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The solution of 4-dimensional Schrodinger equation for Scarf potential and its partner potential constructed By SUSY QM

Wahyulianti, A. Suparmi, C. Cari, and Kristiana N. Wea

Physics Department of Post Graduate Program Sebelas Maret University,
Jl. Ir. Sutami 36A Kentingan Jebres Surakarta 57126, INDONESIA

E-mail: wahyuliantiyuli@yahoo.com

Abstract. The angular part of 4-dimensional Schrodinger equation for Scarf potential was solved by using the Nikiforov-Uvarov method and Supersymmetric Quantum Mechanic method. The determination of the ground state wave function has been used Nikiforov-Uvarov method and by applying the parametric generalization of the hypergeometric type equation. By using manipulation of the properties and operators of the Supersymmetric Quantum Mechanic method the partner potential was constructed. The ground state wave functions of original Scarf potential is different than the ground state wave functions of the construction result potential.

1. Introduction

The D-dimensional Schrodinger equation is one of the wave equations for higher dimensions in quantum mechanics [1-2]. The D-dimensional Schrodinger equation for some potentials are exactly solved by some methods such as Nikiforov-Uvarov (NU) method [3-6], Supersymmetric Quantum Mechanic (SUSY QM) [4,8], Romanovski polynomial method [9], Asymptotic Iteration Method (AIM) [10] and Factorization methods [11]. The NU method is one of the methods with common application, and NU differential equations have been formulated with general parameters of the general hypergeometric type equation [4]. By applying and manipulating the operators of SUSY QM, the properties of SUSY QM and the ground state wave functions of the original potential, the partner potential was constructed.[4]

The purpose of this research is to construct partner potential which is the new potential of the generalized Scarf potential by manipulating the partner potential equations with the ground state wave function of Scarf potential. In this paper, the solution of angular D-dimensional Schrodinger equation for Scarf potential was studied using Nikiforov-Uvarov method with hypergeometric type equations and SUSY QM. The paper is organized as follows. The method is presented in Section 2. The results and discussion are presented in Section 3 and a conclusion in Section 4.

2. Method

In this section, we review the solution of D-dimensional Schrodinger equation with Nikiforov-Uvarov method and Supersymmetry Quantum Mechanics method. The Generalized Schrodinger equation in D-dimensional shown as followed equation [1,12] given as

$$-\frac{\hbar^2}{2\mu}\nabla_D^2\,\psi(r,\Omega)+V(r,\Omega)\psi(r,\Omega)=E\,\psi(r,\Omega) \qquad (1)$$

with

$$\psi(r,\Omega)=\frac{1}{r^{\frac{D-1}{2}}}U(r)Y^{l}_{l_{D-1}\cdots l_1}(\hat{x}) \qquad ;\hat{x}=\theta_1,\theta_2,\cdots\theta_{D-1} \tag{2}$$

The 4-dimensional Schrodinger equation obtained by inserted Eq. (2) into Eq. (1) and solve of using variable separation method with $Y_{\ell'_1,\ldots\ell'_3}(\theta_1,\theta_2,\theta_3)=P(\theta_1)P(\theta_2)P(\theta_3)$ is given as [13]:

$$\frac{\partial^2 U}{\partial r^2}-\left(\lambda_3+\frac{3}{4}\right)\frac{1}{r^2}U+\frac{2\mu}{\hbar^2}\left[E-V(r)\right]U=0 \tag{3}$$

$$\frac{\partial^2 P(\theta_1)}{\partial\theta_1}-V\left(\theta_1\right)P(\theta_1)+\lambda_1 P(\theta_1)=0 \tag{4}$$

$$\frac{1}{P(\theta_2)}\left\{\frac{1}{\sin\theta_2}\left(\frac{\partial}{\partial\theta_2}\sin\theta_2\frac{\partial P(\theta_2)}{\partial\theta_2}\right)\right\}-V\left(\theta_2\right)-\frac{\lambda_1}{\sin^2\theta_2}+\lambda_2=0 \tag{5}$$

$$\frac{1}{P(\theta_3)}\left\{\frac{1}{\sin^2\theta_3}\left(\frac{\partial}{\partial\theta_3}\sin^2\theta_3\frac{\partial P(\theta_3)}{\partial\theta_3}\right)\right\}-V\left(\theta_3\right)+\lambda_3-\frac{\lambda_2}{\sin^2\theta_3}=0 \tag{6}$$

where λ_1,λ_2 and λ_3 is variable separation constant.

In this paper the potential energi of Scarf potential generally given as [14]:

$$V_i\left(\theta_i\right)=\left(\frac{A_i}{\sin^2\theta_i}+\frac{B_i\cos\theta_i}{\sin^2\theta_i}\right),\ \text{for } i=1,2 \text{ and } 3\ . \tag{7}$$

with $A_i=b_i^2+a_i\left(a_i-1\right)$ and $B_i=-2b_i\left(a_i-\frac{1}{2}\right)$.

2.1 Review of NU method

The solution of Schrodinger equation is reduced to the hypergeometric type differential equation by using variable transformation. By using the NU method, the solution of the hypergeometric type differential equation is expressed as

$$\frac{\partial^2\psi(s)}{\partial s^2}+\frac{\bar{\tau}(s)}{\sigma(s)}\frac{\partial\psi(s)}{\partial s}+\frac{\bar{\sigma}(s)}{\sigma^2(s)}\psi(s)=0 \tag{8}$$

where $\bar{\tau}(s)$ is first order polynomial, $\sigma(s)$ and $\bar{\sigma}(s)$ are mostly second order polynomials. By using variable separation method, substitution equation (4) with

$$\psi(s)=\phi(\mathrm{s})\,\mathrm{y(s)} \tag{9}$$

we obtain hypergeometric type as

$$\sigma\frac{\partial^2\,\mathrm{y(s)}}{\partial s^2}+\tau(s)\frac{\partial y(s)}{\partial s}+\lambda y(s)=0 \tag{10}$$

where $\phi(\mathrm{s})$ is logarithmic derivative $\frac{\phi'}{\phi}=\frac{\pi}{\sigma}$ with $\pi(\mathrm{s})$, λ, and τ are

$$\pi=\left(\frac{\sigma'-\bar{\tau}}{2}\right)\pm\sqrt{\left(\frac{\sigma'-\bar{\tau}}{2}\right)^2-\bar{\sigma}+k\sigma} \tag{11}$$

$$\tau=\bar{\tau}+2\pi\,,\quad \lambda=k+\pi'\ \text{and}\qquad \lambda=\lambda_n=-n\tau'-\frac{n(n-1)}{2}\sigma'',\quad n=0,1,2,\ldots \tag{12}$$

The solution of the part $y_n(s)$ by using Rodrigues relation, is given as

$$y_n(s) = \frac{C_n}{w(s)} \frac{d^n}{ds^n}\left(\sigma^n(s)w(s)\right) \tag{13}$$

with C_n is normalization constant, and the weight function $w(s)$ must satisfy the condition

$$\frac{\partial\left(\sigma w\right)}{\partial s} = \tau(s)w(s) \tag{14}$$

The parametric generalization of the hypergeometric type equation in equation (8) is written as

$$\frac{\partial^2\psi(s)}{\partial s^2} + \frac{(c_1 - c_2 s)}{s(1-c_3 s)}\frac{\partial\psi(s)}{\partial s} + \frac{(-\varepsilon_1 s^2 + \varepsilon_2 s - \varepsilon_3)}{s^2(1-c_3 s)^2}\psi(s) = 0 \tag{15}$$

By comparing equation (8) and equation (15), we obtained the energy value equation and eigen function are respectively given as

$$c_2 n - (2n+1)c_5 + (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + n(n-1)c_3 + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0 \tag{16}$$

and

$$\psi(s) = N_{n1} s^{c_{12}} \left(1 - c_3 s\right)^{-c_{12}-(c_{13}/c_3)} P_n^{(c_{10}-1,\,(c_{11}/c_3)-c_{10}-1)}\left(1-2c_3 s\right) \tag{17}$$

where

$$\begin{gathered} c_4 = \frac{1}{2}(1-c_1),\ c_5 = \frac{1}{2}(c_2 - 2c_3),\ c_6 = c_5^2 + \varepsilon_1,\ c_7 = 2c_4 c_5 - \varepsilon_2,\ c_8 = c_4^2 + \varepsilon_3 \\ c_9 = c_3 c_7 + c_3^2 c_8 + c_6,\ c_{10} = c_1 + 2c_4 + 2\sqrt{c_8},\ c_{12} = c_4 + \sqrt{c_8} \\ c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right), c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right) \end{gathered} \tag{18}$$

By using equations (16) and (17) for $n = 0$ the ground state energy and wave function are determined [3-6].

2.2 Review of SUSY QM

Witten defined two charge operators are commute with Hamiltonian (H_{ss}) of the supersymmetry quantum system [4,8] is geven as

$$H_{ss} = \begin{pmatrix} -\frac{d^2}{dx^2} + \frac{d\varphi(x)}{dx} + \varphi^2(x) & 0 \\ 0 & -\frac{d^2}{dx^2} - \frac{d\varphi(x)}{dx} + \varphi^2(x) \end{pmatrix} = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix} \tag{19}$$

with partner Hamiltonian $H_- = H_1$ and $H_+ = H_2$. By setting the new SUSY operators for raising operator $A^+ = -\frac{d}{dx} + \varphi(x)$ and lowering operator $A = \frac{d}{dx} + \varphi(x)$.

The SUSY Hamiltonians of equation (19) are

$$H_-(x) = H_1 = A^+ A, \text{ and } H_+(x) = H_2 = AA^+ \tag{20}$$

and

$$A\psi_0^{(-)} = A\psi_0 = 0 \tag{21}$$

The partners of potential in the SUSY QM method is $V_- = V_1$ and $V_+ = V_2$ are

$$V_-(x) = V_1 = \varphi^2(x) - \varphi'(x) \text{ and } V_+(x) = V_2 = \varphi^2(x) + \varphi'(x) \tag{22}$$

where φ is superpotential. The partner potential V_1 is determined as

$$V_-(x;a_0) = V_1(x;a_0) = V_{ef}(x) - E_0 \tag{23}$$

with V_{ef} is effective potential and E_0 is the ground state energy.

By manipulating equation (22) and by applying equation (17) and equation (23), we construct the new partner potential V_2 as

$$V_2(x) = V_{ef}(x) - E_0 - 2\frac{d}{dx}\left(\frac{d\psi_0 / dx}{\psi_0}\right) \tag{24}$$

where ψ_0 is the ground state wave function [4,8]. Then the new eigen function of new partner potential is also determined by using Nikiforov-Uvarov method.

3. Results and Discussion

In this section, the solution of the Schrodinger equation in 4-dimensional with Nikiforov-Uvarov method and Supersymmetry Quantum Mechanics method .

The solution of the Schrodinger equation in 4-dimensional for θ_1 obtained by inserting Eq. (7) to Eq. (4) is given as

$$\frac{\partial^2 P(\theta_1)}{\partial\theta_1} - \left(\frac{A_1}{\sin^2\theta_1} + \frac{B_1\cos\theta_1}{\sin^2\theta_1}\right)P(\theta_1) + \lambda_1 P(\theta_1) = 0 \tag{25}$$

By substituting Eq. (25) with $\cos\theta_i = (1-2s)$ we obtain

$$\frac{d^2P_1(s)}{ds^2} + \frac{\left(\frac{1}{2}-s\right)}{s(1-s)}\frac{dP_1(s)}{ds} + \frac{1}{s^2(1-s)^2}\left[-(\lambda_1)s^2 + \left(\lambda_1 + \frac{B_1}{2}\right)s - \left(\frac{A_1+B_1}{4}\right)\right]P_1(s) = 0 \tag{26}$$

By comparing Eq. (26) and Eq. (15) and then by applying Eq. (16), Eq. (17) and Eq. (18) we obtained the λ_1 and eigen function are respectively given as

$$\lambda_1 = \frac{1}{2}\left(\frac{3}{4}+A_1\right) + n(n+1) + (2n+1)\left(\frac{1}{2}\sqrt{\frac{1}{4}+A_1-B_1} + \frac{1}{2}\sqrt{\frac{1}{4}+A_1+B_1}\right) + \frac{1}{2}\sqrt{\left(\frac{1}{4}+A_1+B_1\right)\left(\frac{1}{4}+A_1-B_1\right)} \tag{27}$$

and

$$P(\theta_1) = N_0\left(\frac{1-\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1+B_1}}\left(\frac{1+\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1-B_1}} P_{n1}^{\left(\sqrt{\frac{1}{4}+A_1+B_1},\sqrt{\frac{1}{4}+A_1-B_1}\right)}(\cos\theta_1) \tag{28}$$

with

$$c_1 = \frac{1}{2},\ c_2 = c_3 = 1,\ \varepsilon_1 = \lambda_1,\quad \varepsilon_2 = \lambda_1 + \frac{B_1}{2},\quad \varepsilon_3 = \frac{1}{4}(A_1+B_1) \tag{29}$$

The solution of Eq. (27) and Eq. (28) for the ground state $n = 0$, we obtained

$$\lambda_{01} = \frac{1}{2}\left(\frac{3}{4}+A_1\right) + \frac{1}{2}\sqrt{\frac{1}{4}+A_1-B_1} + \frac{1}{2}\sqrt{\frac{1}{4}+A_1+B_1} + \frac{1}{2}\sqrt{\left(\frac{1}{4}+A_1+B_1\right)\left(\frac{1}{4}+A_1-B_1\right)} \tag{30}$$

and

$$P_0(\theta_1) = N_0\left(\frac{1-\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1+B_1}}\left(\frac{1+\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1-B_1}} \tag{31}$$

The partner potential V_2 as new potential constructed from Eq. (31) and Eq. (24) is

$$V_2(\theta_1) = \frac{A_1}{\sin^2\theta_1} + \frac{B_1\cos\theta_1}{\sin^2\theta_1} - E_{01} + \frac{c_{12}}{\left(\frac{1-\cos\theta_1}{2}\right)} - \frac{c_{14}}{\left(\frac{1+\cos\theta_1}{2}\right)} \tag{32}$$

with

$$E_{01} = \lambda_{01}, \quad c_{12} = \frac{1}{4} + \frac{1}{2}\sqrt{\frac{1}{4} + A_1 + B_1} \text{ and } c_{14} = \frac{1}{4} + \frac{1}{2}\sqrt{\frac{1}{4} + A_1 - B_1} \tag{33}$$

The solution of the Schrodinger equation in 4-dimensional for θ_1 partner potensial obtained by inserting Eq. (32) to Eq. (4) is given as

$$\frac{\partial^2 P(\theta_1)}{\partial\theta_1} - \left(\frac{A_1}{\sin^2\theta_1} + \frac{B_1\cos\theta_1}{\sin^2\theta_1} - E_{01} + \frac{c_{12}}{\left(\frac{1-\cos\theta_1}{2}\right)} - \frac{c_{14}}{\left(\frac{1+\cos\theta_1}{2}\right)}\right)P(\theta_1) + \lambda_{01}^{'}P(\theta_1) = 0 \tag{34}$$

By substituting Eq. (34) with $\cos\theta_i = (1-2s)$ we get

$$\frac{d^2P_1(s)}{ds^2} + \frac{\left(\frac{1}{2}-s\right)}{s(1-s)}\frac{dP_1(s)}{ds} + \frac{1}{s^2(1-s)^2}\left[\begin{array}{l} -\left(E_{01}+\lambda_{01}^{'}\right)s^2 + \left(E_{01}+\lambda_{01}^{'}+c_{14}+c_{12}+\frac{B_1}{2}\right)s \\ -\left(\frac{A_1+B_1}{4}+c_{12}\right)\end{array}\right]P_1(s) = 0 \tag{35}$$

By comparing Eq. (35) and Eq. (15) and then by applying Eq. (16), Eq. (17) and Eq. (18) we obtained $\lambda_{01}^{'}$ and eigen function are respectively given as

$$\begin{aligned}\lambda_{01}^{'} = {} & \frac{3}{8} - E_{10} - c_{14} + \frac{1}{2}\left(A_1 + 2c_{12}\right) + \frac{1}{2}\sqrt{\frac{1}{4} + A_1 - B_1 - 4c_{14}} + \frac{1}{2}\sqrt{\frac{1}{4} + A_1 + B_1 + 4c_{12}} \\ & + \frac{1}{2}\sqrt{\left(\frac{1}{4} + A_1 + B_1 + 4c_{12}\right)\left(\frac{1}{4} + A_1 - B_1 - 4c_{14}\right)}\end{aligned} \tag{36}$$

and

$$P_0^{'}(\theta_1) = N_{01}^{'}\left(\frac{1-\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1+B_1+4c_{12}}}\left(\frac{1+\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1-B_1-4c_{14}}} \tag{37}$$

Secondly, the similarly with the steps of θ_1 for solution of θ_2 by applying Eq. (5) as the Schrodinger equation. The solution of θ_2 for ground state *n = 0*, we obtained λ_{02} and eigen function are respectively given as:

$$\lambda_{02} = \frac{1}{2}\left(A_2 + \lambda_{01}\right) + \frac{1}{2}\sqrt{A_2 - B_2 + \lambda_{01}} + \frac{1}{2}\sqrt{A_2 + B_2 + \lambda_{01}} + \frac{1}{2}\sqrt{\left(A_2 + B_2 + \lambda_{01}\right)\left(A_2 - B_2 + \lambda_{01}\right)} \tag{38}$$

and

$$P_0(\theta_2) = N_0 \frac{1}{\sqrt{\sin\theta_2}} \left(\frac{1-\cos\theta_2}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{A_2+B_2+\lambda_{01}}} \left(\frac{1+\cos\theta_2}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{A_2-B_2+\lambda_{01}}} \tag{39}$$

By inserting Eq. (39) to Eq. (24) We get the partner potential θ_2 as

$$V_2(\theta_2) = \frac{A_2}{\sin^2\theta_2} + \frac{B_2\cos\theta_2}{\sin^2\theta_2} + \frac{\lambda_{01}-\frac{1}{4}}{\sin^2\theta_2} - E_{02} + \frac{c_{12}}{\frac{1-\cos\theta_2}{2}} - \frac{c_{14}}{\frac{1+\cos\theta_2}{2}} \tag{40}$$

with

$$E_{02} = \lambda_{02} + \frac{1}{4},\quad c_{12} = \frac{1}{4} + \frac{1}{2}\sqrt{A_2+B_2+\lambda_{01}} \text{ and } c_{14} = -\frac{1}{4} - \frac{1}{2}\sqrt{A_2-B_2+\lambda_{01}} \tag{41}$$

By substituting Eq. (40) to Eq. (5) and then by applying Eq. (15) - Eq. (18) we obtain

$$\begin{aligned}\lambda_{02}^{'} = c_{12} - c_{14} - E_{02} + \frac{1}{2}\left(A_2 + \lambda_{01} + \lambda_{01}^{'} - \frac{1}{4}\right) + \frac{1}{2}\sqrt{A_2 - B_2 + \lambda_{01} + \lambda_{01}^{'} - 4c_{14} - \frac{1}{4}} + \\ \frac{1}{2}\sqrt{A_2 + B_2 + 4c_{12} + \lambda_{01} + \lambda_{01}^{'} - \frac{1}{4}} + \frac{1}{2}\sqrt{\left(A_2 + B_2 + 4c_{12} + \lambda_{01} + \lambda_{01}^{'} - \frac{1}{4}\right)\left(A_2 - B_2 + \lambda_{01} + \lambda_{01}^{'} - 4c_{14} - \frac{1}{4}\right)}\end{aligned} \tag{42}$$

and

$$P_0^{'}(\theta_2) = N_{02}^{'} \frac{1}{\sqrt{\sin\theta_2}} \left(\frac{1-\cos\theta_2}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{A_2+B_2+4c_{12}+\lambda_{01}+\lambda_{01}^{'}-\frac{1}{4}}} \left(\frac{1+\cos\theta_2}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{A_2-B_2+\lambda_{01}+\lambda_{01}^{'}-4c_{14}-\frac{1}{4}}} \tag{43}$$

Finally, the similarly with the steps of θ_1 for solution of θ_3 by applying Eq. (6) as the Schrodinger equation. Then λ_{03} and eigen function for ground state $n = 0$ are respectively given as:

$$\begin{aligned}\lambda_{03} = -\frac{5}{8} + \frac{1}{2}(\lambda_{02} + A_3) + \frac{1}{2}\sqrt{\frac{1}{4} + \lambda_{02} + A_3 - B_3} + \frac{1}{2}\sqrt{\frac{1}{4} + \lambda_{02} + A_3 + B_3} \\ + \frac{1}{2}\sqrt{\left(\frac{1}{4} + \lambda_{02} + A_3 + B_3\right)\left(\frac{1}{4} + \lambda_{02} + A_3 - B_3\right)}\end{aligned} \tag{44}$$

and

$$P_0(\theta_3) = N_0 \frac{1}{\sin\theta_3} \left(\frac{1-\cos\theta_3}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+\lambda_{02}+A_3+B_3}} \left(\frac{1+\cos\theta_3}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+\lambda_{02}+A_3-B_3}} \tag{45}$$

By Substituting Eq. (45) to Eq. (24) we obtain the partner potential θ_3 as

$$V_2(\theta_3) = \frac{A_3}{\sin^2\theta_3} + \frac{B_3\cos\theta_3}{\sin^2\theta_3} + \frac{\lambda_{02}}{\sin^2\theta_3} - E_{03} + \frac{c_{12}}{\frac{1-\cos\theta_3}{2}} - \frac{c_{14}}{\frac{1+\cos\theta_3}{2}} \tag{46}$$

with

$$E_{03} = \lambda_{03} + 1,\quad c_{12} = \frac{1}{4} + \frac{1}{2}\sqrt{\frac{1}{4} + \lambda_{02} + A_3 + B_3} \text{ and } c_{14} = -\frac{1}{4} - \frac{1}{2}\sqrt{\frac{1}{4} + \lambda_{02} + A_3 - B_3} \tag{47}$$

By substituting Eq. (46) to Eq. (6) and then by applying Eq. (15) - Eq. (18) we get:

$$\begin{aligned}\lambda_{03}^{'} = &-\frac{3}{4} - E_{03} - \frac{B_3}{2} - c_{12} - c_{14} + \frac{1}{2}\sqrt{\frac{1}{4} + A_3 - B_3 + \lambda_{02} + \lambda_{02}^{'} - 4c_{14}} + \frac{1}{2}\sqrt{\frac{1}{4} + A_3 + B_3 + \lambda_{02} + \lambda_{02}^{'} + 4c_{12}} \\ &+ \frac{1}{2}\sqrt{\left(\frac{1}{4} + A_3 + B_3 + \lambda_{02} + \lambda_{02}^{'} + 4c_{12}\right)\left(\frac{1}{4} + A_3 - B_3 + \lambda_{02} + \lambda_{02}^{'} - 4c_{14}\right)}\end{aligned} \tag{48}$$

and

$$P_0^{'}(\theta_3) = N_{03}^{'}\frac{1}{\sin\theta_3}\left(\frac{1-\cos\theta_3}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_3+B_3+\lambda_{02}+\lambda_{02}^{'}+4c_{12}}}\left(\frac{1+\cos\theta_3}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_3-B_3+\lambda_{02}+\lambda_{02}^{'}-4c_{14}}} \tag{49}$$

The generalized wave functions are

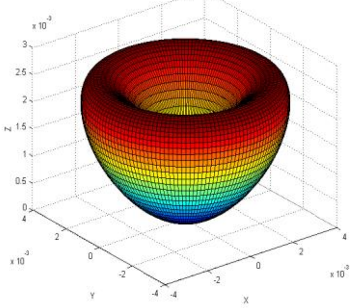
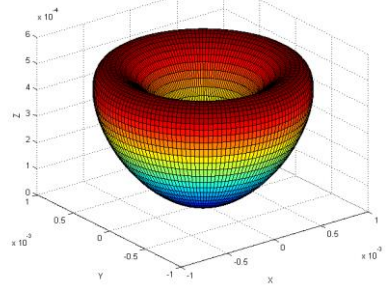
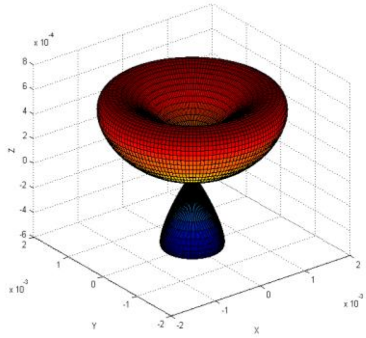
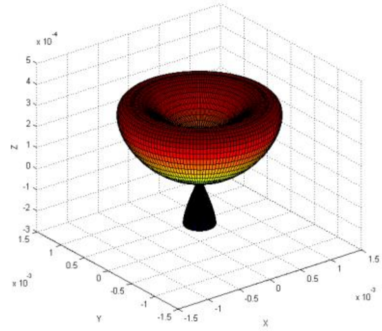
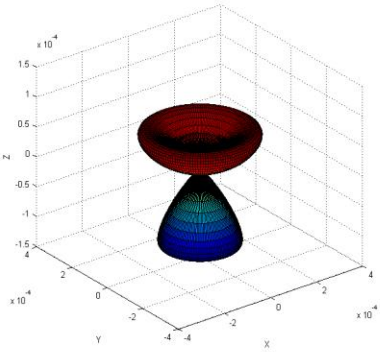
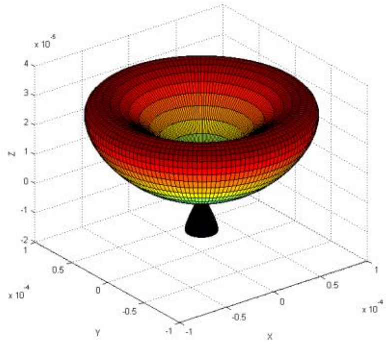
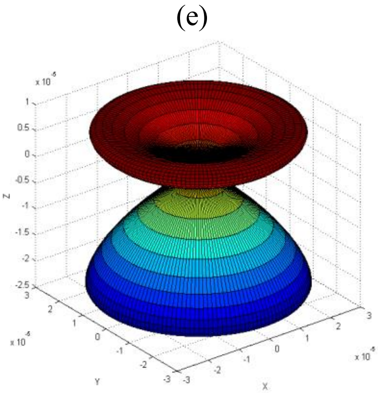
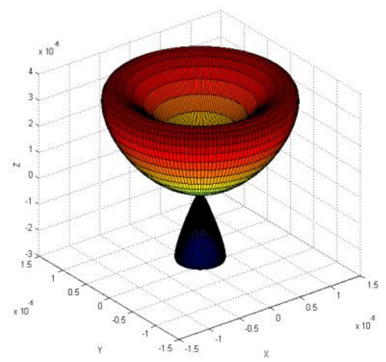
$$\begin{aligned}P(\theta_1) &= N_{n1}\left(\frac{1-\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1+B_1}}\left(\frac{1+\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1-B_1}} P_{n1}^{\left(\sqrt{\frac{1}{4}+A_1+B_1},\sqrt{\frac{1}{4}+A_1-B_1}\right)}(\cos\theta_1) \\ P(\theta_2) &= N_{n2}\frac{1}{\sqrt{\sin\theta_2}}\left(\frac{1-\cos\theta_2}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{A_2+B_2+\lambda_{01}}}\left(\frac{1+\cos\theta_2}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{A_2-B_2+\lambda_{01}}} P_{n2}^{\left(\sqrt{A_2+B_2+\lambda_{01}},\sqrt{A_2-B_2+\lambda_{01}}\right)}(\cos\theta_2) \\ P(\theta_3) &= N_{n3}\frac{1}{\sin\theta_3}\left(\frac{1-\cos\theta_3}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+\lambda_{02}+A_3+B_3}}\left(\frac{1+\cos\theta_3}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+\lambda_{02}+A_3-B_3}} P_{n3}^{\left(\sqrt{\frac{1}{4}+\lambda_{02}+A_3+B_3},\sqrt{\frac{1}{4}+\lambda_{02}+A_3-B_3}\right)}(\cos\theta_3)\end{aligned} \tag{50}$$

and

$$\begin{aligned}P'(\theta_1) &= N_{n1}^{'}\left(\frac{1-\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1+B_1+4c_{12}}}\left(\frac{1+\cos\theta_1}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_1-B_1-4c_{14}}} P_{n1}^{\left(\sqrt{\frac{1}{4}+A_1+B_1+4c_{12}},\sqrt{\frac{1}{4}+A_1-B_1-4c_{14}}\right)}(\cos\theta_1) \\ P'(\theta_2) &= N_{n2}^{'}\frac{1}{\sqrt{\sin\theta_2}}\left(\frac{1-\cos\theta_2}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{A_2+B_2+4c_{12}+\lambda_{01}+\lambda_{01}^{'}-\frac{1}{4}}}\left(\frac{1+\cos\theta_2}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{A_2-B_2+\lambda_{01}+\lambda_{01}^{'}-4c_{14}-\frac{1}{4}}} P_{n2}^{\left(\sqrt{A_2+B_2+4c_{12}+\lambda_{01}+\lambda_{01}^{'}-\frac{1}{4}},\sqrt{A_2-B_2+\lambda_{01}+\lambda_{01}^{'}-4c_{14}-\frac{1}{4}}\right)}(\cos\theta_2) \\ P'(\theta_3) &= N_{n3}^{'}\frac{1}{\sin\theta_3}\left(\frac{1-\cos\theta_3}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_3+B_3+\lambda_{02}+\lambda_{02}^{'}+4c_{12}}}\left(\frac{1+\cos\theta_3}{2}\right)^{\frac{1}{4}+\frac{1}{2}\sqrt{\frac{1}{4}+A_3-B_3+\lambda_{02}+\lambda_{02}^{'}-4c_{14}}} P_{n3}^{\left(\sqrt{\frac{1}{4}+A_3+B_3+\lambda_{02}+\lambda_{02}^{'}+4c_{12}},\sqrt{\frac{1}{4}+A_3-B_3+\lambda_{02}+\lambda_{02}^{'}-4c_{14}}\right)}(\cos\theta_3)\end{aligned} \tag{51}$$

By using the same value of potential parameters, the wave functions for the original Scarf potential and the partner potential as potential construction result are different. For details of the wave function of Equation (50) and Equation (51) can be shown Table 1.

Table 1 Three-dimensional representations of the angular wave function for variation of n_l, a and b.

a	b	n_l	$P(\theta_2)$	$P'(\theta_2)$
4	4	0	 (a)	 (b)
4	4	1	 (c)	 (d)
6	6	1	 (e)	 (f)
8	8	1	 (g)	 (h)

From Table 1, we have wave function for variations of *a, b,* and n_l. The increases of n_l value is given the effect in form of wave functions and the motion range of particle. The wave functions for the original Scarf potential and its the partner potential are different. There is a decrease in the amplitude value at $P\,'(\theta_2)$ beside $P\,(\theta_2)$. An increase in the value of n_l causes a decrease in the amplitude value. The increase of *a* and *b* values causes a decrease in the amplitude value.

4. Conclusion

In this paper, we have presented the solution of 4-dimensional Schrodinger equation of angular part for Scarf potential and partner potential using Nikiforov-Uvarov method and Supersymmetric quantum mechanics method. We obtained the wave functions from angular part solution, which the wave functions depend on the parameters of all components of the composed potential. The partner potential have different wave functions compared to the ground state wave functions of the original potential. This partner potential was considered to be new potential.

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