FUSION REACTIONS IN COLLIDING BEAMS*

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Abstract

Two colliding beam configurations are presented in which fusion reactions can take place between deuterons and tritons. The first is a linear system in which the ion beams travel in the same direction and are focused by a collinear electron beam. In the second configuration the ions travel in a spiral path in a strong magnetic field and are focused by electrons travelling along the lines of force of the magnetic field. The first system yields very little power but the second appears to merit further attention.

1. Introduction

Students of controlled thermonuclear reactions tend to divide feasibility experiments using electromagnetic containment into "ordered" and "equilibrium" systems. The latter category includes virtually all of the magnetically contained plasma systems with which we are familiar. The former category, to which I propose to address myself, is generally regarded with disfavor; it is expected to reduce itself speedily to an equilibrium system by space-charge forces and by coulomb scattering. Furthermore, it usually includes a disproportionate amount of stored energy which, after the order in the system is lost, cannot be recovered.

In this paper I shall present two ordered systems. To the first all the above objections apply. The second seems to be less objectionable, and I present it herewith in the hope that some one will consider it further.

Many fusion reactions are possible candidates for use in production of power. The favorite, on which I shall concentrate, is:

Deuteron + triton → α particle + neutron + 17 MeV. This reaction goes at a relatively low temperature; the reaction cross section has a maximum of about 5 barns at about 100-keV energy for a deuteron bombarding a triton at rest. Most of the energy produced — about 14 MeV — accompanies the neutron.

First, we review briefly the arithmetic of fusion reactions in colliding beams. We consider colliding beams of deuterons with density ρ_D travelling at velocity v_D and tritons with density ρ_T travelling at velocity v_T. Densities are in coulomb per cubic meter; velocities are in meters per second.

In one cubic meter of the triton beam there are 6×10^{18} ρ_T ions. These present to the deuteron beam a cross section of $6 \times 10^{18}\ \rho_T\sigma$, where σ is the fusion cross section. The rate of arrival of deuterons in the triton system is $6 \times 10^{18}\ \rho_D(v_D - v_T)$ per second. Hence the number of collisions per second per cubic meter is

$$36 \times 10^{36}\ \rho_T\rho_D\sigma(v_D - v_T) \quad (1)$$

and the fusion energy liberated is

$$36 \times 10^{36}\ \rho_T\rho_D\sigma(v_D - v_T)E \text{ watts per cubic meter,} \quad (2)$$

where E is the fusion energy liberated per collision.

For 100-keV deuterons bombarding tritons at rest the cross section σ has a maximum value of about 5×10^{-28} m^2. The fusion energy liberated per collision is 17 MeV or 2.7×10^{-12} joules. We assume that the velocities of deuterons and tritons correspond to a relative energy of about 100 keV, whence $v_D - v_T = 3 \times 10^6$ m/sec. We assume further that the densities of the two beams are equal so that $\rho_D = \rho_T = \rho_0$. With these assumptions we find that the fusion power is

$$1.46 \times 10^5\ \rho_o^2 \text{ watts per cubic meter.} \quad (3)$$

If, for example, the system is to yield a fusion power of 1 megawatt per cubic meter, the charge density required is 2.6 coulombs per cubic meter, or about 1.5×10^{13} ions per cubic centimeter.

2. Linear Colliding Beams

The first example we present will be linear colliding beams. The results to be presented will be so absurd that the example will have value only as an indication of problems to be solved.

Suppose that two collinear beams of deuterons and tritons are brought into collision, each having a velocity of 1.5×10^6 m/sec. Going somewhat beyond the state of the art, we assume that both are 1000-ampere beams and that both have cross sections of 1000 square centimeters. This leads to a charge density of 6.7×10^{-3} coulombs per cubic meter and hence, to a fusion yield [from Eq. (3)] of 6.5 watts per cubic meter, or of 0.65 watts per meter of distance along the colliding beams. This seems a depressingly small yield, particularly when we realize that almost 60 MW have gone into producing the two colliding beams.

To improve the situation we make two changes:

a) We will make the two beams travel in the same direction at much higher energy.

b) The beams will be focused by an electron beam travelling in the opposite direction.

By these means the beam will be concentrated in a small cross section and the power level will be increased to more interesting levels.

It will be required that the deuterium and tritium ions differ in velocity by 3×10^6 m/sec as noted in the Introduction. Also the ions will be required to have the same energy so that both beams can come from the same ion source and so that the unreacted ions in both beams can be retarded by the same field to regain their kinetic energy.

These two requirements set velocities of 1.6×10^7 and 1.3×10^7 m/sec for deuterons and tritons respectively; both will have an energy of about 2 MeV.

Space charge and current fields produce a repulsive radial force in the ion beam of

$$E_r - vB_\theta = 2\ Ir(1 - \beta^2)/(\epsilon_o\beta cr_o^2) \text{ volts/m ,} \quad (4)$$

*Work performed under the auspices of the U.S. Atomic Energy Commission.

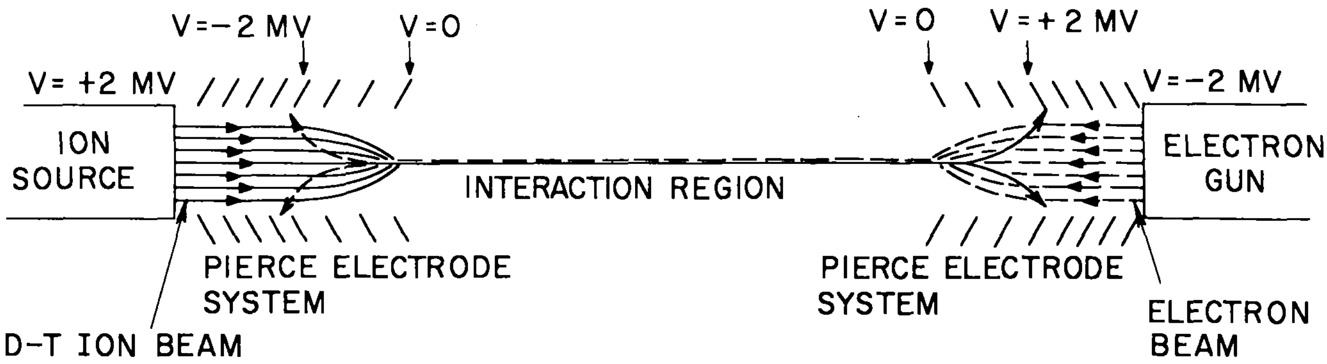

Fig. 1. Linear colliding beam system.

where I is total current (2000 amperes)
βc is ion velocity (about 1.5×10^7)
ϵ_0 is the dielectric constant of free space (1.1×10^{-10} F/m)
r_o is the radius of the beam.

Approximately this force is

$$E_r - vB_\theta = 1200\ Ir/r_o^2 \text{ volts/m} . \qquad (5)$$

If a relativistic electron beam of I_e amperes is now introduced, collinear with the ion beam but travelling in the opposite direction, it will contribute a focusing force of

$$2I_e r(1+\beta\beta_e)/(\epsilon_o\beta_e c r_o^2) \cong 60\ I_e r/r_o^2 \text{ volts/m} , \qquad (6)$$

where $\beta_e c$ is the electron velocity (assumed approximately equal to c). The internal forces in a relativistic beam approximately cancel each other and the electron beam will experience a focusing force due to the ion beam of approximately the strength given by Eq. (5). If the inward forces on ions and electrons are set equal, the electron current required proves to be about 80,000 amperes.

Under the forces just discussed, both beams will collapse to a small diameter determined by the original values of their emittances. If we assume (optimistically) an emittance of 100π cm·mrad for the ion beam, the final beam radius proves to be 1.4 mm. The charge density in each ion beam is about 11 coulombs/m^3 and the fusion power is now 17 MW/m^3. But the beam has become so small that the actual power generated per meter of beam is barely over 100 watts.

In Fig. 1 we present a configuration of electrodes for the linear colliding beam system. This arrangement makes possible the generation of 2-MeV beams of ions and electrons and the deceleration of both beams for recovery of the energy stored. One can safely conclude from the parameters just presented that such a system will never be built.

3. Proposed Configuration

What evidently is required to make a colliding beam system viable is a method for storing the ion beams until they interact. If this can be done the input ion currents become quite reasonable. For an output of 1 MW of fusion power all that is required is an input of 60 mA each of deuterons and tritons.

We note further that a prime requirement of the system is that it include strong restoring forces which will prevent coulomb scattered ions from leaving the system before they have time to take part in a fusion reaction.

The system to be proposed includes coincident deuteron and triton beams of the same momentum circulating in approximately circular paths in a rather high magnetic field, focused by a cylindrical beam of electrons travelling along the lines of force of the field. Figure 2 is a cross-section sketch of the geometry.

To satisfy the relative velocity criterion and to have the same momentum the deuteron energy must be 845 keV; the triton energy will be 564 keV. The deuteron velocity will be 9×10^6 m/sec; the triton velocity will be 6×10^6 m/sec. In a field of 6 tesla, the radius of curvature of these beams will be 3.0 cm.

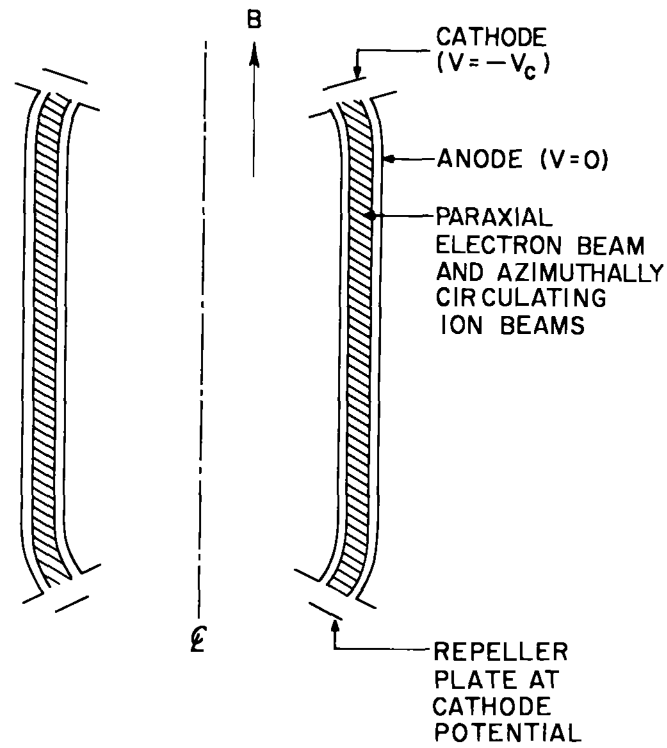

Fig. 2. Cross section through cylindrical colliding beam system.

4. Dynamics of the Electron Beam

We consider first the behavior of the electron beam in the absence of the ion beams. The potential distribution in the beam must satisfy Poisson's equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right)+\frac{\partial^2 V}{\partial z^2}=\frac{4\pi\rho}{\varepsilon} . \quad (7)$$

How the space-charge forces distribute themselves depends on two factors:

First is the boundary conditions. If, for example, we choose to concentrate the space-charge forces in the z-direction we use the continuity of space-charge condition and arrive at the classical $z^{4/3}$ potential distribution. In the present case, however, we choose to eliminate or at least minimize z-variations of potential due to space-charge fields; we can accomplish this by introducing electrodes which hold the outer and inner boundaries of the beam at zero potential (see Fig. 2). Now the primary potential distribution is radial and we can forget the second term on the left-hand side of Eq. (7).

The second factor to be considered is the dependence of electron energy on the potential distribution. If the electrons travel without deviation along the lines of force of the magnetic field, the high retarding space-charge field in the center of the beam will increase the density of the space charge in that region and this fact will need to be taken into acaccount in the solution of Poisson's equation. This problem we shall bypass, and still hope to obtain significant results, by assuming that the electron source has infinitesimal radial extent but that, due to its finite emittance, all electrons are circulating about the lines of force of the magnetic field. The electron orbits will be considered further later in this section. This assumption will permit us to assume a charge density independent of radius and will simplify the solution of Poisson's equation. This solution, with the conditions that it must vanish at the inner radius (r_1) and the outer radius (r_2) of the beam, is rather easily obtained. It is:

$$V=\frac{\pi\rho}{\varepsilon}\left\{r^2-\frac{(r_2^2-r_1^2)\ln r+r_1^2\ln r_2-r_2^2\ln r_1}{\ln(r_2/r_1)}\right\} . \quad (8)$$

If we assume that the radial extent of the electron beam is small we can simplify the expression for the potential by setting

$$r = r_o(1+\delta)$$
$$r_1 = r_o(1-\delta_1)$$
$$r_2 = r_o(1+\delta_2) .$$

Here r_o is a radius lying near the middle of the beam, to be defined more precisely later. With these assumptions, neglecting higher orders of the δ's, Eq. (5) becomes:

$$V=\frac{2\pi\rho\, r_o^2(\delta+\delta_1)(\delta-\delta_2)}{\varepsilon} . \quad (9)$$

The maximum value of V, to this order of approximation, will be for $\delta = (\delta_2 - \delta_1)/2$ ($\simeq 0$).

For example we assume an electron layer 1 mm thick extending from r_1 = 2.95 cm to r_2 = 3.05 cm. For reasons to be presented later we choose a value of ρ of 11.7 coulombs per cubic meter that leads to fusion yields of 20 MW/m^3; substituting for ε the value 1.11×10^{-10} F/m we find that the maximum value of the potential is 165,000 volts. Evidently the primary energy of the electron beam must be a little above this.

The motion of individual electrons is described by the following equations of motion:

$$m\ddot{r}-mr\dot{\theta}^2=eE_r+er\dot{\theta}Bz \quad (10)$$

$$\frac{1}{r}\frac{d}{dt}(mr^2\dot{\theta})=-e\dot{r}B_z \quad (11)$$

$$\dot{z}=\pm\sqrt{2eV_c/m} . \quad (12)$$

Here

$$E_r = [\text{from (9)}]\ \frac{2\pi\rho r_o}{\varepsilon}(2\delta+\delta_1-\delta_2) \quad (13)$$

B_z = constant (6 T for the example to be considered)

V_c = cathode potential (greater than 165 kV in the example given).

The plus or minus sign in Eq. (12) depends on whether the electron is on a primary or reflected path.

From (11)

$$\dot{\theta}=-\frac{eB_z}{2m}\left(1-\frac{r_o^2}{r^2}\right) , \quad (14)$$

where r_o is the radius, somewhere near the middle of the electron sheet, where $\dot{\theta}$ = 0. If we write $r = r_o(1 + \delta)$ etc., Eq. (14) becomes

$$\dot{\theta}=-\frac{eB_z\delta}{m} . \quad (15)$$

Substituting (15) in (10) we obtain

$$mr_o\ddot{\delta}-mr_o(1+\delta)\left(\frac{eB_z\delta}{m}\right)^2$$
$$=\frac{2\pi\rho r_o e}{\varepsilon}(2\delta+\delta_1-\delta_2)-\frac{e^2B_z^2r_o}{m}\delta(1+\delta) . \quad (16)$$

Keeping only first order terms (16) becomes

$$\ddot{\delta}+\delta\left[\left(\frac{eB_z}{m}\right)^2-\frac{4\pi\rho e}{\varepsilon m}\right]=\frac{2\pi\rho e}{\varepsilon m}(\delta_1-\delta_2) . \quad (17)$$

A first integration gives

$$\dot{\delta}^2+\delta^2\left[\left(\frac{eB_z}{m}\right)^2-\frac{4\pi\rho e}{\varepsilon m}\right]$$
$$=\frac{4\pi\rho e}{\varepsilon m}\delta(\delta_1-\delta_2)+\text{integration constant.} \quad (18)$$

If $\dot{\delta}$ = 0 for $\delta = \delta_2$ then the integration constant has the value

$$\delta_2\left[\left(\frac{eB_z}{m}\right)^2\delta_2-\frac{4\pi\rho e}{\varepsilon m}\delta_1\right]$$

and

$$\dot{\delta}^2+(\delta-\delta_2)\left\{(\delta+\delta_2)\left(\frac{eB_z}{m}\right)^2-\frac{4\pi\rho e}{\varepsilon m}(\delta+\delta_1)\right\}=0 .$$

The other value for which $\dot{\delta}$ = 0 must be given by

$$\delta=\frac{-\left(\frac{eB_z}{m}\right)^2\delta_2+\frac{4\pi\rho e}{\varepsilon m}\delta_1}{\left(\frac{eB_z}{m}\right)^2-\frac{4\pi\rho e}{\varepsilon m}} . \quad (19)$$

Evidently singularities will appear in this relation, and the electron sheet will become unstable if $(4\pi\rho/\varepsilon)\cdot(m/eB_z^2)$ approaches unity. For the example we have assumed (B_z = 6 T, ρ = 11.7 coulombs/m^3), this dimensionless quantity has the value 0.21 and the value of δ given by (19) is $0.26\ \delta_1 - 1.26\ \delta_2$. If $\delta_1 \cong \delta_2$, the other value of δ for which $\delta = 0$ is approximately $-\ \delta_1$.

For those familiar with the notation of plasma physics, the quantity $(4\pi\rho/\epsilon)(m/eB_z^2)$ can be recognized as an analog of the plasma β function which is a measure of the ratio of plasma pressure to magnetic pressure. In a plasma β must be kept below unity.

5. Dynamics of the Deuteron and Triton Beams

We assume, initially, that a small number of deuterons and tritons are injected into the space-charge field calculated in the preceding section for the electron sheet. These ions are to move in a flat spiral with negligible velocity in the z-direction. The method of injection into this orbit will be described in the next section.

Motion of the ions will be governed by

$$m_i\ddot{r} - \frac{m_i v_i^2}{r} = eE_r + ev_i B_z \quad , \tag{20}$$

where m_i is the ion mass,
v_i is the ion velocity, given by $m_i v_i = -\ B_z e r_o$,
E_r is given by (13) above.

In the coordinates of the preceding section (20) becomes

$$\ddot{\delta} - \left(\frac{B_z e}{m_i}\right)^2 \frac{1}{1+\delta} = \frac{2\pi\rho e}{\epsilon m_i}(2\delta + \delta_1 - \delta_2) - \left(\frac{B_z e}{m_i}\right)^2 , \tag{21}$$

whence

$$\ddot{\delta} + \delta\left[\left(\frac{B_z e}{m_i}\right)^2 - \frac{4\pi\rho e}{\epsilon m_i}\right] = \frac{2\pi\rho e}{\epsilon m_i}(\delta_1 - \delta_2) \tag{22}$$

whose solution is

$$\delta = \delta_o \sin(\omega t + \varpi) - \frac{2\pi\rho e}{\omega^2 \epsilon m_i}(\delta_1 - \delta_2) , \tag{23}$$

where

$$\omega^2 = \left(\frac{B_z e}{m_i}\right)^2 - \frac{4\pi\rho e}{\epsilon m_i}$$

δ_o and ϖ are determined by initial conditions.

We note that, since ρ represents the density of an electron space charge, the two terms in ω^2 both have the same sign. For deuterons

$$\omega = (8.33 \times 10^{16} + 6.33 \times 10^{19})^{\frac{1}{2}} = 7.96 \times 10^9 .$$

The wavelength of this "betatron oscillation" is 0.71 mm (for deuterons). For tritons, $\omega = 6.50 \times 10^9$ and the betatron wavelength is 0.87 mm. This very short wave oscillation has its frequency determined almost completely by the density of the electronic space charge.

The very strong restoring force provided by the electron cloud should be effective in restoring to their orbits, ions that have undergone coulomb scattering either by other ions or by electrons. This topic is not analyzed in this report but must be given attention in future studies of this device.

6. Ion Injection

Deuterons and tritons are to be injected in such a fashion that they will continue to circulate in the magnetic field and will be unable to escape. During injection they will be given as little axial momentum as possible. Escape of ions at the ends of the devices will be prevented by a local increase in magnetic field. The local increase in axial field will be accompanied by introduction of a radial field component which will serve to reverse the paraxial velocity of the ion beam. Field bumps of this type will be included at both ends of the device as indicated in Fig. 2.

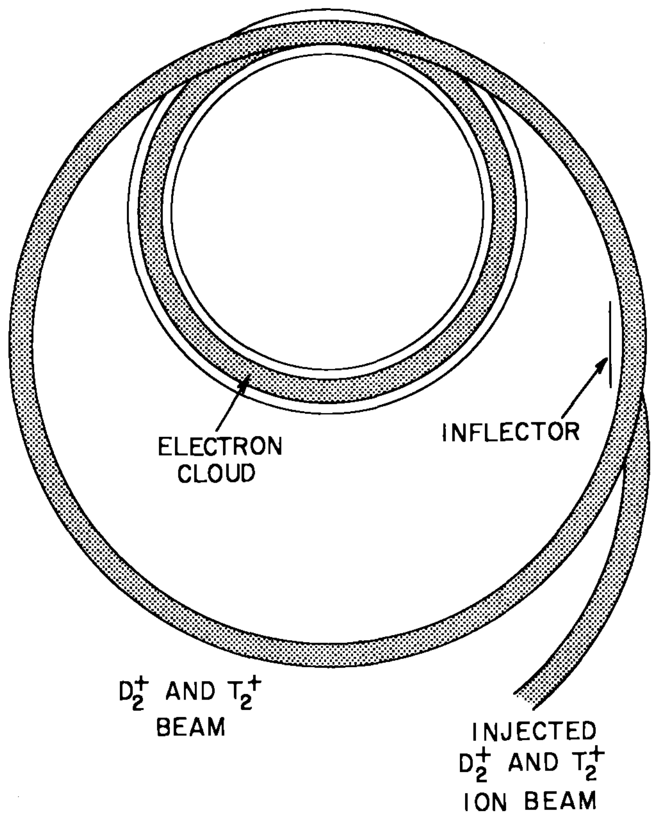

Fig. 3. Injection for cylindrical colliding beam system.

Several methods of injection will occur to the reader. One possible method is illustrated in Fig. 3. This method utilizes molecular ions which pass through an inflector to be deflected onto an orbit that intersects the electron cloud. Something of the order of 10% of the molecular beam should be stripped by electron collisions to become atomic ions which then will switch to orbits through the electron cloud (we assume a stripping cross section of the order of 10^{-16} cm^2). The remainder of the molecular ions will continue on circular orbits and return to the inflector. To prevent their loss by a second deflection, they will make their first entry into the inflector with a small component of paraxial velocity. The inflector is to have a finite axial extent and the paraxial velocity of the ions will be such as to allow the beam to miss the inflector on the second and later revolutions. Thus the molecular beam will re-enter the electron cloud several times until virtually all of the beam has been reduced to atomic ions. This scheme has the virtue of allowing continuous injection. Pulsed injection procedures may, however, prove to be simpler and less demanding of extra magnetic field volume.

Another possible injection method would involve injection of neutral atoms produced by acceleration and stripping of negative ions. These techniques are well established for use in injection into the AGS. The neutral atom beam would be injected tangent to the electron cloud where a fraction of the order of 10% would be ionized and proceed on the desired circular orbits.

The procedure for initially combining the deuteron and triton beams into a single beam involves electrostatic deflection. The two beams, having the same momenta but different energies can be combined by deflection in an electrostatic field.

7. Procedure with High Density Ion Beams

The preceding sections dealt with the motion of ion beams of low intensity in a dense sheet of electrons. It has been shown that, to the first order, the motion of both electrons and ions is stable. There are large restoring forces on the ions which can serve to counteract the undesirable effects of coulomb scattering.

If, now, the electron density is doubled and the ion density is raised to the level of the original electron density, the net density and the electric field pattern will be unchanged. Only the distribution of B_z will be affected by the circulating ion current. For the densities quoted, B_z will drop by about 0.1 T through the thickness. This drop is too small to affect perceptibly the electron or ion motions.

The procedure to increase density would be to raise the injected ion currents. The potential maximum in the electron sheet would then drop and the electron current supply would automatically add electrons to restore the maximum value of the potential.

When the ion density has reached 11.7 coulombs/m^3, the fusion reactions will yield 3600 watts of fusion power per meter length of the system. The deuteron and triton supplies are required to provide only about 200 μA each to maintain this yield.

It would appear that the procedure of pushing ion current and electron current up, maintaining a constant difference between their charge densities, can be continued indefinitely to yield higher and higher levels of fusion power. No doubt, however, instabilities will put a stop to this. The point at which this happens will be difficult to predict theoretically and might more easily be determined experimentally.

DISCUSSION

V. Kelvin Neil (LLL): Your atoms are held in the device electrostatically rather than magnetically?

Blewett: Yes.

V. Kelvin Neil: And each one of these reactions which you're trying to get produces the helium?

Blewett: Yes.

V. Kelvin Neil: And the helium is also then electrostatically held? I'm trying to get to one of the problems in TOKOMAC where the helium is contained and, in effect, quenches the reaction.

Blewett (restating the question): The containment system is essentially an electrostatic containment system and that one of the products of the reaction would be a helium ion and will the helium ions do the same poisoning of the reaction as they do in TOKOMAC reactors? I don't think I can give a very good answer to that, except to say that the helium ions have about 4 MeV of energy which should be enough to kick them out of this region.

Leon Katz (University of Saskatchewan): Is this being proposed as a source of energy or as a source of neutrons for breeders?

Blewett: These are sort of interchangeable, aren't they? I should say that it is being proposed only as an experiment.

Arie Van Steenbergen (BNL): Charge exchange injection of accelerators is not an established fact.