In the previous part of this work preliminary experimental results were reported on radiation acceleration of plasma $H_0^1$ and $H_1^1$ by waves of a circular wave guide. The plasma was created either by a spark gun or by a source with pressure drop. For the diagnostics there are used electrical probes, SV Ch-methods and electrostatic analyzer of particle energies. Containment of the plasma was accomplished by auxiliary magnetic fields. Accelerated ions were produced with energies greater than 10 kev.

In the second part of the work the forces of radiation pressure were derived for the simplest models of the plasma bunches or clots, and the acceleration process was investigated for these bunches or clots by considering relativistic effects.

1. Introduction

The practical realization of the radiation method of plasma acceleration is obviously connected with the use of a different kind of wave guide devices, in which the plasma clot moves, accelerated by an electromagnetic field. Two such wave guide apparatuses were prepared recently in the physics institute of AN USSR, which differed from each other in the type of accelerating wave and in the method of plasma injection. The first experiments show that radia-
tion acceleration of plasma was accomplished in both apparatuses. In the radiotechnical institute of AN USSR a theoretical analysis was conducted at the same time on the possibilities of the radiation method.

Thus the present communication consists of a short report of all these studies.

2. Experimental Results

Both apparatuses used the very same SVCh generator of 10 cm range, operating in a single pulse cycle of 8 µ sec duration. The average density of the power flux per tube crosssection did not exceed $8 \cdot 10^3$ wt/cm$^2$, the KSVN of the entire wave guide system (without plasma) was not less than 1.3. The accelerating wave guides consisted of tubes of circular transverse cross section with stainless steel walls 1 mm thick. The vacuum in the tubes was of the order of $10^{-7}$ to $10^{-6}$ mm Hg.

In the first setup (Fig. 1) the acceleration took place on a wave $H_{01}$, and the plasma source was a spark injector, introduced along the radius to the axis of the tube and equipped with a "symmertrizing counterweight" to decrease distortions of the field. A spark gun (maximum current in an aperiodic cycle 1.8 ka with duration 0.3 µ sec) produced the plasma with a total amount of ions $10^{15}$ to $10^{16}$, containing about 50% ions of atomic hydrogen. The initial concentration of the plasma exceeded $10^{12}$ cm$^{-3}$, and the initial velocity of injection was of the order of $5 \cdot 10^6$ cm/sec.

To reduce plasma leakage to the tube walls there were used either uniform longitudinal or quadrupolar transverse quasi-stationary magnetic fields.
These fields possessed a period of about 1 m sec and, hence, practically did not change during movement of the plasma in the tube (20 µ sec.). During operation of the transverse field, to avoid retardation, the plasma was insulated from the wave guide walls by a glass tube.

Recording of the accelerated ions was accomplished with the help of shielded electric probes, introduced from the face of the wave guide. By comparing the ion current oscillograms on the probe when the SVCh power was turned off and turned on, one could estimate the energy and number of accelerated ions.

Figure 1. Schematic diagram of accelerator on the $H_{01}$ wave. (1) SVCh generator. (2) Ferrite valve. (3) Transformer with type of wave $|H_{10} \rightarrow H_{01}|$. (4) Accelerating waveguide. (5) Spark plasma injector. (6) Magnetic system. (7) Pumping section. (8) Absorbing SVCh, loading. (9) Shielded electric probe. (10) Diagnostic ports. (11) Vacuum-tight SVCh window. (12) Section with two detector heads. (13) High-vacuum pump.

Figure 2. Ion current oscillograms of shielded electric probe. (1) SVCh power turned off. (2) SVCh power turned on.
Figure 2 presents two such matched oscillograms which were taken with a longitudinal field of 300 gauss.

Treatment of the probe measurements showed that the number of known accelerated particles amounts to a value of the order of $10^{12}$, and at the same time this number varied but little with change in operations cycle of the spark current, when the total number of injected particles varied by one order. The maximum velocity of the accelerated ions exceeded $10^8$ cm/sec.

With the help of the SVCh-probe (the detector head), when placed in the diagnostic windows of the tube, it was discovered (Fig. 3) that after shooting of the plasma gun the wave guide is practically shut off. The formation of a "stopper or plug" for the $H_{01}$ wave is confirmed by measurements of the reflected SVCh power: analysis of modulation of the signals (Fig. 4), arriving from two detector heads, shows that reflection of the wave from the plasma amounts to 90%.

![Figure 3. Signals of SVCh probe, placed at the window of the accelerating waveguide. (1) The envelope of the SVCh pulse without plasma. (2) Same during shooting of the plasma gun.](image)

![Figure 4. Signals from two detector heads, displaced by a distance of 10 cm. (1) First head. (2) Second Head.](image)
In the second setup (Fig. 5) the accelerating field was the main
$H_{11}$ wave of the circular tube, and to decrease plasma leakage to the walls
there was used the magnetic field of six straight line conductors with alternating
direction of the currents. The plasma was produced by a source with
differential pressure drop. The plasma column with a concentration above
$10^{12}$ cm$^{-3}$ was formed during discharge between electrodes A-K in a longitudinal magnetic field. The pressure of the admitted hydrogen in the anode
A and cathode K was $10^{-2}$ mm Hg. The pressure in the wave guide was $1 \cdot 10^{-6}$
mm Hg. The difference in the pressures was maintained by four diffusion
pumps.

After creation of the plasma column the voltage on the A-K was turned off and the magnetic field turned on for the straight line conductors (rise time 25 $\mu$ sec). When this field reached a value of the order of $10^3$ gs in the "stoppers" and the plasma was squeezed away from the walls, the SVCh wave was switched on. During motion of the plasma there takes place a transposition or shifting of the standing waves in the tube, recorded by the detector head. The observed modulation of the envelope of the SVCh oscillations is similar to that presented in Fig. 4.

The recording of the accelerated particles is accomplished with the help of an electric probe (unsymmetric double probe), which was shielded from the SVCh field, and which was introduced on the side of the loading and could be shifted along the axis of the wave guide. In Figure 6 oscillograms are presented for the envelope of the SVCh oscillations and the signal from the probe, located at a distance of 30 cm from the place of injection. When the SVCh wave was absent there were no signals from the probe. From the delayed time of plasma arrival one could compute the average velocity of its motion, which in order of magnitude is equal to $10^7$ cm/sec.

Figure 6. Oscillograms. (1) Envelope of the SVCh. (2) Signal from the electric probe.
To measure the energy of the accelerated ions an electrostatic analyzer was used with 127° deflection of the particles, and an electron multiplier in the role of ion detector, located at a distance of 70 cm from the injector. It permitted one to measure the energy within the limits of 1 to 100 kev. Figure 7 presents the curves for the dependence of the detector current on the energy of the recorded ions. When the SVCh power was increased the energy spectrum was shifted to the region of higher energies. When the sensitivity of the scheme was increased, ions were recorded with energies up to 50 kev.

![Figure 7. Energy spectrum of the accelerated ions.](image)


To derive the forces of radiation pressure which act on the plasma bunch or clot, it is easiest to proceed from the laws of conservation.

In a plane electromagnetic wave, propagated in free space, the flux density of the pulse is equal to the average energy density \( w \). Therefore, the total force acting on a stationary bunch (or clot) is equal to

\[
\vec{F} = (S_{\text{scat}} + S_{\text{absorp}}) \vec{wn} - \vec{n}_{\text{scat}} = \vec{F}_{\text{scat}} + \vec{F}_{\text{absorp}},
\]

where \( S_{\text{scat}} \) and \( S_{\text{absorp}} \) are the effective diameters (cross sections?) of
scattering and absorption, \( \vec{n} \) is a unit direction vector at the incident wave, and \( \overrightarrow{I}_{\text{scat}} \) is the total pulse (momentum) carried by the scattered wave.

If the bunch or clot dimensions are small in comparison with the wave length, then with the exception of special values of the frequency, when multipole resonances appear, the problem of finding \( S_{\text{scat}}, S_{\text{absorp}} \) and \( \overrightarrow{I}_{\text{scat}} \) boils down to determining the electric \( \vec{p} \) and magnetic \( \vec{m} \) of the dipole moments induced in the bunch, and subsequent calculation of the field produced by them. Specifically, in this approximation

\[
\overrightarrow{I}_{\text{scat}} = \frac{k}{3} \text{Re} \left[ \vec{p} \cdot \vec{m}^* \right], \quad k = \frac{\omega}{c}
\]

(2)

Further, by way of a model of a small bunch or clot we shall consider a uniform spheroid with an axis of rotation which is parallel to the propagation direction of the wave. For this model we have

\[
\rho_s = V \alpha_\| E_u, \quad \overrightarrow{p}_\perp = V \alpha_\perp E_\perp, \quad m_\| = -V \rho_s H_\|, \quad \overrightarrow{m}_\perp = -V \rho_s \overrightarrow{H}_\perp,
\]

(3)

where \( V \) is the volume of the bunch, and \( \alpha_\| \) and \( \alpha_\perp \) are related with the dielectric "penetrability" (or constant) of the plasma \( E \) by the formulas

\[
\alpha_\| = \frac{1}{4\pi} \frac{\varepsilon - 1}{1 + (\varepsilon - 1)M}, \quad \alpha_\perp = \frac{1}{2\pi} \frac{\varepsilon - 1}{2 + (\varepsilon - 1)(1 - M)}.
\]

(4)

Here \( M \) is their longitudinal coefficient of depolarization \(^7\) (\( M = 1/3 \) for a sphere, \( M \approx \frac{1}{1} \) for a disc, and \( M \ll \frac{1}{1} \) for a needle).

In a plasma without loss \( E = 1 - \omega_0^2 / \omega^2 \) and

\[
\alpha_\| = \frac{1}{4\pi} \frac{\omega^2}{M \omega^2 - \omega_i^2}, \quad \alpha_\perp = \frac{1}{2\pi} \frac{\omega^2}{(1 - M) \omega^2 - \omega_i^2}.
\]

(5)

The zeros in the denominator (5) determine the resonance frequencies, in whose vicinity instead of equation (5) one needs to use more accurate expressions.
(see reference 8), which take into account re-emission of the bunch or clot. It is essential that at resonance itself the polarizability $\alpha$ becomes a purely imaginary quantity.

For $|\varepsilon| < 1$ the magnetic moment of the bunch is small. The diamagnetism of the bunch appears appreciably only if $\omega_0 \gg \omega$ and the depth of penetration $k^{-1}_0 = c/\omega_0$ is small in comparison with the dimensions of the bunch. It behaves then, in the first approximation, like an ideal conductor and

$$\beta_\perp = \frac{1}{4\pi (1-M)}$$

and

$$\beta_\parallel = \frac{1}{4\pi (1+M)}$$

(6)

From formulas (2 through 6) and the general expression of 7 for the diameter of scattering, it follows that in the case of an ideal-conducting spheroid

$$F_{\text{scat}} = \frac{2}{3\pi} \left( 3 + \frac{M^2}{1-M^2} \right) k^\nu \hat{\mathbf{V}} \cdot \hat{\mathbf{w}}$$

(7)

In the other limiting case $\omega_0 \ll \omega$ (rarefied plasma) for any form or shape of bunch we have

$$F_{\text{scat}}' = \frac{1}{6\pi} k^\nu \hat{\mathbf{V}} \cdot \hat{\mathbf{w}}$$

(8)

If the effective frequency of collisions $\hat{\mathbf{V}}$ is small in comparison with $\omega$, then for $\omega_0 \lesssim \omega$ the absorption is mainly determined by the imaginary part $\alpha$. Specifically, for a rarefied plasma ($\omega_0 \ll \omega$) we have

$$F_{\text{absorp}} = \frac{1}{c} \frac{\omega_0}{\omega} \hat{\mathbf{V}} \cdot \hat{\mathbf{w}}$$

(9)

On the other hand, for $\omega \ll \omega_0$ and small depth of penetration the absorption is explained by the imaginary part $\beta$. For example, for a sphere with radius $R$ we have

$$F_{\text{absorp}} = 3\pi R^4 \frac{1}{c} \hat{\mathbf{w}}$$

(10)
But if \( \nu \gg \omega \) ("cold" plasma), then the losses are found according to the usual formulas for skin-effect. Specifically, for a sphere whose radius is large in comparison with the skin-layer thickness \( \delta = \frac{c}{\omega} \sqrt{\frac{2
u}{\omega}} \), we have

\[
F_{\text{absorp}} = 3\pi R^2 \frac{\nu}{\omega} \sqrt{\frac{n\omega}{\omega}}. \tag{II}
\]

In a wave guide without losses the energy and momentum are transmitted only by propagating waves \( (\omega > \omega_{\text{crit}}) \). In each characteristic wave the integrated fluxes of momentum \( \mathcal{P} \) and energy \( J \) through a cross section are related by the equation \( \mathcal{P} = Ju/c^2 \), where \( u \) is a group velocity of the wave: \( u = c \frac{\hbar}{k} \), where \( h \) is a propagation constant.

Let us normalize the characteristic waves of the tube per unit energy flux. Assume that a single wave of the \( \lambda \) th type is incident on a stationary bunch or clot located inside of the wave guide. The currents excited by it in the bunch or clot will appear as a source of secondary waves with amplitudes \( a_n \) (\( n > 0 \) in the direction of acceleration and \( n < 0 \) in the opposite direction). The law of conservation of momentum gives

\[
\dot{u}_t = u_t \left| \dot{q}_t \right|^2 + \sum_{n \neq \lambda} u_n \left| q_n \right|^2 + c^2 \sum_{n \neq \lambda} a_n^2 \tag{12}
\]

(the sign of \( u_n \) agrees with the sign of \( n \)). Furthermore, from the law of conservation of energy it follows that

\[
\{ = \left| \dot{q}_t \right|^2 + \sum_{n \neq \lambda} \left| q_n \right|^2 + Q_t \tag{13}
\]

where \( Q_\lambda \) is the loss in the bunch.

From equations (12) and (13) it follows immediately that for an incident wave of power \( J \) the force accelerating the bunch is equal to

\[
\vec{F} = \frac{J}{c^2} \left\{ u_t Q_t + \sum_n (u_t - u_n) |q_n|^2 \right\}. \tag{14}
\]
The quantities $Q_n$ and $|a_n|^2$ which go into it are energy coefficients of absorption and transformation of the incident wave. For the very simplest models of the bunch or clot they are calculated easily, but in the more complex cases one can use the results derived by numerical method or from experimental data.

In the frequently encountered case in practice, when there are excited in the tube only waves of a single type ($u_\pm = u$, $u_+ = u$, and $u_- = -u$), that is, we have a regime with "a single wave", the formula (13) assumes the form

$$F = \frac{J}{\mathcal{L}} u \left( Q + 2 |a_n|^2 \right).$$

(15)

For a completely reflecting "stopper", equation (15) gives

$$F = 2 \frac{J}{\mathcal{L}} u = \frac{J}{\mathcal{L}} \frac{J}{\mathcal{L}}.$$

(16)

In the case of a small bunch, characterized by the dipole moments $\mathbf{p}$ and $\mathbf{m}$, the calculation carried out according to the general formulas of excitation theory of wave guides gives, for example, for a good conducting spheroid in a circular tube

$$F_{\text{scat}} = 876 \frac{\gamma^2}{\sqrt{1-\gamma^2}} \frac{\sqrt{1}}{(1-M)^2} \frac{J}{\mathcal{L}},$$

(17)

$$F_{\text{absorp}} = 11.8 \frac{\delta S}{S} \cdot G_1 \frac{J}{\mathcal{L}}.$$

(18)

in the field of the $H_{01}$ wave and

$$F_{\text{scat}} = 93 \frac{[\gamma^2 - \gamma^2 (1-M)^2]}{\gamma^2(1-\gamma^2)} \frac{\sqrt{1}}{(1-M)^2} \frac{J}{\mathcal{L}},$$

(19)

$$F_{\text{absorp}} = 19 \frac{\delta S}{S} \cdot G_1 \frac{J}{\mathcal{L}}.$$

(20)
in the field of a rotating (depending on the angle $e^{i\phi}$) $H_{11}$ wave. Here $\eta = \omega_{\text{crit}} / \omega = \frac{1}{k} \sqrt{k^2 - \omega^2}$, $S$ is the area of the transverse cross section of the wave guide, $\beta$ is its radius, $\delta$ is the thickness of the skin-layer, and $\Sigma$ is the surface area of the bunch or clot. The quantities $G_{||}$ and $G_{\perp}$ are dimensionless factors, which are dependent on $M$. For a sphere we have $G_{||} = G_{\perp} = 3/2$. The formulas (17) and (19) hold true for a "single wave" regime.

In contrast to free space, during acceleration in a wave guide there exists a critical achievable energy of the bunch $E_m$, corresponding to the group velocity of the accelerating wave. An exception is the case of acceleration on a main wave of a "not singly connected" wave guide, for which $u = c$.

Although these differences from the point of view of the possibilities of contemporary radiotechnologies do not appear as yet to be actual, however, in the future, for which there are hopes now for the radiation method itself, the more deeply investigated problems of dynamics of relativistic bunches or clots can be of very practical interest. Therefore we shall present here still a few more formulas for the accelerating forces of a thin plasma ring in the known wave field of a coaxial line.

For a rarefied plasma ($\omega_0 \ll \omega$) we have

$$ F_{\text{scat}} = \left( \frac{\omega^2 \sqrt{C_1}}{\pi c^2} \right)^2 \frac{J}{1c \omega}, \quad F_{\text{absorp}} = \frac{2 \omega^2 \sqrt{C_1}}{c^2 \omega} \frac{J}{\omega^2} \quad (21) $$

and for an "ideal-conducting" ring

$$ F = \left( \frac{3 \sqrt{C_1}}{\pi c^2} \right)^2 \frac{\omega^3 J}{2 c} \quad (22) $$
Here $C_1$ is the "linear capacity" of the line and $r$ is the large radius of the ring.

As already mentioned above, in the case of resonance the polarizability becomes a purely imaginary quantity. Therefore in the case of operation in a "single wave" regime, the small bunch or the thin ring (resonance sets in for it at $\omega \approx \omega_0 / \sqrt{2}$) practically "shut off" the wave guide, that is, they behave like a mirror plug or stopper. Utilization of this resonance "shutting" can considerably increase the feasibility of the radiation method of acceleration.

All the expressions presented thus far for the forces pertain to a stationary bunch. By using the Lorentz transformations for the field and the Doppler-effect formulas, one can easily find the forces which act on a moving bunch or clot. Thus, for the bunch in a free state or in a coaxial the force $F$ is related with the force $F_0$, acting on a stationary bunch, by an equation of the form

$$F = \left(\frac{\omega'}{\omega}\right)^n F_0,$$

(23)

where $\omega'$ is the field frequency in the system of coordinates accompanying the bunch, and the values for $n$ are presented in the table:

<table>
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<th>No. of equation</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>21</th>
<th>22</th>
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<tbody>
<tr>
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<td>2</td>
<td>0</td>
<td>2</td>
<td>5/2</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

*Note: In the last column there is given the value of $n$ for a mirror stopper in a coaxial.
If the parameters of the plasma are such that acceleration does in fact take place under the effect of a single force \( F_{\text{scat}} \) or \( F_{\text{absorp}} \), then the equation which relates the energy of the bunch with the traversed path, assumes the form

\[
(\varepsilon + \sqrt{\varepsilon^2 - 1})^{n} \, d\varepsilon = d\zeta \quad (24)
\]

and after integration it gives an implicit dependence of the dimensionless energy \( \varepsilon = E/E_0 \) on the dimensionless coordinate \( \zeta = F_0 \, z/E_0 \). Here \( E_0 \) is the energy at rest of the bunch. In particular, for super relativistic bunches \( (\varepsilon \gg 1 \text{ and } \zeta \gg 1) \quad \varepsilon \sim \zeta^{1/(n+1)} \), and the most optimal, as this can be seen from the table, appear to be the variants or versions which are described by formulas (9) and (21), for which the energy increases linearly with the distance.

In wave guard fields the Doppler-effect formulas have the form

\[
\frac{k'}{k} = \frac{\omega'}{\omega} = \varepsilon - \frac{\sqrt{(-1)(\varepsilon^2 - 1)}}{\varepsilon_m}, \quad \frac{n}{h} = \varepsilon - \varepsilon_m \sqrt{\varepsilon^2 - 1} \quad (25)
\]

and instead of (24), generally speaking, far more complex equations are derived. Thus, for the mirror stopper (16) we have

\[
\xi = \left(\varepsilon^{2} - 1\right) \left\{ \varepsilon_m \ln \frac{(\varepsilon + 1)(\varepsilon^2 - 1)}{\varepsilon_m^2 - 1} + \frac{\sqrt{\varepsilon^2 - 1}}{\varepsilon_m \varepsilon - 1} - 1 \right\} \quad (26)
\]

For \( \xi \gg 1 \), equation (26) converts into \( \varepsilon = \varepsilon_m - (\varepsilon_m^2 - 1)^2 / \xi \).

An exception are the cases of an "ideal-conducting" bunch which can be accelerated by \( H_0 \) or \( E_0 \) waves of a circular wave guide. In both of these cases the energy increases linearly with the distance, until it reaches its own critical value.
LITERATURE

1. V.I. Veksler, report to CERN, Geneva (1956).

2. V.I. Veksler, Atomnaya energiya 2, 427, 1957.


DISCUSSION

G. A. L o e w

I would like to ask Dr. Levin to show a sketch of his waveguide structure for plasma acceleration.

M. L. Levin

The waveguide has a length of about 80 cm, a diameter of roughly 15 cm, and smooth walls (the phase velocity is greater than the speed of light).