Neutron densities from parity-violating elastic electron scattering

O Moreno[1], P Sarriguren[2], E Moya de Guerra[1], JM Udías[2], TW Donnelly[3], I. Sick[4]

[1] Dpto. Física Atómica, Molecular y Nuclear, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Ciudad Universitaria, E-28040 Madrid, Spain
[2] Instituto de Estructura de la Materia, Consejo Superior de Investigaciones Científicas, Serrano 123, E-28006 Madrid, Spain
[3] Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambdrige, MA 02139, USA
[4] Departement für Physik, Universität Basel, CH-4056 Basel, Switzerland

E-mail: `oscar.moreno@iem.cfmac.csic.es`

Abstract. We analyze parity-violating elastic electron scattering as a complementary tool for precise determination of neutron densities in nuclei. In particular we discuss how to extract the ratio between neutron and proton rms radii and monopole form factors from theoretical and experimental asymmetries. The structure of the nuclear target is obtained from a Skyrme Hartree-Fock mean field with pairing interactions in BCS approximation. We focus on the parity-violation asymmetry for ^{208}Pb and for ^{12}C as examples of $N > Z$ and $N = Z$ nuclei. In the latter case we study the influence of nuclear isospin mixing by means of the asymmetry deviation. Distorted wave calculations are shown and are compared to plane wave impulse approximation.

1. Introduction

An improved knowledge of the neutron density distributions in atomic nuclei is one of the most relevant tasks in present nuclear structure activities. Firstly, modern effective nuclear forces are typically constructed without any constraint on neutron density due to the lack of accuracy in the experimental information. An accurate parametrization of the isovector channel of the effective nuclear force turns out to be, however, essential for the description of phenomena such as halos [1] or neutron skins [2] in exotic neutron-rich nuclei. Secondly, constraints in the isospin dependence of the energy functional in nuclear matter would lead to an improved neutron equation of state with important consequences in astrophysics [3], particularly in the structure of neutron stars. Precise neutron density distributions in nuclei are also required to make progress in atomic parity non-conservation experiments [4].

In contrast to the proton distributions, the available experimental data concerning neutrons is currently insufficient. Progress on the determination of neutron densities has been been made through the use of hadronic probes as in hadron scattering, antiprotonic atoms, or excitations of the giant-dipole or spin-dipole resonances. However, these methods are subject to large uncertainties due to the entanglement between the nuclear structure and the reaction mechanism.

This situation has led to a reconsideration of the leptonic probes as tools to determine neutron distributions. Elastic magnetic electron scattering from odd-N nuclei is sensitive to the neutron magnetic moment and information on the odd neutron density can be extracted, but not on the whole neutron density since most of the neutrons are coupled to spin zero and do not contribute to the magnetization. Another possibility is the direct measurement of the neutron form factor from the asymmetry in parity-violating (PV) elastic polarized electron scattering [6]. PV electron-nucleus scattering arises from the interference of electromagnetic and weak neutral amplitudes and it is a clean and powerful tool for measuring the spatial distribution of neutrons in nuclei with unprecedented accuracy. Indeed, electron-nucleus scattering has been in the past an excellent tool to investigate the nuclear structure. Reliable information on electromagnetic form factors and charge density distributions has been accumulated for stable nuclei and it is expected that new experimental facilities will provide information for unstable nuclei as well. Concerning neutron radii, early extractions came from Coulomb energy differences [7] and from neutron pickup reactions [8], but these reactions are mainly sensitive to the tail of the neutron density and model assumptions were needed for the interior density. As a result, no existing measurement of neutron densities or radii has an established accuracy of 1%. In ^{208}Pb, for instance, electron scattering experiments have determined the charge radius to better than 0.001 fm, whereas realistic estimates place the uncertainty in the neutron radius at about 0.2 fm.

2. Theory

Whenever a photon is exchanged between two charged particles, a Z^0 is also exchanged. At the energies of interest in electron scattering the strength of the weak process mediated by the Z^0 boson is negligible compared to the electromagnetic strength. Hence the role played by the exchange of the Z^0 is not significant unless an experiment is set up to measure a parity violating observable. While the electromagnetic interaction conserves parity, the weak interaction does not and this is how we are sensitive to Z^0-exchange in electron scattering.

The degree of parity violation in the process can be measured by means of the parity-violating asymmetry,

$$\mathcal{A} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} \,, \tag{1}$$

which is proportional to the difference between the cross-section of incoming electrons longitudinally polarized parallel and antiparallel to their momentum.

When considering the exchange of a single gauge boson between the nuclear target and the incoming electron, the latter not being affected by the nuclear Coulomb field (i.e. within plane wave Born approximation, PWBA), the parity-violation asymmetry can be written as [9]:

$$\mathcal{A} = \frac{G_F}{2\pi\alpha\sqrt{2}}\; Q^2\, \frac{W^{PV}}{W^{PC}} \tag{2}$$

where G_F and α are the Fermi and fine-structure coupling constants, respectively, W^{PV} and W^{PC} are the parity-violating and parity-conserving responses, and Q is the four-momentum transfer of the scattering ($Q^2 = q^2 - \omega^2$, with q and ω the three-momentum transfer and the energy transfer). The asymmetry thus factorizes into a Standard-Model part and into a nuclear-structure dependent part (the ratio of PV to PC responses), apart from the Q^2 dependence.

By considering only elastic scattering by $J^\pi = 0^+$ nuclear targets, the asymmetry can be written in terms of the Coulomb-type monopole ($C0$) operators, which are the only ones that can induce the elastic transition:

$$\mathcal{A} = \frac{G_F}{2\pi\alpha\sqrt{2}}\; Q^2\, \frac{\widetilde{F}_{C0}(q)}{F_{C0}(q)} = \kappa\, Q^2\, \left[\frac{\widetilde{G}_{E_n}\, F_n^0\, N + \widetilde{G}_{E_p}\, F_p^0\, Z}{G_{E_n}\, F_n^0\, N + G_{E_p}\, F_p^0\, Z}\right] \,, \tag{3}$$

where

$$\kappa = \frac{G_F}{2\pi\alpha\sqrt{2}} \approx 7 \cdot 10^{-6}\ \mathrm{fm}^2. \tag{4}$$

The form factors of protons and of neutrons in Eq. 3 are defined so that $F(q=0)=1$:

$$F_p^0(q) = \frac{1}{Z}\int d^3r j_0(qr)\rho_p(r), \qquad F_n^0(q) = \frac{1}{N}\int d^3r j_0(qr)\rho_n(r) \tag{5}$$

and the nucleon weak-neutral form factors are defined as:

$$\widetilde{G}_{E_p} = \beta_V^p G_{E_p} + \beta_V^n G_{E_n} + \frac{1}{2}\beta_V^{(s)} G_E^{(s)}, \qquad \widetilde{G}_{E_n} = \beta_V^n G_{E_p} + \beta_V^p G_{E_n} + \frac{1}{2}\beta_V^{(s)} G_E^{(s)}, \tag{6}$$

where $\beta_V^p = 0.04$ and $\beta_V^n = -0.5$ (twelve times β_V^p, this difference being the origin of the larger sensitivity of the parity-violating observables to the neutron distribution). The nucleon strangeness content terms contain the strangeness form factor $G_E^{(s)}$ and the factor $\beta_V^{(s)} = -1$. The effect of the strangeness content on the PV asymmetry can be found in Ref. [11] for the maximum value compatible with experiments, but it should be mentioned that the current central value of the experimental range is very close to zero.

By neglecting the strangeness content, $G_E^{(s)}$, and the electric neutron form factor, G_{E_n}, the PV asymmetry can be rewritten as:

$$\mathcal{A} = \frac{\kappa}{2}\, Q^2 \left[4\sin^2\theta_W + \frac{N-Z}{Z} + \frac{N}{Z}\,\mathcal{R}\right] \tag{7}$$

The first two terms within brackets are structure independent isoscalar and isovector contributions while the last term depends on the nuclear structure, with

$$\mathcal{R} = \frac{F_n^0(q)}{F_p^0(q)} - 1 \tag{8}$$

For the heavy, stable nuclei where $N > Z$, the asymmetry can be further approximated as:

$$\mathcal{A} \approx \frac{\kappa}{2}\,\frac{N}{Z}\, Q^2\, \frac{F_n^0(q)}{F_p^0(q)} \tag{9}$$

whereas for $N = Z$ nuclei, assuming pure $T = 0$ ground states ($F_n^0 = F_p^0$, $\mathcal{R} = 0$), the asymmetry reads:

$$\mathcal{A} = 2\,\kappa\,\sin^2\theta_W\, Q^2 \tag{10}$$

3. Results

We show results on PV asymmetry in elastic scattering of polarized electrons by nuclear targets whose ground state structure has been obtained from a Skyrme Hartree-Fock mean field with pairing interactions in BCS approximation (HF+BCS in short).

As an example of the PV asymmetry in elastic electron scattering by a $N > Z$ nuclear target we show in Fig. 1 our results for ^{208}Pb [10]. Three calculations are presented in plane wave (PW) approximation. One of them (dashed line) is the unrealistic case of pure ground state isospin, where $F_n^0(q) = F_p^0(q)$ for every q. The black solid line corresponds to the calculation with the actual HF+BCS nuclear structure, where the form factors of protons and neutrons are different because of the different number of each type of nucleon and also because of the presence of the Coulomb interaction between the protons. The light solid line corresponds to the

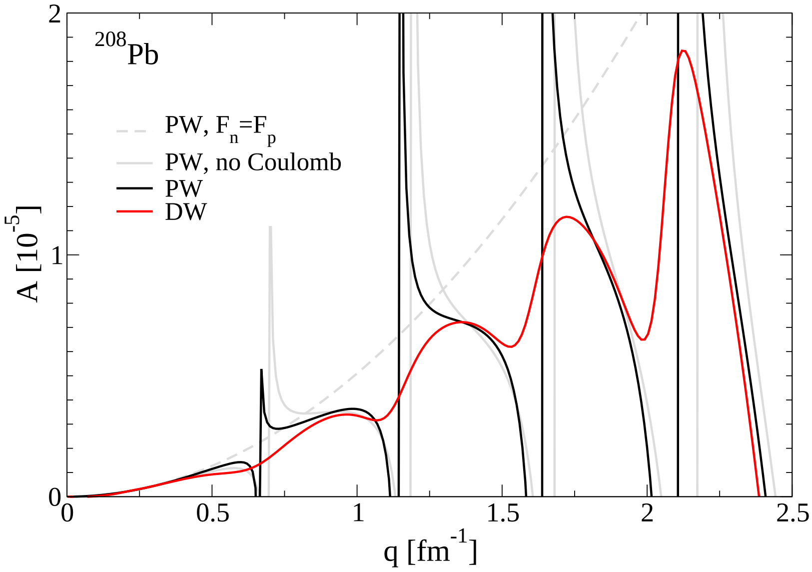

Figure 1. PV asymmetry for ^{208}Pb from a HF+BCS calculation. The solid red line is the full calculation in DW. For comparison we also show PW results in different approximations: taking $\mathcal{R} = 0$ (dashed line), neglecting the Coulomb interaction in the nuclear hamiltonian (light solid line), and full PW calculation (black solid line).

calculation using different proton and neutron form factors but where the Coulomb interaction has been turned off, so that the effect of this interaction on the isospin mixing becomes apparent through a direct comparison with the black solid line. Finally, the full calculation but within the distorted wave (DW) approximation is also shown in Fig. 1. It takes into account the distortion of the incoming and outgoing electron wave function due to the nuclear Coulomb field, which is particularly strong for lead, with a charge of $Z = 82$. The main effect of the distorted calculation is to smooth out or to fill in the divergences of the plane wave calculation.

All the previous results have been obtained neglecting a possible strangeness content of the nucleon, but fully considering the electric form factors of both protons and neutrons and using $Z/N = 0.65$. However, the approximation in Eq. 9 gives similar results and clearly shows that this asymmetry is proportional to the neutron form factor, which is the Fourier transform of the neutron distribution (Eq. 5). The Parity Radius Experiment (PREX) carried out at Jefferson Laboratory takes advantage of this relation between the PV asymmetry and the neutron distribution to measure the neutron radius of ^{208}Pb, as first suggested in [6]. The lead target is arranged as a foil sandwiched between sheets of diamond that help to improve the thermal response of the target. Data on ^{12}C could therefore be acquired in the experiment, and this is precisely the nucleus that we choose here as an example to show the theoretical PV asymmetry for $N = Z$.

In Fig. 2 we show to the left this PV asymmetry in ^{12}C for an unrealistic pure $T = 0$ isospin situation (dashed line), Eq. 10. In this case ($N = Z$ nucleus), switching off the Coulomb interaction automatically yields the pure-isospin structure, $F_n^0(q) = F_p^0(q)$. The solid line corresponds to the asymmetry using the isospin-violating HF+BCS ground state structure, Eq. 7. Both calculations are in plane waves. We show to the right the difference between the previous two curves by means of the asymmetry deviation Γ, defined so that $\mathcal{A} = \mathcal{A}_{T=0}\,(1 + \Gamma)$. Together with the PW calculation, we also show the DW result [11]. This deviation is expressed in this plot as a percentage of the reference value and its value, as can be seen in Eq. 7, is in this

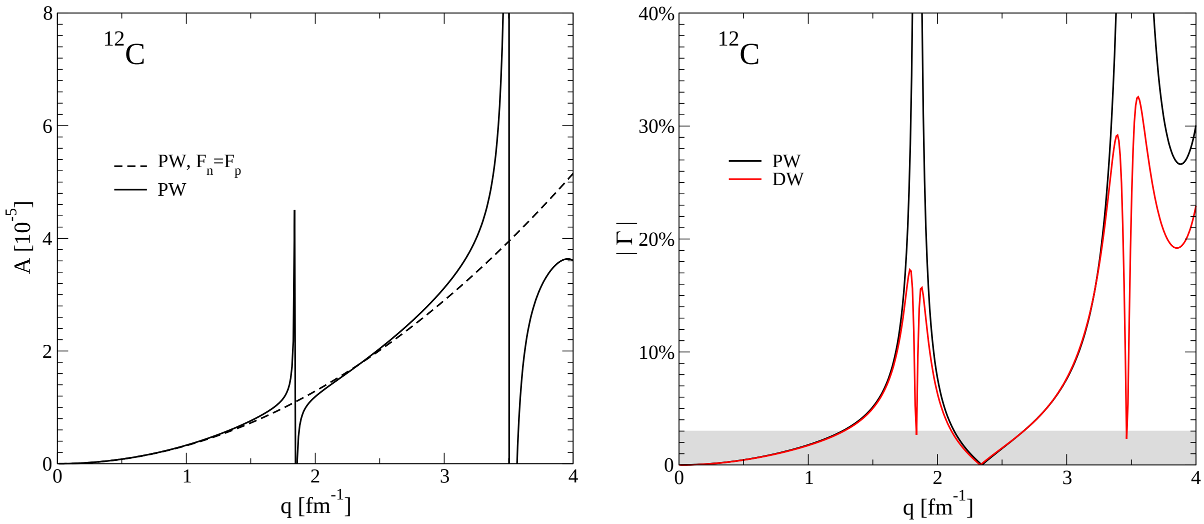

Figure 2. Left: PV asymmetry for ^{12}C with pure $T = 0$ isospin ground state (dashed line) and with isospin mixing as obtained from a HF+BCS calculation (solid line), both in PW. Right: Deviation (in percentage), in PW and also in DW, of the asymmetry with isospin-mixing with respect to the asymmetry for $T = 0$ ground state.

case proportional to $\mathcal{R}$ (Eq. 8). Very small deviations from the reference value are difficult to be isolated experimentally since they are hidden by the relative error of the asymmetry itself. It corresponds to the shady region in the graph (below 3%, but which could reach lower values as the experimental techniques improve). A detailed analysis of the kinematic regions where small effects on the PV asymmetry can be better measured involves the calculation of the figure-of-merit (the larger this quantity, the smaller the relative error of the asymmetry) and, for some purposes, the sensitivity of the asymmetry to small changes in the neutron distribution [11, 12].

Acknowledgements

This work was supported by Ministerio de Ciencia e Innovación (Spain) under Contract FIS2008-01301 and grants FPA-2007-62616 and FPA-2006-07393, by INTAS Open Call grant No 05-1000008-8272, and by UCM and Comunidad de Madrid under grant Grupo de Física Nuclear (910059).

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