New perspective in rotation-vibration interaction

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The study of *shape phase transition* of atomic nuclei has been a subject of great interest. According to the Bohr-Mottelson unified collective model [1], the energy spectrum of the ground band of well deformed even Z-even N nuclei are given by the rotation formula:

$$E=AI(I+1). \quad (1)$$

The coefficient A incorporates the inverse moment of inertia. A host of well deformed nuclei follow this pattern well. If the nucleus is spherical, it executes harmonic vibrations and the level energies are given by the expression:

$$E=aI. \quad (2)$$

In reality, most nuclei lie in between these two limiting symmetries and are called shape transitional, and deviate from either of the above two limiting expressions. The yrast level energies in these nuclei can be described through the composite expression:

$$E(I)=a\,I(I+1) + b\,I. \quad (3)$$

The two terms represent the rotational and the vibrational parts. The energy of any yrast state of the nucleus is a mixture of the two. The ratio of the first part (ROTE) to the total state energy $E(2_1)$ is a measure of the degree of deformation of the nucleus.

A corresponding relation is used in the form of a two parameter Hamiltonian in the Interacting Boson Approximation Model; IBA-1 [2]:

$$H_{IBM} = \varepsilon\, n_d + k\, Q \cdot Q. \quad (4)$$

Here the first term represents the boson energy and corresponds to the vibration term in (3). The second term corresponds to the quadrupole interaction of the bosons and corresponds to the rotation term in (3). In the language of quantum mechanics, each term represents a Hamiltonian and is diagonal in its own basis.

$$H= a\, H_1 + b\, H_2. \quad (5)$$

Since the total Hamiltonian is the sum of the two, any of its eigenstate can be expanded in the basis of either. For example, IBA model employs the U(5) basis. In principle one can also use the SU(3) as a basis, as attempted by Rosensteel [3].

In experiment, in all nuclei, spherical vibrators, deformed rotors and the shape transitional, the level energy ratios $R_{I/2}$ in the ground state band are related to the ratio $R_{4/2}$, as noted very early by Mallmann [4]. From Eq. (3), this relation is determined in the form of Eq. (6)

$$R_{I/2} = R_{4/2}\, I(I-2)/8 – I(I-4)/4. \quad (6)$$

This relation can also be derived from the Mallmann plot of $R_{I/2}$ versus $R_{4/2}$ (see Fig. 1). The ratio of the two sides of the similar triangles in the figure yields Eq. 6. Thus the linear relation (6) is model independent. We shall illustrate its application to a few special cases.

For $R_{4/2}$=2.5, Eq. (6) yields the ratio $R_{6/2}$=9/2. The O(6) symmetry corresponds to the γ-unstable soft rotor, with τ as the O(5) quantum number. The energies in the yrast band ($I=2\tau$) are given by [2]:

$$E_\tau=B\, \tau(\tau+3) + CI(I+1). \quad (7)$$

B and C are constant for a given nucleus.

Eq. (7) yields the relation of $R_{I/2}$ with $R_{4/2}$:

$$R_\tau=R_{I/2}=R_{4/2}\tau(\tau-1)/2 - \tau(\tau-2). \quad (8)$$

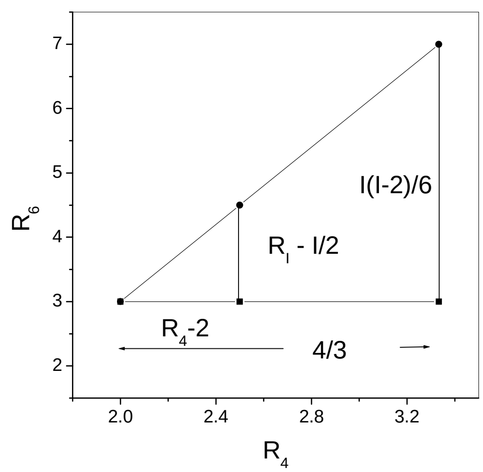

Fig. 1. The linear relation of $R_{I/2}$ to $R_{4/2}$.

For the energy ratio $R_{4/2}$=2.5, the typical value for O(6) symmetry, Eq. (8) yields $R_{6/2}$=9/2, the same as given by Eq. (6).

Next look at the recently identified E(5) critical symmetry [5] point, which lies on the U(5)-O(6) transition class. Here the energy ratio $R_{4/2}$ is 2.2 and the energy ratio $R_{6/2}$ is 3.6. This is also given by the relation (6)! Next we consider the critical symmetry point X(5) [6], which lies on the U(5) to SU(3) transition class and occurs at the edge of the deformed region. From the analytical solution, here the energy ratio $R_{4/2}$ is predicted to be 2.9 and the energy ratio $R_{6/2}$ is predicted to be 5.45 [6], which again is given approximately (within 95%) by Eq. (6).

For I=8, 10 also, the results similar to the above are obtained.

Thus the **important observation** in the present study is that the relation (6), which we shall call a *linear relation*, holds good universally. It incorporates the results of all the five symmetries:- the vibrational, rotational, γ-unstable soft rotor, the II-order phase transition critical point symmetry point E(5) and approximately for the I-order phase transition critical symmetry point X(5) predictions of the ground band energy ratios. In fact the same relation serves as reference to all regular bands, ground state band and higher excited bands in even Z-even N nuclei. The same also holds true for the regular bands of odd-A nuclei, or odd-odd nuclei.

A word of caution is necessary here. In experiment, the dependence of $R_{I/2}$ on $R_{4/2}$ may differ slightly from the predicted ratio in Eq. (6), as happens for any of the above analytical solutions also. In fact the deviations carry their own message; viz. the rotation–vibration interaction introduces the spin dependence. For example, in softly deformed nuclei, in the context of the microscopic treatment in the DPPQ model [7], some spin dependence of the K-admixture of the given state is predicted [8].

The linear relation (6) discussed here is an extension of the Mallmann's observation [4] for the energy relation in the ground band.

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