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Study of five-body leptonic decays of tau at Belle experiment

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Study of five-body leptonic decays of tau at Belle experiment

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Abstract.
To perform an extended test of the Lorentz structure of the charged weak interaction, a study of five-body leptonic decays $\tau^- \to l^- l'^+ l'^- \nu_\tau \bar{\nu}_l$ $(l, l' = e,\, \mu)$ is ongoing with data sample which contains about 0.91×10^9 $\tau^+\tau^-$ pairs, collected at Belle. The Standard Model predicts that the Lorentz structure has a V-A structure and this can be tested through the measurement of Michel parameters. In our study we try to give tighter constraints to Michel parameters through the measurement of branching fraction of five-body leptonic decays of tau. With an embedded formalism of total differential decay width which has recently been published, Monte Carlo event generator has been developed within TAUOLA program. In this paper, we finally report our preliminary result of systematic uncertainties of the branching fractions.

1. Introduction

Standard Model (SM) predicts a maximum asymmetry between left-handed and right-handed fundamental fermions, which indicates that the Lorentz structure of the charged weak current has a V-A structure. The most general, Lorentz invariant, derivative-free and lepton-number-conserving four-lepton point interaction matrix element of the $\tau^- \to l^- \bar{\nu}_l \nu_\tau$[1] decay (its SM Feynman diagram is shown in Fig. 1) is given by:

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{i=S,V,T \\ j,k=L,R}} g^i_{jk} \Big[\bar{u}_j(l^-) \Gamma^i v_\xi(\bar{\nu}_l) \Big] \Big[\bar{u}_\kappa(\nu_\tau) \Gamma_i u_k(\tau^-) \Big],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu). \tag{1}$$

Here, ξ and κ are the chiralities of neutrinos and g^i_{jk} is dimensionless coupling constant. In the SM, $g^V_{LL} = 1$ is the only non-zero constant. The differential decay width of the $\tau^- \to l^- l'^+ l'^- \nu_\tau \bar{\nu}_l$ (its SM diagrams are shown in Fig. 2) is written as [1]:

$$\frac{d\Gamma_5}{d^3p_1 d^3p_2 d^3p_3} \propto \frac{1}{E_1E_2E_3}[(Q_{LL}T^Q_{LL} + Q_{RL}T^Q_{RL} + B_{RL}T^B_{RL} + L \leftrightarrow R) + I_\alpha \Re(T^I_\alpha) + I_\beta \Re(T^I_\beta)], \tag{2}$$

[1] Unless specified otherwise, charge-conjugated decays are implied throughout the paper.

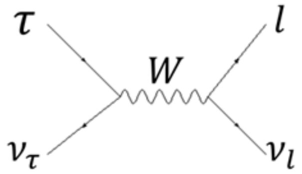

Figure 1: Feynman diagram of the $\tau^- \to l^- \bar{\nu}_l \nu_\tau$ decay in the SM.

Figure 2: Two diagrams for the $\tau^- \to l^- l'^+ l'^- \nu_\tau \bar{\nu}_l$ in the SM.

where (E_1, p_1), (E_2, p_2) and (E_3, p_3) are the energies and momenta of l^-, l'^+ and l'^-, respectively; T^i_{jk} ($i = Q$, B, I; $j, k = L$, R) is a known function of the kinematical variables, Q_{ij} ($i, j = L$, R), B_{LR}, B_{RL}, I_α and I_β are eight Michel parameters [2]. The SM predicts that only $Q_{LL} = 1$ has a non-zero value. If some of the other Michel parameters have non-zero values, we can say there are contributions from New Physics (NP) models. Precision measurement of the branching fractions of $\tau^- \to l^- l'^+ l'^- \nu_\tau \bar{\nu}_l$ decays allows one to constrain Michel parameters. The theoretical formula for the branching fraction (BR) can be obtained integrating Eq. (2):

$$BR_{\text{exp}} = BR_{\text{SM}}[Q_{LL} + bQ_{LR} + cB_{LR} + Q_{RR} + dQ_{RL} + eB_{RL} + \Re(f)I_\alpha + \Re(g)I_\beta] + BR_{\text{NLO}}. \quad (3)$$

Here, BR_{exp} is experimentally measured branching fraction, BR_{SM} is the branching fraction predicted in the SM (see Table 1), BR_{NLO} is radiative correction [3, 4]. The coefficients b, c, d, e, f, and g are the integrated terms of the T^Q_{RL}, T^B_{RL}, T^Q_{LR}, T^B_{LR}, T^I_α, and T^I_β functions, which is renormalized by the BR_{SM}, respectively. BR_{SM} is the value of integrated terms related to T^Q_{LL} functions. Since the T^i_{jk} functions have symmetry under the transformation $L \leftrightarrow R$, the coefficients b, c, d, and e have a relation $b = d$, $c = e$ [1]. Here the BR_{NLO} is negligibly small in our study: BR_{NLO} becomes about $O(0.1)\%$ of BR_{SM} in our study [5]. We therefore do not need to consider the effect of BR_{NLO} for the constraints of Michel parameters.

Table 1: Branching fractions of the $\tau^- \to l^- l'^+ l'^- \nu_\tau \bar{\nu}_l$ decays predicted by the SM.

Mode	BR from Ref. [1]
$\tau^- \to e^- e^+ e^- \bar{\nu}_e \nu_\tau$	$(4.21 \pm 0.01) \times 10^{-5}$
$\tau^- \to \mu^- e^+ e^- \bar{\nu}_\mu \nu_\tau$	$(1.984 \pm 0.004) \times 10^{-5}$
$\tau^- \to e^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau$	$(1.247 \pm 0.001) \times 10^{-7}$
$\tau^- \to \mu^- \mu^+ \mu^- \bar{\nu}_\mu \nu_\tau$	$(1.183 \pm 0.001) \times 10^{-7}$

2. Formula of Branching Fraction Depending on Michel Parameters

To obtain the value of coefficients b, c, d, e, f, and g as introduced in Eq. (3), we perform a numerical integral of Eq. (2).

$$b \sim g = \frac{BR_{\mathrm{NP}}^{b\sim g}}{BR_{\mathrm{SM}}} = \frac{1}{\Gamma_{\mathrm{SM}}} \int d(PS) d\Gamma_{\mathrm{NP}}^{b\sim g}. \tag{4}$$

Here the $d\Gamma_{\mathrm{NP}}^{b\sim g}$ is the differential decay width related to T^i_{jk} function. Eq. (4) is written by:

$$\begin{aligned} b \sim g &= \frac{1}{BR_{\mathrm{SM}}} BR_{\mathrm{NP}}^{b\sim g} = \frac{1}{\Gamma_{\mathrm{SM}}} \int d\Gamma_{\mathrm{NP}}^{b\sim g} d(PS) = \frac{1}{\Gamma_{\mathrm{SM}}} \int \frac{d\Gamma_{\mathrm{NP}}^{b\sim g}}{d\Gamma_{\mathrm{SM}}/\Gamma_{\mathrm{SM}}} [(d\Gamma_{\mathrm{SM}}/\Gamma_{\mathrm{SM}}) d(PS)] \\ &= \frac{1}{\Gamma_{\mathrm{SM}}} \int \frac{d\Gamma_{\mathrm{NP}}^{b\sim g}}{d\widetilde{\Gamma}_{\mathrm{SM}}} [(d\widetilde{\Gamma}_{\mathrm{SM}}) d(PS)] \approx \frac{1}{\Gamma_{\mathrm{SM}}} \frac{1}{N_{\mathrm{gen}}} \sum_{\mathbf{x}\in\Omega} \frac{d\Gamma_{\mathrm{NP}}^{b\sim g}(\mathbf{x})}{d\widetilde{\Gamma}_{\mathrm{SM}}(\mathbf{x})} = \frac{1}{N_{\mathrm{gen}}} \sum_{\mathbf{x}\in\Omega} \frac{d\Gamma_{\mathrm{NP}}^{b\sim g}(\mathbf{x})}{d\Gamma_{\mathrm{SM}}(\mathbf{x})}, \end{aligned} \tag{5}$$

where $d\widetilde{\Gamma}_{\mathrm{SM}} = d\Gamma_{\mathrm{SM}}/\Gamma_{\mathrm{SM}}$ is a normalized differential decay width of the SM, Ω is the allowed phase space (PS), $\mathbf{x}$ follows the distribution of $d\Gamma_{\mathrm{SM}}$, and N_{gen} is the number of generated events. The detail of method is described in the Appendix A. The calculated result for the $\tau^- \to l^- l'^+ l'^- \nu_\tau \bar{\nu}_l$ decays are given as follows.

$$\begin{aligned} BR_{\mathrm{exp}}^{\tau^-\to e^-e^+e^-\bar{\nu}_e\nu_\tau} &= BR_{\mathrm{SM}}^{\tau^-\to e^-e^+e^-\bar{\nu}_e\nu_\tau} \{[Q_{LL} + (1.051 \pm 0.036) Q_{LR} + (-0.2053 \pm 0.1431) B_{LR} \\ &\quad + L \leftrightarrow R] + (0.2416 \pm 0.0002) I_\alpha + (0.8606 \pm 0.0001) I_\beta\}. \end{aligned} \tag{6}$$

$$\begin{aligned} BR_{\mathrm{exp}}^{\tau^-\to \mu^-e^+e^-\bar{\nu}_\mu\nu_\tau} &= BR_{\mathrm{SM}}^{\tau^-\to \mu^-e^+e^-\bar{\nu}_\mu\nu_\tau} \{[Q_{LL} + (1.220 \pm 0.049) Q_{LR} + (-0.8717 \pm 0.1957) B_{LR} \\ &\quad + L \leftrightarrow R] + (181.3 \pm 0.1) I_\alpha + (104.4 \pm 0.1) I_\beta\}. \end{aligned} \tag{7}$$

$$\begin{aligned} BR_{\mathrm{exp}}^{\tau^-\to e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau} &= BR_{\mathrm{SM}}^{\tau^-\to e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau} \{[Q_{LL} + (1.226 \pm 0.001) Q_{LR} + (-0.8456 \pm 0.0001) B_{LR} \\ &\quad + L \leftrightarrow R] + (0.2253 \pm 0.0001) I_\alpha + (0.5231 \pm 0.0001) I_\beta\}. \end{aligned} \tag{8}$$

$$\begin{aligned} BR_{\mathrm{exp}}^{\tau^-\to \mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau} &= BR_{\mathrm{SM}}^{\tau^-\to \mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau} \{[Q_{LL} + (1.216 \pm 0.005) Q_{LR} + (-0.8459 \pm 0.0005) B_{LR} \\ &\quad + L \leftrightarrow R] - (18.00 \pm 0.01) I_\alpha + (197.3 \pm 0.1) I_\beta\}. \end{aligned} \tag{9}$$

The large error in some coefficients in the first two modes derives from a peculiarity in the matrix elements related to factor $1/q^2_{e^+e^-}$ (q, is a 4-momentum of off shell γ ($q = p_{e^+} + p_{e^-}$)). The terms related to $1/q^2_{e^+e^-}$ become very large when the q has small value ($O(m_e) \sim O(\mathrm{MeV})$) and this causes large error. The explicit expressions through the coupling constants g^i_{jk} can be found in Appendix B.

3. Selections, background

The selection process is organized in two stages. The first-stage selection suppresses beam background and rejects most of the background from the non-$\tau^+\tau^-$ processes, and select the candidate of signal-events. In the second stage, we select the samples enriched with the signal events. Each event is divided into two hemispheres using the plane perpendicular to the direction of the thrust axis in the center-of-mass system (c.m.s.). One charged track is reconstructed in one hemisphere (tag side) and three charged tracks identified as electrons or muons - in the other hemisphere (signal side).

Full MC simulation of the signal modes and the $\tau^+\tau^-$ generic MC sample were used to study detection efficiencies of the signal events and background contamination from the other τ decays, see Table 2. Recently published analytical formalism for the $\tau^- \to l^- l'^+ l'^- \bar{\nu}_l \nu_\tau$ differential decay width [1] was used to develop Monte Carlo event generator in the framework of the KKMC+TAUOLA generator [6, 7]. A Monte Carlo (MC) sample of 4 million signal decays was used for evaluating the background and calculate efficiencies. The detector response is simulated

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by a GEANT3-based program [8].
Expected numbers of the signal events with the whole Belle statistics are calculated taking into account MC detection efficiencies, branching fractions, and the number of the $\tau^+\tau^-$ pairs at Belle ($N_{\tau\tau} = 0.91 \times 10^9$). Figure 4 shows the distribution of the invariant mass of three charged outgoing leptons for the selected events.
For the selection, we use a parameter $\sum_{i<j} \cos\theta_{ij}$ which was introduced in Ref. [9], is a sum of $\cos\theta_{ij}$ where θ_{ij} is an angle between two leptons in the signal-hemisphere (Fig. 3).

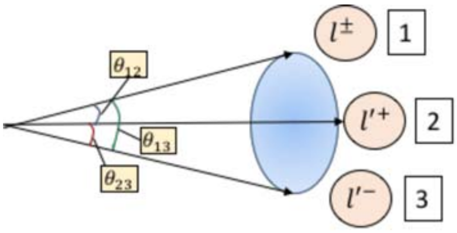

Figure 3: Explanation of θ_{ij}.

We apply the cut $\sum_{i<j} \cos\theta_{ij}$ should be larger than 2.90, 2.93, 2.70, and 2.85 for $e^-e^+e^-\bar{\nu}_e\nu_\tau$, $\mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$, $e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$, and $\mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$, respectively. For the modes $\tau^- \to e^-e^+e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$, this selection suppresses the backgrounds such as $\tau^- \to \pi^-\pi^0\nu_\tau \to \pi^-(\gamma\gamma)\nu_\tau \to \pi^-((e^+e^-)\gamma)\nu_\tau$ and $\tau^- \to \pi^-\pi^0\nu_\tau \to \pi^-(e^+e^-\gamma)\nu_\tau$. For the modes $\tau^- \to e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$, this selection suppresses the backgrounds such as $\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$ and $\tau^- \to \pi^-\pi^+\pi^-\pi^0\nu_\tau$. In these backgrounds, the pion is misidentified as electron or muon. The particle identification (PID) we used is described in Appendix C.
Because of the gamma conversion in the material of the detector, a radiative leptonic decay $\tau^- \to l^-\bar{\nu}_l\nu_\tau\gamma$ becomes a dominant background for the $\tau^- \to e^-e^+e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$ modes. To suppress this background, we apply the cut that the reconstructed gamma conversion point r_{xy} (is a distance to the beam axis (z-axis)) should be smaller than 1.6 cm and 1.5 cm, respectively.
Since two muons in the modes $\tau^- \to e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$ are generated by off-shell photon conversion, the invariant mass of $\mu^+\mu^-$ tends to be smaller. To suppress $\tau^- \to \pi^-\pi^0\nu_\tau \to \pi^-(\gamma\gamma)\nu_\tau \to \pi^-((e^+e^-)\gamma)\nu_\tau$ and $\tau^- \to \pi^-\pi^+\pi^-\nu_\tau$ processes whose branching fraction is large, we require the invariant mass of $\mu^+\mu^-$ should be smaller than 0.4 GeV only for $\tau^- \to e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$.
Also the number of photons are used. For the $\tau^- \to e^-e^+e^-\bar{\nu}_e\nu_\tau$ and the $\tau^- \to e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$ modes, we require the number of photons in the signal-hemisphere should be less than one, and when the case of one we require the energy of photon should be less than 0.5 GeV. For the $\tau^- \to \mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$, we require the number of photons in the (signal+tag)-hemisphere should be less than five, and the sum of energy of photon in the signal-hemisphere should be less than 0.3 GeV. For the $\tau^- \to \mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$, we require there should be no photon in the signal-hemisphere. Since the number of selections we use is many, we introduced some important selections to suppress the backgrounds. Table 2 shows the result after applying all selections.

Table 2: Summary of the signal detection efficiencies and background contaminations.

τ^- decay mode	$e^-e^+e^-\bar{\nu}_e\nu_\tau$	$\mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$	$e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$	$\mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$
Detection efficiency	(1.769±0.004)%	(1.204±0.003)%	(3.561±0.006)%	(1.674±0.004)%
Main backgrounds	$e^-\bar{\nu}_e\nu_\tau\gamma$ $\rightarrow e^-\bar{\nu}_e\nu_\tau(e^+e^-)$ $\pi^-\pi^0\nu_\tau$ $\rightarrow \pi^-(\gamma\gamma)\nu_\tau$ $\rightarrow \pi^-((e^+e^-)\gamma)\nu_\tau$ (mis-ID π as e) $e^-\bar{\nu}_e\nu_\tau$	$\mu^-\bar{\nu}_\mu\nu_\tau\gamma$ $\rightarrow \mu^-\bar{\nu}_\mu\nu_\tau(e^+e^-)$ $\pi^-\pi^0\nu_\tau$ $\rightarrow \pi^-(e^+e^-\gamma)\nu_\tau$ $\pi^-\pi^0\pi^0\nu_\tau$ $\rightarrow \pi^-(\gamma\gamma)(\gamma\gamma)\nu_\tau$ $\rightarrow \pi^-((e^+e^-)\gamma)(\gamma\gamma)\nu_\tau$ (mis-ID π as μ)	$\pi^-\pi^0\nu_\tau$ $\rightarrow \pi^-(\gamma\gamma)\nu_\tau$ $\rightarrow \pi^-((e^+e^-)\gamma)\nu_\tau$ $\pi^-\pi^+\pi^-\nu_\tau$ (mis-ID π as μ, e)	$\pi^-\pi^0\nu_\tau$ $\rightarrow \pi^-(\gamma\gamma)\nu_\tau$ $\rightarrow \pi^-((e^+e^-)\gamma)\nu_\tau$ $\pi^-\pi^+\pi^-\nu_\tau$ (mis-ID π as μ)
Expected number of signal events	1300	430	8	4
Fraction of the signal	47%	50%	37%	16%

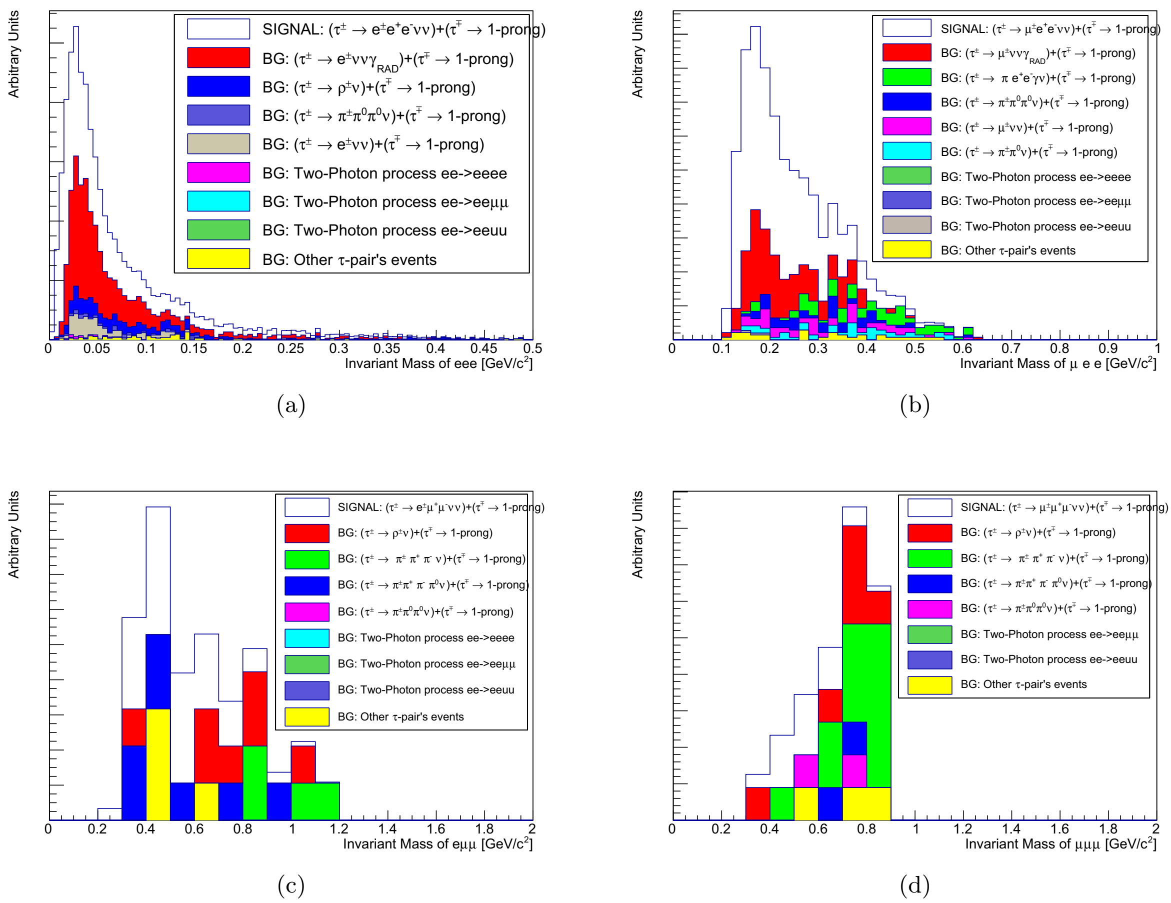

Figure 4: Distribution of the invariant mass of three charged outgoing leptons, $M_{ll'l'}$, for the selected events: (a) $\tau^- \rightarrow e^-e^+e^-\bar{\nu}_e\nu_\tau$, (b) $\tau^- \rightarrow \mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$, (c) $\tau^- \rightarrow e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$, and (d) $\tau^- \rightarrow \mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$. Open histograms show signal events, hatched histograms - background contributions.

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4. Preliminary Result of Systematic Uncertainties of Branching Fractions

Following systematic uncertainties are taken into account:

- **Particle Identification (PID) correction**
  This error is considered because the efficiency of PID is different between MC samples and experimental data.
- **Tracking efficiency**
  This error considers the effect of difference of charged track's efficiency between MC samples and experimental data.
- **Trigger Correction**
  We also consider the difference of trigger efficiency's correction between MC samples and data.
- **Tag-side (1-prong side)**
  This error is for tag-side.
- **Luminosity**
  This error is for an uncertainties of luminosity.
- **Background (BG)**
  The systematic error is estimated as the statistical error of backgrounds.
- **Selection Cut**
  The systematic error is estimated from the fluctuation of the number of events between experimental data and MC by shifting the cut point of selection.

Table 3, 4, 5, and 6 show the preliminary result of systematic uncertainties of branching fraction of $\tau^- \to e^-e^+e^-\bar{\nu}_e\nu_\tau$, $\tau^- \to \mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$, $\tau^- \to e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$, and $\tau^- \to \mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$, respectively. Since we have mainly two kinds of data set called SVD1 and SVD2, we show the systematic uncertainties for each data sets. The "Selection Cut" has not been estimated yet. For this estimation, we used the data of SVD1's integrated luminosity about 140 fb^{-1} ($\Upsilon(4S)$ on-resonance) and the data of SVD2's integrated luminosity 560 fb^{-1} ($\Upsilon(4S)$ on-resonance) [2].

5. Conclusion

With the purpose of constraining the Michel parameters, we study the five-body leptonic decays of tau $\tau^- \to l^-l'^+l'^-\bar{\nu}_l\nu_\tau$ at Belle. As our results, we obtained the formula of branching fraction of $\tau^- \to l^-l'^+l'^-\bar{\nu}_l\nu_\tau$ expressed by the Michel parameters. Also the selections and background's contamination are determined through the full Monte Carlo simulation. The estimation of systematic uncertainties of branching fractions is ongoing. After finishing the estimation, we measure the branching fractions and constrain the Michel parameters.

6. Acknowledgements

We strongly appreciate Pablo Roig and Denis Epifanov for the helpful discussion.

[2] In Belle, there are mainly two data sets called SVD1 and SVD2. In this study, we used only $\Upsilon(4S)$ on-resonance data in SVD1 and SVD2 whose total integrated luminosity is about 700 fb^{-1}. Including the other data not $\Upsilon(4S)$ on-resonance, the total integrated luminosity becomes about 1000 fb^{-1}.

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Table 3: Systematic uncertainties of the $\tau^- \to e^- e^+ e^- \bar{\nu}_e \nu_\tau$ (preliminary)

contents	syst. error (SVD1)	syst. error (SVD2)
PID correction	7.3%	5.0%
Tracking efficiency	1.1%	1.1%
Trigger correction	0.1%	0.1%
Tag-side	0.35%	0.35%
Luminosity	1.4%	1.4%
Background	5.5%	2.8%
Selection Cut	–	–
Total	12.1%	6.0%

Table 4: Systematic uncertainties of the $\tau^- \to \mu^- e^+ e^- \bar{\nu}_\mu \nu_\tau$ (preliminary)

contents	syst. error (SVD1)	syst. error (SVD2)
PID correction	6.9%	6.4%
Tracking efficiency	1.1%	1.1%
Trigger correction	0.1%	0.1%
Tag-side	0.35%	0.35%
Luminosity	1.4%	1.4%
Background	9.6%	4.8%
Selection Cut	–	–
Total	13.0%	8.2%

Table 5: Systematic uncertainties of the $\tau^- \to e^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau$ (preliminary)

contents	syst. error (SVD1)	syst. error (SVD2)
PID correction	8.7%	7.4%
Tracking efficiency	1.1%	1.1%
Trigger correction	0.1%	0.1%
Tag-side	0.35%	0.35%
Luminosity	1.4%	1.4%
Background	71.0%	35%
Selection Cut	–	–
Total	72.0%	36.0%

Table 6: Systematic uncertainties of the $\tau^- \to \mu^- \mu^+ \mu^- \bar{\nu}_\mu \nu_\tau$ (preliminary)

contents	syst. error (SVD1)	syst. error (SVD2)
PID correction	6.2%	8.4%
Tracking efficiency	1.1%	1.1%
Trigger correction	0.1%	0.1%
Tag-side	0.35%	0.35%
Luminosity	1.4%	1.4%
Background	71.0%	35%
Selection Cut	–	–
Total	72.0%	36.0%

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Appendix A. Formula of Differential Decay Width and Coefficients $b \sim g$

To perform the numerical integration, we generate the MC events which follow a distribution of SM-case. The differential decay width of $\tau^- \to l^- l'^+ l'^- \nu_\tau \bar{\nu}_l$ decays given by [1]:

$$\frac{d\Gamma_5}{dp_1 d\Omega_1 dp_2 d\Omega_2 dp_3 d\Omega_3} = \frac{p_1^2 p_2^2 p_3^2}{3 \cdot 2^{18} \cdot \pi^{10} M E_1 E_2 E_3} |\mathcal{M}_{\text{total}}|^2, \tag{A.1}$$

Including the NP parts, $|\mathcal{M}_{\text{total}}|^2$ is given by [1]:

$$|\mathcal{M}_{\text{total}}|^2 = e^4 |G_{ll'}|^2 [(Q_{LL} T_{LL}^Q + Q_{RL} T_{RL}^Q + B_{RL} T_{RL}^B + L \leftrightarrow R) + I_\alpha \Re(T_\alpha^I) + I_\beta \Re(T_\beta^I)], \tag{A.2}$$

where $|G_{ll'}|^2$ is a scale factor, and T_{jk}^i are the known function of kinematical variables as described in Sec. 1. In the SM, only $Q_{LL} T_{LL}^Q$ remains ($Q_{LL} = 1$), i.e., $T_{LL}^Q = T_{\text{SM}}$. In Ref. [1], $T_{RL}^Q, T_{RL}^B, T_\alpha^I$, and T_β^I are given by,

$$T_{RL}^Q = \frac{1}{3}(16 T_{RL}^S - T_{RL}^V), T_{RL}^B = \frac{4}{3}(T_{RL}^V - 4 T_{RL}^S), T_\alpha^I = 4 T_{LRRL}^{SV}, T_\beta^I = 2 T_{LLRR}^{SV}. \tag{A.3}$$

where the $T_{RL}^S, T_{RL}^V, T_{LRRL}^{SV}$, and T_{LLRR}^{SV} are known functions which are given in Ref. [1]. We calculate the integral of $T_{RL}^Q, T_{RL}^B, T_\alpha^I$, and T_β^I from the integral of $T_{RL}^S, T_{RL}^V, T_{LRRL}^{SV}$, and T_{LLRR}^{SV}. For the convenience of calculation, we perform a transformation:

$$dp_1 d\Omega_1 dp_2 d\Omega_2 dp_3 d\Omega_3 \to dp_{3\text{body}} d\Omega_{3\text{body}} d\widetilde{M}_{3\text{body}}^2 d\widetilde{\Omega}_3 d\widetilde{M}_{2\text{body}}^2 d\widetilde{\Omega}_2, \tag{A.4}$$

where, $p_{3\text{body}}$ is a momentum of three leptons frame ($l^- l'^+ l'^-$) in a tau-rest frame, $\Omega_{3\text{body}}$ is a solid angle of three leptons frame ($l^- l'^+ l'^-$) in a tau-rest frame, $\widetilde{M}_{3\text{body}}$ is a invariant mass of three leptons frame ($l^- l'^+ l'^-$), $\widetilde{\Omega}_3$ is a solid angle of two leptons frame ($l'^+ l'^-$) in a 3-body ($l^- l'^+ l'^-$) rest frame, $\widetilde{M}_{2\text{body}}$ is a invariant mass of two leptons frame ($l'^+ l'^-$), and $\widetilde{\Omega}_2$ is a solid angle of one lepton (l'^+ or l'^-) in a 2-body ($l'^+ l'^-$) rest frame.
The Jacobian is calculated as this,

$$\left| \frac{\partial(p_1, \Omega_1, p_2, \Omega_2, p_3, \Omega_3)}{\partial(p_{3\text{body}}, \Omega_{3\text{body}}, \widetilde{M}_{3\text{body}}^2, \widetilde{\Omega}_3, \widetilde{M}_{2\text{body}}^2, \widetilde{\Omega}_2)} \right| = \frac{E_1 E_2 E_3}{p_1^2 p_2^2 p_3^2} \frac{p_{3\text{body}}^2}{(E_1 + E_2 + E_3)} \frac{\widetilde{p}_{2\text{body}}}{\widetilde{M}_{3\text{body}}} \frac{\widetilde{p}_{1\text{body}}}{\widetilde{M}_{2\text{body}}}. \tag{A.5}$$

After the transformation, the differential decay width is written by:

$$\begin{aligned} \frac{d\Gamma_5}{dp_1 d\Omega_1 dp_2 d\Omega_2 dp_3 d\Omega_3} \left| \frac{\partial(p_1, \Omega_1, p_2, \Omega_2, p_3, \Omega_3)}{\partial(p_{3\text{body}}, \Omega_{3\text{body}}, \widetilde{M}_{3\text{body}}^2, \widetilde{\Omega}_3, \widetilde{M}_{2\text{body}}^2, \widetilde{\Omega}_2)} \right| &= \frac{e^4 |G_{ll'}|^2}{3 \cdot 2^{18} \cdot \pi^{10} M} \frac{p_{3\text{body}}^2}{(E_1 + E_2 + E_3)} \\ \times \frac{\widetilde{p}_{2\text{body}}}{\widetilde{M}_{3\text{body}}} \frac{\widetilde{p}_{1\text{body}}}{\widetilde{M}_{2\text{body}}} [(Q_{LL} T_{\text{SM}} + Q_{RL} T_{RL}^Q &+ B_{RL} T_{RL}^B + L \leftrightarrow R) + I_\alpha \Re(T_\alpha^I) + I_\beta \Re(T_\beta^I)]. \end{aligned} \tag{A.6}$$

We introduce some expressions;

$$d\Gamma_{\text{SM}} = F T_{\text{SM}} \tag{A.7}$$
$$d\Gamma_{RL}^S = F T_{RL}^S \tag{A.8}$$
$$d\Gamma_{RL}^V = F T_{RL}^V \tag{A.9}$$
$$d\Gamma_{LRRL}^{SV} = F T_{LRRL}^{SV} \tag{A.10}$$
$$d\Gamma_{LLRR}^{SV} = F T_{LLRR}^{SV}. \tag{A.11}$$

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Here $F = \frac{e^4|G_{ll'}|^2}{3\cdot 2^{18}\cdot\pi^{10}M}\frac{p^2_{\rm 3body}}{(E_1+E_2+E_3)}\frac{\widetilde{p}_{\rm 2body}}{\widetilde{M}_{\rm 3body}}\frac{\widetilde{p}_{\rm 1body}}{\widetilde{M}_{\rm 2body}}$. From the expressions of Eq. (A.3), we can calculate the constants $b \sim g$ through the calculation of integral of $d\Gamma^S_{RL}, d\Gamma^V_{RL}, d\Gamma^{SV}_{LRRL}$, and $d\Gamma^{SV}_{LLRR}$.

$$\begin{aligned}
d(b) &= d(d) = d\Gamma^Q_{RL}/\Gamma_{\rm SM} = \frac{1}{3\Gamma_{\rm SM}}(16d\Gamma^S_{RL} - d\Gamma^V_{RL}),\\
d(c) &= d(e) = d\Gamma^B_{RL}/\Gamma_{\rm SM} = \frac{4}{3\Gamma_{\rm SM}}(d\Gamma^V_{RL} - 4d\Gamma^S_{RL}),\\
d(f) &= d\Gamma^I_{\alpha}/\Gamma_{\rm SM} = 4d\Gamma^{SV}_{LRRL}/\Gamma_{\rm SM},\\
d(g) &= d\Gamma^I_{\beta}/\Gamma_{\rm SM} = 2d\Gamma^{SV}_{LLRR}/\Gamma_{\rm SM}. \qquad \text{(A.12)}
\end{aligned}$$

This calculation is performed by using the similar formula to Eq. (5), for example,

$$b = \frac{\Gamma^Q_{RL}}{\Gamma_{\rm SM}} = \frac{1}{N_{\rm gen}}\sum_{\mathbf{x}\in\Omega}\frac{16d\Gamma^S_{RL}(\mathbf{x}) - d\Gamma^V_{RL}(\mathbf{x})}{3d\Gamma_{\rm SM}(\mathbf{x})} \qquad \text{(A.13)}$$

We generate each variable $(p_{\rm 3body}, \Omega_{\rm 3body}, \widetilde{M}_{\rm 3body}, \widetilde{\Omega}_3, \widetilde{M}_{\rm 2body}, \widetilde{\Omega}_2)$ randomly so that its variables follow the distribution of $d\Gamma_{\rm SM}$.

Appendix B. Formula of Branching Fraction with Coupling Constants

The formalisms Eq. (6), (7), (8), and (9) can be rewritten to the formalism expressed through the coupling constants g^i_{jk} as follows.

$$\begin{aligned}
BR^{e^-e^+e^-\bar{\nu}_e\nu_\tau}_{\rm exp} = BR^{e^-e^+e^-\bar{\nu}_e\nu_\tau}_{\rm SM}\{&|g^V_{LL}|^2(1+\frac{|g^S_{LL}|^2}{4|g^V_{LL}|^2}) + (0.2501\pm0.0001)|g^S_{RL}|^2 + (0.8465\pm0.1073)|g^V_{RL}|^2\\
&+(2.693\pm0.215)|g^T_{RL}|^2 - (0.1540\pm0.1073)g^S_{RL}g^{T*}_{RL} + (0.4303\pm0.0001)g^S_{LL}g^{V*}_{RR}\\
&+(0.06039\pm0.00004)g^S_{LR}g^{V*}_{RL} + (0.3623\pm0.0002)g^V_{LR}g^{T*}_{RL} + L\leftrightarrow R\}.\\
BR^{\mu^-e^+e^-\bar{\nu}_\mu\nu_\tau}_{\rm exp} = BR^{\mu^-e^+e^-\bar{\nu}_\mu\nu_\tau}_{\rm SM}\{&|g^V_{LL}|^2(1+\frac{|g^S_{LL}|^2}{4|g^V_{LL}|^2}) + (0.2506\pm0.0001)|g^S_{RL}|^2 + (0.3484\pm0.1468)|g^V_{RL}|^2\\
&+(1.699\pm0.294)|g^T_{RL}|^2 - (0.6538\pm0.1468)g^S_{RL}g^{T*}_{RL} + (52.20\pm0.01)g^S_{LL}g^{V*}_{RR}\\
&+(45.33\pm0.01)g^S_{LR}g^{V*}_{RL} + (272.0\pm0.1)g^V_{LR}g^{T*}_{RL} + L\leftrightarrow R\}.\\
BR^{e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau}_{\rm exp} = BR^{e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau}_{\rm SM}\{&|g^V_{LL}|^2(1+\frac{|g^S_{LL}|^2}{4|g^V_{LL}|^2}) + (0.2536\pm0.0001)|g^S_{RL}|^2 + (0.3802\pm0.0001)|g^V_{RL}|^2\\
&+(1.775\pm0.001)|g^T_{RL}|^2 - (0.6342\pm0.0001)g^S_{RL}g^{T*}_{RL} + (0.2616\pm0.0001)g^S_{LL}g^{V*}_{RR}\\
&+(0.05633\pm0.00001)g^S_{LR}g^{V*}_{RL} + (0.3380\pm0.0001)g^V_{LR}g^{T*}_{RL} + L\leftrightarrow R\}.\\
BR^{\mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau}_{\rm exp} = BR^{\mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau}_{\rm SM}\{&|g^V_{LL}|^2(1+\frac{|g^S_{LL}|^2}{4|g^V_{LL}|^2}) + (0.2512\pm0.0001)|g^S_{RL}|^2 + (0.3704\pm0.0001)|g^V_{RL}|^2\\
&+(1.745\pm0.015)|g^T_{RL}|^2 - (0.6344\pm0.0004)g^S_{RL}g^{T*}_{RL} + (98.67\pm0.01)g^S_{LL}g^{V*}_{RR}\\
&-(4.510\pm0.001)g^S_{LR}g^{V*}_{RL} - (27.060\pm0.006)g^V_{LR}g^{T*}_{RL} + L\leftrightarrow R\}.
\end{aligned} \qquad \text{(B.1)}$$

Appendix C. Particle Identification (PID) in the Selection

To identify electron and muon, we use following method. A likelihood ratio cut $\mathcal{P}_\mu = \mathcal{L}_\mu/(\mathcal{L}_\mu + \mathcal{L}_\pi + \mathcal{L}_K) > 0.7$ is applied to select muons [10]. To identify electrons the likelihood ratio parameter $\mathcal{P}_e = \mathcal{L}_e/(\mathcal{L}_e + \mathcal{L}_{\rm other})$ is constructed [11]. To select $\tau^- \to e^-e^+e^-\bar{\nu}_e\nu_\tau$ events, we require $\mathcal{P}_{e^-} > 0.7$ and $\mathcal{P}_{e^-}$, $\mathcal{P}_{e^+} > 0.5$. For the $\tau^- \to \mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$ events $\mathcal{P}_{e^-}$, $\mathcal{P}_{e^+} > 0.5$; while for the $\tau^- \to e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$ events $\mathcal{P}_{e^-} > 0.7$.

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References

[1] A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP **1604**, 185 (2016) [arXiv:1508.01822 [hep-ph]].
[2] W. Fetscher, H. J. Gerber and K. F. Johnson, Phys. Lett. B 173, 102 (1986).
[3] M. Fael and C. Greub, arXiv:1611.03726 [hep-ph].
[4] G. M. Pruna, A. Signer and Y. Ulrich, arXiv:1611.03617 [hep-ph].
[5] Private communication of Matteo Fael.
[6] S. Jadach, B. F. L. Ward and Z. Was, Comput. Phys. Commun. **130** (2000) 260.
[7] Z. Was, Nucl. Phys. Proc. Suppl. **98**, 96 (2001).
[8] R. Brun *et al*, GEANT 3.21, CERN Report No. DD/EE/84-1 (1984).
[9] M. S. Alam *et al*, Phys. Rev. Lett. 76, 2637 (1996).
[10] A. Abashian *et al.*, Nucl. Instr. Meth. A **491**, 69 (2002).
[11] K. Hanagaki *et al.*, Nucl. Instr. Meth. A **485**, 490 (2002).