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NEW PREDICTIONS FOR ELECTROPRODUCTION AMPLITUDES IN THE RESONANCE REGION*

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ABSTRACT

The role played by orbital angular momentum L_z in parton models at $P=\infty$ is delineated. By postulating similar behavior for parton wave functions of equal L_z, we are able to relate the electroexcitation form factors of the low-lying resonances to the elastic form factors of the nucleon.

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The electromagnetic form factors of the hadrons have been considered within the context of parton [1] models by several authors [2-4]. In the absence of more specific properties of the wave functions involved, one cannot extract much information beyond the Drell-Yan-West [2, 3] relation. I would like in this note to propose a statement of approximate universality (eq. (4)) for transition form factors which is suggested (but not proved!) by writing these in terms of parton wave functions in the infinite momentum frame.

In constructing form factors in the parton model, it will suffice (as discussed in ref. [2]) for us to consider the matrix element $<h_2\lambda_2\underset{\sim}{P}'|J_0|h_1\lambda_1\underset{\sim}{P}>$, where $\lambda_1\underset{\sim}{P}$ $(\lambda_2\underset{\sim}{P}')$ denote the helicity and momentum of hadron h_1 (h_2), and $\underset{\sim}{P}'=\underset{\sim}{P}+\underset{\sim}{Q}=P\vec{i}_z + Q\vec{i}_x$. The parton model then leads [2] to a statement that

$$<h_2\lambda_2\underset{\sim}{P}'|J_0|h_1\lambda_1\underset{\sim}{P}> \underset{P\to\infty}{\approx} \sum_a (\psi', j_0^a\, \psi) \tag{1}$$

where j_0 is the bare charge density operator of parton type a, and $\psi(\psi')$ is the initial (final) hadron state expanded onto a Hilbert space of many-parton wave functions at infinite momentum [2]. In momentum space, ψ and ψ' carry as arguments the longitudinal fractions η_i, the transverse momenta $\underset{\sim}{K}_i$ and the z components of spin S_{iz} of the partons at infinite momentum. (The S_{iz} differ from parton helicities only by corrections of $0(Q/P)$.) <u>It is a simple fact (and essential to our discussion) that at infinite momentum the bare charge density j_0 cannot flip the spin of the parton from which it scatters.</u> From the expression (1) it then follows that none of the parton spins are flipped during the contact with the current. Therefore, if the physical matrix element $<h_2\lambda_2\underset{\sim}{P}'|J_0|h_1\lambda_1\underset{\sim}{P}>$ does not vanish at infinite momentum for $\lambda_2\neq\lambda_1$, there is of necessity a flip in L_z, the total z-component of orbital angular momentum of the partons; this consideration implies the presence of components with different L_z's, including

$L_z \neq 0$, in the ∞-momentum parton wave functions of any hadron with spin. As an example, it can be seen that the Pauli form factor of the nucleon F_2 is proportional to such a flip matrix element. The nonvanishing of F_2 then necessitates a classification scheme at $P=\infty$ in which L_z and S_z are not separately diagonal. This is the parton model realization of the conclusion of Dashen and Gell-Mann [5].

Let us expand the hadron ket in terms of eigenstates $|L>$, $|S>$ of L_z, S_z respectively

$$|h, P, J_z = \lambda> = \sum_L C_L^{h\lambda} \, |L> \, |S_z = \lambda - L> \tag{2}$$

with $\sum_L |C_L^{h\lambda}|^2 = 1$. The kets $|L>$ and $|S>$ should display additional labelling to indicate the intermediate angular, momenta which couple to give the final values of L and S, but we omit this for the sake of typographic clarity.

It will now prove convenient to project the state vectors $|L>$ onto kets $|\underset{\sim}{X}_1 \ldots \underset{\sim}{X}_n; \eta_1 \ldots \eta_n>$ which provide a basis set in transverse position space. It is then a straightforward exercise, starting from momentum space and Fourier transforming, to write the form factor in terms of the transverse position space wave functions. We shall only state the result here:

$$\lim_{P\to\infty} P^{-1} <h_2 \underset{\sim}{P}' \lambda_2 | J_0 | h_1 \underset{\sim}{P} \lambda_1>$$

$$= \sum_{n=1}^{\infty} \sum_a e_a \int \prod_{i=1}^{n} (d\eta_i/\eta_i) \, d^2\underset{\sim}{X}_i' \, \delta(\Sigma \eta_i \underset{\sim}{X}_i') \, \delta(\Sigma \eta_i - 1)$$

$$\sum_L \left(C_{L+\Delta\lambda}^{h_2\lambda_2}\right)^* \left(C_L^{h_1\lambda_1}\right) \left(\psi_{L+\Delta\lambda}^{h_2}(\eta_1 \ldots \eta_a \ldots \eta_n; \underset{\sim}{X}_1' \ldots \underset{\sim}{X}_a' \ldots \underset{\sim}{X}_n')\right)^*$$

$$e^{i\underset{\sim}{Q} \cdot \underset{\sim}{X}_a'} \left(\psi_L^{h_1}(\eta_1 \ldots \eta_a \ldots \eta_n; \underset{\sim}{X}_1' \ldots \underset{\sim}{X}_a' \ldots \underset{\sim}{X}_n')\right) \tag{3}$$

with $\Delta\lambda \equiv \lambda_2 - \lambda_1$.

In eq. (3), "a" labels the struck parton, with charge e_a. The $\underset{\sim}{X}'_i$ are equal to $\underset{\sim}{X}_i - \underset{\sim}{X}_0$, the displacement of $\underset{\sim}{X}_i$ from the "center-of-mass" $\underset{\sim}{X}_0 = \Sigma \eta_i \underset{\sim}{X}_i$. All the spin parts have dotted out, conforming to the previous discussion; and the independence of the ψ's of the momenta $\underset{\sim}{P}$ and $\underset{\sim}{P}'$ is a result of the Galilean invariance in the infinite momentum frame.

Needless to say, we know little or nothing about the wave functions ψ. However, let us explore the consequences of a most naive possibility: we conjecture that for equal values of L there is enough similarity in the wave functions such that the Q^2 behavior of the integrals $\int \psi_{L+\Delta\lambda}^{h_2 *} e^{i\underset{\sim}{Q}\cdot\underset{\sim}{X}_a} \psi_L^{h_1}$, for Q not too small, is not very different for any of the nonstrange low-lying baryons with given charge. $|\underset{\sim}{Q}|$ should be at least large enough to deaccentuate the orthogonality properties of the wave functions for $h_1 \neq h_2$, $\lambda_1 \neq \lambda_2$.

We also assume that the representations at $P \to \infty$ are sufficiently mixed such that each of the different L_z values is reasonably represented in the wave functions of all the low-lying nonstrange baryons — more concisely, the $C_L^{h\lambda}$'s for given λ are nonzero over the whole spread of L's for any of the low-lying nonstrange baryons.

<u>The end result of such an assumption is that the matrix element</u> $\lim_{P \to \infty} \langle h_2 \underset{\sim}{P} + \underset{\sim}{Q} \lambda_1 + \lambda \,|J_0|\, h_1 \underset{\sim}{P} \lambda_1 \rangle$ <u>can be written as</u> $F_{\Delta\lambda}(Q^2) \times G(Q^2)$, <u>where</u> <u>$G(Q^2)$ contains all the dependence on</u> $h_1, h_2, \lambda_1, \lambda_2$, <u>but is a very slowly varying function of Q^2.</u>

In this paper we shall deal with the nonstrange baryons and $\Delta\lambda = 0, -1$. In addition, we shall consider separately the isovector and isoscalar currents, so that the operational form of our ansatz is as follows:

Define for each N*

$$\lim_{P\to\infty} \frac{\langle N^* \underset{\sim}{P}' \pm 1/2 \,|J_0^{V,S}|\, N \underset{\sim}{P}\, 1/2\rangle}{\langle N \underset{\sim}{P}' \pm 1/2 \,|J_0^{V,S}|\, N \underset{\sim}{P}\, 1/2\rangle} = r_{\pm}^{V,S}(Q^2) \qquad (4)$$

and hypothesize that for Q^2 away from zero (say $Q^2 \gtrsim 0.5$ GeV^2)

(i) r is a very slowly varying function of Q^2 (i.e., much more slowly varying than the form factors themselves) and

(ii) perhaps $r(Q^2) \sim 1$. This is a guess based on the common normalization of the wave functions. A gross failure of this condition (say $r \gtrsim 10$ or $\lesssim 0.10$) would indicate that something has gone awry in our reasoning, namely that the wave functions are not all that similar in the IMF. So this criterion will play an important role in the initial evaluation of predictions in the model.

We now turn to some examples.

$\underline{\gamma_V N \to P_{33}(1236)}$

In terms of standard invariants, we have for the nucleon

$$\langle NP'\lambda'|J_\mu|NP\lambda\rangle = \bar{u}_{\lambda'}(P')\left[\left(\tau_3 F_1^V + F_1^S\right)\gamma_\mu + i\sigma_{\mu\nu}q^\nu\left(\tau_3 F_2^V + F_2^S\right)/2m\right]u_\lambda(P) \qquad (5)$$

while for the Δ transition we simplify matters by keeping only the magnetic dipole transition (this being a very good approximation to the data [7]):

$$\langle \Delta P'\lambda'|\, J_\mu\,|NP\lambda\rangle = iF^*(Q^2)\bar{\psi}^\alpha_{\lambda'}(P')\,\epsilon_{\mu\alpha\beta\gamma}P'^\beta q^\gamma u_\lambda(P) \quad . \qquad (6)$$

The spinors are normalized to $\bar{u}(P)\,u(P) = 2m$, $\bar{\psi}^\alpha\psi_\alpha = 2m$, where M is the mass of the Δ. F* is related to $G_M^*(Q^2)$ measured by Bartel et al. [8] through the relation

$$G^*_M(Q^2) = \sqrt{\frac{2}{3}}\ m \sqrt{(M+m)^2 + Q^2}\ F^* \qquad (7)$$

Working in the infinite momentum frame [2] $P^\mu = (P + m^2/2P, 0, 0, P)$, $q^\mu = (m\nu/P, Q, 0, 0)$, $q^2 \approx -Q^2$, and ignoring isoscalar pieces, we can process eqs. (5) and (6) through eq. (4) and use eq. (7) to obtain our working relation

$$G^*_M(Q^2)/\left[2\sqrt{1+Q^2/(M+m)^2}\ F_2^V(Q^2)\right] \approx r_-^V(Q^2) \qquad (8)$$

In fig. 1 is plotted the LHS of eq. (8) for the data in ref. [8] with $0.5\ \mathrm{GeV}^2 < Q^2 \leq 2.34\ \mathrm{GeV}^2$. It is seen that $r_-^V(Q^2)$ is indeed very slowly varying over this range of Q^2. (Statistically, in fact, it is quite consistent with being constant ($\simeq 0.76$) with a $\chi^2 = 8$ for 11 degrees of freedom.) Both the average value of $r_-(Q^2)$ and its behavior as a function of Q^2 are in remarkable agreement with our hypotheses (i) and (ii).

$\gamma_V N \rightarrow D_{13}(1525)$

In this case, we work with the three c.m. helicity amplitudes g_1, g_2, g_3 introduced by Bjorken and Walecka [9]. Our ansatz leads to the equations

$$\left(2\omega^*(M+m) - Q^2\right) g_1^{V,S} - 3M(M+m)\, g_2^{V,S}$$

$$+ M(2\omega^* + M+m)\, g_3^{V,S} \approx 2\sqrt{6}\ r_+^{V,S}\ F_1^{V,S}/Q^2 \qquad (9)$$

$$-Q^3(2\omega^* + M+m)\, g_1^{V,S} + 3MQ^3 + QM\left(2\omega^*(M+m) - Q^2\right) g_3^{V,S}$$

$$\approx \sqrt{6}\ r_-^{V,S}\ QF_2^{V,S}/m \qquad (10)$$

with $\omega^* = (M^2 - m^2 - Q^2)/2M$. The ratios $r_\pm^{V,S}$ should satisfy our hypotheses (i) and (ii). [They are also different from the r's in eq. (8).]

The normalization of the g's is such that the c. m. helicity peak cross sections are given by

$$\begin{Bmatrix} \sigma_{1/2} \\ \sigma_{3/2} \\ \sigma_L \end{Bmatrix} = K \begin{Bmatrix} \frac{1}{3} & |g_3|^2 \\ & |g_2|^2 \\ \frac{1}{6}\frac{Q^2}{M^2} & |g_1|^2 \end{Bmatrix}$$

$$\sigma_{tot} = \frac{1}{2}\left(\sigma_{3/2} + \sigma_{1/2}\right) + \epsilon\sigma_L$$

where

$$K = 32\pi\alpha\ M^4\ (E^* + m)\ q^{*4}/(\Gamma(M^2 - m^2) \tag{11}$$

Given our ignorance of the separate isospin amplitudes, we form the linear combinations $g_i^V + g_i^S$ to obtain a single pair of equations for the electroproduction amplitudes from protons. These are similar in form to eqs. (9) and (10) with $g_i^{V,S} \rightarrow g_2^p$, and the right hand sides being replaced by $2\sqrt{6}\left(r_+^V F_1^V + r_+^S F_1^S\right)$ and $\sqrt{6}\left(r^V F_2^V + r_-^S F_2^S\right)$, respectively. Since $F_2^S \ll F_2^V$, we shall especially consider the new version of eq. (9)

$$-Q^3(2\,\omega^* + M+m)\ g_1^p\ +\ 3MQ^3\ g_2^p + QM\left(2\omega^*(M+m) - Q^2\right) g_3^p$$

$$\approx\ \sqrt{6}\ Qr_-^V\ F_2^V/m \tag{12}$$

At $Q^2=0$, eq. (12) reduces to an identity. For $Q^2 > 0$, the amplitude analysis of the cross section data is at present in a very confused state. Recent measurements on $\gamma_V p \rightarrow p\eta^o$ [10, 11] allow one to estimate with some confidence the peak cross section for $\gamma_V p \rightarrow S_{11}(1525)$. Subtracting this from the cross section for $\gamma_V p \rightarrow$ (1525 peak) [12] one can obtain an estimate for $\sigma(\gamma_V p \rightarrow D_{13}(1525))$. Three data points obtained in this way are shown in fig. 2.

We shall examine the implications of eq. (12) by testing three extreme possibilities:

(a) $\sigma_L = \sigma_{3/2} = 0, \quad \sigma_{1/2} \neq 0$.

This allows us to solve eq. (12) for g_3. However the solution is unphysical in that $\sigma_{1/2}$ would develop a pole when $2\omega^*(M+m) - Q^2 = 0$ which happens for $Q^2 = 1.04\ \text{GeV}^2$.

(b) $\sigma_L = \sigma_{1/2} = 0, \quad \sigma_{3/2} \neq 0$.

Solving eq. (12) for g_2, we substitute into eq. (11), to obtain the cross section. The "best" fit in this case (with constant r_-^V) is shown in fig. 2 as a dashed curve. Not only is the fit poor ($\chi^2 = 25$ for 2d/f) but the required value of r_-^V is on the low side ($r_-^V = 0.19$). An increase by more than a factor of 2 in r_-^V over the Q^2 range of the data (0.5 - 1.5) would be required in order to obtain a good fit. This is not satisfactory.

(c) $\sigma_{1/2} = \sigma_{3/2} = 0, \quad \sigma_L \neq 0$.

In this case, we can solve eq. (12) for g_1, and obtain σ from eq. (11), with $\epsilon \simeq 1$. An excellent fit to the data ($\chi^2 = 0.37$ for 2d/f) is obtained for constant $r_-^V = 0.47$. This is shown as the solid curve in fig. 2. The value of r_-^V is entirely in accord with our hypothesis.

Thus our model distinctly favors a strong longitudinal component in the electro-excitation of the D_{13}. This result is favored by at least one recent data analysis [13], but is completely rejected in various versions of the relativistic quark model [14].

One must note, however, that since the factor $2\omega^*(M+m) - Q^2$ multiplying g_3 in eq. (12) is very small in the region $Q^2 \sim 1\ \text{GeV}^2$, a fair amount of $\sigma_{1/2}$ can be tolerated by our model if σ_L is substantial in this region of Q^2.

It is of interest to examine the high Q^2 ($Q^2 >> (M+m)^2$) behavior of eqs. (19) and (10). If G_E and G_M continue to scale at such values of Q^2, we have $F_1^V \sim F_1^S \sim Q^{-4}$, $F_2^V \sim Q^{-6}$. Then it is easy to see that a consistent solution to (9) and (10) in this limit implies

$$g_2(Q^2) \lesssim \text{const} \cdot g_3(Q^2)$$
$$g_1(Q^2) \lesssim \text{const} \cdot g_3(Q^2)/Q^2 \qquad (13)$$

From eqs. (13) and (14), we have the predictions that for large Q^2

$$\sigma_L/\sigma_T \lesssim \text{const}/Q^2$$
$$\sigma_{3/2}/\sigma_{1/2} \lesssim \text{const} \qquad (14)$$

So the dominance by σ_L in the region $Q^2 \sim 1$ discussed above is predicted to be a temporary phenomenon, with σ_T taking over at large Q^2. This is again contrary to the predictions of the quark model [14].

$\gamma_V N \rightarrow S_{11}(1525)$

We may repeat the whole preceding discussion, with the simplification in the present case of dropping the g_2 amplitude.

The analogues of eqs. (9) and (10) are

$$Q^2(M-m)\, g_1^p + Q^2 M\, g_3^p \approx 2\left(r_+^V F_1^V + r_+^S F_1^S\right) \qquad (15)$$

$$-Q^2 g_1^p + M(M-m)\, g_3^p \approx r_-^V F_2^V/m \qquad (16)$$

with

$$\sigma_T = 1/2\ \sigma_{1/2} = 8\pi\alpha\ M^5\ q^{*4}\, |g_3|^2 / \left(\Gamma(M^2-m^2)(E^*+m)\right)$$

and (17)

$$\sigma_L = 8\pi\alpha\ M^5\ q^{*4}\, |g_1|^2/\left(\Gamma(M^2-m^2)(E^*+m)\right)$$

Exploring now the possibilities of total dominance by $\sigma_{1/2}$ or σ_L, we find the following:

(a) $\sigma_{1/2} = 0, \quad \sigma_L \neq 0$.

The "best fit", plotted in fig. 3 as a dashed line against the data of Kummer et al. [10], is obtained for $r_-^V = 0.48$, with $\chi^2 \simeq 25$ for 3 degrees of freedom. The fit is unsatisfactory, the required variation in r_-^V in order to accommodate the data being a factor of 2.3 over our range of Q^2.

(b) $\sigma_L = 0, \quad \sigma_{1/2} \neq 0$.

Plotted as a solid line in fig. 3, this fit is obtained for constant $r_-^V = 0.39$, with $\chi^2 = 1.5$ for 3 degrees of freedom.

So in this case, our model shows a definite preference for an electric dipole excitation, this time in accord with the quark model [14].

As in the case of the D_{13}, one finds in the high Q^2 limit that $\sigma_L/\sigma_T <$ const/Q^2.

To conclude, we have conjectured a form of universality for excitation form factors of the low-lying baryons which is amenable to experimental verification. The model can provide very good fits to the data in the region $Q^2 \simeq 1$ GeV^2 if the electro-excitation of the $P_{33}(1236)$ is almost pure M1; of the $D_{13}(1525)$, almost pure Coulomb; and of the $S_{11}(1525)$, almost pure E1.

Kinematic details, the derivation of eq. (3), as well as further discussion will be given in a subsequent publication.

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FIGURE CAPTIONS

1. Ratio $G_M^*(Q^2)/\left[2\left(1+Q^2/(M+m)^2\right)^{1/2} F_2^V(Q^2)\right]$ vs Q^2. Data taken from ref. [8].

2. Theoretical fits to total D_{13} excitation cross sections. Data extracted from fig. 7 in A. B. Clegg, ref. [11].

3. Theoretical fits to total S_{11} excitation cross section. Data taken from ref. [10], based on a branching ratio $(S_{11} \to \eta p)/(S_{11} \to \text{all}) = 0.55$.

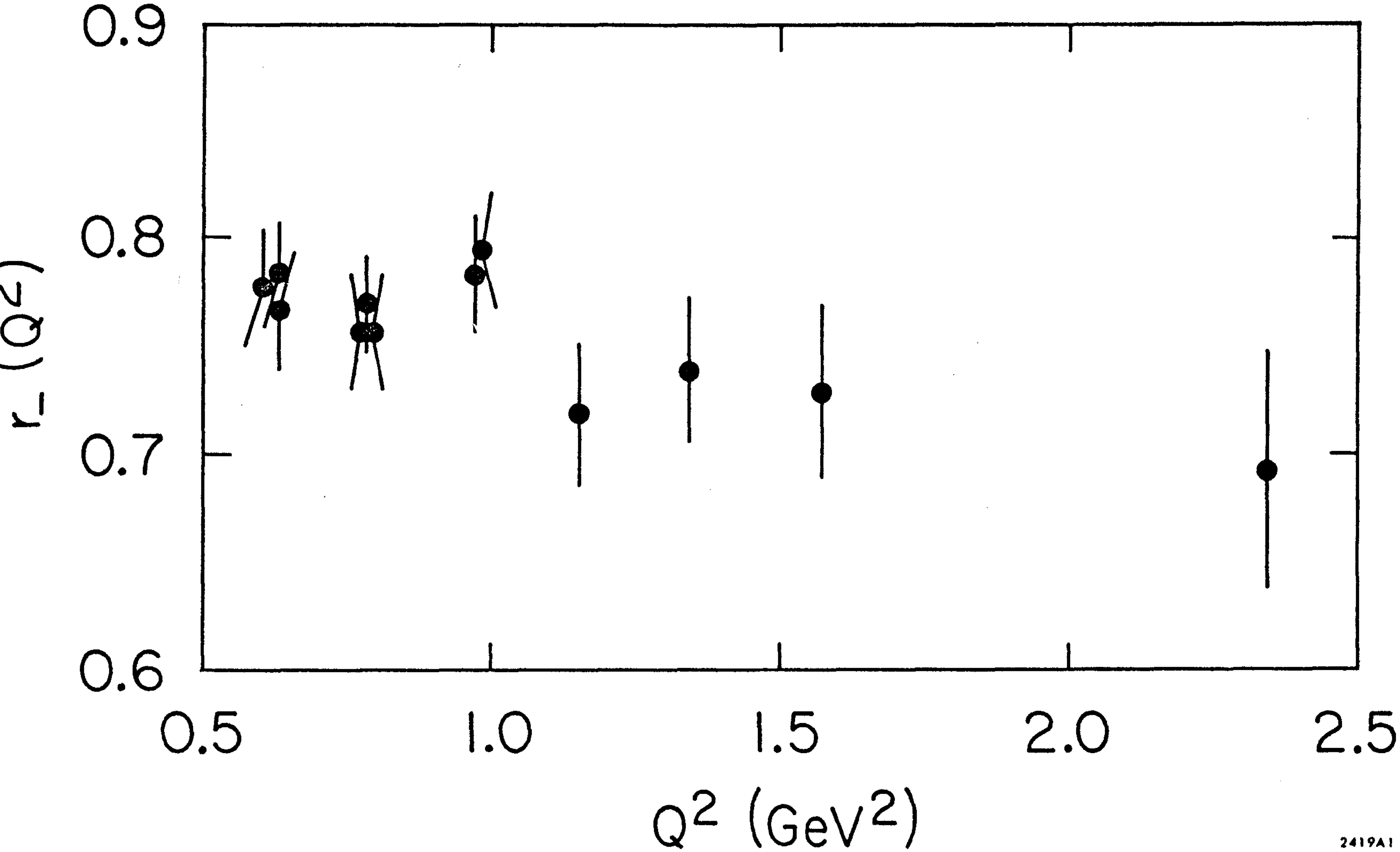


Fig. 1

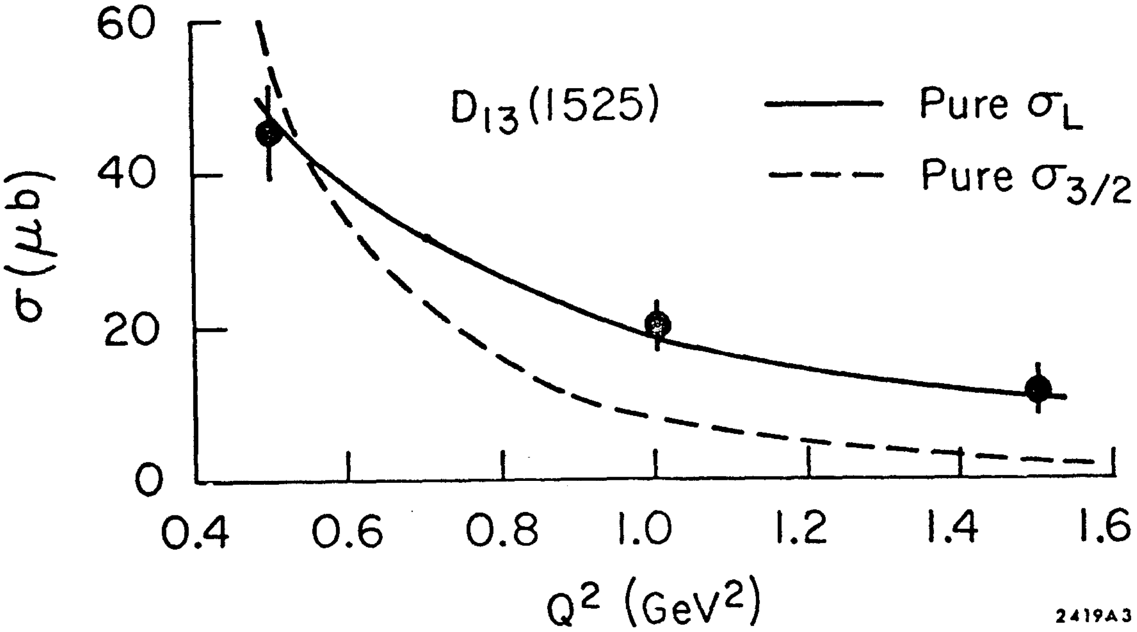
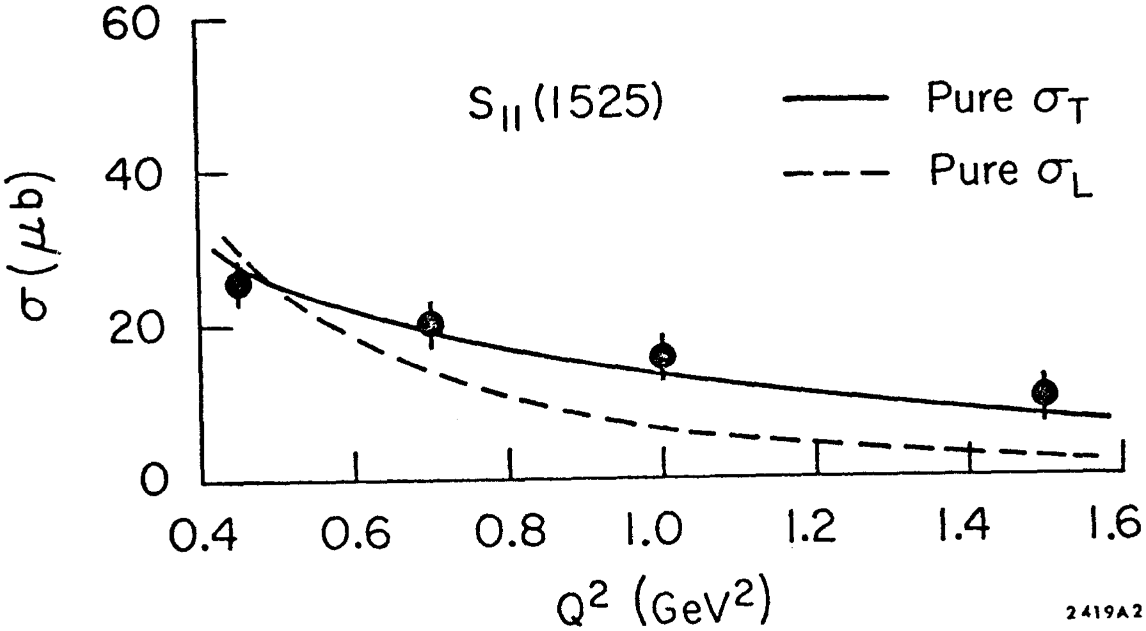

60
40
20
0
σ (μb)
D13(1525)
Pure σL
Pure σ3/2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Q2 (GeV2)
2419A3

Fig. 2


60
40
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σ (μb)
S11(1525)
Pure σT
Pure σL
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Q2 (GeV2)
2419A2