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FIRST NNLL PREDICTION OF $\mathcal{B}(\bar{B} \to X_S\gamma)$

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We discuss the first NNLL prediction of the $\bar{B} \to X_s\gamma$ branching ratio, including important additional subtleties due to non-perturbative corrections and logarithmically-enhanced cut effects.

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Among the flavour-changing current processes, the inclusive $b \to s\gamma$ mode is still the most prominent. The stringent bounds obtained from this mode on various non-standard scenarios are a clear example of the importance of clean FCNC observables in discriminating new-physics models. Its branching ratio has already been measured by several independent experiments using semi-inclusive or fully inclusive methods [1,2,3,4,5]. The world average of those five measurements (performed by the Heavy Flavour Averaging Group (HFAG) [6]) for a photon energy cut $E_\gamma > 1.6$ GeV reads

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm exp} = = \left(3.55 \pm 0.24\, ^{+0.09}_{-0.10} \pm 0.03\right) \times 10^{-4} \quad (1)$$

where the errors are combined statistical and systematic, systematic due to the extrapolation, and due to the $b \to d\gamma$ fraction.

On the theory side, perturbative QCD contributions to the decay rate are dominant and lead to large logarithms $\alpha_s(M_W)\times \log(m_b^2/M_W^2)$, which have to be resummed in order to get a reasonable result. Resumming all the terms of the form $(\alpha_s(m_b))^p\, \alpha_s^n(m_b)\, \log^n(m_b/M)$ (with $M = m_t$ or $M = m_W$, $n = 0, 1, 2, \ldots$) for fixed p corresponds for $p = 0$ to leading-log (LL), for $p = 1$ to next-to-leading-log (NLL), and for $p = 2$ to next-to-next-to-leading-log (NNLL) precision. The previous NLL prediction, based on the original QCD calculations of several groups [7,8,9,10,11,12,13,14,15,16,17], had an additional charm mass renormalization scheme ambiguity, first analysed in Ref.[18]. For an energy cut $E_\gamma > 1.6$ GeV it reads [20]

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm NLL} = = (3.61^{+0.24}_{-0.40} \pm 0.02 \pm 0.25 \pm 0.15) \times 10^{-4} \quad (2)$$

where the errors are due to the charm scheme dependence, CKM input, further parametric dependences, and to perturbative scale dependence. The dominant uncertainty related to the definition of m_c was taken into account by varying m_c/m_b in the conservative range $0.18 \le m_c/m_b \le 0.31$, which covers both, the pole mass value (with its numerical error) and the running mass value $\bar{m}_c(\mu_c)$ with $\mu_c \in [m_c, m_b]$; for the central value $m_c/m_b = 0.23$ was used [20]. However, the renormalization scheme for m_c is an NNLL issue. It was shown that a complete NNLL calculation reduces this large uncertainty at least by a factor of 2 [21].

Within a global effort, such a NNLL calculation was quite recently finalized [22]. The tedious calculational steps were performed by various groups [24,25,26,31,28,27,29,?,23,32]. One crucial piece is the calculation of the three-loop matrix elements of the four-quark operators, which was first made within the so-called large-β_0 approximation [23]. A calculation that goes beyond this approximation by employing an interpolation in the charm quark mass m_c from $m_c > m_b$ to the physical m_c value has just been completed [32]. It is that part of the NNLL calculation where

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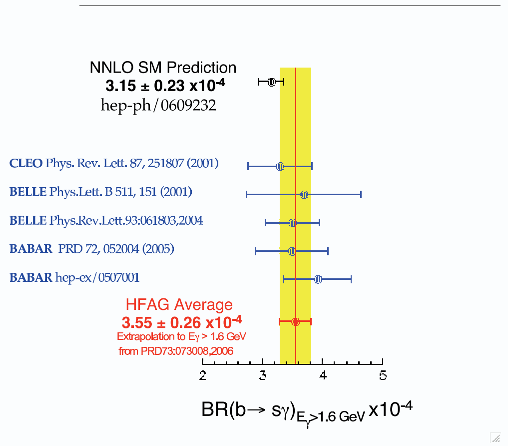

Fig. 1. New NNLL prediction versus HFAG average.

there is still space for improvement. All those results lead to the first estimate of the $\bar{B} \to X_s\gamma$ branching ratio to NNLL precision. It reads for a photon energy cut $E_\gamma > 1.0$ GeV [22]:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm NNLL} = (3.27 \pm 0.23) \times 10^{-4}. \qquad (3)$$

The overall uncertainty consists of non-perturbative (5%), parametric (3%), higher-order (3%) and m_c-interpolation ambiguity (3%), which have been added in quadrature. For higher photon energy cut we have the following numerical fit:

$$\left(\frac{\mathcal{B}(E_\gamma > E_0)}{\mathcal{B}(E_\gamma > 1.0\ {\rm GeV})}\right) \simeq 1 - 0.031y - 0.047y^2, \qquad (4)$$

where $y = E_0/(1.0\,{\rm GeV}) - 1$. This formula coincides with the NNLL results up to $\pm 0.1\%$ for $E_0 \in [1.0,\ 1.6]$ GeV. The error is practically E_0-independent in this range. For $E_\gamma > 1.6$ GeV the NNLL prediction reads [22]:

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm NNLL} = (3.15 \pm 0.23) \times 10^{-4}. \qquad (5)$$

Compared with the HFAG average, given in Eq.(1), the NNLL prediction is 1.2σ below the experimental data (see Fig. 1 [19]).

The reduction of the renormalization-scale dependence at the NNLL is clearly seen in Fig. 2. The most important effect occurs for the charm mass $\overline{\rm MS}$ renormalization scale μ_c that was the main source of uncertainty at the NLL. The current uncertainty of $\pm 3\%$ due to higher-order ($\mathcal{O}(\alpha_s^3)$) effects is estimated from the NNLL curves in Fig. 2. The reduction factor of the perturbative error is more than a factor 3. The central value of the NNLL prediction is based on the choices $\mu_b = 2.5$GeV and $\mu_c = 1.5$ GeV.

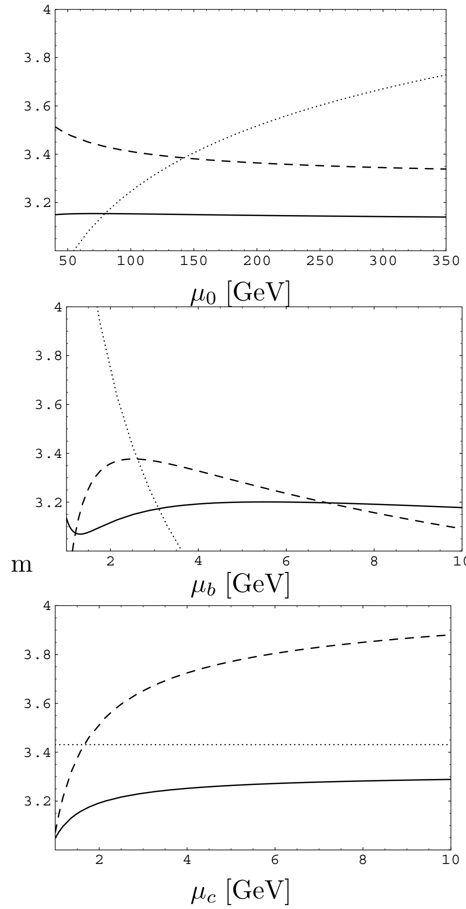

Fig. 2. Renormalization-scale dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in units 10^{-4} at the LL (dotted lines), NLL (dashed lines) and NNLL (solid lines). The plots describe the dependence on the matching scale μ_0, the low-energy scale μ_b, and the charm mass renormalization scale μ_c.

There are some perturbative NNLL corrections which are not included yet in the present NNLL estimate, but are expected to be smaller than the current uncertainty: the virtual- and bremsstrahlung contributions to the $(\mathcal{O}_7, \mathcal{O}_8)$ and $(\mathcal{O}_8, \mathcal{O}_8)$ interferences at order α_s^2, the NNLL bremsstrahlung contributions in the large β_0-approximation beyond the $(\mathcal{O}_7, \mathcal{O}_7)$ interference term (which are already available [33]), the four-loop mixing of the four-quark operators into the operator $\mathcal{O}_8$ (see recent work [31]), the exact m_c dependence of the various matrix elements beyond the large β_0 approximation (see [34]) and perturbative logarithmically-enhanced cut effects (see discussion below and [53]).

Nevertheless, the final result includes subdominant contributions such as the perturbative electroweak two-loop corrections of order -3.7% [35,17,36,37] and the non-perturbative corrections scaling with $1/m_b^2$ or $1/m_c^2$ of order $+1\%$ and $+3\%$ respectively [38,41,42,43,44].

It is well known, that the local operator product expansion (OPE) for the decay $\bar{B} \to X_s\gamma$ has certain limitations if one takes into account other operators than the leading $\mathcal{O}_7$, as was already shown within the analysis of $1/m_c^2$ power corrections. The additional error of 5% in the NNLL prediction corresponds to non-perturbative corrections, which scale with $\alpha_s \Lambda/m_b$. Quite recently, a specific piece of the additional non-perturbative corrections was estimated [45]. Because the overall sign of the whole effect is still unknown, this partial estimate is not included in the central value of the present NNLL prediction [22].

However, there are more subtleties. There is an additional sensitivity to non-perturbative physics, due to necessary cuts in the photon energy spectrum to suppress the background from other B decays (see Fig. 3). This leads to a breakdown of the local OPE, which can be cured by partial resummation of these effects to all orders into a non-perturbative shape-function [46,47,48]. Those shape-function effects are taken into account

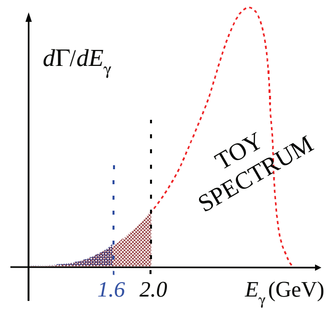

Fig. 3. Cut in the photon energy spectrum

in the experimental value by the HFAG [6] and the corresponding theoretical uncertainties due to this model dependence is reflected in the extrapolation error in the experimental number quoted above in Eq. (1). Here, one should keep in mind that the experimental energy cuts in the last experiments are at 1.8 GeV or 1.9 GeV or even higher. The extrapolation down to 1.6 GeV is done using three different theoretical schemes to calculate the extrapolation factor [49,17,50,51] and averaging those results (inspite of the fact that the previous parametrisation of the shape-function in [17] needs to be improved in view of the recent experimental findings on the photon spectrum).

Moreover, it was argued that a cut around 1.6 GeV might not guarantee that a theoretical description in terms of a local OPE is sufficient because of the sensitivity to the scale $\Delta = m_b - 2E_\gamma$ [52]. A multiscale OPE with three short-distance scales m_b, $\sqrt{m_b\Delta}$, and Δ was proposed to connect the shape function and the local OPE region. Quite recently, such additional cutoff-related effects were numerically estimated using (model-independent) SCET methods [53,54,55]. Those perturbative effects due to the additional scale are negligible for 1.0 GeV but lead to an effect of order 3% at 1.6 GeV [53]. The size of these effects at 1.6 GeV is at the same level as the 3% higher-order uncertainty in the present NNLL prediction. It is suggestive that in the future those additional perturbative cut effects get analysed and combined together with those already included in the experimental average of the HFAG.

There are also other claims for non-negligible cut effects at 1.6 GeV [56] which, however, are based on models of the non-perturbative shape function. Moreover, there is an alternative approach to the cut effects in the photon energy spectrum based on dressed gluon exponentiation and incorporating Sudakov and renormalon resummations [57,58]. It should be emphasized that the higher predictive power of this approach is related in part to the assumption that non-perturbative power corrections associated with the shape function follow the pattern of ambiguities present in the perturbative calculation [59].

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