Journal of Physics: Conference Series **335** (2011) 012031 doi:10.1088/1742-6596/335/1/012031

Nearest Neighbour Interactions and Symmetries

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Abstract. We study the minimal possible deviations from the Hermitic Nearest-Neighbour Interations (NNI) texture in the quark sector such that it is possible to accommodate the actual experimental data. We also show that the NNI structure can be obtained through the implementation of an Abelian discrete flavour symmetry at the Lagrangian level, where the minimal realisation is Z_4, requiring at least the presence of two Higgs doublets. Finally, we explore the consequences on the leptonic sector of this Z_4 flavour symmetry, in the context of $SU(5)$ Grand Unification with the standard fermionic content plus three right-handed neutrinos and two Higgs quintets.

1. Introduction

A huge effort has been made in the last decades in understanding the pattern of fermion masses and mixings. In the Standard Model, the Yukawa interaction terms that describe the fermion masses are unexplained. Many attempts have been made to find a framework where the fermion masses and mixings can be explained. One possible approach is to impose some zeroes on the mass matrix elements, for instance the well known Fritzsch Ansatz [1]. Such Ansatz combine the Nearest-Neighbour Interaction (NNI) [2] form with the Hermiticity condition. The NNI structure, where entries $(1,1)$, $(1,3)$, $(2,2)$ and $(3,1)$ vanish, can always be achieved for quark mass matrices M_u, M_d in the SM [2] through a weak basis transformation. However, it is no longer true in other contexts as is the case of two-Higgs doublet model. One can find many works in literature where a symmetry leads to the NNI structure, for example [6, 7] or [3] where a Z_4 flavour symmetry was implemented in the context of two Higgs doublets. Some of those models are based on Grand Unification Theories (GUT), which are very appealing to implement flavour symmetries once the SM fermions are unified in large multiplets. For example [8, 5] models based on the $SU(5)$ gauge group [9].

This work is organised as follows. In section 2 we study the minimal deviations from the Hermitic NNI structure such that both up- and down-quark mass matrices accommodate the actual quark data. In section 3 we implemet an Abelian discrete flavour symmetry that leads to quark mass matrices in the NNI form, where the minimal realisation in the context of two Higgs doublets is Z_4. This Z_4 flavour symmetry is implemented in the context of the $SU(5)$ GUT model in section 4 where the leptonic sector is analysed.

2. Minimal Deviations from Hermiticity: Quark Sector

In this section, we study the minimal deviations from the Hermitic NNI structure such that it is possible to accommodate the experimental data on both up- and down-quark mass matrices, M_u, M_d.

In the NNI basis, the quark mass matrices can be written as,

$$M_u = \begin{pmatrix} 0 & A_u & 0 \\ A'_u & 0 & B_u \\ 0 & B'_u & C_u \end{pmatrix}, \qquad M_d = K_q \begin{pmatrix} 0 & A_d & 0 \\ A'_d & 0 & B_d \\ 0 & B'_d & C_d \end{pmatrix}, \tag{1}$$

where $(A,\, A',\, B,\, B',\, C)_{u,d}$ are taken real without loss of generality and K is a diagonal phase matrix, $K_q = \mathrm{diag}(e^{i\kappa_1}, e^{i\kappa_2}, 1)$.

Since we are interested in determining the minimal possible deviations from the Hermitic NNI structure, we introduce the parameters ϵ^u_a, ϵ^u_b, ϵ^d_a and ϵ^d_b,

$$\epsilon^{u,d}_a \equiv \frac{A'_{u,d} - A_{u,d}}{A'_{u,d} + A_{u,d}}, \qquad \epsilon^{u,d}_b \equiv \frac{B'_{u,d} - B_{u,d}}{B'_{u,d} + B_{u,d}}. \tag{2}$$

To better determine the minimal values of $\epsilon^{u,d}_{a,b}$ we will work with the Hermitian matrices, H_u, H_d, defined as $H_{u,d} = M_{u,d}\, M^\dagger_{u,d}$. The complex H_d matrix can be written as,

$$H_d = K\, H^0_d\, K^\dagger, \tag{3}$$

where H^0_d is a real matrix and the phases κ_1, κ_2 are given by

$$\kappa_1 = arg(H_{d_{13}}), \qquad \kappa_1 = arg(H_{d_{23}}). \tag{4}$$

With H_u and H^0_d being Hermitian real matrices they can be diagonalised by real orthogonal matrices O_u and O_d respectively as,

$$H_u = O_u\ \mathrm{diag}(m_u^2, m_c^2, m_t^2)\, O_u^\top, \qquad H^0_d = O_d\ \mathrm{diag}(m_d^2, m_s^2, m_b^2)\, O_d^\top, \tag{5}$$

then the quark mixing matrix, the so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix [10, 11], V, is given by,

$$V = O_u^\top\, K\, O_d. \tag{6}$$

The deviations $\epsilon^{u,d}_{a,b}$ of Eq. (2) can be related to the quark masses through the invariants of H_u and H_d (see Ref. [3]), hence if the deviations $\epsilon^{u,d}_{a,b}$, the quark masses and the phases κ_1, κ_2 are given, the matrices M_u, M_d and consequently H_u, H_d can be fully reconstructed.

The CKM matrix is computed by Eq. (6) after diagonalization of H_u and H^0_d.

In our numerical calculations, we have performed a scan of the deviations $\epsilon^{u,d}_{a,b}$, the phases κ_1, κ_2 and the running quark masses at the electroweak scale within their errors. We have accepted solutions that lead to a CKM matrix in agreement with the actual data [12] of the CKM moduli, the angles of the unitary triangle and the strength of CP violation I. We sketched on Table 1 these input data together with the running quark masses.

To measure the global deviation from the Hermiticity of the pair of quark mass matrices, M_u, M_d, we defined the parameter ε_q,

$$\varepsilon_q \equiv \frac{1}{2}\sqrt{(\epsilon^u_a)^2 + \left(\epsilon^u_b\right)^2 + (\epsilon^d_a)^2 + \left(\epsilon^d_b\right)^2}, \tag{7}$$

Table 1. Values of the running up- and down-quark masses, CKM element moduli $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and the angles β and γ of the unitarity triangle and the strength of CP violation I, at the scale $M_Z = 91.1876 \pm 0.0021\,\mathrm{GeV}$ [12]. The quark masses are calculated [13] to M_Z scale through the renormalisation group equations for QCD in the $\overline{MS}$ [14, 15, 16, 17, 18] scheme, at 4-loop level.

$m_u = 1.4 \pm 0.5\,\mathrm{MeV}$	$m_c = 0.62^{+0.06}_{-0.07}\,\mathrm{GeV}$	$m_t = 170.2 \pm 1.0\,\mathrm{GeV}$
$m_d = 2.9 \pm 0.5\,\mathrm{MeV}$	$m_s = 58^{+16}_{-12}\,\mathrm{MeV}$	$m_b = 2.86^{+0.16}_{-0.06}\,\mathrm{GeV}$
$\lvert V_{us}\rvert = 0.2253 \pm 0.0007$	$\lvert V_{ub}\rvert = \left(3.47^{+0.16}_{-0.12}\right) \times 10^{-3}$	
$\lvert V_{cb}\rvert = \left(41.0^{+1.1}_{-0.7}\right) \times 10^{-3}$	$\sin 2\beta = 0.673 \pm 0.023$	
$\gamma = \left(73^{+22}_{-25}\right)^{\circ}$	$I = \left(2.91^{+0.19}_{-0.11}\right) \times 10^{-5}$	

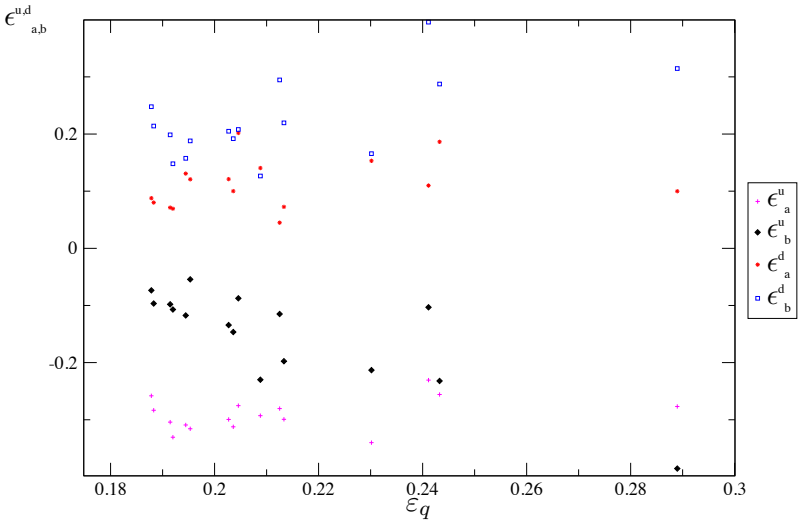

Figure 1. Set of solutions for $\epsilon^{u,d}_{a,b}$ corresponding to the constraint $\varepsilon_q \leq 0.3$.

The numerical results are shown in Fig. 1 where the deviations $\epsilon^{u,d}_{a,b}$ are plotted as a function of the parameter ε_q. From our search one sees that very small values of $\epsilon^{u,d}_{a,b}$ are not possible. Furthermore, if we consider $\epsilon_{a,b} = 0$ in one sector the deviations in the other become very large.

As the minimal global deviation is $\varepsilon_q = 0.188$ we conclude that within 20% it is possible to accomodate the actual experimental quark data on Hermitian NNI mass matrices. The values of a numerical example can be found in Ref. [3].

3. NNI from a Discrete Flavour Symmetry

In this section we show that it is possible to obtain the up- and down-quark mass matrices M_u, M_d in the NNI form through the implementation of an Abelian discrete flavour symmetry at the Lagrangian level. We shall show that the minimal realisation on cyclic groups is Z_4, requiring the presence of at least two Higgs doublets.

As we are interested in a minimal flavour symmetry, such that the NNI structure appears in the quark mass matrices, we restrict our search to Abelian and discrete symmetries and thus avoid the presence of Nambu-Goldstone bosons. We will search for symmetries of Z_n type. Under such symmetries a field,Ψ_j, with charge $\mathcal{Q}(\Psi_j)$ transforms as

$$\Psi_j \longrightarrow \Psi'_j = e^{i\frac{2\pi}{n}\mathcal{Q}(\Psi_j)}\,\Psi_j\,. \tag{8}$$

Let us start with the most general quark Yukawa Lagrangian,

$$-\mathcal{L}_Y = Y_u\,\overline{Q}_{L_i} u_{R_j} \Phi^u_{ij} \,+\, Y_d\,\overline{Q}_{L_i} d_{R_j} \Phi^d_{ij} \,+\, \text{H.c.}\,. \tag{9}$$

where$\Phi^{\,u}_{ij}$ and$\Phi^{\,d}_{ij}$ correspond generically to the Higgs fields that couple to the (i,j)-entries of the up- and down-quark sector. To obtain the NNI form in the quark mass matrices, we must require zero charges for the trilinear matrix elements corresponding to non-zero mass matrix entries

$$\mathcal{Q}(\overline{Q}_{L_i} u_{R_j} \Phi^u_{ij}) = 0\,, \qquad \mathcal{Q}(\overline{Q}_{L_i} d_{R_j} \Phi^d_{ij}) = 0\,, \tag{10}$$

and non-zero charges for the trilinear matrix elements corresponding to zero mass matrix entries

$$\mathcal{Q}(\overline{Q}_{L_i} u_{R_j} \Phi^u_{ij}) \neq 0\,, \qquad \mathcal{Q}(\overline{Q}_{L_i} d_{R_j} \Phi^d_{ij}) \neq 0\,. \tag{11}$$

Writing down Eqs. (10) and (11) for all entries and solving for the charges of the Higgs doublets$\Phi^{\,u}_{ij}\,, \Phi^d_{ij}$, we find that at least two Higgs doublets are required$\Phi_{\,1}$ and$\Phi_{\,2}$ with charges ϕ_1 and ϕ_2 respectively (see details in Ref. [3]). The quark field charges are then given by the following assignment,

$$\begin{aligned} (q_1, q_2) &= (q_3 + \phi_1 - \phi_2, q_3 - \phi_1 + \phi_2)\,, \\ (u_1,\, u_2,\, u_3) &= (q_3 - \phi_1 + 2\phi_2, q_3 + \phi_1, q_3 + \phi_2)\,, \\ (d_1,\, d_2,\, d_3) &= (q_3 - 2\phi_1 + \phi_2, q_3 - \phi_2, q_3 - \phi_1)\,, \end{aligned} \tag{12}$$

where $\mathcal{Q}(Q_{Li}) \equiv q_i$, $\mathcal{Q}(u_{Ri}) \equiv u_i$, $\mathcal{Q}(d_{Ri}) \equiv d_i$ with Q_{Li} the left-handed quark doublets.

Hence, the quark Yukawa Lagrangian in Eq. (9) becomes

$$-\mathcal{L}_Y = Y_u^1\,\overline{Q}_L \widetilde{\Phi}_1\, u_R \,+\, Y_u^2\,\overline{Q}_L \widetilde{\Phi}_2\, u_R \,+\, Y_d^1\,\overline{Q}_L \Phi_1\, d_R \,+\, Y_d^2\,\overline{Q}_L \Phi_2\, d_R \,+\, \text{H.c.}\,, \tag{13}$$

where $\widetilde{\Phi}_j \equiv i\,\sigma_2\,\Phi_j^*$ and $j = 1, 2$. After spontaneous symmetry breaking, the quark mass matrices are given by,

$$M_u = v_1^*\,Y_u^1 + v_2^*\,Y_u^2\,, \qquad M_d = v_1\,Y_d^1 + v_2\,Y_d^2\,, \tag{14}$$

taking precisely the NNI form, with $\langle \Phi_i \rangle \equiv v_i$ and $\langle \widetilde{\Phi}_i \rangle \equiv v_i^*$ and $Y^{1,2}_{u,d}$ the Yukawa matrices.

At this point we have found that two Higgs doublets are enough to achieve the NNI structure for both up- and down-quark mass matrices and the general Z_n quark charges are given by Eq. (12). Let us now find the minimal Z_n symmetry in which this implementation is possible.

From the bilinears, $\overline{Q}_{L_i} u_{R_j}$,

$$\begin{pmatrix} -2\phi_1 + 3\phi_2 & \phi_2 & -\phi_1 + 2\phi_2 \\ \phi_2 & 2\phi_1 - \phi_2 & \phi_1 \\ -\phi_1 + 2\phi_2 & \phi_1 & \phi_2 \end{pmatrix}\,, \tag{15}$$

and $\overline{Q}_{L_i} d_{R_j}$,

$$\begin{pmatrix} -3\phi_1 + 2\phi_2 & -\phi_1 & -2\phi_1 + \phi_2 \\ -\phi_1 & \phi_1 - 2\phi_2 & -\phi_2 \\ -2\phi_1 + \phi_2 & -\phi_2 & -\phi_1 \end{pmatrix} \tag{16}$$

one sees that neither Z_2 nor Z_3 are compatible with the NNI form for both up- and down-quark sectors. For instance, taking $(1,1)$-element of Eq. (15) one gets,

$$-2\phi_1 + 3\phi_2 \,=\, \phi_2 \pmod 2\,, \qquad -2\phi_1 + 3\phi_2 \,=\, \phi_1 \pmod 3\,, \tag{17}$$

as the up-quark sector couples with $\widetilde{\Phi}_1$ and $\widetilde{\Phi}_2$, where $\mathcal{Q}(\widetilde{\Phi}_i) = -\phi_i$; this entry becomes non-zero in the mass matrix, which is not desired. Hence, in the context of two Higgs doublets, the minimal symmetry that makes the NNI structure achievable is Z_4. In fact this conclusion is true even if the number of Higgs doublets is three or more.

4. NNI in the context of $SU(5)$

In this section we extend the Z_4 flavour symmetry, discussed in the last section, in the context of $SU(5)$ Grand Unification.

In the $SU(5)$ model the three generations of 10, 5^* fermionic multiplets are completely filled by the left-handed SM fields $(q, u^c, d^c, \ell\,, e^c)_i$, where $i = 1, 2, 3$ is the generation index,

$$\mathsf{10}_i = (q, u^c, e^c)_i\,, \quad \mathsf{5}^*_i = (\ell, d^c)_i\,, \tag{18}$$

Beyond the standard field content we have added, to this $SU(5) \times Z_4$ model, three right-handed neutrinos, $\nu_{R\,i}$, and two Higgs quintets. The $SU(5)$ fields assignment stated in Eq. (18) implies the follow charge relations

$$\mathcal{Q}(\mathsf{10}_i) = \mathcal{Q}(Q_{L_i}) = -\mathcal{Q}(u_{R_i}) = -\mathcal{Q}(e_{R_i})\,, \tag{19a}$$
$$\mathcal{Q}(\mathsf{5}^*_i) = \mathcal{Q}(\ell_{L_i}) = -\mathcal{Q}(d_{R_i})\,, \tag{19b}$$

from which one gets $\phi_2 = -2q_3$. The Z_4 charges of the $SU(5)$ multiplets are then given as:

$$\mathcal{Q}(\mathsf{10}) = (3q_3 + \phi_1,\, -q_3 - \phi_1,\, q_3)\,, \qquad \mathcal{Q}(\mathsf{5}^*) = (q_3 + 2\phi_1,\, -3q_3,\, -q_3 + \phi_1)\,. \tag{20}$$

Writting down the bilinear $\mathsf{10}_i\mathsf{10}_j$ and $\mathsf{10}_i\mathsf{5}^*_j$ one verifies that the NNI structure is achieved.

Let us now explore the leptonic sector. Due to the $SU(5)$ field assignments, Eq. (18), the charged lepton mass matrix, m_ℓ, gets the NNI form. However, since the right-handed neutrino fields, ν_{R_i}, are singlets under $SU(5)$, their Z_4 charges are free. Such freedom leads to a non parallel structure in the leptonic sector.

In this model, neutrinos acquire Majorana masses through type-I seesaw [19, 20, 21, 22]. The effective neutrino mass matrix, m_ν, can be computed from the seesaw formula, well approximated by:

$$m_\nu = -m_D\, M_R^{-1}\, m_D^\top\,, \tag{21}$$

where the symmetric Majorana mass matrix, M_R, is determined directly by the right-handed neutrino charges, ν_i, while the Dirac mass matrix, m_D, is determined in a similar way to the quark mass matrices M_u, M_d in Eq. (14). As mentioned before, the charged lepton mass matrix gets the NNI form and can be written as,

$$m_\ell = K_\ell^\dagger \begin{pmatrix} 0 & A_\ell & 0 \\ A'_\ell & 0 & B_\ell \\ 0 & B'_\ell & C_\ell \end{pmatrix}\,, \tag{22}$$

where the constants $(A,\, A',\, B,\, B',\, C)_\ell$ are taken to be real and positive and the diagonal phase matrix is $K_\ell = \mathrm{diag}(e^{i\sigma_1}, e^{i\sigma_2}, 1)$.

Performing a scan of the charges ϕ_1, q_3 and ν_i one finds that only six effective neutrino mass matrix textures are possible. From the counting of the number of parameters one finds that only two of those textures are physically viable. The remaining ones lead to small mixing angles, which is not acceptable. The two textures zeroes left,

$$a)\ \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu\, e^{i\varphi} \end{pmatrix}\,, \qquad b)\ \begin{pmatrix} A_\nu & B_\nu & C_\nu \\ B_\nu & 0 & 0 \\ C_\nu & 0 & D_\nu\, e^{i\varphi} \end{pmatrix}\,, \tag{23}$$

have a total of twelve parameters which are sufficient to explain the twelve physical ones (the six lepton masses, the three mixing angles, one Dirac phase and two Majorana phases).

The lepton mixing matrix or Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [23, 24, 25], U, is given by

$$U = O_\ell^\top \, K_\ell^\dagger \, U_\nu \,, \tag{24}$$

where U_ν is the matrix that diagonalise the effective neutrino mass matrix, a) and b) of Eq. (23). To determine the matrix O_ℓ that diagonalises the charged lepton mass matrix it is convenient to introduce the Hermitian mass matrix h_ℓ,

$$h_\ell = \, m_\ell \, m_\ell^\dagger \,. \tag{25}$$

Since the charged lepton mass matrix has NNI form, one can parameterise it in the same way as is done for quark sector, Eq. (2),

$$\epsilon_a^\ell \equiv \frac{A'_\ell - A_\ell}{A'_\ell + A_\ell} \,, \qquad \epsilon_b^\ell \equiv \frac{B'_\ell - B_\ell}{B'_\ell + B_\ell} \,, \tag{26}$$

and also define the global deviation parameter for the charged lepton mass matrix, ε_ℓ,

$$\varepsilon_\ell \equiv \frac{1}{2} \sqrt{\left(\epsilon_a^\ell\right)^2 + \left(\epsilon_b^\ell\right)^2} \,, \tag{27}$$

In our numeric procedure we have computed the PMNS matrix and taken the solutions

Table 2. Values [26] of the charged lepton masses, mixing angles θ_{13}, θ_{12} and θ_{23} and neutrino mass squared differences,Δ m^2_{21}, $|\Delta m^2_{31}|$. The charged lepton masses are calculated [17, 18] at M_Z scale, $M_Z = 91.1876 \pm 0.0021\,\text{GeV}$ [12], through the renormalisation group equations for QED in the $\overline{MS}$ scheme at 1-loop level.

$$m_e(M_Z) = 0.486661305 \pm 0.000000056 \text{ MeV}$$

$$m_\mu(M_Z) = 102.728989 \pm 0.000013 \text{ MeV}$$

$$m_\tau(M_Z) = 1746.28 \pm 0.16 \text{ MeV}$$

$$\Delta m^2_{21} = \left(7.59^{+0.23}_{-0.18}\right) \times 10^{-5}\,\text{eV}^2 \qquad \left|\Delta m^2_{31}\right| = \left(2.40^{+0.12}_{-0.11}\right) \times 10^{-3}\,\text{eV}^2$$

$$\sin^2\theta_{12} = 0.318^{+0.019}_{-0.016} \qquad \sin^2\theta_{23} = 0.50^{+0.07}_{-0.06} \qquad \sin^2\theta_{13} < 0.035 \;\text{ at } 90\%\text{ C.L.}$$

that are in aggreement with the neutrino oscillation data [26]. We made a scan of all input parameters within their allowed range, namely the neutrino mass squared differences, m^2_{21} , $|\Delta m^2_{31}|$ ($\Delta m^2_{ij} \equiv m^2_i - m^2_j$), the lightest neutrino mass (m_1 for normal hierarchy (NH) or m_3 for inverted hierarchy (IH)), the phases ϕ,$\sigma_{\,1}$, σ_2, the charged lepton masses, the parameters ϵ_a^ℓ, ϵ_b^ℓ and D_ν is taken as a free parameter. The lightest neutrino mass matrix was scanned for different magnitudes below 2 eV. We sketch in Table 2 the experimental data used in the numerical procedure.

In addition to the restriction imposed by the neutrino oscillation data, one has to further consider the constraints on the effective Majorana mass, m_{ee}, proportional to the neutrinoless double beta decay amplitude [27], the constraint from Tritium β decay [12], m_{ν_e}, and the bound on the sum of light neutrino masses from cosmological and astrophysical data [28]. We found that an effective neutrino mass matrix of the form a) is only compatible with experimental data in the case of normal neutrino mass spectrum and b) only in the case of inverted neutrino mass

Table 3. Summary of the predictions for texture a) and b).

a)	b)
NH	NH
—	$\sin^2\theta_{13} > 0.010$
$0.0013\,\text{eV} \leq m_1 \leq 0.016\,\text{eV}$	$0.0042\,\text{eV} \leq m_3 \leq 0.011\,\text{eV}$
$6.4\times10^{-4}\,\text{eV} < \lvert m_{ee}\rvert < 2.2\times10^{-3}\,\text{eV}$	$0.015\,\text{eV} < \lvert m_{ee}\rvert < 0.022\,\text{eV}$
$\varepsilon_\ell > 0.0011$	$\varepsilon_\ell > 0.0013$

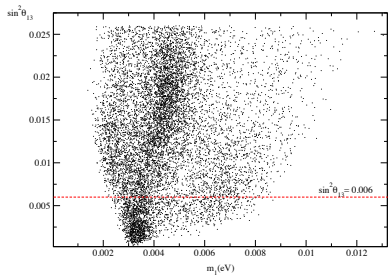

Figure 2. Plot of $\sin^2\theta_{13}$ over m_1 for texture a) and normal hierarchy.

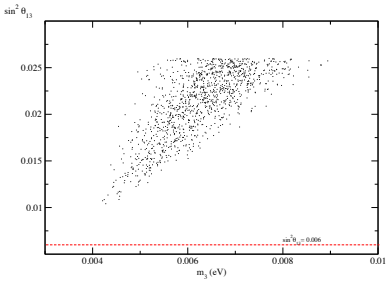

Figure 3. Plot of $\sin^2\theta_{13}$ over m_3 for texture b) and inverted hierarchy.

spectrum. The limits of the lightest neutrino mass for both cases are bounded. We plot in Figures 2 and 3 $\sin^2\theta_{13}$ as a function of the lightest neutrino mass m_1 for NH and m_3 for IH, respectively.

Our results are in agreement with the other constraints considered. We summarise in Table 3 the most important results of this $SU(5)\times Z_4$ GUT model, for more details see Ref. [5].

5. Conclusions

In this work we have studied the minimal deviations from the Hermitic NNI structure, such that the quark mass matrices M_u and M_d could accommodate the experimental quark data. It is shown in Figure 1 the deviations for both sectors as a function of the global deviation, ε_q. One sees that the deviations $\epsilon^{u,d}_{a,b}$ can not be very small, if they are zero for one sector we obtain large deviations in the other sector. It is possible to accommodate the actual quark data on mass matrices in the NNI form with a deviation of order of 20%.

We have shown also that it is possible to obtain the NNI structure for the both up- and down-quark mass matrices through the implementation of Z_4 flavour symmetry at the Lagrangian level in the context of two Higgs doublet model.

Then, with Grand Unification in mind, we extend this Z_4 flavour symmetry in the context of $SU(5)$, where we added right-handed neutrinos and two Higgs quintets. Due to the $SU(5)$ field assignments, the charged lepton mass matrix takes the NNI form while the effective neutrino mass matrix does not, leading to two viable textures a) and b) in Eq. (23). After analising the physical viability of those effective neutrino mass matrix textures we resume in Table 2 the relevant results.

Acknowledgments
We would like to thank the organizers of *DISCRETE'10 - Symposium on Prospects in the Physics of Discrete Symmetries* for the opportunity to present this work. We also thank David Emmanuel-Costa for helping to prepare the talk and the proceeding manuscript. This work was supported by Fundação para a Ciência e a Tecnologia (FCT, Portugal), under the contract SFRH/BD/61623/2009 and the projects CERN/FP/109305/2009 and CFTP-FCT Unit 777 partially funded through POCTI(FEDER) and by the Marie Curie Initial Training Network UNILHC PITN-GA-2009-237920.

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