DYNAMIC APERTURE STUDIES OF THE nuSTORM FFAG RING *

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Abstract

FFAG rings with a racetrack configuration are very promising as their flexible design allow for dedicated spaces for injection/extraction, RF cavities etc. A racetrack FFAG is considered as an option for the nuSTORM facility, which aims to deliver neutrino beams produced from the decay of muons stored in a ring with long sections pointing towards detectors. In this paper we discuss the definition of dynamic aperture in these machines and use the PyZgoubi framework to compute the many turn motion in the nuSTORM ring. The roles of machine imperfections and symmetry are discussed.

INTRODUCTION

The idea of using a muon decay to produce a neutrino beam with a perfectly known spectrum and flux composition was invented a long time ago, but gained more attention only when the neutrino oscillation phenomenon was discovered. The concept was developed further into the Neutrino Factory proposal, which was then addressed in several dedicated R&D studies culminating recently with the International Design Study for the Neutrino Factory (IDS-NF) [1].

In order to allow for a start of the neutrino physics experiments based on muon decay using conventional accelerator technology, the nuSTORM project was proposed [2]. In nuSTORM high energy pions produced at the target will be directed into the ring where they will decay to form muons. nuSTORM is recognized as the only facility, that can precisely measure neutrino cross-sections, including the ones for the electron neutrino for which almost no data exists. This is of high importance for all long baseline neutrino experiments as the cross-section uncertainties are a major source of systematic errors. In addition nuSTORM is capable of contributing to the search for sterile neutrinos, in particular by resolving the long standing LSND-MiniBooNE anomaly [3]. nuSTORM, if approved, will be equipped with near detectors for neutrino interaction physics and flux measurement, and far a detector with ≈2 km baseline length for sterile neutrino searches. As the required pion momentum is currently set at 5 GeV/c with a large momentum spread of 10% to achieve a circulating muon momentum of 3.8 GeV/c, a relatively high proton energy at the target is required, which could be obtained from existing proton drivers like the Main Injector at FNAL or SPS at CERN.

As the flux is one of the key elements for a successful neutrino experiment, so it is required that the momentum acceptance of the ring is pushed to ±10% or even ±16%. Although the design based on a standard accelerator lattice with separated function magnets has been proposed, the design based on scaling FFAG lattice, shown in Figure 1, is being developed in parallel. The scaling FFAG allows zero chromaticity with large dynamical acceptance, which enables large momentum spread of the beam with low losses by avoiding dangerous resonances.

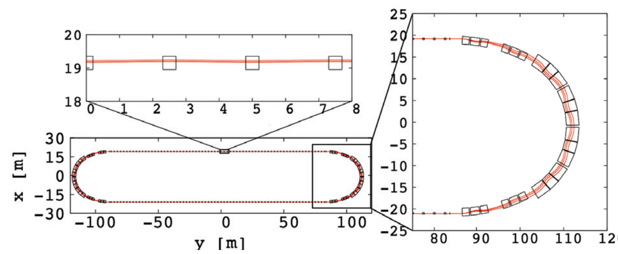

Figure 1: nuSTORM FFAG lattice layout, with insets showing the straight cell and arc sections.

The nuSTORM FFAG lattice is a 500 m circumference racetrack with scaling FFAG magnets in the arcs, novel straight FFAG magnets in the straights and matching sections at the end of the straights. The main parameters are given in Table 1. The arc cells use the traditional scaling FFAG transverse field profile

$$B_z(r) = B_0(r/r_0)^k, \quad (1)$$

where B_0 is the field at the reference radius, r_0, and k is the field index. To achieve the same scaling properties in a straight section this must be modified to

$$B_z(x) = B_0 e^{m(x-x_0)}, \quad (2)$$

where m is the normalised field gradient [4]. The nuSTORM FFAG lattice is discussed further in [5].

The nuSTORM FFAG has previously been studied with a private tracking code [6]. In this paper we describe the implementation of the nuSTORM ring design using PyZgoubi and validate the result against these previous studies. We then calculate the dynamic aperture (DA) for the lattice.

PYZGOUBI

PyZgoubi [7] is an accelerator design framework built around the Zgoubi [8] particle tracking code. Zgoubi is widely used for the study of FFAG accelerators due to its ability to handle large complex magnets including fringe fields and particles with large deviations of position and momentum. PyZgoubi is written in Python and built on the standard scientific libraries NumPy and SciPy. It extends

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Table 1: Main Parameters of the nuSTORM FFAG Ring

Muon Momentum		3.8 GeV/c ± 16 %
Circumference		500 m
Ring Tune		Q_h=7.07 , Q_v=4.15
Arc	r_0	17.6 m
	k	6.043
	B_{0F}	2.07 T
	B_{0D}	2.83 T
Straight	x_0	36.2 m
	m	5.5 m^{-1}
	B_{0F}	0.61 T
	B_{0D}	0.68 T

Zgoubi to allow scripting and optimisation, and contains methods for analysing a lattice, for example to find the lattice functions and the DA. PyZgoubi will allow further optimisation of the nuSTORM design as well as simulating the effect of misalignments on the DA.

nuSTORM FFAG Lattice

The arc and arc matching cells of nuSTORM are standard scaling FFAG FDF cells, and so were implemented using the FFAG element of Zgoubi for each triplet.

Zgoubi does not currently have a specific element for the exponential field straight FFAG magnets used in the straight DFD cell and straight match sections. Instead these were implemented using the DIPOLES element with coefficients found by fitting a multipole expansion (up to x^6) to the field profile over a 1 m transverse range around the closed orbit. Enge functions were used to model the magnet fringe fields. Figure 2 shows the resulting straight cell mid-plane field and closed orbits.

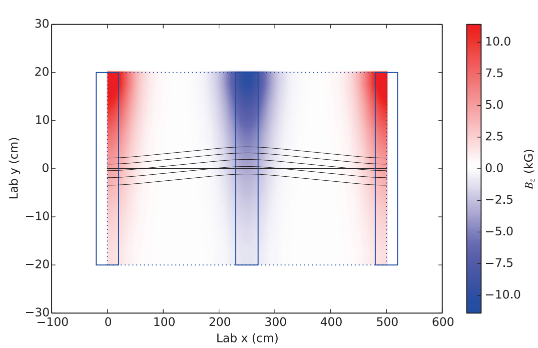

Figure 2: The mid-plane field in the straight cell, with closed orbits for 3.8 GeV ±16%.

Zgoubi can overlap the fringe fields of neighbouring magnets by linear superposition, as long as the magnets are contained within the same element. To achieve this each cell of the lattice is represented by a single 'DIPOLES' element with multiple sub-elements. The fringe fields are truncated at the end of the arc cells in both the previous and PyZgoubi models. Lattice parameters are read from a text table so that they can be easily adjusted for new lattice revisions.

Comparison to Previous Analysis

The field profiles were compared between the PyZgoubi and Lagrange's previous model along the closed orbits at a range of energies. Figure 3 shows the comparison in a section containing a straight cell and the straight matching cell. Differences are around 2%, due to slight difference in the closed orbit.

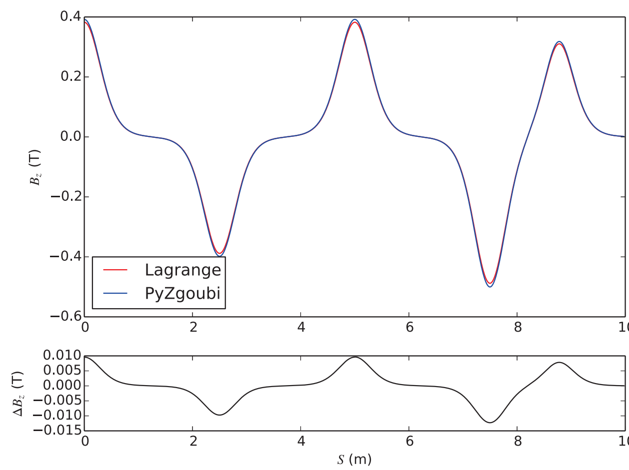

Figure 3: Comparison of magnetic field in PyZgoubi and the previous model along the closed orbit for the reference energy.

The PyZgoubi model gives a small difference in tune at the matching energy of 3.54 GeV: Q_h=7.0466, Q_v=4.1686 compared to Q_h=7.07, Q_v=4.15 in the previous model. This is likely due to slight differences in the tracking method and modelling of the fringe field. PyZgoubi can be used to re-optimise the lattice in order to correct the tune. The field index in the focusing and defocusing magnet in the straight section were varied. Adjustments at the 1% level were sufficient to match the tune to Q_h=7.0681, Q_v=4.1476. Figure 4 shows the tune before and after matching.

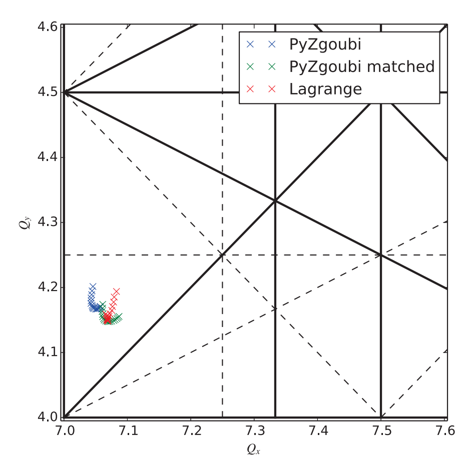

Figure 4: Comparison of ring tune between the previous results and PyZgoubi before and after matching the tune.

Figure 5 shows good agreement of the beta functions between the models at the matching energy, with a small

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difference due to an unresolved mismatch between the arc and straight sections in the PyZgoubi model.

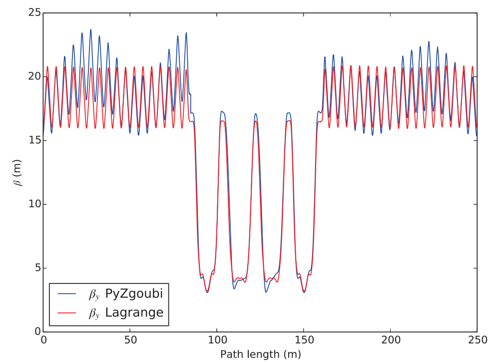

Figure 5: Comparison of horizontal beta function between PyZgoubi and the previous model.

DYNAMIC APERTURE

The focusing provided by the magnets in an accelerator serves to provide a restoring force for particles that deviate transversely from the reference trajectory. Non-linearities in the magnets due to their finite length, edge angles, higher multipoles and errors, may cause transverse motion to become unstable at high amplitudes. This causes beam loss after many turns, as particles drift away from the reference trajectory. It is found that there is an approximate boundary between stable and unstable motion, and the particle amplitude at which this boundary occurs is termed the dynamic aperture by analogy with the real physical machine aperture.

The muons in the nuSTORM ring have a relativistic γ of 36, and so a mean lifetime of 79 µs in which they can travel 23.6 km or 47 turns. We therefore consider trajectory to be stable if it survives 100 turns around the ring. While this method may not measure true long term stability, it is sufficient to calculate losses that will be significant for nuSTORM.

DA is sometimes treated independently in each of the two transverse planes x and y. However in order to be sensitive to coupled resonances we must consider both planes simultaneously. We consider DA as a function of real-space angle θ_{DA}, so that for a given action amplitude, the transverse components of amplitude are given by

$$J_x = J_{\mathrm{DA}} \cos(\theta_{\mathrm{DA}})$$
$$J_y = J_{\mathrm{DA}} \sin(\theta_{\mathrm{DA}}). \quad (3)$$

At large amplitudes the transverse phase space trajectories can be distorted from the ellipse found in the linear regime. If a search for the edge of stability is only made in a single phase space direction then it can give a misleading estimate of the stable area. In order to avoid this PyZgoubi uses particles with a range of phase space angles. The DA algorithm used in PyZgoubi is described in greater detail in [9].

The DA in the nuSTORM FFAG ring was measured over 100 turns at 11 real-space angles. The minimum DA was found to be 387 mm mrad at 36° with horizontal and vertical DA of 1172 and 913 mm mrad respectively, as shown in Figure 6. Figure 7 shows the phase space for horizontal and vertical motion.

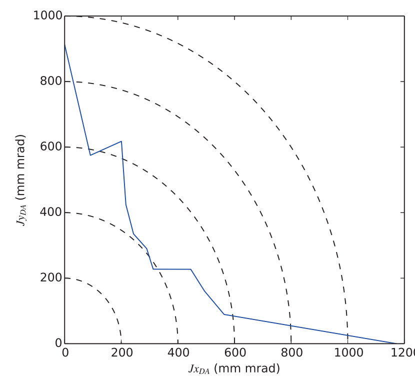

Figure 6: Dynamic aperture as a function of angle for nuSTORM FFAG in PyZgoubi.

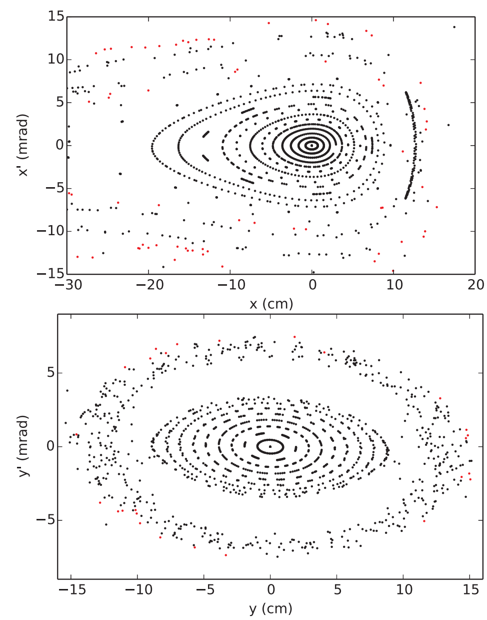

Figure 7: Horizontal (top) and vertical (bottom) phase space. Black points show trajectories that survive 100 turns.

CONCLUSION

We have implemented the nuSTORM FFAG lattice in the PyZgoubi code, including the novel straight scaling FFAG magnets. The close agreement in fields, optics and DA validates the previous results. We currently find a minimum DA of 387 mm mrad. This is comparable to previous results, but may be improved once the optics are resolved. The reduction in DA away from pure horizontal and vertical amplitudes shows the importance of this method.

PyZgoubi will allow us to carry out further advanced studies, for example simulating the effect of magnet errors and misalignments that would be present in the constructed lattice.

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