QCD pomeron in the small x physics

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Abstract

The theory of the BFKL pomeron is reviewed. It is shown, that the equations for compound states of several reggeized gluons are exactly solvable in the multi-colour QCD. The gauge-invariant effective action for high energy processes with quasi-multi-Regge kinematics for produced gluons is constructed in terms of the Wilson contour integrals. After integrating over the physical degrees of freedom the reggeon action is derived.

Résumé

La théorie du poméron BFKL est passé en revue.

1. Introduction

The theoretical description of the deep inelastic ep scattering at small Bjorken variable $x = Q^2/(2m\nu)$ is based on the GLAP [1] and BFKL [2] evolution equations governing correspondingly the Q^2 and x dependence of the parton distributions $n_i(x)$:

$$\frac{\partial n_i(x)}{\partial \ln Q^2} = -W_i n_i(x) + \sum_k \int_x^1 \frac{d\,x'}{x}\, W_{k\to i}(\frac{x}{x'})\, n_k(x')\,, \tag{1}$$

$$\frac{\partial g(k_\perp)}{\partial \ln \frac{1}{x}} = 2\omega(k_\perp^2)\, g(k_\perp) + \int d^2\, k'_\perp\, K(k_\perp, k'_\perp)\, g(k'_\perp)\,. \tag{2}$$

The gluon inclusive probability $g(x, k_\perp)$ depends on the longitudinal Sudakov component x of the gluon momentum k and on its transverse projection $k_\perp$ in the infinite momentum frame of the proton $\left|\overrightarrow{p}_A\right| \to \infty$ and can be expressed in terms of the imaginary part of the gluon-gluon scattering amplitude $A(s,t)$ at $t = 0$ in the Regge regime of high energies $\sqrt{s} = \sqrt{2p_A p_B}$ and fixed momentum transfers $q = \sqrt{-t}$. The most probable process at large s is the gluon production in the multi-Regge kinematics for final state particle momenta $k_0 = p_{A'}, k_1 = q_1 - q_2, ..., k_n = q_n - q_{n+1}, k_{n+1} = p_{B'}$:

$$s \gg s_i = 2k_{i-1}k_i \gg t_i = q_i^2 = (p_A - \sum_{r=0}^{i-1} k_r)^2\,, \tag{3}$$

$$\prod_{i=1}^{n+1} s_i = s \prod_{i=1}^{n} k_i^2\ ,\ k_\perp^2 = -k^2.$$

In the leading logarithmic approximation (LLA) the production amplitude in this kinematics has the multi-Regge form [2]:

$$A_{2\to 2+n}^{LLA} = A_{2\to 2+n}^{tree} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}\ . \tag{4}$$

Here $s_i^{\omega(t_i)}$ are the Regge-factors appearing from the radiative corrections to the Born production amplitude $A_{2\to 2+n}^{tree}$. The gluon Regge trajectory in LLA is $j = 1 + \omega(t)$, where

$$\omega(t) = -\frac{g^2 N_c}{16\pi^3} \int d^2k \frac{q^2}{k^2(q-k)^2}\ ,\ t = -q^2\ . \tag{5}$$

Infrared divergencies in the Regge factors cancel with analogous divergencies in σ_{tot} from the contribution of real gluons. The production amplitude in the

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tree approximation has the following factorised form

$$A^{tree}_{2\to 2+n} = 2gT^{c_1}_{A'A}\Gamma_1 \tfrac{1}{t_1} gT^{d_1}_{c_2c_1}\Gamma^1_{2,1}\tfrac{1}{t_2}.... \qquad (6)$$

$$...gT^{d_n}_{c_{n+1}c_n}\Gamma^n_{n+1,n}\frac{1}{t_{n+1}} gT^{c_{n+1}}_{B'B}\Gamma_2 \,.$$

Here A, B and A', B', d_r $(r = 1, 2...n)$ are colour indices for initial and final gluons correspondingly. $T^c_{ab} = -if_{abc}$ are generators of the gauge group $SU(N_c)$ and g is the Yang-Mills coupling constant. Futher,

$$\Gamma_1 = \frac{1}{2} e^{\lambda}_{\nu} e^{\lambda' *}_{\nu'} \Gamma^{\nu\nu' +}, \ \Gamma^r_{r+1,r} = -\frac{1}{2}\Gamma_{\mu}(q_{r+1}, q_r) e^{\lambda_r}_{\mu}(k_r) \qquad (7)$$

are the reggeon-particle-particle (RPP) and reggeon-reggeon-particle (RRP) vertices correspondingly. The quantities $\lambda_r = \pm 1$ are the s -channel helicities of gluons in the c.m.system. They are conserved for each of two colliding particles: $\Gamma_1 = \delta_{\lambda'\lambda}$ (note, that it is not valid in the one loop approximation [3]). The tensor $\Gamma^{\nu\nu'+}$ can be written as a sum two terms:

$$\Gamma^{\nu\nu'} = \gamma^{\nu\nu' +} - q^2 (n^+)^{\nu} \frac{1}{p^+_A} (n^+)^{\nu'}, \qquad (8)$$

where we introduced the light cone vectors $n^- = \frac{p_A}{E}, n^+ = \frac{p_B}{E}, E = \sqrt{s}/2, n^+n^- = 2$ and the light cone projections $k^{\pm} = k^{\sigma} n^{\pm}_{\sigma}$ of the Lorentz vectors k^{σ}. The first term is the light cone component of the Yang-Mills vertex:

$$\gamma^{\nu\nu' +} = (p^+_A + p^+_{A'})\delta^{\nu\nu'} - 2p^{\nu'}_A (n^+)^{\nu} - 2p^{\nu}_{A'}(n^+) \qquad (9)$$

Similarly, the effective RRP vertex $\Gamma(q_2, q_1)$ can be presented as [2]

$$\Gamma^{\sigma}(q_2, q_1) = \gamma^{\sigma-+} - 2q_1^2 \frac{(n^-)^{\sigma}}{k_1^-} + 2q_2^2 \frac{(n^+)^{\sigma}}{k_1^+}, \qquad (10)$$

where

$$\gamma^{\sigma+-} = 2q^{\sigma}_2 + 2q^{\sigma}_1 - 2(n^-)^{\sigma} k^+_1 + 2(n^+)^{\sigma} k^-_1 \qquad (11)$$

is the light-cone component of the Yang-Mills vertex. Note, that Γ^{σ} has the important property:

$$(k_1)^{\mu}\Gamma_{\mu}(q_2, q_1) = 0, \qquad (12)$$

which gives us a possibility to chose an arbitrary gauge for each of the produced gluons. In the left (l) light cone gauge where $p_A e^l(k) = 0$ the polarization vector $e^l(k)$ is parametrised in terms of the two-dimensional vector $e^l_{\perp}$

$$e^l = e^l_{\perp} - \frac{k_{\perp} e^l_{\perp}}{kp_A} p_A \qquad (13)$$

and the reggeon-reggeon-particle vertex Γ takes an especially simple form

$$\Gamma^1_{2,1} = Ce^* + C^* e, \ \ C = \frac{q_1^* q_2}{k_1^*}, \qquad (14)$$

if we introduce the complex components $e = e_x + ie_y$, $e^* = e_x - ie_y$ and $k = k_x + ik_y$, $k^* = k_x - ik_y$ for transverse vectors $e_{\perp}, k_{\perp}$. The complex representation was used in [4] to construct an effective scalar field theory for the multi-Regge processes. This theory was derived recently from the Yang-Mills one by integrating over the fields describing the highly virtual particles [5].

The effective action describing multi-Regge processes can be written in the form invariant under the abelian gauge transformations $\delta V^a_{\mu} = i\partial_{\mu}\chi^a$ for the physical fields V_{μ} provided that the fields $A_{\pm}$ corresponding to the reggeized gluons in the crossing channel are gauge invariant ($\delta A_{\pm} = 0$):

$$S_{mR} = \int d^4x \{ \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(\partial_{\perp\sigma} A^a_+)(\partial_{\perp\sigma} A^a_-) + \qquad (15)$$

$$+\frac{1}{2} g \,[-A^a_+ (F_{-\sigma} T^a \, i\partial^{-1}_- F_{-\sigma}) - A^a_- (F_{+\sigma} T^a \, i\partial^{-1}_+ F_{+\sigma}) +$$

$$(\partial^{-1}_- F^a_{-\sigma})(A_- T^a \, i\partial_{\sigma} A_+) + + (\partial^{-1}_+ F^a_{+\sigma})(A_+ T^a \, i\partial_{\sigma} A_-) +$$

$$i(\frac{1}{\partial_+}\frac{1}{\partial_-} F^a_{+-})(\partial_{\sigma} A_+) T^a (\partial_{\sigma} A_-) + iF^a_{+-}(A_- T^a A_+)] \},$$

where $F_{\mu\nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu}$ and $N_{\pm} = N_0 \pm N_3$ are the light-cone components of the vectors N_{μ} .

The total cross-section calculated in LLA using the above expressions for production amplitudes grows very rapidly as s^{ω} for $\omega = (g^2 N_c/\pi^2) \ln 2$ and violates the Froissart bound $\sigma_{tot} < c \ln^2 s$ [2]. One of the possible ways of the unitarization of the LLA results is to use the above effective field theory [4,5].

2. Multi-reggeon compound states

Another more simple (but not so general) method is related to the solution of the BKP equations [6] for the compound states of n reggeized gluons:

$$E\Psi = \sum_{i<k} H_{ik}\Psi \,, \qquad (16)$$

where the energy E is proportional to the position $\omega = j - 1$ of the singularity of the t channel partial wave:

$$E = -\frac{16\pi^2}{g^2 N_c}\omega \,, \qquad (17)$$

and the pair Hamiltonian H_{ik} has the remarkable property of the holomorthic separability [7]:

$$H_{ik} = -\frac{2T^a_i T^a_k}{N_c}(h_{ik} + h^*_{ik}). \qquad (18)$$

where T^a_i are the gauge group generators acting on colour indices of the gluon i. The holomorphic pair hamiltonian is [8]

$$h_{ik} = \frac{1}{p_i}\ln(\rho_{ik}) p_i + \frac{1}{p_k}\ln(\rho_{ik}) p_k + \ln(p_i p_k) + 2\gamma. \ (19)$$

Here $\gamma = -\psi(1)$ is the Euler constant and we introduced the complex coordinates $\rho_k = x_k + iy_k$ for impact parameters $\vec{\rho}$ of n gluons and their momenta $p_k = i\frac{\partial}{\partial(\rho_k)}$ (note,that $\rho_{ik} = \rho_i - \rho_k$). It is invariant [7] under the Möbius transformations:

$$\rho'_k = \frac{a\,\rho_k + b}{c\,\rho_k + d} \tag{20}$$

for any complex values of a, b, c, d. In the multi-colour QCD ($N_c \to \infty$) the total hamiltonian H can be written as a sum of the mutually commuting holomorphic and anti-holomorphic operators [8]:

$$H = h + h^*, [h, h^*] = 0, \tag{21}$$

where h contains only the interaction of the neighbouring gluons

$$h = \sum_{i=1}^{n} h_{i,i+1}, . \tag{22}$$

Therefore in this case the solution of the Schrödinger equation has the property of the holomorphic factorization:

$$\Psi = \sum_k c_k \psi_k \psi_k^* \tag{23}$$

where ψ and $\psi*$ are correspondingly analitic and anti-analitic functions of their arguments and the sum is performed over all degenerate solutions of the Schrödinger equations in the holomorthic and anti-holomorthic subspaces:

$$\epsilon\psi = h\psi, \epsilon^*\psi^* = h^*\psi^*, E = \epsilon + \epsilon^*. \tag{24}$$

These equations have nontrivial integrals of motion depending on the spectral parameter θ [9]:

$$t(\theta) = tr\,T(\theta)\,,\ [t(u), t(v)] = [t(\theta), h] = 0, \tag{25}$$

The transfer matrix $t(\theta)$ is expessed as a trace of the monodromy matrix $T(\theta)$ constructed from the product

$$T(\theta) = L_1(\theta)L_2(\theta)...L_n(\theta) \tag{26}$$

of the $L-$operators:

$$L_k(\theta) = \begin{pmatrix} \theta + i\rho_k\partial_k & i\partial_k \\ -i\rho_k^2\partial_k & \theta - i\rho_k\partial_k \end{pmatrix}. \tag{27}$$

Thus, the solution of the Schrödinger equation is reduced to a pure algebraic problem of the finding the representation of the Yang-Baxter commutation relations:

$$T_{i_1 i'_1}(u)\, T_{i_2 i'_2}(v)\ (v - u + iP_{12}) = \tag{28}$$

$$= (v - u + iP_{12})\, T_{i_2 i'_2}(v)\, T_{i_1 i'_1}(u)\ ,$$

where the operator P_{12} in the left and right hand sides of the equation interchanges correspondingly the right and left indices of the matrices $T(u)$ and $T(v)$. Moreover [10], the holomorphic Hamiltonian for the Schrödinger equation coincides with the Hamiltonian for a completely integrable Heisenberg model with the spins belonging to an infinite dimensional representation of the noncompact Möbius group and its eigen values and eigen functions can be expressed in terms of the Baxter function $Q(\lambda)$ satisfying the equation:

$$t(\lambda)Q(\lambda) = (\lambda + i)^n Q(\lambda + i) + (\lambda - i)^n Q(\lambda - i), \tag{29}$$

where $t(\lambda)$ is an eigen value of the transfer matrix. The solution of this equation is known for $n = 2$. In a general case $n > 2$ one can present it as a linear combination of the solutions for $n = 2$, obtaining a reccurence relation for the coefficients. In particular for $n = 3$ this relation takes the form:

$$Ad_k(A) = \tag{30}$$

$$= \frac{k(k+1)}{2(2k+1)}(k - m + 1)(k + m)(d_{k+1}(A) + d_{k-1}(A))$$

with the initial conditions $d_0 = 0, d_1 = 1$. For integer values of the conformal weight m the quantization condition for eigen values A is $d_{m-1}(A) = 0$. Although the orthogonality relation

$$\sum_A \frac{d_k(A)\, d'_k(A)}{d_{m-2}(A)\, d'_{m-1}(A)} = \tag{31}$$

$$= \delta_{k,k'} \frac{k(k+1)(k-m+1)(k+m)}{2(2k+1)},$$

and the completeness condition

$$\sum_{k=1}^{m-1} \frac{2(2k+1)\, d_k(A)\, d_k(A')}{k(k+1)(k-m+1)(k+m)} = \tag{32}$$

$$= \delta_{A,A'}\, d_{m-2}(A)\, d'_{m-1}(A)$$

for the polynomials $d_k(A)$ are known, their theory is not constructed yet, which does not allow us to calculate analitically the Odderon intercept.

All these results are based on calculations of effective Reggeon vertices and the gluon Regge trajectory in the first nontrivial orders of perturbation theory. Up to now we do not know the region of applicability of LLA including the intervals of energies and momentum transfers fixing the scale for the QCD coupling constant.

Therefore it is needed to generalise the effective field theory of ref. [4] to processes for which the final state particles are separated in several groups consisting of an arbitrary number of gluons with a fixed invariant mass; each group is produced with respect to others in the multi-Regge kinematics. These conditions correspond to the quasi-multi-Regge kinematics of Ref. [11] where only one additional group consisting of two gluons was considered.

3. Quasi-elastic processes

Let us begin with the quasi-elastic process in which the final state contains apart from the particle B' with momentum $p_{B'} \simeq p_B$ also several gluons with a fixed invariant mass in the fragmentation region of the initial gluon A. It is convenient to denote the colour indices of the produced gluons by $a_1, a_2, ...a_n$ leaving the index a_0 for the particle A. Further, the momenta of the produced gluons and of the particle A are denoted by $k_1, k_2, ...k_n$, and $-k_0$ correspondingly. $q = -\sum_{i=0}^{n} k_i$ is the momentum transfer. Omitting the polarization vectors $e_{\nu_i}(k_i)$ for the gluons $i = 0, 1, ...n$ we can write the production amplitude related with the single gluon exchange in the tensor representation

$$A^{\nu_0\nu_1...\nu_n}_{a_0a_1...a_nB'B} = -\phi^{\nu_0\nu_1...\nu_n+}_{a_0a_1...a_nc} \frac{1}{t} g\, p_B^- T^c_{B'B} \delta_{\lambda_{B'},\lambda_B} . \tag{33}$$

Here the form-factor ϕ depends on the invariants constructed from the momenta $k_0, ..k_n$.

For the simplest case of the production of one additional gluon ϕ was calculated in the Born approximation in [11]. We present this result in the form:

$$\phi^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} = g^2\{\Gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} - \tag{34}$$
$$-T^a_{a_1a_0}T^c_{a_2a}\frac{\gamma^{\nu_1\nu_0\sigma}(k_1,-k_0)\Gamma^{\nu_2\sigma+}(k_2,k_2+q)}{(k_0+k_1)^2} -$$
$$-T^a_{a_2a_0}T^c_{a_1a}\frac{\gamma^{\nu_2\nu_0\sigma}(k_2,-k_0)\Gamma^{\nu_1\sigma+}(k_1,k_1+q)}{(k_0+k_2)^2} -$$
$$-T^a_{a_2a_1}T^c_{a_0a}\frac{\gamma^{\nu_2\nu_1\sigma}(k_2,-k_1)\Gamma^{\nu_0\sigma+}(k_0,k_0+q)}{(k_1+k_2)^2}\}.$$

The last three terms in the brackets correspond to the Feynman diagram contributions constructed from the gluon propagator combining the usual Yang-Mills vertex γ and the effective RPP vertex Γ. The first term can be written as

$$\Gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} = \gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} + \Delta^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} , \tag{35}$$

where γ is the light-cone projection of the usual quadrilinear Yang-Mills vertex

$$\gamma^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c} = T^a_{a_1a_0}T^c_{a_2a}(\delta^{\nu_1\nu_2}\delta^{\nu_0+} - \delta^{\nu_1+}\delta^{\nu_0\nu_2}) + \tag{36}$$
$$+T^a_{a_2a_0}T^c_{a_1a}(\delta^{\nu_2\nu_1}\delta^{\nu_0+} - \delta^{\nu_2+}\delta^{\nu_0\nu_1}) +$$
$$+T^a_{a_2a_1}T^c_{a_0a}(\delta^{\nu_2\nu_0}\delta^{\nu_1+} - \delta^{\nu_2+}\delta^{\nu_1\nu_0})$$

and Δ is a new induced vertex

$$\Delta^{\nu_0\nu_1\nu_2+}_{a_0a_1a_2c}(k_0^+, k_1^+, k_2^+) = \tag{37}$$
$$= -t(n^+)^{\nu_0}(n^+)^{\nu_1}(n^+)^{\nu_2}\{\frac{T^a_{a_2a_0}T^c_{a_1a}}{k_1^+\ k_2^+} + \frac{T^a_{a_2a_1}T^c_{a_0a}}{k_0^+\ k_2^+}\}.$$

In the general case of the production of n additional gluons in the fragmentation region of the initial particle A for the gauge invariance of the production amplitude one should introduce an infinite number of the effective vertices Δ for the gluon-reggeon interactions.

4. Effective action for high energy processes in QCD

It turns out [12], that one can construct the effective action, reproducing the vertices for the diffractive processes:

$$S_{diffr} = -\int d^4x\ tr[\frac{1}{2}G^2_{\mu\nu} + j_-(V)A_+ + j_+(V)A_-], \tag{38}$$

where $G_{\mu\nu} = \frac{1}{g}[D_\mu, D_\nu]$ and $D_\mu = \partial_\mu + gV_\mu$ is the covariant derivative for the Yang-Mills field $V = t_aV^a$, ($[t_a, t_b] = f_{abc}t_c$) which describes the real gluons. The reggeons are described by the fields $A_\pm = t_aA^a_\pm$. The currents $j_\pm$ are given below:

$$j_\pm(V) = j^{mYM}_\pm(V) + j^{ind}_\pm(V) \tag{39}$$

where the modified Yang-Mills current j^{mYM} and the induced current j^{ind} equal correspondingly:

$$j^{mYM}_\pm = U^{-1}(V_\pm)\, j^{YM}_\pm\, U(V_\pm)\ ,\ j^{ind}_\pm = -\partial^2_\perp\partial_\pm U(V_\pm). \tag{40}$$

Here

$$j^{YM}_\pm = -[D_\mu, G_{\mu\pm}] \tag{41}$$

and

$$U(V_\pm) = P\ \exp(-\frac{g}{2}\int_{-\infty}^{x^\pm} dx'^{\pm}V_\pm) = \frac{1}{1+g\partial_\pm^{-1}V_\pm} \tag{42}$$

is the path-ordered Wilson exponent. According to equations of motion for the field V we have $j^{YM} = 0$ and therefore $j_\pm = j^{ind}_\pm$. It means, that one can express the effective action for double quasi-elastic processes after integrating over $A_\pm$ in terms of the action for a two-dimensional σ-model:

$$S_{double} = S^{YM}(V) - \frac{2}{g^2}\int d^2x_\perp\ tr\ (\partial_{\perp\sigma}T_-)(\partial_{\perp\sigma}T_+), \tag{43}$$

where

$$T\pm = P\ \exp(-\frac{g}{2}\int_{-\infty}^{\infty} dx^\pm V_\pm). \tag{44}$$

This σ -model was derived earlier by E.Verlinde and H.Verlinde [13] using other arguments.

For the general case of the reggeon-gluon interaction local in the rapidity interval $(y-\eta, y+\eta)$ where $y = \frac{1}{2}\ln\frac{k^+}{k^-}$ the effective action has the form [12]:

$$S_{eff} = S^{YM}(v) - \int d^4x\ tr\ L, \tag{45}$$

$$L = (A_+^{reg}(v) - A_+)\partial^2_{\perp\sigma} A_- + (A_-^{reg}(v) - A_-)\partial^2_{\perp\sigma} A_+$$

where

$$A_\pm^{reg}(v) = -\frac{\partial_\pm}{g} U(v_\pm) = v_\pm - g\, v_\pm \frac{1}{\partial_\pm} v_\pm + ... \quad (46)$$

is the composite reggeon operator and $A_\pm$ is the reggeon operator, satisfying the kinematical restriction: $\partial_\pm A_\mp = 0$ which is important for the gauge invariance of S_{eff}. After calculating the functional integral over v one can obtain the pure reggeon effective action ,describing all possible processes of production and annihilation of reggeons in the t-channel. Because $A_\pm^{reg}$ contains the terms linear in $v_\pm$ there is a non-trivial solution of the Euler-Lagrange equations for S_{eff}. This solution can be constructed as a series in $A_\pm$. By calculating the quantum fluctuations around it one can find the gluon Regge trajectory and multi-reggeon vertices in upper orders of the perturbation theory. The subsequent integration over $A_\pm$ corresponds to the solution of the reggeon field theory acting in the two-dimensional impact parameter subspace with the time coinciding with the rapidity. It is important, that the t-channel dynamics of the reggeon theory is in the agreement with the s-channel unitarity for the initial Yang-Mills model. In the Hamiltonian formulation of this reggeon calculus the wave function will contain the components with an arbitrary number of reggeized gluons. Nevertheless, one can hope that at least some of the remarkable properties of the BFKL equation would remain in the general case of the non-conserving number of reggeized gluons.

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