MUON LIFETIME MEASUREMENT

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Table of Contents

1. What is Muon

The Muon is the second lightest lepton in the Standard Model with an approximate mass of 106 MeV. It has a spin 1/2 and a charge of ± 1. Its position in the classification of fermions can be seen in Table 1.1. The mean lifetime of the muon is around 2.2 μs. Muon has a high penetrating power and its energy loss through matter is less than that of electron's.

Table 1.1: The Fermions

$\smile$	1^{st} generation	2^{nd} generation	3^{rd} generation
LEPTONS	$Q=-1$ $S=1/2$ e $M \approx 0.511 MeV/c^2$ *electron*	$Q=-1$ $S=1/2$ μ $M \approx 106 MeV/c^2$ *muon*	$Q=-1$ $S=1/2$ τ $M \approx 1.78 GeV/c^2$ *tau*
	$Q=0$ $S=1/2$ ν_e $M \leq 15$ eV e *neutrino*	$Q=0$ $S=1/2$ ν_μ $M \leq 0.17$ MeV μ *neutrino*	$Q=0$ $S=1/2$ ν_τ $M \leq 24$ MeV τ *neutrino*
QUARKS	$Q=2/3$ $S=1/2$ u $M \approx 0.3 GeV/c^2$ *up*	$Q=2/3$ $S=1/2$ c $M \approx 1.5 GeV/c^2$ *charm*	$Q=2/3$ $S=1/2$ t $M \approx 174 GeV/c^2$ *top*
	$Q=-1/3$ $S=1/2$ d $M \approx 0.3 GeV/c^2$ *down*	$Q=-1/3$ $S=1/2$ s $M \approx 0.5 GeV/c^2$ *strange*	$Q=-1/3$ $S=1/2$ b $M \approx 4.7 GeV/c^2$ *bottom*

2. How Are Cosmic Muons Created

Muons were first identified in cosmic ray experiments by Anderson and Neddermeyer in 1936. The cosmic radiation, which consists of high energy particles that are mostly protons, enters the earth's atmosphere and interacts with the atmospheric nuclei resulting in secondary particles (Figure 2.1). Collisions of cosmic rays with atoms in the upper atmosphere produce mostly neutral and charged pions. Each neutral pion decays into a pair of gamma rays in less than 10^{-6} seconds. The charged pions each decay, within less than 30 ns, into a muon and a muon neutrino. So, the cosmic muons produced by the decay of pions, which are produced high up in the atmosphere (typically on 20,000 meters of amplitute). The pion decay, characterized by weak interaction is;

$$\pi \rightarrow \mu + \nu_\mu$$

The muons then decay in about 2 μs into an electron, a muon neutrino and an electron anti-neutrino. Decay of charged pions into muons is as shown in Figure 2.1. It is primarily these muons that are observed by the detector.

Since the muon has a relatively long life time and the reaction mechanism is characterized by weak interaction, a large amount of the muons produced on high altitutes will reach the surface of the earth.

In matter there are two competing processes for muon:

- Decay (Figure 2.2)

$$\mu \rightarrow e + \nu_e + \nu_\mu$$

- Capture by the nucleus (Figure 2.3)

$$\mu + p \rightarrow \nu_\mu + n$$

Due to Coulomb repulsion, nuclear capture is not very likely for the positive muons. So, for μ^+ decay is the dominant process. For the negative muon ,however, capture by the nucleus is the most possible process. This leads to the shorter life time for μ^-.

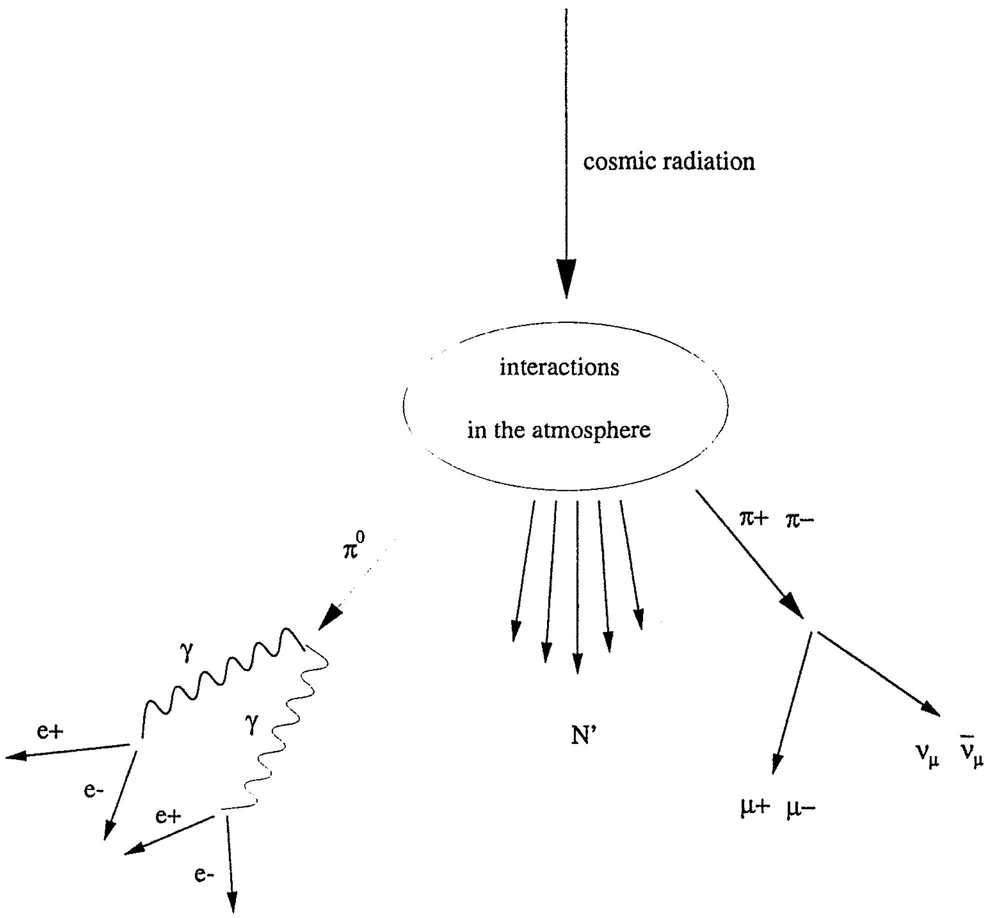

Figure 2.1: creation

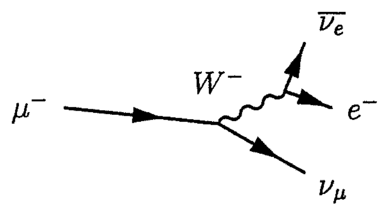

Figure 2.2: Decay of μ^-

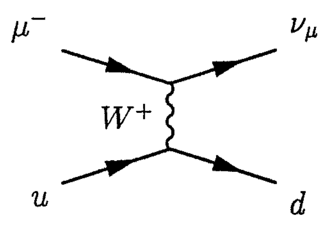

Figure 2.3: Capture of μ^-

3. How To Measure the Lifetime of the Muon

3.1. Mean Range of Muons

Muons are assumed to decay spontaneously and therefore the probability of a muon decaying during any time interval dt is the same, and this probability is independent of the history of the muon.

Consider N muons at the time $t = 0$. The average number of muons still intact at a time t is denoted by $N(t)$; the number of muons decaying in the interval t, $t + dt$ is

$$-dN(t) = N(t)\frac{dt}{\tau_0} \tag{3.1}$$

Integrating equation (1) we find

$$N(t) = Ne^{\frac{-t}{\tau_0}} \tag{3.2}$$

Here τ_0 is the mean life of the muon.

To obtain the mean range of a fast muon, one has to distinguish clearly between apparent life and proper life. The lifetime of a muon is equal to τ_0 in the rest system of the muon. Due to relativistic time dilation, a muon with a velocity V appears to have a lifetime

$$\tau_0' = \tau_0\gamma, \quad \gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{3.3}$$

The mean range of a muon with velocity V is therefore

$$R(V) = V \cdot \tau_0' = V \cdot \tau_0 \cdot \gamma \tag{3.4}$$

or

$$R(V) = \frac{\tau_0}{m_\mu} \cdot p \tag{3.5}$$

$$p = mV, \quad m = \frac{m_\mu}{\sqrt{1 - \frac{V^2}{c^2}}} = m_\mu \cdot \gamma \; .$$

where m_μ and p are rest mass and momentum of the muon. The range R of a muon incerases therefore more rapidly than its velocity.

The following remark may be useful. In the system of reference where the muon is moving, the apparent life of the muon is prolonged due to time dilation and therefore the muon can cover a range R, which could not be covered during the time τ_0. In the rest-system of the muon, however, the time available for the muon is only equal to τ_0. The range which was found to be equal to R in the system where the muon was moving appears to be contracted by a factor $\frac{1}{\gamma}$ in the rest-system. Therefore the muon has to live only for the time needed to pass the contracted range, R/γ.

3.2. Muons Traversing an Absorber

We calculate the mean range of a muon passing through an absorber. Consider muons with momentum p entering an absorber. Due to absorption the muons reaching a depth η will have a momentum:

$$p(\eta) < p\,. \tag{3.6}$$

Denote the average number of muons reaching η by $N(\eta)$. The average number decaying in the interval η, $\eta + d\eta$ is obtained from (1.1) and (1.3)

$$-dN = N(\eta)\frac{m_\mu}{\tau_0}\frac{d\eta}{p(\eta)} \tag{3.7}$$

Integrating over η we find $N(\eta)$:

$$N(\eta) = N(0) \cdot e^{\frac{m_\mu}{\tau_0}} \int_0^\eta \frac{d\eta}{p(\eta)} \tag{3.8}$$

3.2.1. Muons Traversing a Homogeneous Absorber

For fast muons ($p >> m_\mu c$) we may assume

$$\frac{dp}{d\eta} = constant = \frac{m_\mu \cdot c}{a} \tag{3.9}$$

and therefore

$$p(\eta) = p_0 - \frac{\eta}{\eta_1}(p_0 - p_1) \tag{3.10}$$

where p_0 is the momentum of the incident muons and p_1 is the momentum of the muons which have reached a depth $\eta = \eta_1$.

Using Equation 2.9 and Equation 2.10 we find

$$N(\eta_1) = N(0)(\frac{p_0}{p_1})^{\frac{-m_\mu \eta_1}{\tau_0(p_0-p_1)}}. \tag{3.11}$$

3.2.2. Muons Nearing the End of Their Range

We may use the approximate range-momentum relation which comes from energy losses of charged particles due to inelastic collisions. (Rossi and Greisen (1941))

$$-\frac{dE}{d\theta} = \frac{A}{(\frac{V}{c})^2} \tag{3.12}$$

with

$$A = 0.153 \cdot \frac{Z}{A} \cdot \begin{cases} 20.2 + 3log\frac{p}{m_e c} - 2\log z & for\ electrons \\ 20.5 + 4log\frac{p}{m_e c} - 2\log z & for\ protons,\ muons \end{cases}$$

Neglecting the terms $log\frac{p}{m_e c}$, A can be regarded as constant, and introducing $E = mc^2(\gamma - 1)$ and $(\frac{V}{c})^2 = 1 - \frac{1}{\gamma^2}$ the equation (1.12) can be integrated

$$R(E) = \frac{E^2}{A(E + mc^2)} \tag{3.13}$$

The following two approximations are useful

- Nonrelativistic Case:

$$R(E) \sim \frac{E^2}{Amc^2} \sim \frac{p^4}{4Am^3c^2} \tag{3.14}$$

- Extreme Relativistic Case:

$$R(E) \sim \frac{1}{A}(E - mc^2) \tag{3.15}$$

Taking into account non-relativistic case we find

$$R(p) \sim \frac{1}{4}(\frac{p}{m_\mu c})^4 a \tag{3.16}$$

So from (1.8)

$$N(\eta) = N(0) \cdot e^{-\frac{a}{3c\tau_0} \cdot \frac{p(0)^3 - p(\eta)^3}{(m_\mu c)^3}} \tag{3.17}$$

The fraction of muons with initial momentum $m_\mu c$ which decay only after being brought to rest is therefore:

$$N(p=0)/N(p=m_\mu c) = e^{-\frac{a}{3c\tau_0}}$$

In the case of air we have approximately;

$$\frac{dp}{d\eta} = 2.5 \cdot 10^{-3}$$
$$a = 4 \cdot 10^4, \quad c\tau_0 = 6.3 \cdot 10^4$$

therefore,

$$N(p = 0)/N(p = m_\mu c) = 0.81.$$

Thus most slow muons moving in air will be brought to rest before decaying.

3.2.3. Muons Traversing the Free Atmosphere

The change in air density with height has to be taken into account. For simplicity we assume the density distribution to be exponential.

$$\theta(x) = \theta_0 e^{-\frac{x}{x_0}} \tag{3.18}$$

where x_0 is the height of the homogeneous atmosphere. $\theta(x)$ is the absorption equivalent of the atmosphere at the height x. θ_0 is the absorption equivalent of the whole atmosphere.

Fast muons arriving at sea-level with a momentum p_1 must be supposed to have a momentum

$$p(x) = p_1 + \mu c \frac{x_0}{a}(1 - e^{-\frac{x}{x_0}}) \tag{3.19}$$

at the height x. Denoting the loss of momentum throughout the whole of the atmosphere by

$$p_A = \frac{m_\mu \cdot c \cdot x_0}{a} \tag{3.20}$$

we find

$$N(0) = N(x) \left(\frac{p_A}{p_1} \cdot \frac{p(x)}{p_A + p_1 - p(x)} \right)^{-\frac{x_0}{c\tau_0} \cdot \frac{m_\mu c}{p_1 + p_A}}.$$

3.3. Direct Measurement of the Lifetime of the Muon

A classical arrangement (Figure 3.1) to measure the lifetime was used by Maze (1941), Rasetti (1941) and Rossi (1942). The absorber S is placed below a coincidence arrangement I-II. A set of anticoincidence counters A is placed below S. Anticoincidences I, II-A are caused mainly by particles coming from above and stopping in S. To reduce the effect of showers the top counters are surrounded by lead and a 10 cm. thick lead block is placed between the coincidence counters. Due to this selection the anticoincidences are mainly due to muons stopping in S.

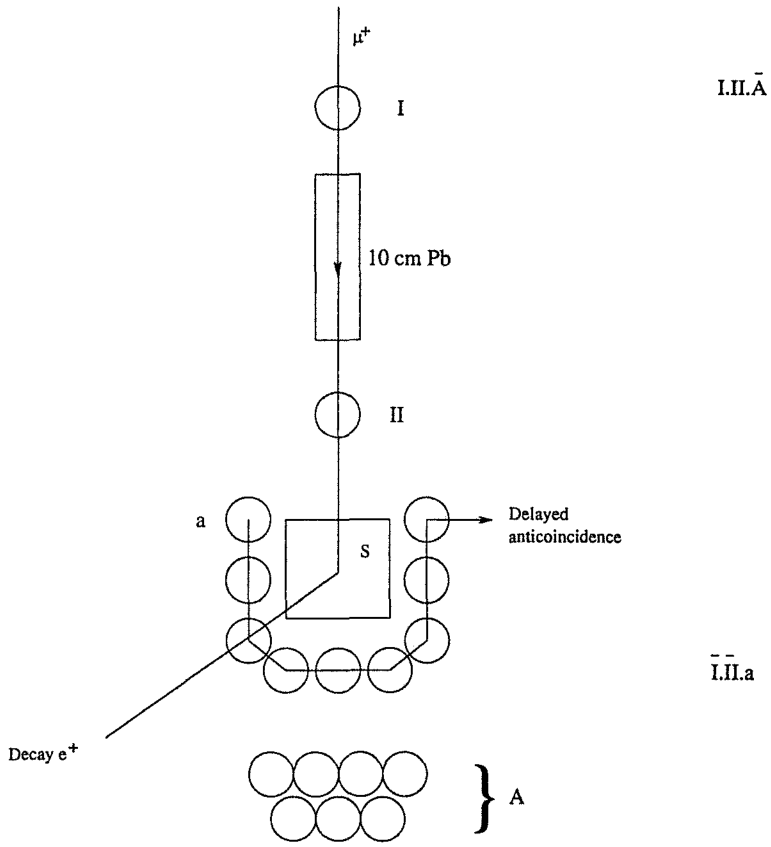

Figure 3.1: Setup of Rossi

The sides of the absorber S are covered with a counter battery. If any of the counters a are discharged shortly after the coincidence pulse, then the pulse is recorded and the time delay between the anticoincidence and the pulse of the counter a is also recorded. Such delayed pulses are mainly due to muons stopping in S and emitting a decay electron afterwards. The registered time delay can be regarded as the time spent by muon in the absorber before it decayed.

As a result of extensive measurements it was found that the rate of delayed pulses corresponding to delays greater than t is given by:

$$\zeta(> t) \sim e^{-t/\tau_0}, \tag{3.21}$$

$$\tau_0 = \begin{cases} 1.5 \pm 0.4\mu S & \text{Rasetti} \\ 2.2 \pm 0.2\mu s & \text{Maze} \\ 2.15 \pm 0.15\mu s & \text{Rossi} \\ 2.33 \pm 0.15\mu s & \text{Conversi} \end{cases}$$

whereas the latest measurements give an average of 2.19703 ± 0.00004 μs.

4. Experimental Apparatus

The main parts of the experimental set-up are three scintillation counters and a thick aluminium block. The experimental arrangement is shown in Figure 4.7. The scintillators are for detecting the incoming and outgoing particles, the aluminium plate causes some of the incoming cosmic muons to stop. Aluminium was chosen as a stopper because it has a long radiation length, it is non-magnetic and it is also relatively cheap. The scintillator counters (Figure 4.2) consist of a scintillator(Figure 4.1) connected to the photo multiplier tube (pmt) through a light guide. As the particles pass through the scintillators electrons of the doped molecules are excited. Then these excited electrons emit photons during deexcitation. Since the scintillator plates are covered by aluminium foil no light can escape from side walls and in addition to that they are covered by black tape which prevents any escape or intake of light. In this way all the photons are directed to the light guide and then to the photocathode of the pmt where the photoelectrons are formed and then amplified by negative high voltage to produce negative signal output.

Since the aim is to find the lifetime of the muon, we need to book the time interval between each incoming muon and outgoing electron. Then fitting the data using Equation 3.2 will give us the lifetime.

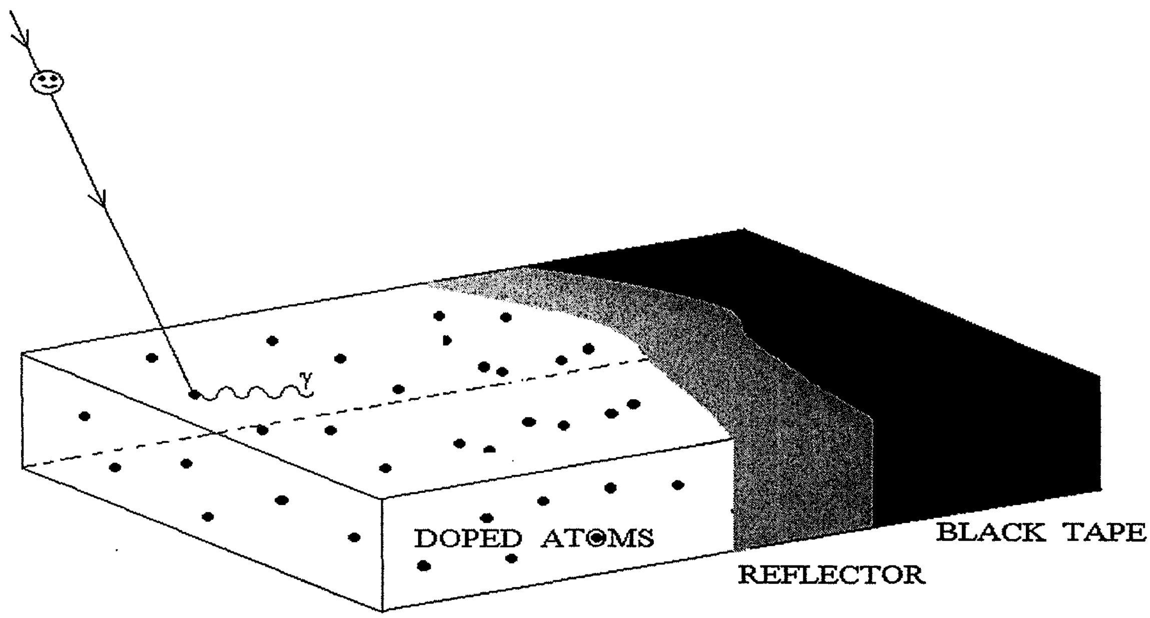

Figure 4.1: Scintillator

4.1. Hardware Instruments

Other than the detector itself, we use a NIM (Nuclear Instrument Modules) and a CAMAC (Computer Automated Measurement And Control) crate with appropriate cards and modules. All algorithm and tunings of the experiment are done within the NIM crate and CAMAC is used for the read-out. The NIM modules to be used can be summarized as follows:

- Discriminator:
  Accepts negative input signals. If the input has a larger magnitude than its adjustable threshold, it gives out a -0.8 V square output (NIM level) with a desired width.

- FI/FO (Fan-in Fan-out):
  In general is used to sum several channels and invert the result if necessary. It doesn't use NIM level, it has an adjustable DC off-set.

- TAC (Time to Amplitude Converter):
  Works like a clock to calculate the time between two successive events (start/stop). It has an adjustable interval range for which an output of 0-10 V (positive) is generated with a fixed width of 2 μs. It accepts start/stop signals in NIM level.

- Coincidence:
  Uses NIM level both for output and input. Gives out a signal with an adjustable width if the input signals coincide (overlap in their widths) for a period.

- Dual Timer:
  Allows to change the width of a NIM standard input and also give very long delays (upto seconds).

- Counter:
  Counts the number of NIM level signals for a given time interval.

The CAMAC crate is connected to a PC through GPIB (General Purpose Interface Board) for the read-out. An ADC (Analog to Digital Converter) card which has a gate and signal input is placed in the crate. It can integrate a maximum value of 12.5 V.ns which has to be considered while adjusting the width of the gate signal that has to be in NIM level.

4.2. Getting Started

In this section we will be familiarizing ourselves with the experimental setup. It is important to go through all the steps mentioned below before you start the experiment, especially if it is your first time with such instruments.

4.2.1. Analog signal

There are three layers of scintillating counters in our experiment. In this part, the top counter will be used to see the signal on the analog oscilloscope. It is crucial to see the level of signal and level of noise, in order to determine the threshold to be set for the discriminator. Make sure that you have the below connections made, and observe the behaviour of the PMT output.

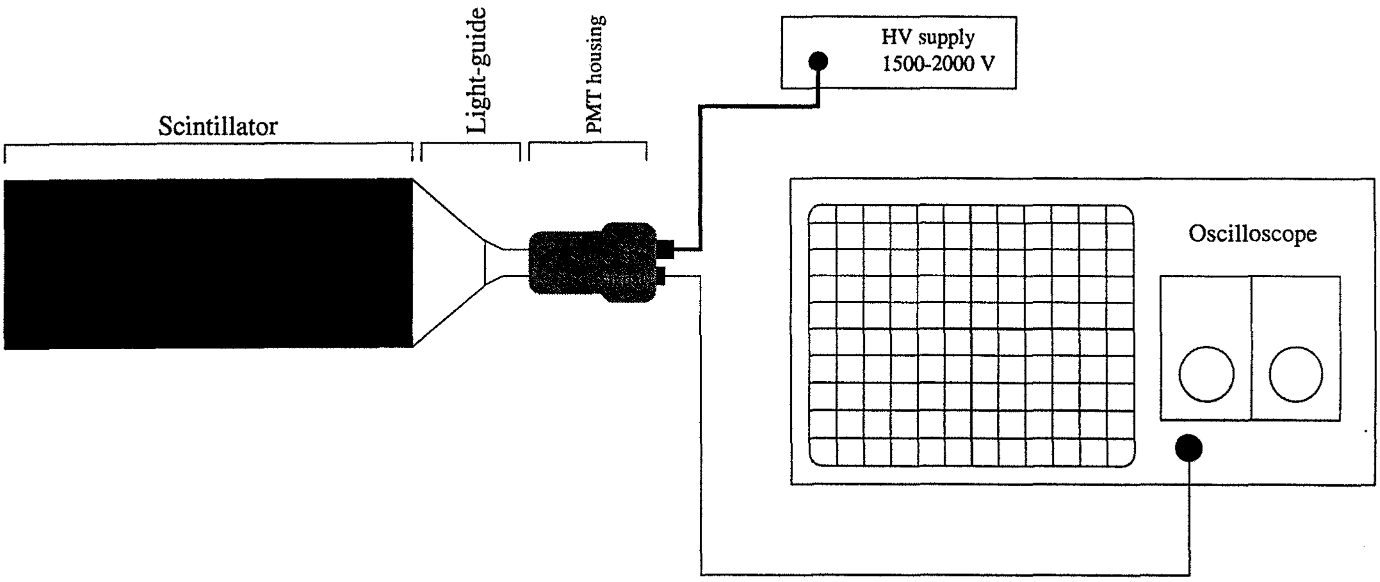

Figure 4.2: set1

4.2.2. Plato Curve Measurement

Every scintillating counter has a dependence on the high-voltage (HV) applied to the PMT. To explore the behaviour of the number of counts as a function of the applied

HV, you will take data with the below settings. The threshold of the discriminator must be set to the value that you have determined in the previous part. Adjust the time interval of the counter to 10 seconds and take data at every 25 Volts between 1700-2500 Volts, wait for about 10 seconds after changing the voltage for stabilazation. Record and then plot your results on the sheet at the back. You should observe a stable region (plato) on your graph.

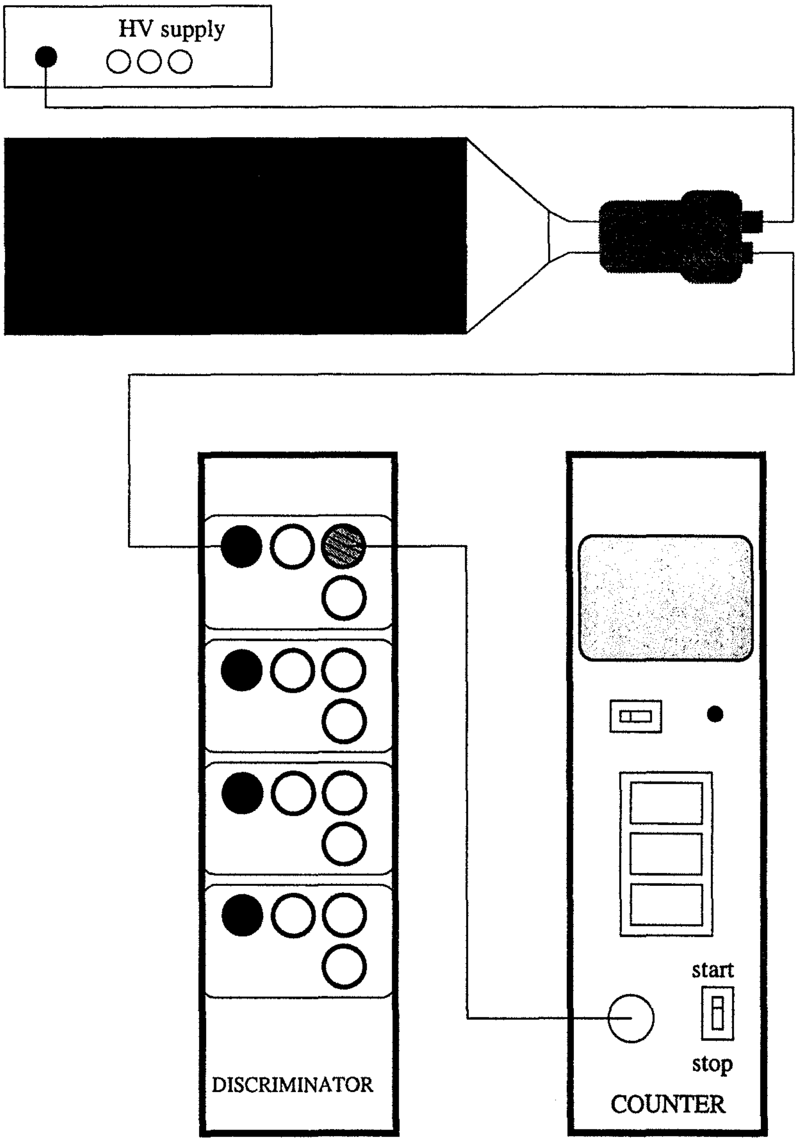

Figure 4.3: set2

HV (volts)	RATE (Hz)	HV (volts)	RATE (Hz)	HV (volts)	RATE (Hz)	HV (volts)	RATE (Hz)

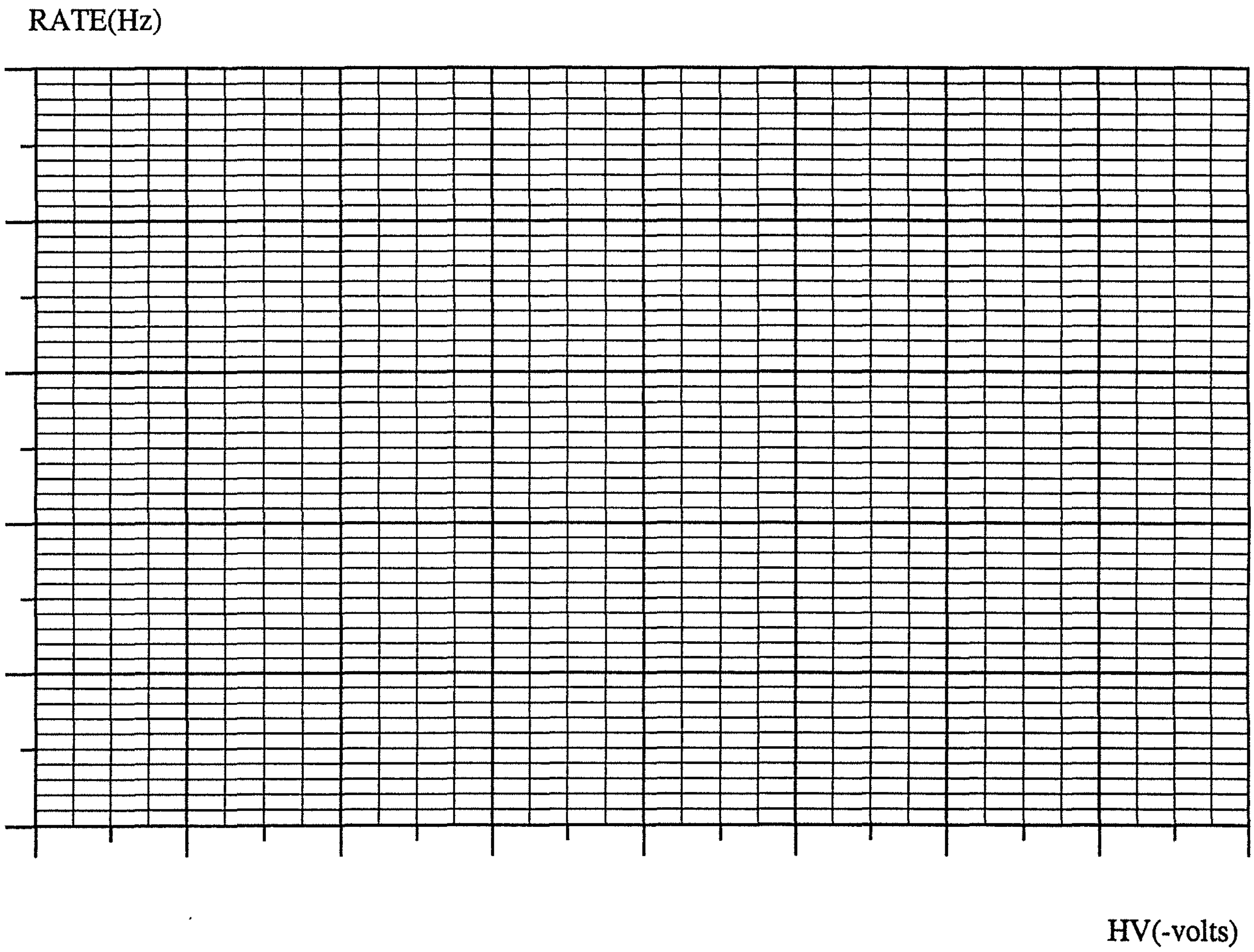

Figure 4.4: Plato

4.2.3. TAC Calibration

This part is performed to show the linear dependence of the TAC output to its input. In order to check this, we must generate two successive signals (start/stop) with a known time difference and record the output amplitude of TAC. If this is done for several time differences the results can be plotted to see the behaviour. Carefully connect the cicuit below and set the TAC range to 10 μs. Follow your assistants instructions to perform the data read-out.

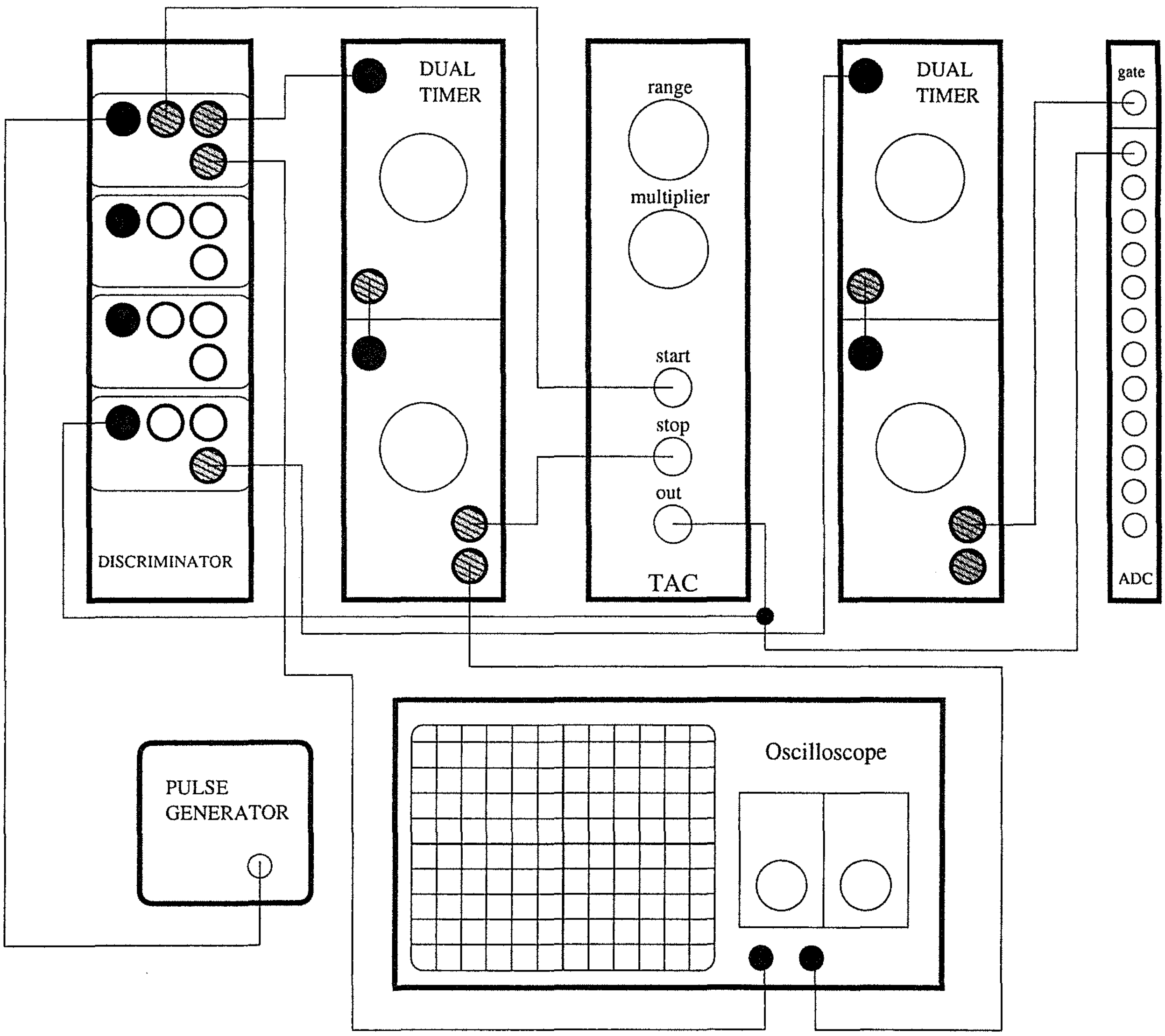

Figure 4.5: set3

Time difference (μs)										
TAC output (-Volts)										

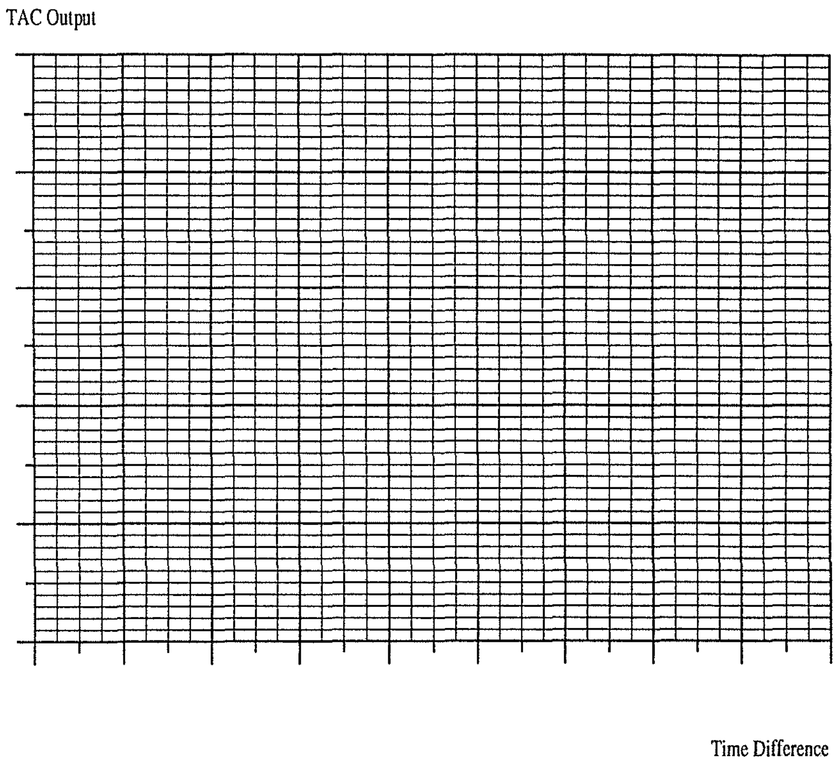

Figure 4.6: TAC

4.3. Experimental Set-up

Our set-up consists of three layers of scintillator counters each with an area of ≈ 0.4 m^2 placed on top of each other with a spacing of $\approx$ 15 cm. In between the middle and bottom layers, an aliminium block of 7 cm thick is placed. This aliminium medium is used to stop the incoming muons and cause them to decay (Figure 2.2). The incoming muon will be the start for the TAC and the decay positron will be the stop, hence the algorithm (Figure 4.7) is set as follows: The start signal is generated when there is a signal from both top (A) and middle (B) counters, but nothing from the bottom (C), this enables us to identify a muon that stops in the aliminium. The stop signal is generated when there is a signal from C but nothing from A and B which shows us that this event belongs to the decay positron that proceeds to the forward cone of the direction of the incoming muon.

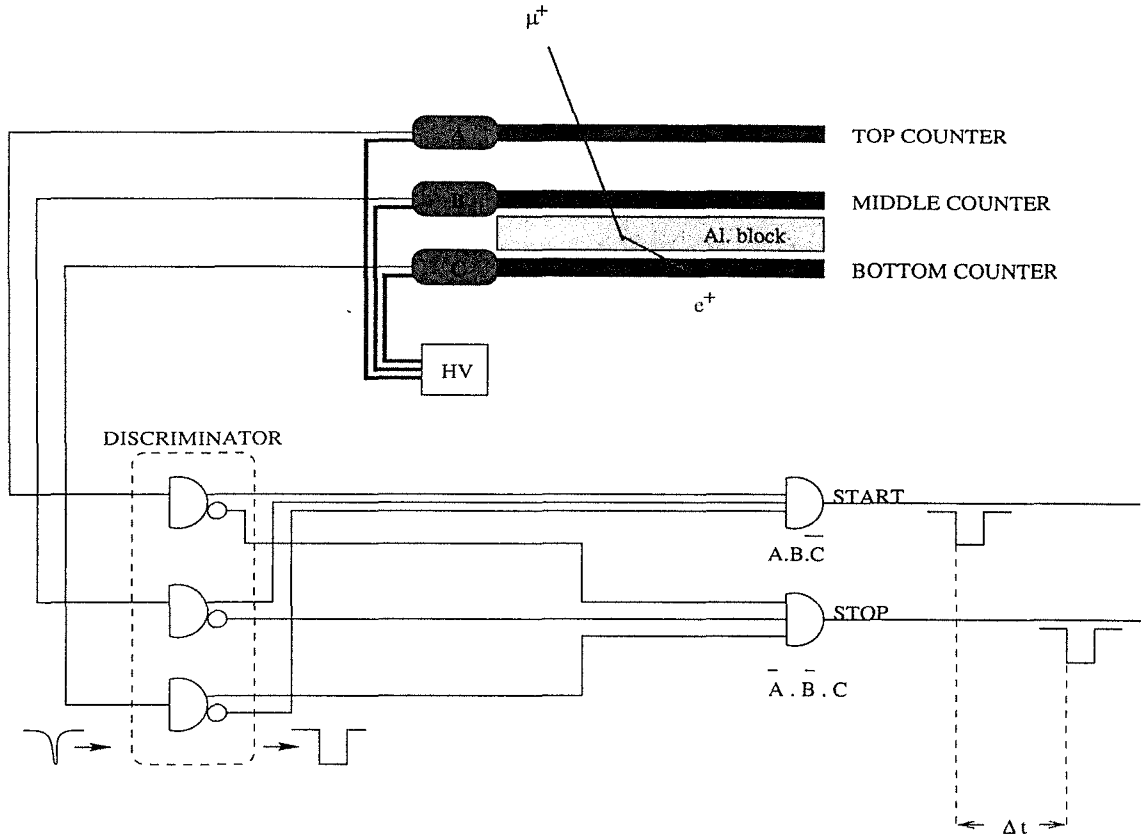

Figure 4.7: Algorithm

All detectors, modules and necessary connections for our experiment are summarized in Figure 4.8. Try to make the connections and understand the purpose of each. Before you start data taking show your setup to the instructor and make sure that you adjust the following operational values which we had optimized for the best performance:

Detector	A	B	C
Threshold	-40mV	-28mV	-22mV
Width	10 ns	10 ns	10 ns
High Voltage	-1900V	-1900V	-1750V

Gate threshold	-18.8mV
Gate width	14 ns
Gate delay	400 ns

TAC Range	100 ns
TAC Multiplier	100

Table 4.1: Operating values

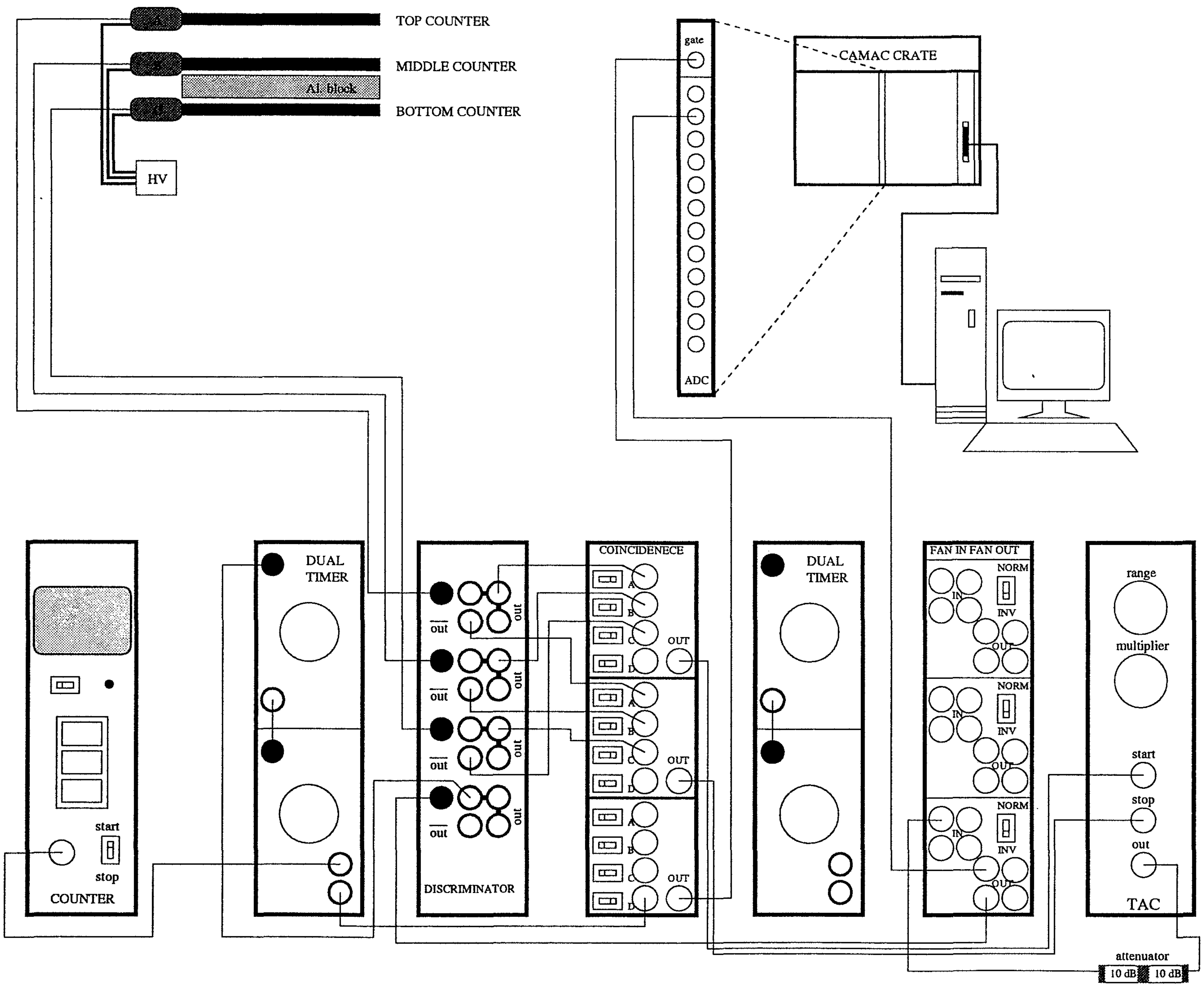

Figure 4.8: setup

5. Data Taking and Analysis

Before starting the real run, it is best to check some event rates with the detector. The first thing is determining the event rates for each layer (A, B, and C) individually, after that the START, STOP and trigger rates can also be checked. You should use the counter on the crate and make the necessary connection to get the rates:

Table 5.1: Event Rates

Top detector (A)	Hz
Middle detector (B)	Hz
Bottom detector (C)	Hz
Start	Hz
Stop	Hz
Trigger	Hz

The rate of trigger, as you have just measured, does not supply us a good statistics within the few hours that you are expected to perform this experiment. At least 10.000 trigger events are necessary for a reliable analyses. For this reason, you can analyse an old set of data given to you by using the following function to make the fit:

$$N(t) = P_1.e^{-t/P_2} + P_3.e^{-t/P_4} + Constant \tag{5.1}$$

Note that you have to use two separate exponentials in the function which, in fact, is a special case for our set-up. The exponentials are used to differentiate the μ^+ decays from the μ^- captures. The rate of μ^- capture by the nucleus is significantly higher in aliminium than in any other stopping medium, this enforces us to introduce the second exponential. An example of the fit to a set of data ($\approx$14000 events) is shown in Figure 5.1. Note the logarithmic scale and the error bars on each point. It is important to estimate the level of random trigger that introduces the constant level to our function, and put that constant by hand. You can now start data taking using the read-out program named "*cosmic_run*".

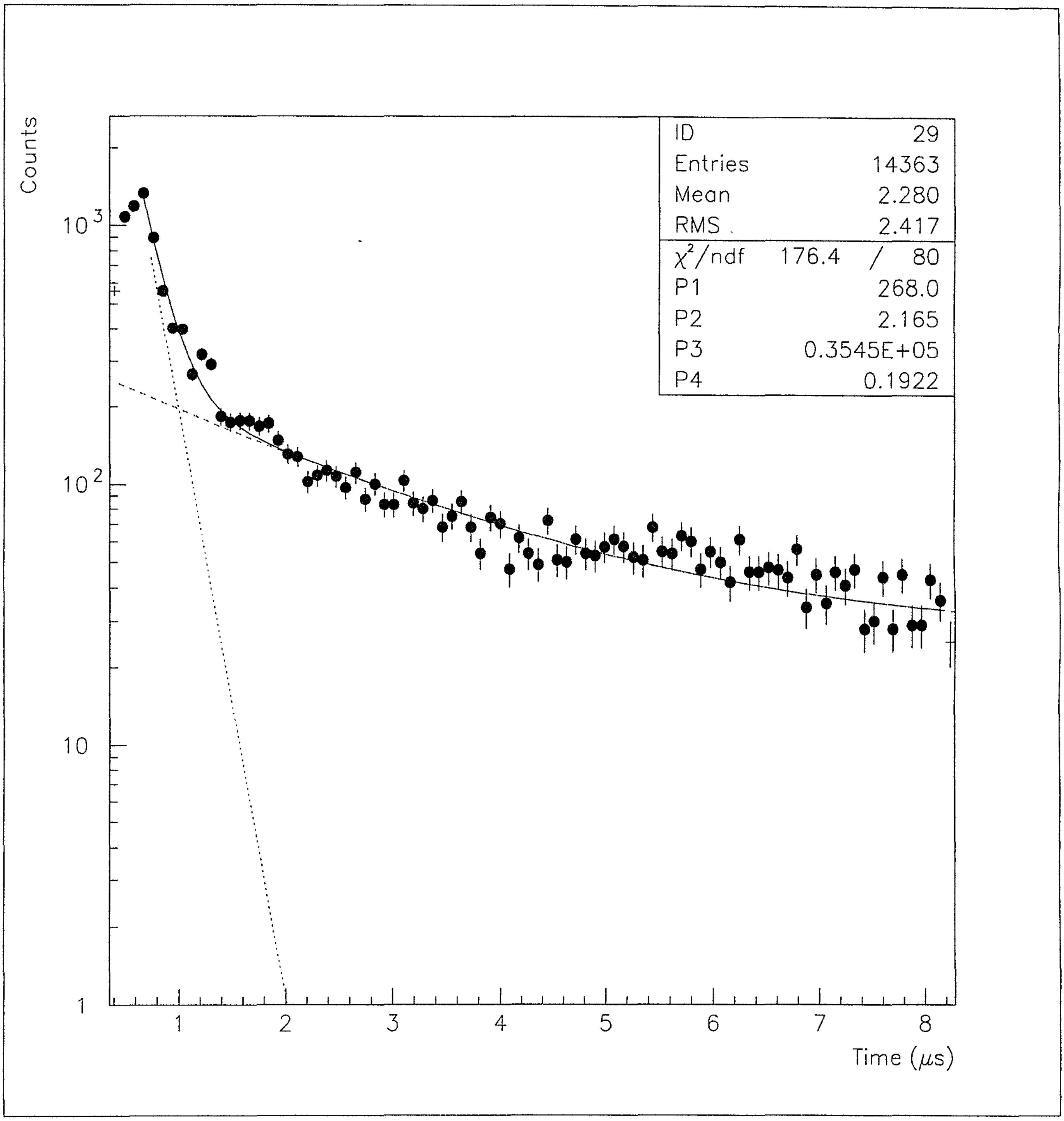

Figure 5.1: Data Analysis

6. Simulation with GEANT

Simulation of the decay of the cosmic muon is examined by using *GEANT* simulation package of CERN. GEANT is developed for designing and optimizing detectors in high energy physics.

The GEANT program simulates the passage of elementary particles through the matter and the particles trajectories and detectors can be visualized either interactively or in a batch mode. We used a Motif-based graphical user-interface (GUI) version of GEANT++.

All user subroutines with data describing the experimental setup were written in FORTRAN 77 programming language. The user routines are classified as follows:

- In UGINIT subroutine the user opens files and initialize GEANT
- In UGEOM the user define needed materials, compounds and volumes
- GUKINE routine has used to generate kinematics of the initial track
- GUSTEP is called at the end of each tracking step and provides the user by hits in detectors
- GDXYZ is used to visualized the tracks during the transport process

At interaction of the incoming particle with matter the each physics process is generated by Lund event generator will accepted by GEANT for further analysis.

The decay of cosmic muons is simulated by using GEANT in which first of all volumes have to be determined. The outermost medium is defined to be *universe*. The Master Reference System is attached to it. All detectors and stopper are positioned inside the *universe*. The *concrete* of the building is also included in consideration. As in the case of real muonscope, there are three scintillator plates and a stopper. After the definition of the volumes, kinematic properties of the particles are defined in the related subroutines. The incoming particles are positively charged muons having the average momentum of the order of 3 GeV when arriving at the *concrete*. The *concrete* is taken with thickness $\approx$ 1.5 m will filter out the incoming muons and therefore the only 0.75 – 0.9 GeV momentum interval is important for study the decay of muons in *Al* absorber. Then the conditions have to be given for selecting events which will create START and STOP signals for measuring the lifetime of cosmic muons. The logic goes as follows: cosmic muons pass through the *universe*, *concrete*, top plate, middle plate and stopper. In the stopper they decay into positrons and these positrons are supposed to be detected in bottom plate. In GEANT dotted blue lines represent gammas, solid red line stands for charged particles, except muons, black blank/dotted line is for neutral hadrons and neutrinos, dashed green line for muons. The result of GEANT simulation for 5 muon events is shown in Figure 5.1.

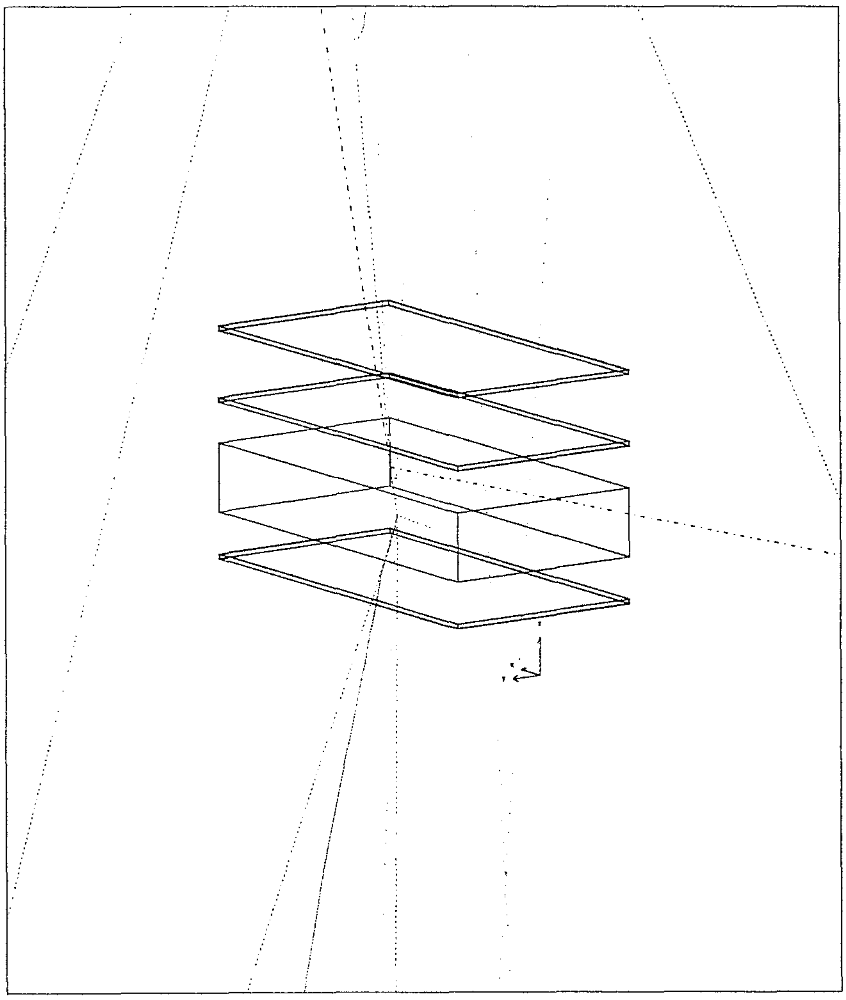

Figure 6.1: GEANT Simulation

In the figure incoming green lines are μ^+'s. The red line represents the decayed positron. Our studies show that out of 150 muon events the only one of them fulfills our condition, namely the incoming muon decays into positron and neutrinos in the stopper and reaches bottom plate.

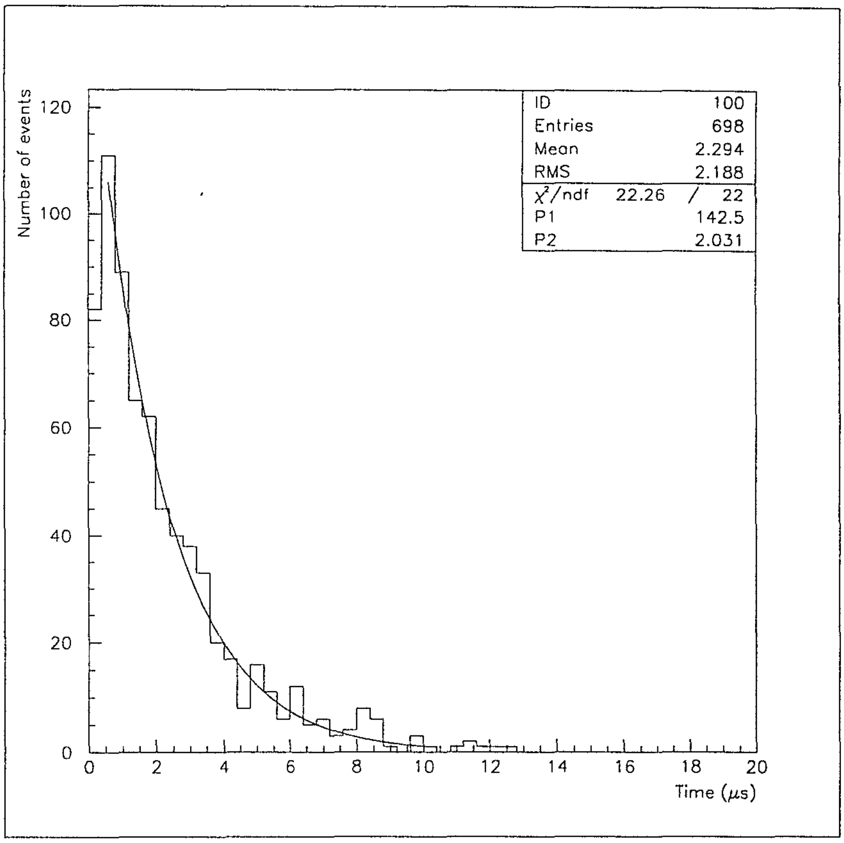

Figure 6.2: Distribution in lifetime of μ^+

The distribution in lifetime of positive muons is presented in Figure 5.2. The result of fitting the simulation data with single exponent gives mean lifetime $2.03 \pm 0.1 \mu s$.

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