COSMOLOGICAL CONSTRAINTS ON NEUTRINOS AND OTHER "INOS" AND THE "MISSING LIGHT" PROBLEM

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ABSTRACT

The arguments favoring non-baryonic dark matter are summarized. Cosmological constraints are presented on the masses of neutrinos and other "inos", where "ino" represents any candidate particle for the dark matter. A neutrino mass $10 eV \lesssim m_\nu \lesssim 25 eV$ is favored for the most massive eigenstate, where the upper limit may be extended to 100 eV if one assumes weaker constraints on the age of the universe.

I. INTRODUCTION

Over a decade ago massive neutrinos were suggested as possible candidates for the "missing mass" in the haloes of galaxies[1]. On larger and larger scales, less and less of the dynamically inferred mass can be accounted for by luminous matter[2]. Some form of dark matter must reside in the haloes of galaxies and clusters of galaxies. The density of "ordinary" nucleonic matter (faint stars, gas clouds, etc.) that can contribute to the missing mass is restricted by primordial nucleosynthetic arguments to $0.01 \leq \Omega_b \leq 0.14$ [3]. [Here Ω_b is the ratio of baryonic matter density ρ_b to critical density $\rho_c = \frac{3H_o^2}{8\pi G} = 1.88 \times 10^{-29} h_o^2$ gm/cm^3 $= 8.1 \times 10^{-11} h_o^2$ eV^4 where the Hubble parameter $H_o = 100\ h_o$ kms^{-1}Mpc^{-1}. We will also use the notation Ω_ν to represent the ratio of neutrino density ρ_ν to critical density and simply Ω for the ratio of total energy density ρ of the universe to critical density.] Schramm and Steigman[4]

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stressed massive neutrinos as the least "ad hoc" possibility for the dark matter; other candidates include primordial black holes, axions, and supersymmetric particles like photinos.

Experimental evidence for neutrino masses[5] and neutrino oscillations[6] as well as theoretical work in grand unified theories (for a review, see P. Langacker[7]) further stimulated interest in massive neutrinos. The original Lubimov et al.[5] result of 14 eV $\lesssim m_{\nu_e} \lesssim$ 42 eV has been revised to $m_{\nu_e} = 35 \pm 5$ eV, although because of uncertainties in the molecular effects, the mass may be as low as $m_{\nu_e} = 0$. In any case, these results have not been verified in other laboratories, and several experiments are in progress. The negative results from oscillation experiments place strict limits on the squared mass difference of neutrino species and the mixing angle.

We will examine constraints on neutrino masses due to generalized cosmological arguments as well as specific models and look for masses consistent with all the arguments. After reviewing the necessary background material, a discussion of mass-to-light ratios and density constraints, we discuss fermion densities appropriate to neutrinos and to supersymmetric "inos", and find mass constraints from phase space arguments[8] and restrictions on the age of the universe[9,10,11]. We examine the evolution in a neutrino-dominated universe of the adiabatic density perturbations that may be responsible for galaxy formation, in particular with large-scale structures forming first. Using the concept of the Jeans mass, the minimum mass that can collapse under its own self-gravity, we find constraints on masses of neutrinos and other "inos". The alternative hierarchical view with "cold" dark matter where small scales form first is also discussed. Among the alternative to massive neutrinos that have been proposed for the nonbaryonic dark matter are supersymmetric particles[12], axions[13], and primordial black holes[14]. We will argue that if the role of the large-scale structure is dominant as implied by the large voids and large clusters, dark matter should not differ significantly from massive neutrinos, either in mass or decoupling temperature. Although we will argue that low mass neutrinos are preferred for the large scales we will also point out that they encounter problems in understanding the smaller-scale structures as indicated by the correlation function[15] and the relative dynamical equilibration timescales of different size systems. Possible scenarios[16] which avoid these problems are discussed. Finally we discuss experimental implications and summarize the results. This talk summarizes the detailed results presented in the paper by Freese and Schramm[32].

II. COSMOLOGICAL MASS DENSITIES

In this section we will summarize the cosmological density arguments. Since the deceleration parameter q_o in the standard hot big bang model with zero cosmological constant ($\lambda = 0$) is estimated to range from 0 ± 0.5 to 1.5 ± 0.5[17], an extreme upper limit to Ω with $\Lambda = 0$ is $\Omega \lesssim 4$. Dividing the mass of a bound system (obtained by application of the virial theorem) by its luminosity, one can obtain mass-to-light ratios (M/L) and estimates of matter contributions on different scales. Many authors[18] find evidence for M/L increasing linearly with scale from M/L $\sim$ (1-2) for stars to M/L $\sim$ $(300\text{-}800)h_o$ for rich clusters (see Table I, drawn largely from Faber and Gallagher[2]).

Table I

Mass-to-Light Ratios

Object	$\frac{M}{L} / (\frac{M}{L})_\odot$	Ω
stars	1-4	$(0.7\text{-}2.9) \times 10^{-3}\, h_o^{-1}$
spiral galaxies	$(8\text{-}12)h_o$	$(5.7\text{-}8.6) \times 10^{-3}$
elliptical and SO galaxies	$(10\text{-}20)h_o$	$(0.7\text{-}1.4) \times 10^{-2}$
binaries and small groups	$(60\text{-}180)h_o$	$(0.4\text{-}1.3) \times 10^{-1}$
clusters of galaxies	$(280\text{-}840)h_o$	0.2-0.6

Multiplying M/L on a given scale by an average luminosity density (uncertain by a factor of 2) for the universe[19],

$$\mathcal{L} \simeq 2 \times 10^8 h_o (L_\odot/\mathrm{Mpc}^{-3}), \tag{2.1}$$

one obtains a mass density (also listed in Table I) implied by assuming M/L on that scale applies to the average light of the universe. Davis et al.[20] have suggested that the M/L curve may be approaching an asymptotic limit (perhaps $\Omega = 1$) on the scales of superclusters, while other authors[21] believe that the curve flattens already on smaller scales. In any case the consensus is that some form of dark matter dominates the dynamics of objects on scales larger than 100 kpc and, as shown by flat rotation curves, may be important on scales larger than 10 kpc.

The above arguments are independent of whether or not the matter is baryonic. Big Bang Nucleosynthesis[3] provides density constraints on the baryonic components. A lower limit on the amount of baryonic matter in the universe can be derived from combined D and ^{3}He abundances, $\Omega_b h_o^2 \gtrsim 0.01$, and the observed abundances of ^{4}He (mass fraction $Y \lesssim 0.25$), D, and ^{7}Li result in an upper limit to baryonic matter density, $\Omega_b h_o^2 \lesssim 0.034$. These arguments restrict baryonic matter to the range

$$0.01 \lesssim \Omega_b \lesssim 0.14. \tag{2.2}$$

Helium abundances from Big Bang Nucleosynthesis also constrain the number of neutrino species; at most four low mass ($\lesssim 1$ MeV), long-lived neutrino species are compatible with $Y \lesssim 0.26$, and only three with the best observational limit of $Y \lesssim 0.25$[22]. We know two of these experimentally, namely the ν_e where $m_{\nu_e} \lesssim 60$ eV and the ν_μ where $m_{\nu_\mu} \lesssim 570$ keV. The experimental mass limit on the ν_τ is $m_{\nu_\tau} \lesssim 250$ MeV[23]. In this case we may already know all the neutrinos and other low mass "inos" which interact with the strength of neutrinos. Note, however, that the limit of three increases if the particle couples more weakly than the neutrino[24] and thus decouples in the early universe at a temperature $\gtrsim 100$ MeV.

If M/L really keeps increasing on scales larger than binaries and small groups, then Ω exceeds the upper limit on Ω_b and we are forced to say that the bulk of the matter in the universe is non-baryonic. While the general trend towards larger Ω exists, and in this paper we will assume that Ω is $\gtrsim 0.15$, the situation is by no means settled; there is still a possibility that $\Omega \leq 0.15$ and everything is baryonic, as emphasized by Gott et al.[25] One should note that inflation[26] explicitly predicts $\Omega = 1.000...$ and thus requires non-baryonic matter. On the basis of "simplicity" we believe that a nonbaryonic universe with $\Omega \gtrsim 0.15$ should satisfy $\Omega = 1$.

III. NEUTRINO (AND OTHER INO) DENSITIES AND MASSES

The equilibrium number density of a relativistic fermionic species (subscript f) is given by

$$n_f = \frac{1}{2\pi^2} \int dp\, p^2/[\exp(p - \mu_f)/T_f) + 1] \tag{3.1}$$

(throughout we take $\hbar = c = k_B = 1$). The fermions fall out of chemical equilibrium at temperature T_D when the reaction rates for their production (e.g. $e^+e^- \to f\bar{f}$) can no longer keep up with the expansion of the universe. By entropy conservation it can be shown[27] that neutrino (T_ν) and photon (T_γ) temperatures after e^+e^- annihilation are related by $T_\nu = (4/11)^{1/3} T_\gamma$, whereas the temperature of other fermions is given by $T_f = T\,(3.9/g_{*f})^{1/3}$ for $T_\gamma \ll m_e$, where g_{*f} is the number of relativistic species at decoupling.[24]

The value of the number density in the present epoch for a species of neutrinos which are relativistic at decoupling is given by $n_{\nu_i} = 109\ \text{cm}^{-3} \left(\frac{T_{\gamma o}}{2.7K}\right)^3$ ($i = e, \mu, \tau$ and $T_{\gamma o}$ is photon temperature today), and the energy density in units of the closure density by

$$\Omega_{\nu_i} = \frac{\rho_{\nu_i}}{\rho_c} = \frac{m_{\nu_i}}{97\ \text{eV}}\, h_o^{-2} \left(\frac{T_{\gamma o}}{2.7K}\right)^3 \qquad (3.2)$$

where m_{ν_i} is the mass of a neutrino species. If the sum of the masses of different neutrino species exceeds $\sim 100\ h_o^2$ eV the universe is closed. Requiring $\Omega \lesssim 4$ and $h_o \lesssim 1$ gives only the weak limit, $\sum m_\nu \lesssim 400$ eV; we will see that age of the universe constraints can strengthen this limit. The ratio of neutrino to baryonic matter is given by

$$\frac{\Omega_\nu}{\Omega_b} \gtrsim \frac{\sum_i m_{\nu_i}}{2.4\text{eV}} \qquad (3.3)$$

where the equality sign corresponds to the largest value of baryonic matter density consistent with element abundances from primordial nucleosynthesis, $\Omega_b h_o^2 \lesssim 0.034$. Hence if the sum of the neutrino masses exceeds a few eV, neutrinos are the dominant matter in the universe and must play an important role in galaxy formation.

The above discussion of fermion number densities assumes the fermions are relativistic at decoupling. For neutrinos more massive than a few MeV or for other fermions whose mass exceeds their decoupling temperature, the mass density falls roughly as $m_f^{-1.85}$ [28]. Thus total density limits $\Omega \lesssim 4$ can be satisfied for sufficiently massive ($m_\nu \gtrsim 1$ GeV) particles, while if $\sum_i m_{\nu_i} \gtrsim 20$ GeV the density has fallen so low that neutrinos cannot be the dominant matter. Krauss[29] and Goldberg[30] have recently shown that for certain supersymmetric particles the annihilation rates can be slower than those for neutrinos, and the mass limits are pushed to even higher values. Following Gunn _et al._[31] for these very massive particles the appropriate mass density one has to worry about exceeding is the density of matter in groups of galaxies ($\Omega \leq 0.13$) not the total density of the universe (since as we will see these massive particles must cluster on small scales, in ways such that baryons are good tracers of their presence. This yields $m_\nu \gtrsim 6$ GeV for neutrinos and corresponding higher limits for photinos.

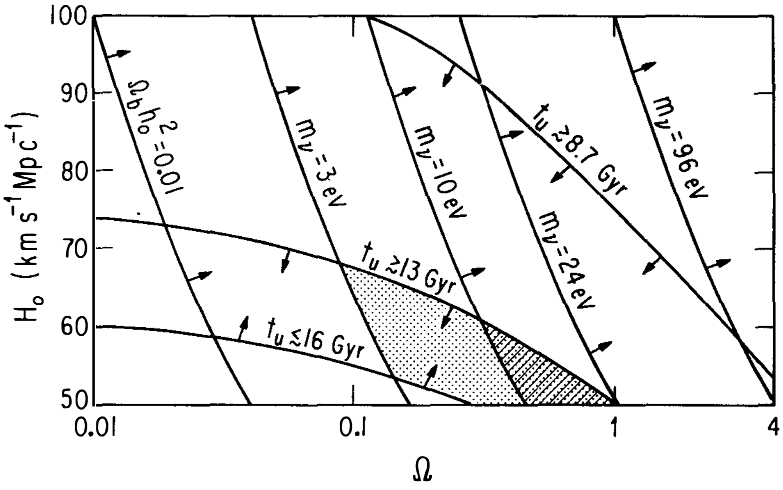

Figure 1: On a plot of Hubble parameter H_o vs. energy density Ω we have drawn curves for several values of the age of the universe ($t_u = f(\Omega) H_o^{-1}$), for the energy density in baryons ($\Omega_b h_o^2 \geq 0.01$), and for the total energy density of the universe with several values of the neutrino mass $\Omega_{total} =$

$$\Omega_b + \Omega_\nu \geq (0.01 + \frac{\sum m_\nu}{97 \text{ eV}}) h_o^{-2} \text{(for } T_{\gamma o} = 2.7\text{K)}.$$

The firm upper limit to the age of the universe $t_u < 8.7$ Gyr restricts $\sum m_\nu \lesssim 100$ eV, while an age range consistent with dynamics and globular clusters 13 Gyr $< t_u <$ 19 Gyr requires $\sum m_\nu \lesssim 25$ eV. The dotted region indicates the range of neutrinos massive enough to serve as the dark matter in clusters ($m_\nu > 3$ eV) yet consistent with all the age arguments. The smaller hatched region indicates the range of neutrinos which may be responsible for the formation of large-scale structure in the adiabatic picture ($m_\nu \gtrsim 10$ eV, cf. § 4).

Figure 1 is a plot of total energy density in the universe ($\Omega \lesssim 4$) vs. Hubble parameter ($0.5 \lesssim h_o \lesssim 1$). The total energy density is the sum of baryon density ($\Omega_b h_o^2 \gtrsim 0.01$) and neutrino density ($\Omega_{\nu_i} h_o^2 \simeq m_{\nu_i}/97$ eV); we have plotted this sum for several values of neutrino mass. We have also plotted curves for several values of the age of the universe, which can be parameterized (for $\lambda = 0$) as $t_u = f(\Omega)H_o^{-1}$ (where $f(\Omega)$ is a monotonically decreasing function of Ω with values between 1 and ½ in the range of interest). Several arguments[10)] have been used to restrict the age of the universe: certainly it must exceed the age of the solar system $t_u > 4.6$ Gyr (1 Gyr = 10^9 yr), dynamical arguments ($h_o \gtrsim 0.5$) restrict $t_u \lesssim 20$ Gyr, the age of the globular clusters combined with an upper limit on ^{4}He fraction $Y \lesssim 0.26$ restricts 13 Gyr $\lesssim t_u \lesssim$ 19 Gyr[14)], and nucleocosmochronology requires 8.7 Gyr $\lesssim t_u \lesssim$ 19 Gyr. The range (13-16) Gyr is simultaneously consistent with all arguments, while the widest range allowed by the most stringent limits is (8.7 - 19) Gyr. Consistency with the widest range allowed as well as the restrictions $\Omega \lesssim 4$ and $h_o > ½$ requires

$$\sum_i m_{\nu_i} \lesssim 100 \text{ eV} \quad (8.7 \text{ Gyr} < t_u). \tag{3.4}$$

Consistency with the "best fit" range of ages restricts

$$\sum_i m_{\nu_i} \lesssim 25 \text{ eV} \quad (13 \text{ Gyr} < t_u) \tag{3.5}$$

(see also 9)), where $\Omega = 1$ is achieved only for $\sum_i m_{\nu_i} \simeq 25$ eV. As Schramm[11)] noted, this best fit age also limits $h_o^{\frac{1}{}} < 0.7$ to have concordance. By the inversion of this age argument, an actual neutrino mass gives an upper limit to the age of the universe. For example, if Lubimov et al.[5)] are correct and $m_{\nu_e} \gtrsim 30$ eV, then the universe must be younger than 12 Gyr.

Similar constraints can also be found on the masses of other fermions which decouple while still relativistic and are candidates for the dark matter; for this analysis see Freese and Schramm[32)] as well as Olive and Turner[33)]. For all standard unified models with $g_{*f} \lesssim 161$, the limit from our best fit age argument does not allow m_f to exceed 400 eV, contrary to the limits in previous papers(12).

Tremaine and Gunn[8)] have used phase space arguments to obtain a restriction on neutrino masses. The smaller the scale on which neutrinos are confined, the larger the velocity dispersion, and the easier it is for neutrinos to escape from the region. A necessary (but not sufficient) condition for trapping neutrinos on the scale of clusters is the requirement $m_\nu \gtrsim 5h_o^{\frac{1}{2}}$ eV, on the scale of binaries and small groups $m_\nu \gtrsim 14h_o^{\frac{1}{2}}$ eV, and in galaxies $m_\nu \gtrsim 20$ eV. If massive neutrinos are to solve the missing mass problem they must be trapped at least on scales of clusters of galaxies, i.e., $m_\nu \gtrsim 3$ eV. Of course to actually trap them requires some cluster formation scenarios. Possibilities will be addressed in the next two sections, where it will be shown that this lower limit can probably be strengthened for any realistic scenario.

IV. ADIABATIC PERTURBATIONS

The formation of galaxies requires the clumping of baryons; i.e., enhancements $\delta_b = \frac{\delta\rho_b}{\rho_b}$ in the baryon density over the background value must grow from small values in the early universe to nonlinearity ($\delta_b > 1$) by the present-day to achieve the formation of bound structure. In the adiabatic mode the baryon perturbations δ_b are accompanied by radiation perturbations δ_γ, whereas in the isothermal mode initially $\delta_\gamma << \delta_b$. In general any primordial fluctuation scheme for galaxy formation can be treated as a superposition of these two independent modes. Thus in the adiabatic theory of galaxy formation, initially $\delta_\gamma = \delta_\nu = \delta_{\bar{\nu}} = \frac{4}{3}\delta_b$ (where $\delta_i = \delta\rho_i/\rho_i$ describes the density enhancement of particle species i in a perturbation over the background value). These fluctuations grow together outside the horizon, and once inside the horizon their evolution depends on the value of the Jeans mass.

The Jeans mass is the smallest mass unstable to gravitational collapse. It is given by the rest mass of particles in a sphere of radius equal to the Jeans length λ_J, the scale on which radiation pressure forces just balance gravitational forces. In Figure 2 we have plotted the evolution of neutrino (M_J) and baryon (M_{Jb}) Jeans masses in a neutrino-dominated universe[34]. The neutrino Jeans mass reaches its peak value[35),]

$$M_{\nu M} \simeq 1.8\ m_{pl}{}^3/m_\nu{}^2 \simeq 3 \times 10^{18}\ M_\Theta/(m_\nu/eV)^2 \qquad (4.1)$$

at $z_M \simeq 1900\ m_\nu(eV)$, where $m_{pl} = G^{-\frac{1}{2}} = 1.2 \times 10^{19}$ GeV is the Planck mass. Neutrino perturbations on scales $\gtrsim M_{\nu M}$ can grow once the neutrinos become the dominant matter. However, Bond et al.[34] have shown that neutrino perturbations on scales smaller than $M_{\nu M}$ are strongly damped by free-streaming of the neutrinos out of dense regions (Landau damping). Only perturbations on scales larger than $M_{\ M}$ can survive and grow to nonlinearity[36]. To enable the formation of large-scale structure, we require this damping scale to be smaller than the largest structure observed, superclusters of mass $\simeq 10^{16}\ M_\Theta$, i.e., $M_{\nu M} \lesssim M_{sc} \simeq 10^{16}\ M_\Theta$. Eq. (4.1) is only approximate; folding an initial power spectrum $|\delta_k|^2 \propto k^n$ with a transfer function to describe damping by neutrino diffusion, Bond, Szalay, and Turner[36] obtain an n-dependent power spectrum. Although the peak of the power structure is the scale on which perturbations first go nonlinear, significant power may exist on somewhat smaller or larger scales. We take the

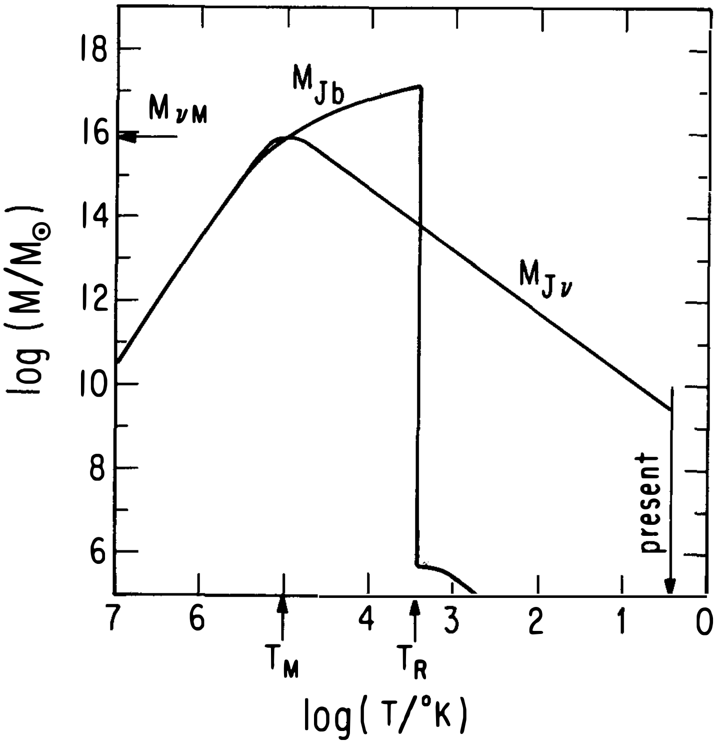

Figure 2: Neutrino and baryon Jeans masses as a function of temperature for $m_\nu \simeq 20$ eV. During the radiation-dominated era, the Jeans mass is approximately the comoving mass inside the horizon and grows as $(1 + z)^{-3}$. Once neutrinos become the dominant matter at $T_M \simeq 10^5$ K, the neutrino Jean mass peaks at $M_{\nu M} \simeq 1.8\ m_{pl}{}^3/m_\nu^2 \simeq 7.5 \times 10^{15}\ M_\odot$ for $m_\nu = 20$ eV and thereafter falls as $(1 + z)^{3/2}$. The exact shape of $M_{J\nu}$ near its peak value has been calculated by Bond, Efstathiou, and Silk (1980) and is merely approximate here. M_{Jb} drops at recombination ($T_R \simeq 2700$K) to $\simeq 5 \times 10^5 M_\odot$.

least restrictive limit, the smallest mass for physically plausible values of n that has significant power, and find that

$$M_{\nu M} \simeq \frac{9 \times 10^{17}\ M_\odot}{(m_\nu/\mathrm{eV})^2} \lesssim 10^{16}\ M_\odot\ . \qquad (4.2)$$

In a universe with one massive neutrino species this requires

$m_\nu \gtrsim 10$ eV.

If there are three species of neutrinos with equal mass, the mass of each species must satisfy $m_{\nu_i} \gtrsim 16$ eV, giving a sum of masses $\sum_i m_{\nu_i} \gtrsim 48$ eV. This is not compatible with the requirement $\sum_i m_{\nu_i} \lesssim 25$ eV from consistency of all the arguments restricting the age of the universe. A "best fit" model does not allow all the neutrino masses to be equal. Of course if we relax our age constraint to $t_u > 8.7$ Gyr then equal masses are allowed. If larger scales than $M_{\nu M}$ in Eq. (4.3) reach nonlinearity first and tidally strip the smaller scales, the limit on the masses only becomes more restrictive.

In this adiabatic picture with massive neutrinos, the smallest scales to form initially are large clusters, and smaller scales come from later cooling and fragmentation. The alternative model of galaxy formation, where small scales form first and cluster hierarchically onto larger and larger scales, has not been shown to give rise to the observed structure on large scales, namely the large voids seen by Kirshner et al.[38] as well as large clusters and filaments. Also, dark matter which clusters first on small scales will yield a constant M/L; however $\Omega \simeq 1$ for the universe is possible only if M/L continues to increase with scale, or if there are regions with M/L larger than any measured value. If, indeed, small scale damping is required for the formation of large-scale structure, we also get a lower limit on the Jeans mass. Note that the necessity for small-scale damping has not been rigorously proven. The failure of the attempts of Frenk, White, and Davis[15] to have hierarchical pictures work and the ease with which their small-scale damped models succeed in giving rise to observed large-scale structure is certainly suggestive if not compelling. Damping those scales smaller than cluster sizes,

$$M_{\nu M} \simeq \left(\frac{g_f}{2}\right) \left(\frac{10.75}{g_{*f}}\right) \frac{1.8\ m_{pl}^3}{m_f^2} \gtrsim 10^{14}\ M_\odot \qquad (4.3)$$

requires

$$m_f \lesssim 200\ \mathrm{eV}\ (g_f/2)^{1/2} \qquad (4.4)$$

for the mass of any non-interacting particle proposed for the dominant matter (for an alternative approach to similar results see (39)). For particles decoupling earlier than neutrinos the number density is lower, and hence to conserve the inequality in Eq. (4.4), the restriction on the mass only becomes tighter. If valid, this argument rules out the high mass branch, $m_f \gtrsim 6$ GeV for the dominant matter (if $\Omega \simeq 1$), since such high mass particles would cluster first on very small scales and would not explain the large voids or large M/L (this does not mean such particles cannot exist; it merely means that their contribution to Ω must be small, $\lesssim 0.2$). This argument would also rule out gravitinos or other supersymmetric particles in the keV mass range.

Since our primary motivation for non-baryonic matter is to get a large Ω, this upper limit on the mass of our candidate particle becomes quite constraining. As we mentioned in Section III, the number density of any species, n_f, decreases roughly stepwise with increasing decoupling temperature T_D for that particle[24]. Given low particle masses m_f, in order to keep a high Ω, where $\Omega \simeq m_f n_f/\rho_c$, the decoupling temperature T_f must also be low. Quantitatively, the constraint $m_f \lesssim 200$ eV and $\Omega \sim 1$ argues that

$$n_f \gtrsim 61\ h_o^{\ 2}\ (g_f/2)^{-\frac{1}{2}}\ \text{cm}^{-3}.$$

Comparing this with Fermi-Dirac number density Eq. (3.2), we find that the number of relativistic species at decoupling must satisfy $g_{*f} \leq 16 h_o^{-2} (g_f/2)^{3/2} (\frac{T_{\gamma o}}{2.7K})^3$, i.e. for $g_f \leq 2$ and $T_{\gamma o}$ = 2.7K T_D must be $\lesssim$ 0(100 MeV). In other words, in the favored model where Ω is large ($\Omega \simeq 1$) and large-scale structure forms first, the universe is probably dominated by particles which behave in all ways like massive neutrinos.

Any candidate particle for the dominant matter decoupling at $T_D \lesssim 100$ MeV is constrained by Big Bang Nucleosynthesis arguments in exactly the same way as neutrinos. We have seen in Section II that it is very difficult to add to the three known neutrino species a full additional neutrino type (or equivalently $g_* = 7/4$ additional degrees of freedom, where one bosonic spin state contributes $g_* = 1$ and one fermionic spin state contributes $g_* = 7/8$) without violating limits on the observed ^{4}He abundance. Thus it is probable that the dominant matter is a neutrino and not some other "ino". Axions have been proposed for the dark matter,[13] but since they have a low Jeans mass they cannot be the dominant matter if the large voids require large scales forming first. These arguments would also prevent small (planetary mass) black holes from being the dominant matter unless they were able to stimulate Ostriker-Cowie[40] explosions[14].

The formation of structure in the universe requires the growth of perturbations from small amplitudes in the early universe to nonlinearity ($\delta\rho/\rho > 1$) by the present day. In the standard adiabatic picture of galaxy formation without massive neutrinos, the coupling of baryons to photons before recombination damps the perturbations on scales smaller than the Silk mass, $M_S \simeq 3 \times 10^{13}\Omega_B^{-1/2}\Omega_\nu^{-3/4}h_o^{-5/2}M_\odot$ [34,41], and prevents perturbations from growing on larger scales. After recombination at $1+z_R \simeq 1000$ the surviving perturbations in an Einstein-de-Sitter universe grow as $(1 + z)^{-1}$, allowing a growth factor of only about 1000 between recombination and the present day. To reach nonlinearity by today, the perturbations must have been $(\delta\rho/\rho)_b \simeq 10^{-3}$ at recombination. The resulting microwave anisotropy $\delta T/T \simeq 1/3(\delta\rho/\rho)_b \simeq 1/3 \times 10^{-3}$ would be larger than the observed large-scale anisotropy, $\lesssim 2 \times 10^{-4}$. This dilemma is resolved in a universe dominated by massive neutrinos or other "inos".[41] After the neutrinos become the dominant matter at $1 + z_M \simeq 1900\ m_\nu$ (time t_M), neutrino perturbations grow as $(1 + z)^{-1}$ on scales larger than the maximum neutrino Jeans mass, $M \geq M_{\nu M}$. Baryon perturbations are tied to the photons until after the time of recombination (t_R), when they can rapidly catch up to the neutrino perturbations. This extra growth period for neutrino perturbations allows nonlinearity by the present epoch with $\delta_\nu(t_R) \simeq 10^{-3}$ and

$$\delta_b(t_R) \simeq \delta_\nu(t_R)\left(\frac{1 + z_R}{1 + z_M}\right) \simeq \frac{10^{-3}}{1.9m_\nu} \simeq 5 \times 10^{-4} m_\nu^{-1} . \qquad (4.5)$$

The microwave anisotropy, now only $\delta T/T \simeq 2 \times 10^{-4} m_\nu^{-1}$, no longer violates the observed limit as long as $m_\nu \gtrsim 1$ eV.

Before leaving the adiabatic mode, it is worthwhile to remember that GUT s naturally allow adiabatic perturbations but not isothermal ones[42]. All attempts at producing isothermals in the context of GUT s have been very ad hoc and unnatural[43]. In fact inflationary scenarios seem naturally to give rise to adiabatic perturbations[44] with a Zel'dovich spectrum, i.e. fluctuations come within the horizon at constant although model-dependent amplitude.

HIERARCHICAL CLUSTERING

The alternative to condensing large scales first is to build up from small scales. We discuss this because the adiabatic pancake scheme of the previous section does not by itself lead to the small scale structure. Such structure requires subsequent cooling and fragmentation of the baryons while leaving the non-interacting neutrinos on the large scale where they produce the large M/L. These cooling and fragmentation processes are not well understood, so many *ad hoc* assumptions are required. On the other hand, isothermal perturbations naturally produce these small scales down all the way to the scale of globular clusters. The hierarchical picture, whether produced by isothermals or by adiabatics with a small "ino" Jeans mass so that small scales form first, does correctly fit the 2 and 3 point galaxy correlation functions[15] up to cluster scales of $\sim$ 5 Mpc. To fit this in the simple pancake picture (with no cooling or fragmentation assumptions) requires a fine tuning of the parameters or at least a very small Jeans mass that is not needed in a hierarchical scheme. If neutrionos make the large scales first then galaxies might not form until redshifts $z < 1$, but quasars are seen with $z \sim 3.5$. In addition, the equilibration time scale can be determined from the dynamics of galaxies on various scales and it is found that the largest scales are not in dynamical equilibrium yet, whereas small scales are. For example, the core of the Virgo Cluster as well as the Coma Cluster are well virialized, whereas the Virgo Supercluster is not. This might argue in favor of isothermals or at least a hierarchical picture with small ($\lesssim 10^8$ $M_\odot$) Jeans mass as given by axions, GeV mass photinos, or planetary mass black holes. But as we've already seen GUT s argues against isothermals and the large scale voids, superclusters, and $\Omega \sim 1$ argue against any hierarchical picture on the largest scales (hierarchical models produce constant M/L).

One might suggest a solution where the adiabatic picture holds but there are two significant nonbaryonic particles, the dominant one a 10 to 25 eV neutrino and the second a particle such as the axion or massive gravitino which has a small Jeans mass and yields the small scale structure. Such a model is certainly possible but seems very *ad hoc* in that we require new particles to solve each problem. Alternatively, some new hierarchical model may be developed with tidal stripping and a power spectrum which, contrary to previous attempts, does produce the large scale structure and enables $\Omega \sim 1$. We feel the most likely model is one in which the adiabatic picture holds and cooling and fragmentation of the baryons is enhanced at the intersection of caustics[16]. In this case galaxy formation occurs in filaments. Fry[46] has shown that the galaxy correlation functions for filaments agrees with the observed 2 and 3 point galaxy correlation functions as well as the hierarchical case. To meet the time scale constraints one merely has to argue that within the filaments distant regions will take longer to equilibrate than central ones. Of course, a low density universe with $\Omega < 0.15$ and only baryons is also a possibility. Scenarios of mixed isothermal and adiabatic components are also possible[47]; since the adiabatic mode grows more quickly, the resulting large-scale structure is essentially that of the adiabatic picture.

VI. DISCUSSION AND SUMMARY

We have tried to be very explicit about the assumptions required for each cosmological "ino" mass argument, and then we synthesized the arguments to obtain some very powerful simultaneous constraints.

Table II - Cosmological Matter

	Light emitting matter (glowing baryons)	Dark Matter (no dissipation into galactic cores)			
		COLD[1]		WARM[2]	HOT[3]
		Baryons	Others		
Examples	stars	Jupiters or stellar mass black holes[4]	Axions, (10-100)GeV mass "inos", planetary mass black holes	10eV-400 eV[7] "inos"	10 eV - 25 eV[7] "inos" (if $N_\nu=3$ then ν_e, ν_μ or ν_τ)
Maximum Jeans Mass	Irrelevant	Depends on fluctuation spectrum	$\lesssim M_\odot$	$\sim 10^{12} M_\odot$ - $10^{16} M_\odot$	$\sim 10^{15}$-$10^{16} M_\odot$
Ω_i	~ 0.01	~ 0.01 - 0.14	$\lesssim 0.6$	$\gtrsim 1$ [6]	$\gtrsim 1$ [6]
M/L	$1 - 10h_o$	$\lesssim 150\ h_o$	$\lesssim 800\ h_o$ [5]	increases with scale	
dark halos of dwarf spheroidals	--	O K		marginal on phase-space, unlikely from Jeans mass	NO
galaxy correlation function	observed	O K		requires special heating and cooling of baryons so mass anti-correlates with light	
large-scale filaments and voids	NO	no existing hierarchical model yields this structure		marginally natural	natural
$\frac{\delta T}{T} \lesssim 10^{-4}$ with adiabatic fluctuations	NO	NO	YES	YES	
galaxy formation epoch	$z \sim 100$	$z \sim 100$	$z \sim (100-1)$	$z \sim 10$	$z \sim 1$

1 "cold": decouples while non-relativistic.
2 "warm": decouples while relativistic, present temperature $T_f < T_\nu \simeq 2K$.
3 "hot": decouples while relativistic, present temperature $T_f \simeq T_\nu \simeq 2K$.
4 stellar black holes were baryons at Big Bang Nucleosynthesis.
5 it is assumed that low Jeans mass will result in non-dissipative clustering with baryons; no self-consistent calculation shows otherwise.
6 since these don't fit small scales well, prime motivation is large scales and high Ω's.
7 assumes $t_u > 13$ Gyr or equivalently $\Omega h_o^2 < 0.25$, i.e. can have $\Omega = 1$ only for $h_o = 1/2$.

Table II gives a summary of the arguments for different possible constituents of the universe. A universe with $\Omega \gtrsim 0.15$ must probably be dominated by non-baryonic matter, although the observational evidence pointing in this direction is not without loopholes. The galaxy formation mode compatible with GUT s, namely the adiabatic mode, produces disagreement with the 3K background unless the universe is dominated by non-baryonic matter. We also mentioned that Big Bang Nucleosynthesis constrains the number of neutrinos to at most 4, probably only 3.

Merely from limits on the total mass density of the universe, neutrino masses are restricted to two ranges, $\sum_i m_{\nu_i} \lesssim 400$ eV or $m_{\nu_i} \gtrsim 6$ GeV (corresponding arguments for other "inos" depend on their decoupling temperatures). However, applying an additional age constraint at $t_u > 8.7$ Gyr, we find a stricter limit $\sum_i m_{\nu_i} \leq 100$ eV ($\sum_{ino} m_{ino} \lesssim 2$ keV) while the range $m_{\nu_i} \gtrsim 6$ GeV remains unchanged. If we further restrict the age by requiring consistency between globular cluster ages, observed helium abundances and Big Bang Nucleosynthesis, then $t_u > 13$ Gyr and $\sum_i m_{\nu_i} \leq 25$ eV ($\sum_{ino} m_{ino} \lesssim 400$ eV) (remember age arguments assume $\Lambda = 0$).

We mentioned that the Tremaine and Gunn[8] phase space argument gives a necessary (but not sufficient) limit of $m_\nu \gtrsim 3$ eV if neutrinos are to cluster on cosmologically significant scales. In the GUT s favored adiabatic scenario, we showed that a maximum neutrino Jeans mass small enough to allow the formation of superclusters requires $m_\nu > 10$ eV. For three neutrino species of equal mass, the limit becomes more restrictive, $m_{\nu_i} > 16$ eV ($\sum_i m_{\nu_i} > 48$ eV).

Hence three equal neutrino masses are allowed only for a universe younger than 12 Byr, an age excluded by the best fit age arguments. Based on the conclusions of Frenk, White, and Davis[15] that the formation of large-scale structure required small-scale damping, a potentially far-reaching argument was presented that the mass of the dominant particle be less than ~ 200 eV. In a high density universe of $\Omega \simeq 1$, with $T_{\gamma o} = 2.7$K, a particle with this mass and $g_f \leq 2$ decouples at $T_D \lesssim 100$ MeV; in other words the dominant matter is similar to a massive neutrino. The nucleosynthetic constraint that there are probably only 3 neutrinos leads to the conclusion that not only does the dominant particle act like a massive neutrino but it probably is one. The mass for this best fit neutrino is $10 \lesssim m_\nu \lesssim 25$ eV. While the adiabatic picture with massive neutrinos successfully gives the large scale structure, some problems exist on smaller scales. Various speculative models are given which might solve the small scale problems while retaining a solution to the large scale. We feel the most promising is the scheme of Bond, Centrella, Szalay and Wilson[16], which investigates the cooling and fragmentation of pancakes into galaxies. It will be interesting to see whether subsequent work verifies this model.

In the most likely scenario the most massive neutrino would have a mass 10 eV $\lesssim m_\nu \lesssim$ 25 eV while the other neutrinos would have negligible masses. If the Lubimov *et al.*[5] result holds then we may have found the answer, massive ν_e with m_{ν_μ} and $m_{\nu_\tau} \lesssim$ 3 eV. Although one might naïvely expect the ν_τ to be the most massive, because these are weak rather than mass eigenstates, depending on the mixing matrix any combination of leptons might be involved in the most massive eigenstate. Ushida *et al.*[64] in an experiment looking for $\nu_\mu - \nu_\tau$ oscillations sensitive to mixing angles $\sin^2 2\alpha \gtrsim 0.013$ found a limit $|m_{\nu_\mu}{}^2 - m_{\nu_\tau}{}^2| < 3.0\,\mathrm{eV}^2$ (90% confidence level). This suggests that either the mixing angle is very small, that limits on the age of the universe from globular clusters must be reevaluated (although all current possible corrections go towards longer, not shorter ages), or that there is some form of dark matter other than the three known neutrino species. Unfortunately, if the mass eigenstates is close to ν_τ and the mixing angle is small, the experimental detection of the dominant matter in the universe may take a long time.

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