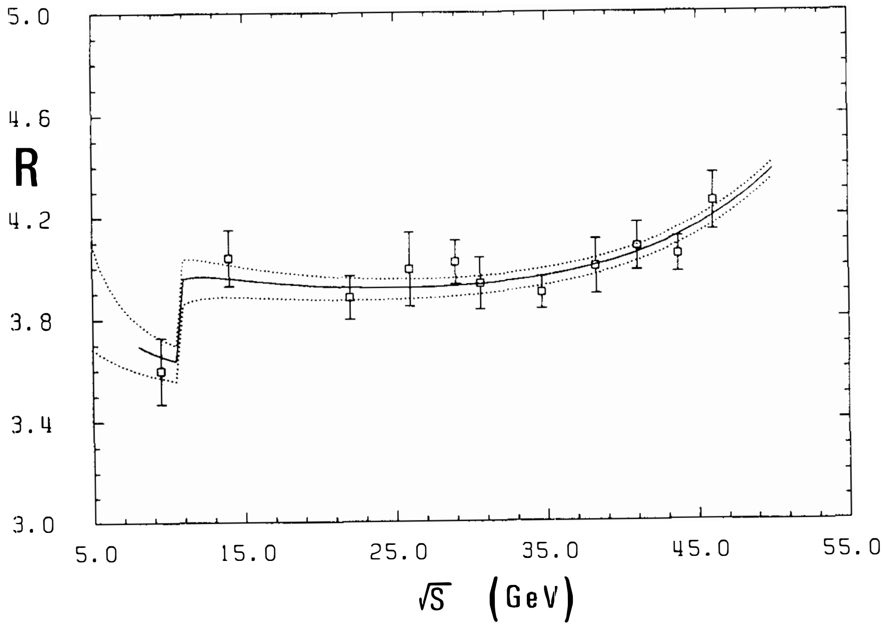

Figure 3 : Average R values as function of the center of the mass energy. Combined data from CESR, DORIS, PEP and PETRA. The dotted lines show the ± 1σ variation of α_s from the fitted value.