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New Weyl-invariant vector-tensor theory for the cosmological constant

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Abstract. We introduce a new Weyl-invariant and generally-covariant vector-tensor theory with higher derivatives. This theory can be induced by extending the mimetic construction to vector fields of conformal weight four. We demonstrate that in gauge-invariant variables this novel theory reduces to the Henneaux-Teitelboim description of the unimodular gravity. Hence, compared with the standard general relativity, our new higher derivative vector-tensor theory has only one new global degree of freedom — the cosmological constant. Finally we discuss potential extensions of this vector-tensor theory.

Keywords: dark energy theory, modified gravity

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1 Introduction

The cosmological constant problem remains an unsolved mystery, for reviews see e.g. [1-4]. One of the cornerstones of this problem is a fine-tuning or (un)naturalness of the value of the observed acceleration of our expanding universe. However, any discussion of naturalness, fine-tuning, and especially related anthropic reasoning [5] at least implicitly assumes an ensemble of theories or solutions where the cosmological constant or vacuum energy can take different values. One of the setups where such an ensemble is realized by different solutions is the so-called *unimodular qravity* which was first proposed by Einstein almost a century ago in [6].¹ In unimodular gravity the dynamics of spacetime is given by the trace-free part of the standard Einstein field equations. If one assumes that matter energy-momentum tensor is conserved, the value of the cosmological constant is given by an integration constant, for recent discussion see e.g. [7, 8]. This integration constant is not related to the Planck and electroweak scales or any other parameters and coupling constants of the Standard Model. This property does not solve the cosmological constant problem, but puts it in a rather different perspective. There are already quite a few different action principles reproducing the dynamics of the unimodular gravity, see e.g. [9-14]. The most relevant for this work is the theory by Henneaux and Teitelboim (HT) [10]. The main advantage of this formulation is that it is manifestly generally covariant and has a slightly simpler formulation than in [13]. To ensure the general covariance the HT action contains a vector field.

On the other hand, recently another construction leading to the trace-free equations of motion for the metric was proposed by Chamseddine and Mukhanov in [15] under the name *Mimetic* Gravity. This construction is dynamically equivalent to irrotational dust minimally coupled to standard General Relativity [15–17], Hence it is more interesting for modeling dark matter. Similarly to HT theory the energy density of this mimetic irrotational dark matter is a Lagrange multiplier. One of the main features of this mimetic construction is that the theory is Weyl-invariant. This Weyl-invariance with respect to $h_{\mu\nu} = \Omega^2(x) h'_{\mu\nu}$ originates from the ansatz of the composite metric

$$g_{\mu\nu} = h_{\mu\nu} \cdot h^{\alpha\beta} \partial_{\alpha} \phi \, \partial_{\beta} \phi \,, \tag{1.1}$$

¹For more recent discussions see [1].

into the Einstein-Hilbert action. Here it is assumed that the scalar field ϕ is Weyl-invariant. Different mimetic constructions with vector fields were considered in [17–20]. All of these theories use Weyl-invariant vector fields and no one of these constructions corresponds to the unimodular gravity. Motivated by the HT vector-field formulation of the unimodular gravity we search for a novel nontrivial Weyl-invariant generalization of the mimetic ansatz for the composite metric (1.1) containing a vector field V^{μ} .

2 Mimetic vector field of conformal weight four

In this paper we propose a new extension of the *mimetic* construction [15] to a vector field, V^{α} , namely we propose to use the ansatz

$$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla^{h}_{\alpha} V^{\alpha}\right)^{1/2} , \qquad (2.1)$$

where the covariant derivative, ∇_{α}^{h} , is the Levi-Civita connection compatible with the auxiliary metric $h_{\mu\nu}$

$$\nabla^{h}_{\alpha}h_{\mu\nu} = 0. \qquad (2.2)$$

We will call the metric $g_{\mu\nu}$ the *physical metric*. In contrast to [20] the vector field V^{μ} is not a gauge potential / connection. However, similarly to [20], with this particular form of the conformal factor in front of $h_{\mu\nu}$ the resulting theory becomes Weyl-invariant. Indeed, the Weyl transformation of the auxiliary metric $h_{\mu\nu}$

$$h_{\mu\nu} = \Omega^2 (x) \, h'_{\mu\nu} \,, \tag{2.3}$$

performed along with the corresponding transformation of the vector field

$$V^{\mu} = \Omega^{-4} (x) \, V^{\prime \mu} \,, \tag{2.4}$$

keeps the metric $g_{\mu\nu}$ invariant. This is easy to check using

$$\nabla_{\alpha}^{h}V^{\alpha} = \frac{1}{\sqrt{-h}}\partial_{\alpha}\left(\sqrt{-h}V^{\alpha}\right) = \frac{1}{\Omega^{4}}\frac{1}{\sqrt{-h'}}\partial_{\alpha}\left(\sqrt{-h'}V'^{\alpha}\right) = \Omega^{-4}\nabla_{\alpha}^{h'}V'^{\alpha}.$$
 (2.5)

Unlike the constructions in [17–20] the vector field V^{μ} has conformal weight four under the Weyl transformations. Another crucial difference from these works and from the original mimetic construction [15] is that the map (2.1) from $h_{\mu\nu}$ to $g_{\mu\nu}$ is not algebraic, but contains derivatives² of the auxiliary metric $h_{\mu\nu}$ as

$$g_{\mu\nu} = \frac{h_{\mu\nu}}{(-h)^{1/4}} \cdot \left(\partial_{\alpha}\sqrt{-h}V^{\alpha}\right)^{1/2} . \tag{2.6}$$

Substituting the ansatz (2.1) into any action functional $S[g, \Phi_m]$ (with some matter fields Φ_m) induces a novel Weyl-invariant theory with the action functional

$$S[h, V, \Phi_m] = S[g(h, V), \Phi_m].$$
 (2.7)

²This does not allow to use the inverse function theorem. The complications due to appearance of $h_{\mu\nu,\alpha}$ are mentioned in [21].

There is also an obvious ancillary gauge invariance with respect to

$$V_{\mu} = V'_{\mu} + \partial_{\mu}\theta$$
, where $\Box \theta = 0$, (2.8)

which is similar to residual gauge redundancy in the Lorenz gauge.

Now we can plug in the ansatz (2.1) into the Einstein-Hilbert action to obtain an action for a higher-derivative vector-tensor theory³

$$S_{g}[h,V] = -\frac{1}{2} \int d^{4}x \sqrt{-h} \left[\left(\nabla^{h}_{\alpha} V^{\alpha} \right)^{1/2} R(h) + \frac{3}{8} \cdot \frac{\left(\nabla^{h}_{\mu} \nabla^{h}_{\alpha} V^{\alpha} \right)^{2}}{\left(\nabla^{h}_{\sigma} V^{\sigma} \right)^{3/2}} \right].$$
(2.9)

This is clearly a novel scalar-vector theory going beyond Horndeski and other more recent constructions. For details see [22, 23]. The gravitational part of the whole theory can be more conveniently written as

$$S_{g}[h,V] = -\frac{1}{2} \int d^{4}x \sqrt{-h} \left[\sqrt{DR}(h) + \frac{3}{8} \cdot \frac{h^{\alpha\beta} D_{,\alpha} D_{,\beta}}{D^{3/2}} \right], \qquad (2.10)$$

where we introduce the notation for the four-divergence

$$D = \nabla^{h}_{\alpha} V^{\alpha} \,. \tag{2.11}$$

Under the Weyl transformations this scalar quantity has conformal weight four

$$D = \Omega^{-4} D' \,. \tag{2.12}$$

It should be stressed that as a result of this procedure all matter fields acquire a universal coupling to the vector field V^{α} due to the substitution (2.7). The total action is $S[h, V, \Phi_m] = S_g[h, V] + S_m[h, V, \Phi_m]$.

3 Equations of motion

Let us derive equations of motion for our novel vector-tensor theory.

$$\delta S = \frac{1}{2} \int d^4 x \sqrt{-g} \left(T_{\mu\nu} - G_{\mu\nu} \right) \delta g^{\mu\nu} + \text{Boundary terms}, \qquad (3.1)$$

where $G_{\mu\nu}$ is the Einstein tensor for $g_{\mu\nu}$ and the energy momentum tensor of matter is defined as usual through

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \cdot \frac{\delta S_m}{\delta g^{\mu\nu}} \,. \tag{3.2}$$

The variation of the contravariant metric yields

$$\delta g^{\mu\nu} = \frac{\delta h^{\mu\nu}}{\sqrt{D}} - \frac{1}{2} g^{\mu\nu} \frac{\delta D}{D} , \qquad (3.3)$$

where the variation of the divergence (2.11) can be expressed as

$$\delta D = \nabla^{h}_{\alpha} \delta V^{\alpha} - \frac{1}{2} h_{\alpha\beta} V^{\lambda} \nabla^{h}_{\lambda} \delta h^{\alpha\beta} . \qquad (3.4)$$

³We use: the standard notation $\sqrt{-h} \equiv \sqrt{-\det h_{\mu\nu}}$, the signature convention (+, -, -, -), and the units $c = \hbar = 1$, $M_{\rm Pl} = (8\pi G_{\rm N})^{-1/2} = 1$.

Integrating by parts, neglecting the boundary terms and using $\sqrt{-g} = D\sqrt{-h}$ we obtain equation of motion for the vector field

$$\frac{1}{\sqrt{-h}} \cdot \frac{\delta S}{\delta V^{\mu}} = \frac{1}{4} \partial_{\mu} \left(T - G \right) = 0, \qquad (3.5)$$

along with the equation of motion for the auxiliary metric

$$\frac{1}{\sqrt{-h}} \cdot \frac{\delta S}{\delta h^{\alpha\beta}} = \frac{\sqrt{D}}{2} \left[T_{\alpha\beta} - G_{\alpha\beta} - \frac{1}{4} g_{\alpha\beta} \left(T - G - \frac{1}{D} V^{\lambda} \partial_{\lambda} \left(T - G \right) \right) \right] = 0, \qquad (3.6)$$

where $T = T_{\alpha\beta}g^{\alpha\beta}$ and $G = G_{\alpha\beta}g^{\alpha\beta}$. Using the equation of motion (3.5) for the vector V^{α} , the equation of motion for the metric $h_{\mu\nu}$ transforms to the trace-free part of the Einstein equations

$$G_{\alpha\beta} - T_{\alpha\beta} - \frac{1}{4}g_{\alpha\beta}\left(G - T\right) = 0.$$
(3.7)

Crucially, both equations of motion (3.5) and (3.6) are manifestly invariant with respect to the Weyl transformations of $h_{\mu\nu}$ and V^{α} , as V^{λ}/D = inv and all other quantities are expressed through manifestly gauge invariant objects $g_{\mu\nu}$ and matter fields Φ_m .

For the later it is convenient to consider the Weyl-invariant vector

$$W^{\mu} = \frac{V^{\mu}}{\nabla^{h}_{\alpha} V^{\alpha}} \,. \tag{3.8}$$

Considered as an equation on original variables, the equation of motion for the vector (3.5) has fourth derivatives of $\{h_{\mu\nu}, V^{\alpha}\}$ while the trace-free part of the g-Einstein equations (3.7) has up to third derivatives of these original variables. However, one can write both equations of motion (3.5) and (3.7) in the gauge where $g_{\mu\nu} = h_{\mu\nu}$ so that the dependence on auxiliary vector field V^{μ} completely disappears from (3.5) and (3.7) so that the latter becomes the standard trace-free part of the Einstein equations on standard variables.

In fact, these equations of motion are those of the so-called *unimodular* gravity. The only difference from the standard GR is that the cosmological constant is an integration constant. Indeed, integrating the equation of motion (3.5) for the vector V^{α} one obtains

$$G - T = 4\Lambda = \text{const}.$$
(3.9)

Substituting this solution into the trace-free part of the Einstein equations one derives the standard Einstein equations with the cosmological constant Λ

$$G_{\alpha\beta} = \Lambda g_{\alpha\beta} + T_{\alpha\beta} \,. \tag{3.10}$$

Hence one can say that our construction provides *Mimetic Dark Energy* or *Mimetic Cosmological Constant*.

We could guess that our mimetic theory describes unimodular gravity by observing that in the coordinate ${\rm frame}^4$ where

$$V^{\mu}(x) \doteq \frac{1}{4} \frac{x^{\mu}}{\sqrt{-h}},$$
 (3.11)

⁴If equality holds only in a particular frame we use " \doteq " instead of "=".

the determinant of the physical metric is unity, see (2.6) and all quantities depend on $h_{\mu\nu}$ through

$$g_{\mu\nu} \doteq \frac{h_{\mu\nu}}{(-h)^{1/4}}$$
. (3.12)

Another notable option is a less symmetric frame with the timelike vector

$$V^{\mu}(x) \doteq t \, \frac{\delta^{\mu}_{t}}{\sqrt{-h}} \,, \tag{3.13}$$

where t is a time coordinate. For a nice discussion on how one can construct other unimodular coordinates where $\sqrt{-g} = 1$ see [9].

4 Gauge invariant variables and scalar-vector-tensor formulation

Now we can follow a similar procedure as in [24] and upgrade D to an independent dynamical variable in order to eliminate the second derivatives from the action, so that

$$S[h, D, V, \lambda] = -\frac{1}{2} \int d^4x \sqrt{-h} \left[\sqrt{DR}(h) + \frac{3}{8} \cdot \frac{h^{\alpha\beta} D_{,\alpha} D_{,\beta}}{D^{3/2}} + \lambda \left(D - \nabla^{h}_{\alpha} V^{\alpha} \right) \right].$$
(4.1)

Hence, we introduced a constraint with the corresponding Lagrange multiplier and promoted theory (2.9) to a vector-tensor-scalar theory in this way. This theory should be Weyl-invariant, as it was the case with the original action (2.9). This requirement forces the Lagrange multiplier, λ , to be invariant under the Weyl transformations. In this way all matter fields acquire a universal coupling to the scalar field D.

One can further canonically normalize the kinetic term by defining a new scalar field of conformal weight one

$$D = \left(\frac{\varphi^2}{6}\right)^2,\tag{4.2}$$

so that the action (4.1) takes the form

$$S[h,\varphi,V,\lambda] = \int d^4x \sqrt{-h} \left[-\frac{1}{2} \left(\partial \varphi \right)^2 - \frac{1}{12} \varphi^2 R(h) - \frac{\lambda}{72} \varphi^4 + \frac{\lambda}{2} \cdot \nabla^h_\alpha V^\alpha \right] .$$
(4.3)

The first three terms correspond to the Dirac's theory of the Weyl-invariant gravity [25], see also [26]. These terms are also the starting point for the so-called Conformal Inflation [27]. In our sign convention the scalar field φ has a ghost-like kinetic term. Importantly, in contrast to [25] the would be coupling constant λ is a Lagrange multiplier field. All other matter fields are coupled to the physical metric

$$g_{\mu\nu} = \frac{\varphi^2}{6} \cdot h_{\mu\nu} \,. \tag{4.4}$$

The form of the action is closely related, but not identical to those studied in [28]. The main difference is the full diffeomorphism invariance of our action (4.3) whereas the theories studied in [28] were only invariant with respect to transverse diffeomorphisms preserving the value of $\sqrt{-h}$. It seems that the vector field V^{μ} (absent in [28]) in our construction works⁵ as

⁵In another [13] generally-covariant formulation of unimodular gravity by Kuchař instead of the vector V^{μ} there are four compensator scalar fields X^{A} representing general unimodular coordinates. Formula (3.11) represents one possible set of them and can be useful to show canonical equivalence between our and Kuchař formulations.

a Stückelberg, Freiherr von Breidenbach zu Breidenstein und Melsbach field (also colloquially known as a compensator field) restoring the full diffeomorphism invariance. However, the form of the action suggests that the Weyl symmetry is in a sense empty (or as sometimes called fake) in our construction and corresponds to the Noether current which is identically vanishing, see [29, 30]. We leave the clarification of this issue for a future work.

The dynamical variables $\{h_{\mu\nu}, V^{\mu}, \lambda, D,\}$ transform as

$$h_{\mu\nu} = \Omega^{2} (x) h'_{\mu\nu}, \qquad (4.5)$$

$$D = \Omega^{-4} (x) D', \qquad (4.5)$$

$$V^{\mu} = \Omega^{-4} (x) V'^{\mu}, \qquad \lambda = \lambda'.$$

Instead of these dynamical variables one can introduce a new set of independent dynamical variables $\{g_{\mu\nu}, W^{\mu}, \Lambda, D\}$, where the first three

$$g_{\mu\nu} = D^{1/2} h_{\mu\nu},$$
 (4.6)
 $W^{\mu} = D^{-1} V^{\mu},$
 $\Lambda = \frac{\lambda}{2},$

are gauge invariant. This field-redefinitions resemble the Weyl transformations with $\Omega = D^{1/4}$, except we do not transform D and consequently do not reduce the dimensionality of the phase space. Hence this transformation is different from fixing the gauge where D = 1, even though the variables W^{μ} and $g_{\mu\nu}$ are equal to the corresponding variables in this gauge.

In this way the divergence transforms

$$\nabla^{h}_{\alpha} V^{\alpha} = \frac{D}{\sqrt{-g}} \partial_{\alpha} \left(\sqrt{-g} W^{\alpha} \right) = D \nabla^{g}_{\mu} W^{\mu} \,. \tag{4.7}$$

Performing this field redefinition in (4.1) one obtains

$$S[g, W, \Lambda, \Phi_m] = \int d^4x \sqrt{-g} \left[-\frac{1}{2} R(g) + \Lambda \left(\nabla^{g)}_{\mu} W^{\mu} - 1 \right) \right] + S_m [g, \Phi_m] .$$
(4.8)

This action functional does not depend anymore on the scalar field D, but only on gauge invariant dynamical variables (4.6). In fact, as a result of this transformation we obtained the Henneaux-Teitelboim representation [10] of the unimodular gravity. Indeed, the variation of this action with respect to the vector field W^{μ} implies that Λ is a constant of integration (global degree of freedom):

$$\frac{1}{\sqrt{-g}} \cdot \frac{\delta S}{\delta W^{\mu}} = -\partial_{\mu}\Lambda = 0, \qquad (4.9)$$

while the variation with respect to the metric gives the Einstein equations with the cosmological constant Λ

$$\frac{2}{\sqrt{-g}} \cdot \frac{\delta S}{\delta g^{\mu\nu}} = T_{\mu\nu} + \Lambda g_{\mu\nu} - G_{\mu\nu} = 0.$$
 (4.10)

Finally there is a constraint

$$\frac{1}{\sqrt{-g}} \cdot \frac{\delta S}{\delta \Lambda} = \nabla^{g)}_{\mu} W^{\mu} - 1 = 0.$$
(4.11)

The constraint equation (4.11) per construction becomes identity in terms of original fields $\{h_{\mu\nu}, V^{\alpha}\}$ and does not provide any new information regarding the dynamics. In electrodynamics one faces a similar situation with $\nabla^{g}_{\mu}F^{\mu\nu}$ which is identically conserved per construction.

The constraint equation (4.11) can be considered as a non-conservation of the current W^{μ} . As it is discussed in [10] the constraint equation (4.11) only allows to find the evolution of the corresponding charge

$$\mathscr{T}(t) = \int_{\Sigma} d^3 \mathbf{y} \sqrt{-g} W^t(t, \mathbf{y}) , \qquad (4.12)$$

which is often called "cosmic time" or four-dimensional spacetime volume. To derive this expression we first use the ADM 3+1 decomposition [31, 32] defining a foliation:

$$ds^{2} = N^{2}dt^{2} - \gamma_{ik} \left(dy^{i} + N^{i}dt \right) \left(dy^{k} + N^{k}dt \right) \,. \tag{4.13}$$

Here the 3d induced metric is given by

$$\gamma_{ik} = -g_{\mu\nu} \frac{\partial x^{\mu}}{\partial y^{i}} \frac{\partial x^{\nu}}{\partial y^{k}}, \qquad (4.14)$$

where y^i are coordinates defined on the hypersurfaces of constant time t. Following, [32] the unit normal one form to hypersurfaces of constant time is given by

$$n_{\mu} = N \,\partial_{\mu} t = N \delta^t_{\mu} \,. \tag{4.15}$$

Then, applying the 4d Gauss-Stokes theorem⁶ we can transform the integral over a spacetime volume \mathscr{V} between two hypersurfaces Σ_1 and Σ_2 of constant time t with $t_2 > t_1$ as

$$\int_{\mathscr{V}} d^4x \sqrt{-g} \,\nabla^{g)}_{\mu} W^{\mu} = \int_{\Sigma_2} d^3 \mathbf{y} \sqrt{\gamma} \, n_{\mu} W^{\mu} + \int_{\Sigma_1} d^3 \mathbf{y} \sqrt{\gamma} \, n_{\mu} W^{\mu} \,, \tag{4.16}$$

where for Σ_2 the unit normal one form $n_{\mu}(t_2)$ is given by (4.15) while for Σ_1 we have $n_{\mu}(t_1) = -N\delta^t_{\mu}$. Now using these expressions for the normal, the constraint (4.11) and the standard ADM expression $\sqrt{-g} = N\sqrt{\gamma}$ we obtain

$$\int_{\mathscr{V}} d^4x \sqrt{-g} = \int_{\Sigma_2} d^3\mathbf{y} \sqrt{-g} W^t - \int_{\Sigma_1} d^3\mathbf{y} \sqrt{-g} W^t = \mathscr{T}(t_2) - \mathscr{T}(t_1) .$$
(4.17)

Vice versa we could start from (4.12) guessing this expression by using the analogy with the electric change, see e.g. page 275 [33],⁷ and only use 3d Stokes theorem to calculate

$$\dot{\mathscr{T}}(t) = \int d^{3}\mathbf{y}\partial_{t}\left(\sqrt{-g}W^{t}(t,\mathbf{y})\right) = \int d^{3}\mathbf{y}\left(\sqrt{-g} - \partial_{i}\left(\sqrt{-g}W^{i}\right)\right) = \int d^{3}\mathbf{y}\sqrt{-g} - \oint_{\mathcal{B}} ds_{i}\sqrt{-g}W^{i}, \qquad (4.18)$$

⁶Under the usual assumption that the flow of W^{μ} vanishes at special (timelike) boundaries.

⁷It is important to stress that there is a substantial difference between the ADM spatial metric used here $\gamma_{ik} = -g_{ik}$ which is just the induced metric on the hypersurface and the Landau-Lifshitz approach which actually utilizes $\gamma_{ik}^{LL} = \gamma_{ik} + N_i N_k / (N^2 - N^i N_i)$. This difference appears, as the ADM spatial distances between points labeled by different coordinates **x** are taken at the same moment of time (which therefore cannot be measured) while Landau and Lifshitz as a physical distance use a half of the proper time a light signal needs for a return travel between these points. Because of this difference, in ADM and here one uses $N = 1/\sqrt{g^{tt}}$ instead of $\sqrt{g_{tt}}$ of Landau-Lifshitz. Despite of these differences the physically relevant expression for the total charge like the one we use in (4.12) is the same in both approaches due to $\sqrt{-g}$.

where the last integral is taken over the boundary surface \mathcal{B} of the three-dimensional space. As we have already assumed, there is no flux of W^i through the boundary surface, so that integrating the above expression we get

$$\mathscr{T}(t_2) - \mathscr{T}(t_1) = \int_{t_1}^{t_2} dt \int d^3 \mathbf{y} \sqrt{-g} \,. \tag{4.19}$$

It is worth noting that one can write the "cosmic time" $\mathscr{T}(t)$ in terms of $\{h_{\mu\nu}, V^{\alpha}\}$:

$$\mathscr{T}(t) = \int d^3 \mathbf{y} \sqrt{-h} V^t(t, \mathbf{y}) , \qquad (4.20)$$

as the tensor density $\sqrt{-h}V^{\mu}$ is invariant under the Weyl transformations (4.5) and remains invariant under the field redefinition (4.6). In the special coordinate system (3.13) the charge expression takes a particularly simple form

$$\mathscr{T}(t) \doteq t \int d^3 \mathbf{y} \,. \tag{4.21}$$

Clearly there is still a lot of gauge redundancy in the action (4.8), as it does not allow to find all components of W^{μ} , but only the global mode $\mathscr{T}(t)$. To find the evolution of the global mode one has to specify conditions for normal components of W^i to the spatial boundary surface \mathcal{B} at all times and initial $W^t(t_1, \mathbf{y})$ (or final $W^t(t_2, \mathbf{y})$) charge density. Of course one can specify both, the initial $W^t(t_1, \mathbf{y})$ and the final $W^t(t_2, \mathbf{y})$, though in that case the boundary conditions for W^i should be chosen consistently so that the flux of the current W^i could compensate for the charges in the charge additional to the four-volume of the spacetime between two Cauchy hypersurfaces:

$$\mathscr{T}(t_2) - \mathscr{T}(t_1) = \int_{t_1}^{t_2} dt \int d^3 \mathbf{y} \sqrt{-g} - \int_{t_1}^{t_2} dt \oint_{\mathcal{B}} ds_i \sqrt{-g} W^i.$$
(4.22)

Of course very different charge densities $W^{t}(t, \mathbf{y})$ can still correspond to the same global charge $\mathscr{T}(t)$.

5 Scalar instead of vector?

It is worth commenting on the need of using a vector field W^{μ} and not a more minimalistic scalar field in the HT construction. One can try to use a quite often employed decomposition of the vector field W^{μ} into the longitudinal and transverse parts

$$W^{\mu} = W^{\mu}_{L} + W^{\mu}_{T}, \qquad (5.1)$$

where

$$W_L^{\mu} = \nabla^{\mu} \sigma$$
, and $\nabla_{\mu} W_T^{\mu} = 0$, (5.2)

with some scalar field σ . In the HT action the transverse part obviously disappears, so that it is tempting to employ σ as a dynamical variable. In that case this scalar field satisfies equation

$$\Box_q \sigma = \nabla_\mu W^\mu = 1. \tag{5.3}$$

Clearly the decomposition (5.1) and (5.2) does not allow one to determine σ and W_T^{μ} uniquely, as the d'Alembertian has zero modes — free solutions of the wave equation. It is easy to

see that one cannot use just the scalar σ in HT construction. Indeed, let us substitute the decomposition (5.1) and (5.2) directly into the action, so that omitting external matter one obtains

$$S_{\text{other}}\left[g,\sigma,\Lambda\right] = \int d^4x \sqrt{-g} \left[-\frac{1}{2}R\left(g\right) + \Lambda\left(\Box_g\sigma - 1\right)\right], \qquad (5.4)$$

resulting in the equation of motion for the former Lagrange multiplier

$$\frac{1}{\sqrt{-g}} \cdot \frac{\delta S}{\delta \sigma} = \Box_g \Lambda = 0, \qquad (5.5)$$

which is drastically different from (4.9). The only way to reconcile the solutions of this wave equation with the correct desired result, $\Lambda = const$, is to put extremely fine-tuned, poorly motivated and nonstandard boundary and initial conditions. Moreover, now both equations of motion for Λ and for σ are hyperbolic second order PDE. These equations are only coupled through gravity and describe two independent local degrees of freedom: a ghost $\alpha = (\Lambda + \sigma)/\sqrt{2}$ and a normal scalar field $\beta = (\Lambda - \sigma)/\sqrt{2}$. This is drastically different from the original HT setup where both equations of motion are in form of the constraints — first order PDE, and there are no local degrees of freedom except of the usual gravitons.

Last but not least, it is unclear how to introduce a scalar field instead of our vector V^{μ} in our mimetic construction from the section (2) and keep the desired Weyl-invariance. Hence we conclude that one cannot reduce the number of variables in the action and just use the scalar field σ instead of the vector W^{μ} , even so the transverse part of this vector disappears from the action.

6 Conclusion and discussion

We proposed a new vector-tensor theory (2.9) with a vector field V^{μ} of conformal weight four:

$$S_{g}[h,V] = -\frac{1}{2} \int d^{4}x \sqrt{-h} \left[\left(\nabla^{h}_{\alpha} V^{\alpha} \right)^{1/2} R(h) + \frac{3}{8} \cdot \frac{\left(\nabla^{h}_{\mu} \nabla^{h}_{\alpha} V^{\alpha} \right)^{2}}{\left(\nabla^{h}_{\sigma} V^{\sigma} \right)^{3/2}} \right].$$
(6.1)

Notably the Weyl-symmetry is often considered to be a desirable and intriguing property in not only in condensed matter physics, but also in gravity and in particle physics, for recent discussions see e.g. [34, 35]. This higher-derivative Weyl-invariant theory is highly degenerate and has only one global degree of freedom (4.20)

$$\mathscr{T}(t) = \int d^3 \mathbf{x} \sqrt{-h} V^t(t, \mathbf{x}) , \qquad (6.2)$$

whose canonical momentum is the cosmological constant Λ . This global degree of freedom is Weyl-invariant. We obtained this theory by making a mimetic substitution

$$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla^{h}_{\alpha} V^{\alpha}\right)^{1/2} , \qquad (6.3)$$

into the Einstein-Hilbert action.

Further we reformulated this theory as a Weyl-invariant scalar-vector-tensor gravity (4.3), which closely resembles the Dirac's theory of the Weyl-invariant gravity [25]. However, our formulation has an additional constraint and a vector field of different conformal weight. Then we introduced gauge-invariant local variables (4.6) and found that our theory reduces to the generally covariant Henneaux-Teitelboim representation [10] of unimodular gravity. In contrast to other formulations of unimodular gravity our action (2.9) has manifest i) Weyl-invariance and ii) general covariance, while there are iii) no explicit constraints imposed using Lagrange multipliers. The price for the combination of all these three properties is the presence of higher derivatives in the action. Despite of these higher derivatives the theory does not suffer from the Ostrogradsky ghosts [36] in the standard sense. Indeed, as it is in the standard Ostrogradsky prescription, the Henneaux-Teitelboim theory is linear in the canonical momentum Λ , but for each solution the momentum stays constant in the whole spacetime, see (4.9), on all solutions. Hence it is impossible to use this canonical momentum to decrease the energy.

Vector-tensor theories are quite popular in the context of modeling dark energy and dark matter phenomena, for recent reviews see e.g. [22, 23]. Clearly our vector field is not a U(1) gauge potential. Our theory goes beyond Horndeski's most general construction for the U(1) vector fields with second order equations of motion [37]. Neither can one find our theory in more general p-form constructions [38–40]. Moreover our construction goes beyond popular generalized Proca vector-tensor theories [41, 42] and Einstein aether models [43] where the U(1) invariance is broken, and goes even beyond further extended vector-tensor theories [44, 45]. Also our construction is principally different from other mimetic vector models [17–20]. A crucial difference is that our contravariant vector field has conformal weight four contrary to the ordinary Weyl-invariant covariant vector fields of weight zero used in the previous constructions. Another difference is that we have derivatives of the metric inside of the mimetic transformation (6.3).

Another interesting feature of our formulation of the unimodular gravity, which is common with [10, 13], is a spontaneous breaking of the Lorentz symmetry. Indeed, in our construction (6.3) vanishing V^{μ} corresponds to a singularity. In Weyl-invariant or HT formulation the persistent presence of W^{μ} is enforced by the constraint (4.11). Clearly this Lorentzsymmetry breaking is not relevant as far as it does not propagate to the Standard Model fields. Interestingly, one can reproduce the dynamics of the original Chamseddine-Mukhanov scalar mimetic dark matter via Lorentz-symmetry breaking in a so-called pre-geometric setup [46], where the spacetime manifold appears only via Lorentz-symmetry breaking. One can wonder whether a different pre-geometric setup can provide our *Mimetic Cosmological Constant* or maybe evolving *Mimetic Dark Energy* along with their initial data. One can also use this spontaneous symmetry breaking for further extensions, as in [47].

Different formulations of the same classical theory can correspond to distinct quantum theories due to anomalies. As our formulation is Weyl-invariant, potentially, the so-called conformal or Weyl anomaly may play a crucial role. However, this important issue goes beyond the scope of this paper where we study only classical properties. For unimodular gravity a relevant discussion on this point can be found in e.g. [14, 48–55]. These differences can be important for quantum vacuum energy and for the UV structure of the theory. Moreover, formulations of the same theory in terms of different dynamical variables are relevant for potential modifications and extensions. In particular, these modifications are interesting in any attempt to dynamically compensate the cosmological constant. On the other hand, extensions can model deviations from an exact cosmological constant to novel forms of evolving vacuum energy. Hence suggesting another formulation of the unimodular gravity can be useful also in this regard.

Finally we would like to mention further ways of generalizing our setup. Scalar field mimetic models can be extended by plugging in the mimetic ansatz into actions already containing different scalar-field operators. In particular, this procedure yields phenomenologically interesting theories for the operators $V(\phi)$ (see e.g. [56, 57]) and $\gamma(\phi) (\Box \phi)^2$, see e.g. [56, 58–61]. The latter operators are rather constrained phenomenologically [61–63] especially as they can introduce mild ghost instabilities, see [63] and [64–68]. Different extensions are also interesting, as they can point out directions to embed or UV complete the theory. To extend our vector-tensor theory, one can start from any progenitor theory with dynamical variables $\{g_{\mu\nu}, W^{\mu}\}$ and some matter fields Φ_m and perform *simultaneous* transformation of metric (6.3) and of the vector field (3.8):

$$W^{\mu} = \frac{V^{\mu}}{\nabla^{h}_{\alpha} V^{\alpha}} \,. \tag{6.4}$$

After substituting these transformed composite objects into the progenitor tensor (or vectortensor or even scalar-vector-tensor) theory we obtain a new vector-tensor theory with the action

$$S[h, V, \Phi_m] = S[g(h, V), W(h, V), \Phi_m].$$
(6.5)

This induced novel vector-tensor theory (with some external matter fields Φ_m) is Weylinvariant per construction. After transition back to the Weyl-invariant variables (4.6) one just adds a constraint term (4.11), $\Lambda \left(\nabla^{g}_{\mu} W^{\mu} - 1 \right)$, to the original action $S \left[g, W, \Phi_m \right]$. For example, extending our model by adding the standard kinetic term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ with the usual field tensor $F_{\mu\nu} = \partial_{\mu}W_{\nu} - \partial_{\nu}W_{\mu}$ to the progenitor Einstein-Hilbert action and making the combined mimetic ansatz (6.3), (6.4) generates a new Weyl-invariant gauge theory preserving even the residual gauge symmetry (2.8). One can also use a different (e.g. with curvature corrections) progenitor or seed gravitational Lagrangian instead of the standard Einstein-Hilbert one.

Interestingly, one can consider the HT action as an example of theories with two measures [69]. In this way the cosmological constant enforces the equality between these two measures $\sqrt{-g}$ and $\partial_{\mu} (\sqrt{-g} W^{\mu})$. The latter measure can be written in terms of the original variables as $\partial_{\mu} (\sqrt{-h} V^{\mu})$ which is Weyl-invariant. Modifying, measure for different matter degrees of freedom using the Weyl-invariant measure $\partial_{\mu} (\sqrt{-h} V^{\mu})$ instead of $\sqrt{-h}$ may give further interesting extensions of our setup.

One can further expand the story by using various tensor fields of other conformal weights. For each progenitor field Ψ which we want to transform to a field of conformal weight k one should make a substitution of a composite field with $\Psi = \psi \left(\nabla_{\alpha}^{h} V^{\alpha} \right)^{-k/4}$. Also the unusual vector field V^{μ} can be a composite object [70].

Another interesting generalization is an extension of the conformal mimetic ansatz (6.3) to more general *disformal* transformations [71] where instead of the usual $\partial_{\mu}\phi$ one exploits the vector field V^{μ} , see e.g. [72] and [19, 73, 74]. For instance the transformation

$$g_{\mu\nu} = h_{\mu\nu} \cdot \left(\nabla^{h}_{\alpha} V^{\alpha}\right)^{1/2} + \left(\nabla^{h}_{\alpha} V^{\alpha}\right)^{-1} V_{\mu} V_{\nu} , \qquad (6.6)$$

still generates new Weyl-invariant theories. Whether the induced theories can describe interesting physics, similarly to the setup presented in the paper, remains to be seen and is an interesting open question.

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