

The growth factor of matter perturbations in $f(R)$ gravity

Xiangyun Fu^{1,2}, Puxun Wu^{1,2}, Hongwei Yu^{1,2,a}

¹Department of Physics and Institute of Physics, Hunan Normal University, Changsha, Hunan 410081, China

²Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University, Changsha, Hunan 410081, China

Received: 12 March 2010 / Revised: 31 March 2010 / Published online: 29 April 2010
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Abstract The growth of matter perturbations in the $f(R)$ model proposed by Starobinsky is studied in this paper. Three different parametric forms of the growth index are considered respectively, and constraints on the model are obtained at both the 1σ and 2σ confidence levels, by using the current observational data for the growth factor. It is found, for all the three parametric forms of the growth index examined, that the Starobinsky model is consistent with the observations only at the 2σ confidence level.

1 Introduction

The present cosmic accelerating expansion [1–29] is one of the key challenges in fundamental physics and cosmology. There are basically two kinds of options to explain this mysterious acceleration. One is the well known dark energy [30–35], an energy component, which has a sufficient negative pressure to induce a late-time accelerated expansion; the other is the modified gravity, which originates from the idea that our understanding of gravity is incorrect in the cosmic scale and general relativity needs to be modified. One of the popular modified gravities is the $f(R)$ theory (see [36–39] for a review), where R is the Ricci scalar and $f(R)$ is an arbitrary function of R . For an $f(R)$ model, its action takes the form

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G_N} f(R) + L_m \right], \quad (1)$$

where g is the trace of the metric $g_{\mu\nu}$, G_N is a bare Newton gravity constant and L_m is the Lagrangian of matter. Considering a spatially flat Friedman–Lemaître–Robertson–Walker universe, whose metric is $ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$, and varying the above action with respect to $g_{\mu\nu}$, one can

obtain

$$3FH^2 = 8\pi G_N (\rho_m + \rho_{\text{rad}}) + \frac{1}{2}(FR - f) - 3H\dot{F}, \quad (2)$$

$$-2F\dot{H} = 8\pi G_N \left(\rho_m + \frac{4}{3}\rho_{\text{rad}} \right) + \ddot{F} - H\dot{F}, \quad (3)$$

where $R = 6(2H^2 + \dot{H})$, an over-dot stands for a derivative with respect to the cosmic time t , $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter and $F \equiv \frac{df(R)}{dR}$.

Originally, Capozziello [40] proposed an $f(R)$ model, $f(R) = R - \alpha/R^m$ ($\alpha > 0, m > 0$), to explain the present accelerating expansion. However, this model was plagued with some problems, which are related to the solar-system constraints [41], the instabilities [42], a viable cosmic evolution history with an accelerating expansion [43] and a standard matter-dominated stage [44]. The main reason this model does not work is that $f_{,RR} \equiv \partial^2 f / \partial R^2 < 0$, which gives a negative mass squared for the scalaron field. Soon, the aforementioned problems were solved, for example, the instabilities and the inconsistency with the solar-system constraint were solved in [45, 46], and the problem of matter dominance was solved in [47–49]. Later, Amendola et al. [54] gave the conditions to obtain a viable $f(R)$ model. Some models satisfying these conditions, the Starobinsky model, for an example, have been proposed [54–62]. Moreover, it is interesting to note that there are some models [45, 50–53] in $f(R)$ gravity, which can not only explain the present accelerating expansion successfully, but, at the same time, can also yield an inflation in the early era of our universe without a scalar field.

Let us note that both the dark energy and $f(R)$ gravity can explain the present accelerating expansion. However, although different models can give the same late-time expansion, they may produce different growths of matter perturbations [63]. Thus, the studies of the linear growth of matter perturbations [64–100] provide a particular method

^a e-mail: hwyu@hunnu.edu.cn

to discriminate different models. Defining the growth function $\delta(z) \equiv \delta\rho_m/\rho_m$ (ρ_m is the energy density of matter) and the growth factor $f \equiv \frac{d \ln \delta}{d \ln a}$, the authors in [101, 102] found that f can be parameterized as

$$f \simeq \Omega_m^\gamma, \quad (4)$$

where γ is called the growth index and Ω_m is the fractional energy density of matter. If γ is treated as a constant, its theoretical value can be obtained by expanding the equation of γ around $\Omega_m \simeq 1$, which is a good approximation at the high redshift. Then different models lead to different theoretical values of γ [80–99], for example, $\gamma_\infty \simeq 6/11$ [80, 82] for Λ CDM model and $\gamma_\infty \simeq 11/16$ [80, 81] for flat DGP model. Therefore, it is possible to distinguish them. By comparing the theoretical value of γ with the observed one, one can hopefully single out the model which is consistent with the observations.

However, the growth index is, in general, a function of redshift. Some works have been done on the evolutionary form of $\gamma(z)$. In [89–96], the authors studied $\gamma(z)$ with a linear expansion, $\gamma \approx \gamma_0 + \gamma'_0 z$, and found that this form gives a very good approximation at the low redshift $z < 0.5$ and for different models γ'_0 is different. Thus, an accurate measurement of γ'_0 could provide another characteristic discriminative signature to discriminate different models. In [100], we proposed a parametrization $\gamma(z) = \gamma_0 + \gamma_1 z/(1+z)$, and obtained that, for w CDM and DGP models, this form approximates the growth factor f very well both at the low and high redshift regions.

In this paper, we aim to examine the growth factor of matter perturbations in $f(R)$ gravity and we take the Starobinsky $f(R)$ model as an example. Let us note that the density perturbations of the Starobinsky $f(R)$ model have been studied systematically in the literature [91–95]. But what we plan to do here is to examine different parametric forms of growth index and study the observational constraints from the growth factor data.

2 Starobinsky model

The Starobinsky model has the form

$$f(R) = R + \lambda_s R_0 \left[\left(1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right], \quad (5)$$

where λ_s and $n > 0$ are two positive constants, and R_0 corresponds essentially to the present value of the Ricci scalar R . This model has been studied in the literature [62, 91–95] and it has been found that, when $n \geq 2$, all known laboratory and solar-system tests of gravity can be satisfied [62]. In

this paper, we will let $n = 2$ for simplicity. Constant curvature solutions (for example: de Sitter solution: $R = \text{const} = x_1 R_0 > 0$) are the roots of the algebraic equation [62]

$$Rf'(R) = 2f(R). \quad (6)$$

Substituting the expression of $f(R)$ given in (5) into the above equation, one can obtain

$$\lambda_s = \frac{x_1(1+x_1^2)^{n+1}}{2[(1+x_1^2)^{n+1} - 1 - (n+1)x_1^2]}. \quad (7)$$

In order to satisfy the stability conditions of the system, the following inequality must be satisfied [62]

$$(1+x_1^2)^{n+2} > 1 + (n+2)x_1^2 + (n+1)(2n+1)x_1^4. \quad (8)$$

Setting $n = 2$ and solving the above inequality, one gets $x_1 > \sqrt{\sqrt{13}-2}$, which leads to $\lambda_s > 0.94$. We use $\lambda_s = 0.95$ in this paper, without loss of generality.

3 The growth of matter perturbations

As shown in [117, 118], the background evolution of a viable $f(R)$ is very complicated. Here, we neglect all higher-derivative and non-linear terms, and we then obtain the equation governing the growth of matter perturbations on subhorizon scales as follows [99]:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}}\rho_m\delta = 0, \quad (9)$$

where G_{eff} is an effective Newton gravity constant and for an $f(R)$ model, it can be expressed as [103]

$$G_{\text{eff}} = \frac{G_N}{F} \frac{1 + 4\frac{k^2 F'}{a^2 F}}{1 + 3\frac{k^2 F'}{a^2 F}}. \quad (10)$$

Defining the growth factor $f \equiv d \ln \delta / d \ln a$, (9) becomes

$$\frac{df}{d \ln a} + f^2 + \frac{1}{2} \left(1 - \frac{d \ln \Omega_m}{d \ln a} \right) f = \frac{3}{2} \frac{G_{\text{eff}}}{G_N} \Omega_m. \quad (11)$$

Obviously, the growth factor is scale dependent, which leads to a dispersion of growth index [104]. Here we consider the wavenumber k within the range

$$0.01 \text{ h Mpc}^{-1} \leq k \leq 0.2 \text{ h Mpc}^{-1}, \quad (12)$$

which is relevant to the galaxy power spectrum [116]. In scale smaller than 0.2 h Mpc^{-1} , non-linear effects are obvious and for scale larger than 0.01 h Mpc^{-1} the current observations are not so accurate.

3.1 A constant γ

In this subsection, we discuss the parameterized form $f \equiv \frac{d \ln \delta}{d \ln a} \simeq \Omega_m^\gamma$ with a constant γ . Usually, the theoretical value of γ can be obtained by expanding the equation of γ around $\Omega_m \simeq 1$, which is a good approximation at the high redshift. In principle, we can also obtain the theoretical value of γ by solving (11) numerically and using the value of Ω_{m0} given by current observations. Since the observational results on Ω_{m0} for Starobinsky model is not obtained yet, we use $\Omega_{m0} = 0.278^{+0.024}_{-0.023}$ at the 68% confidence level given in [105] with a model independent method. Solving (11) to obtain $f(0)$ numerically and using the relation $f(0) = \Omega_{m0}^{\gamma_0}$ with Ω_{m0} taking the best fit value 0.278, we find $\gamma_0 \simeq 0.42$, which seems to be almost independent of the value of k .

In order to discriminate different models with the growth factor, we must compare the theoretical value and the observational one of γ . The current observations give 12 data points of the growth factor [106–113]. Let us note that although the data given in [112, 113] are measured without ‘any’ bias, other data points are obtained by assuming a flat Λ CDM model with $\Omega_{m,0}$ taking a specific value, for example, $\Omega_{m,0} = 0.25$ or 0.30. So, caution must be exercised when using these data. With this caveat in mind, it may still be worthwhile to apply the data to fit models [81, 114, 115]. Using these 12 data, we find that, for a constant γ_0 and $\Omega_{m0} = 0.278$, $\chi^2 = 4.6$ and $\gamma = 0.63^{+0.17+0.47}_{-0.14-0.33}$ at the 1σ and 2σ confidence levels. It is easy to see that the Starobinsky model is allowed only at the 2σ confidence level. However, by comparing f and $\Omega_m^{\gamma_0}$, we can see that the error rate is larger than 10% as shown in Fig. 1, which means that the result obtained with a constant γ may be biased. This bias arises from the fact that γ is a function of redshift instead of a constant. More recently, the authors in [91–95] discussed a linearized form of γ with $\gamma = \gamma_0 + \gamma_1 z$, where $\gamma_1 \equiv \gamma'_0 = \frac{d\gamma}{dz}(z=0)$. In the subsequent subsection, we will examine this varying form of γ in detail.

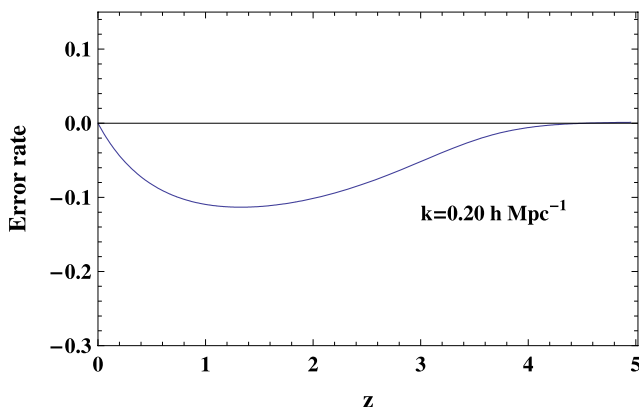


Fig. 1 The relative difference between the growth factor f and $\Omega_m^{\gamma_0}$ with $\Omega_{m,0} = 0.278$

3.2 $\gamma = \gamma_0 + \gamma_1 z$

This linearized form of γ has been studied in the w CDM, DGP and $f(R)$ gravity, and it gives a very good approximation at the redshift region $z < 0.5$. In [96], we found that the constraints on γ_0 and γ_1 from three low redshift observational data cannot rule out the DGP model at 1σ confidence level. Here, we want to see what happens for the Starobinsky’s model, where we have

$$\gamma_1 = [\ln \Omega_{m,0}^{-1}]^{-1} \left[-\Omega_{m,0}^{\gamma_0} - 3 \left(\gamma_0 - \frac{1}{2} \right) \left(-1 - \frac{2\dot{H}_0}{3H_0^2} \right) + \frac{3}{2} \frac{G_{\text{eff}}}{G_N} \Omega_{m,0}^{1-\gamma_0} - \frac{1}{2} \right]. \quad (13)$$

When $\Omega_{m,0} = 0.278$, we obtain that $\gamma_0 \simeq 0.41$. At the same time, we find that, for different k , the variation of γ_1 is small, for example, γ_1 varies from -0.20 to -0.24 when k is from $k = 0.01 \text{ h Mpc}^{-1}$ to 0.2 h Mpc^{-1} . In Fig. 2, we give the relative difference between the growth factor f and $\Omega_m^{\gamma_0 + \gamma_1 z}$ with $\Omega_{m0} = 0.278$ and find that, at low redshifts, the error is below 2%, which means that this linearized form gives a better approximation.

Now we discuss the constraints on γ_0 and γ_1 from the observations. Since this linearized form is valid in the low redshifts, only three low redshift data points can be used. Figure 3 shows the results. From this figure, one can see again that only at the 2σ confidence level is the Starobinsky model consistent with the observations and it can be ruled out at the 1σ confidence level. This is in contrast with the DGP model [96].

However, the approximate form $f \simeq \Omega_m^{\gamma_0 + \gamma_1 z}$ is only valid at the low redshifts. In order to use all the current observational data, we need to find a new approximate expression of f , which can give a good approximation in all redshift regions.

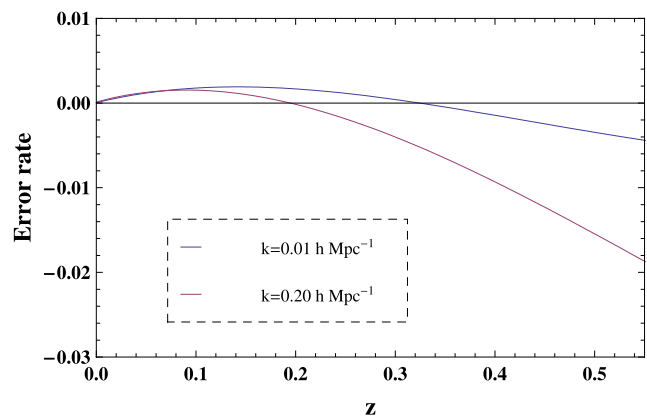


Fig. 2 The relative difference between the growth factor f and $\Omega_m^{\gamma_0 + \gamma_1 z}$ with $\Omega_{m,0} = 0.278$

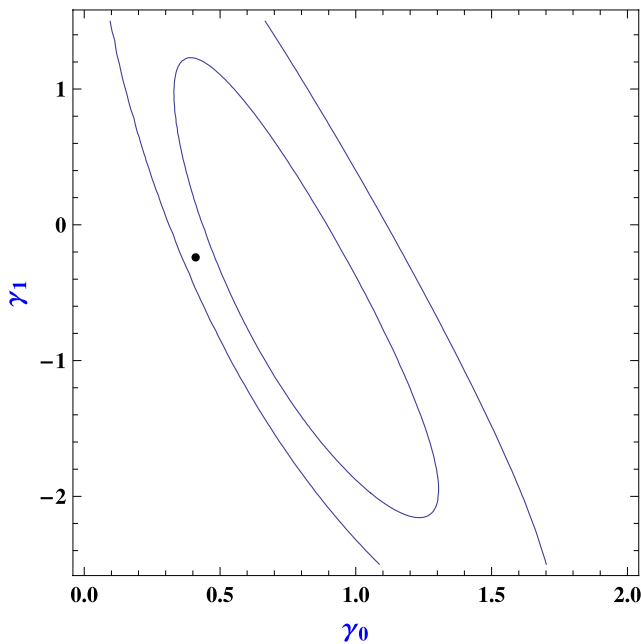


Fig. 3 The 1σ and 2σ contours of γ_0 and γ_1 by fitting the Starobinsky model with the three low redshift growth factor data

3.3 A new approximation of f

From the Fig. 2 in [91–95], which gives the evolution of f , one can see that f is larger than 1 in the region of $1 < z < 3.5$. Since in a flat universe Ω_m is always less than one, according to the usual approximation $f \simeq \Omega_m^{\gamma(z)}$ one cannot obtain $f > 1$ if $\gamma(z)$ is positive in the region $1 < z < 3.5$. Thus the usual parameterized form of f is hard to give a good approximation.

From the definition of G_{eff} and the fact that F is close to one for $z > 1$, one has at the redshift region $z > 1$

$$\frac{G_{\text{eff}}}{G_N} \simeq 1 + \frac{\frac{k^2 F'}{a^2 F}}{1 + 3 \frac{k^2 F'}{a^2 F}}. \quad (14)$$

We assume an approximation of f by multiplying $\Omega_m^{\gamma_0}$ with a factor similar to the above expression, i.e., we assume

$$f \simeq \left(1 + \alpha \frac{1}{(1+z)^2 + 3}\right) \Omega_m^{\gamma_0}, \quad (15)$$

where α is a constant. We find that when α is about equal to 0.85, for different wavenumbers k , the error rate, which is defined as $(1 + \alpha \frac{1}{(1+z)^2 + 3}) \Omega_m^{\gamma_0} / f - 1$, is about less than 5%. The result is shown in Fig. 4. Therefore, with this new parameterized form of f , one may use all the observational data. After numerical calculations, we obtain that γ_0 is about 0.57, which seems to be almost independent of the value of wavenumbers k , when $\alpha = 0.85$. Using the 12 observational data points of the growth factor, we place the constraints on α and γ_0 , which are shown in Fig. 5. From this figure, we

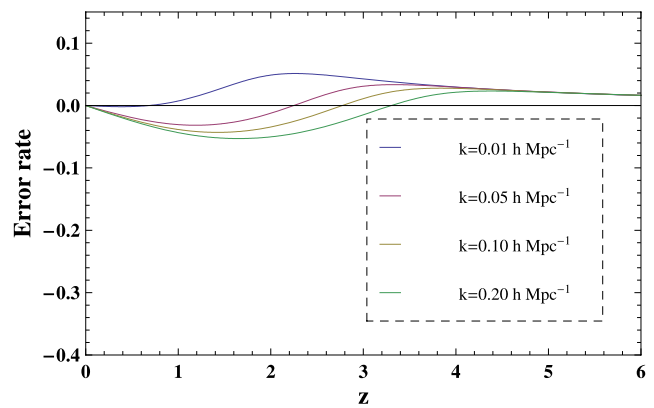


Fig. 4 The relative difference between the growth factor f and $(1 + \alpha \frac{1}{(1+z)^2 + 3}) \Omega_m^{\gamma_0}$ with $\Omega_{m,0} = 0.278$

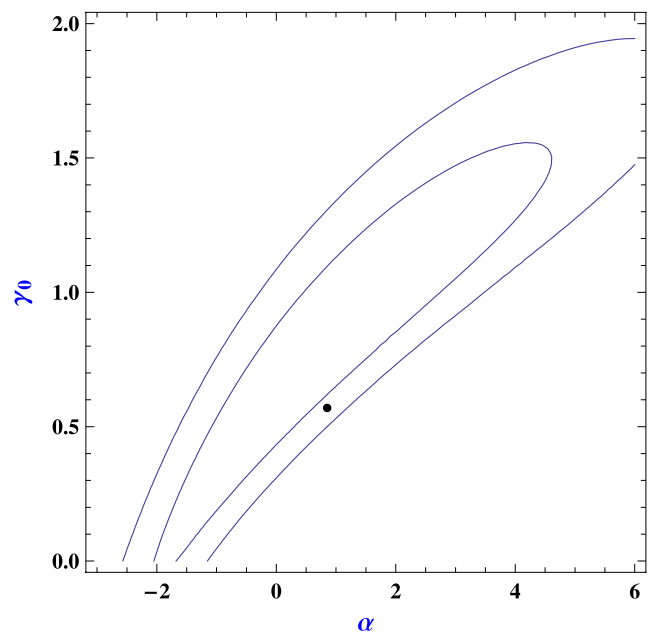


Fig. 5 The 1σ and 2σ contours of γ_0 and α by fitting the Starobinsky model with the current growth factor data

still find that the Starobinsky model is allowed by the current observations only at the 2σ confidence level.

4 Conclusions

In this paper, we study the growth of matter perturbations in an $f(R)$ model proposed by Starobinsky. Firstly, we discuss the case of a constant growth index. By comparing the theoretical value and the observational one, we find that the Starobinsky model is allowed by the current observations only at the 2σ confidence level. However, in this case, the error rate between the growth factor f and $\Omega_m^{\gamma_0}$ is larger than 10%, so, the result obtained with a constant γ may be biased. Then, a linear expansion of growth index,

$\gamma = \gamma_0 + \gamma_{1z}$, is studied, which is valid at the low redshift region $z < 0.5$ and gives a better approximation at these redshifts. With three low redshift observational data, we find again that the Starobinsky model is allowed only at the 2σ confidence level. Finally, in order to use all the present data, we propose a new approximate form of f , and show that this new form gives a reasonable approximation both at low and high redshift regions. For different scales, the largest error is less than 5%. With this new proposed form of f , we still find that the Starobinsky model is consistent with the observations only at 2σ confidence level. So, our results seem to suggest that although the Starobinsky $f(R)$ model is excluded by the current growth factor data at 1σ confidence level, it is still allowed at 2σ level.

It should be pointed out that, in our discussion of the growth of matter perturbations, the higher-derivative terms were discarded. Recently, it has been found, that with the covariant perturbation theory (see [119] for a recent review), which offers the simplest way to describe the evolution of the perturbations, these higher-derivatives terms can be kept in the analysis of matter growth. So, it remains an interesting topic to examine what happens when the effects of these terms are taken into account.

Acknowledgements Xiangyun Fu is grateful to R. Gannouji and Yungui Gong for their very helpful communications. This work was supported in part by the National Natural Science Foundation of China under Grants Nos. 10775050, 10705055 and 10935013, the SRFDP under Grant No. 20070542002, the Programme for the Key Discipline in Hunan Province, the FANEDD under Grant No. 200922, the Program for NCET (No. 09-0144), the program for PCSIRT, and the Hunan Provincial Innovation Foundation for Postgraduate No. CX2009B101.

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