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Theory status for $K \to \pi\nu\bar{\nu}$ modes

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Abstract. Motivated by the future prospects of the NA62 and KOTO experiments, which aim to measure the rare kaon decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, respectively, we discuss the present uncertainties of the Standard Model theory prediction. We also examine which calculations could further improve the theoretical accuracy of the charged and neutral decay modes.

1. Introduction

Flavour Changing Neutral Currents are very attractive channels to look for physics beyond the Standard Model. Of particular interest are the golden $K \to \pi\nu\bar{\nu}$ decay modes. Within the framework of the standard model (SM), these observables can be treated at leading order from W-box and Z-penguin Feynman diagrams, and are strongly suppressed both by the Glashow-Iliopoulos-Maiani (GIM) mechanism, and by hierarchy of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Consequently, their branching ratios are dominated by short-distance contributions, making the rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ theoretically clean and very sensitive to possible new flavour dynamics.

The determination of these decay modes is an enormous challenge due to the presence of neutrinos in the final state. The most accurate measurement of the charged mode was obtained by the E787 and E949 experiments at the Brookhaven National Laboratory (BNL) [1]

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = \left(17.3^{+11.5}_{-10.5}\right) \times 10^{-11}. \tag{1}$$

In the near future, the NA62 experiment at CERN will determine this branching ratio with a 10% precision. This experiment is currently taking data, and recently the NA62 collaboration presented its first result in Moriond 2018 [2]

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = \left(28^{+44}_{-28}\right) \times 10^{-11}, \tag{2}$$

which is based on the observation of one event. The experimental uncertainties are still quite large, but they are expected to be significantly reduced within the next few years. On the other hand, the experimental determination of the neutral mode is even more challenging, since the only observable particles are two photons from the π^0 decay. In addition, the initial momentum

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and its decay vertex are unknown, making the signal selection very difficult. The experimental upper limit on the branching ratio was determined by the KEK E391a experiment to be [3]

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu})_{\rm exp} \leq 2.6 \times 10^{-8}. \tag{3}$$

In the future, the KOTO experiment at the J-PARC laboratory in Japan aims to achieve a sensitivity close to the branching ratio predicted by the SM, and provide the first measurement of this neutral channel.

In view of the future prospect of these important experiments, which aim to measure the branching ratios of the two golden modes $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ with unprecedented accuracy, we discuss here the status of the SM theory prediction, and motivate further improvements for the theoretical determination of the charged and neutral decay modes. This document is organised as follows: section 2 describes the theoretical framework and status of the short-distance contribution. An error analysis for the different contributions in the theory prediction of the corresponding branching ratios is provided in section 3, motivating the computation of higher order QCD calculations for the top-quark contribution. Finally, section 4 summarises the main points of this document.

2. Structure of $K \to \pi\nu\bar{\nu}$ at NLO

Theoretically, the $K \to \pi\nu\bar{\nu}$ transitions can be described by an effective theory, since the external momenta are of the order m_K, and are therefore much smaller than the mass of the W and Z gauge bosons and the top quark. Below the charm scale, $\mu < m_c$, the corresponding effective Hamiltonian is given by

$$\mathrm{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \frac{\alpha_e}{2\pi \sin^2\theta_W} \sum_{l=e,\mu,\tau} \Big[\lambda_c X^l(x_c) + \lambda_t X(x_t)\Big] (\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_{lL}\gamma^\mu \nu_{lL}) + \text{h.c.}, \tag{4}$$

where G_F is the Fermi constant, α_e is the electromagnetic coupling, and θ_W is the weak mixing angle. The parameters λ_q, with $q = c, t$, are defined in terms of the CKM matrix elements as $\lambda_q = V^*_{qs}V_{qd}$, and the subscript L denotes left-handed fermion fields. Moreover, the functions $X^l(x_c)$ and $X(x_t)$, with $x_q = m_q^2(\mu_q)/M_W^2$ and $m_q^2(\mu_q)$ the quark $\overline{\rm MS}$-mass, correspond to the matching contributions of internal charm and top quarks, respectively, to the operator of equation (4).

The charm-quark contribution is only relevant for the CP conserving mode $K^+ \to \pi^+\nu\bar{\nu}$, and this function has been determined up to next-to-leading order (NLO) electroweak corrections [4], and next-to-next-to-leading order (NNLO) QCD corrections [5]. For a proper determination, a complete renormalisation group analysis has to be performed to sum large logarithms $\ln \mu_c^2/\mu_W^2$, and the threshold corrections to the Wilson coefficients originating at $\mu_b = \mathcal{O}(m_b)$ also have to be included. For phenomenological reasons the following function is defined

$$P_c(x_c) = \frac{1}{\lambda^4}\left[\frac{2}{3}X_c^e(x_c) + \frac{1}{3}X_c^\tau(x_c)\right], \tag{5}$$

with $\lambda = |V_{us}| = 0.2248$ is the Cabibbo angle. As claimed in reference [5], the inclusion of the NNLO corrections essentially removes the entire sensitivity of P_c to the non-physical scale μ_c, and on higher order terms in α_s that affect the evaluation of $\alpha_s(\mu_c)$ from $\alpha_s(M_Z)$. The theoretical prediction for the charm-quark short-distance function is

$$P_c(x_c) = 0.372 \pm 0.015 \tag{6}$$

with the residual error of P_c completely dominated by the parametric uncertainty from $m_c(m_c)$. Hence a better determination of $m_c(m_c)$ is an important theoretical goal in connection with

$K^+ \to \pi^+\nu\bar{\nu}$. In contrast, the $X(x_t)$ function can be calculated within fixed-order perturbation theory, since only one energy scale is involved and the anomalous dimension matrix vanishes. This top-quark contribution has been determined with NLO QCD corrections [6, 7] and two-loop electroweak corrections [8], for which a numerical update gives

$$X(x_t) = 1.469 \pm 0.022. \tag{7}$$

The inclusion of $\mathcal{O}(\alpha_s)$ corrections reduced the leading-order uncertainty due to the top quark matching scale $\mu_t = \mathcal{O}(m_t)$ from 9% down to 2%. Here the uncertainty is dominated by the experimental error in the top-quark mass.

3. Theory prediction and error budget

Within the SM the branching ratios of the $K \to \pi\nu\bar{\nu}$ decay modes are proportional to the top-quark contribution $X(x_t)$, which accounts for almost 100% and 63% of the total rates, respectively. The corresponding expressions for the branching ratios can be written in the following compact forms:

$$\begin{aligned}
\mathrm{Br}\left(K^0 \to \pi^0\nu\bar{\nu}\right) &= \kappa_L \left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X(x_t)\right)^2 , \\
\mathrm{Br}\left(K^+ \to \pi^+\nu\bar{\nu}\right) &= \kappa_+ (1+\Delta_{\mathrm{EM}}) \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X(x_t)\right)^2 \right. \\
&\quad \left. + \left(\frac{\mathrm{Re}\lambda_c}{\lambda}(P_c(X) + \delta P_{c,u}) + \frac{\mathrm{Re}\lambda_t}{\lambda^5} X(x_t)\right)^2\right] ,
\end{aligned} \tag{8}$$

where the accurately determined terms κ_L and κ_+ include the hadronic matrix elements, which can be directly extracted from the well measured semileptonic decay $K^+ \to \pi^0 e^+ \nu$. A detailed analysis of these contributions within the framework of Chiral Perturbation Theory (ChPT) has been performed in reference [9], and the corresponding numerical values are

$$\kappa_L = (2.229 \pm 0.017) \times 10^{-10} \left[\frac{\lambda}{0.225}\right]^8 \tag{9}$$

$$\kappa_+ = (0.5173 \pm 0.0025) \times 10^{-10} \left[\frac{\lambda}{0.225}\right]^8 . \tag{10}$$

In the second equation of (8) the factor $\Delta_{\mathrm{EM}} = -0.003$ attempts to include long distance QED corrections. Moreover $\delta P_{c,u}$ parametrises the dimension-eight four-fermion operators generated at the charm scale, and the long distance contributions from the up quark loops. All of these effects can be described in ChPT and have been calculated in reference [10] to be

$$\delta P_{c,u} = 0.04 \pm 0.02. \tag{11}$$

In the future, Lattice QCD will be able to determine these effects, reducing the residual error of δP_c. Such prospects have been considered in references [11] and [12].

In the following we present an error analysis and the theory predictions for the $K \to \pi\nu\bar{\nu}$ decay modes. For this purpose we use the Wolfenstein parametrisation to express the CKM matrix elements as a function of four fundamental parameters (λ, A, $\bar{\rho}$, $\bar{\eta}$), such that the real and imaginary parts of λ_q are

$$\mathrm{Re}\lambda_c = -\lambda\left(1 - \frac{\lambda^2}{2}\right), \quad \mathrm{Re}\lambda_t = -\left(1 - \frac{\lambda^2}{2}\right) A^2\lambda^5(1-\bar{\rho}), \quad \mathrm{Im}\lambda_t = \bar{\eta}A^2 \frac{\lambda^5}{\left(1 - \frac{\lambda^2}{2}\right)}, \tag{12}$$

For this evaluation we only consider the results from the last global CKM analysis obtained by means of the fit package CKMfitter [13]. With this, the SM theory prediction for the branching ratios of the $K \to \pi\nu\bar{\nu}$ decays is

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = \left(8.345^{+0.222}_{-0.323} \pm 0.357\right) \times 10^{-11}, \tag{13}$$

and

$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})_{\mathrm{SM}} = \left(2.874^{+0.145}_{-0.181} \pm 0.087\right) \times 10^{-11}, \tag{14}$$

where the first and second errors stem from the parametric and the theory prediction uncertainties, respectively. The former are dominated by the uncertainty in the determination of the CKM parameters V_{ub}, V_{cb} and γ. On the other hand, the different contributions to the theory uncertainty for the charged mode are $(\kappa_+ : 0.5\%, X(x_t) : 2.1\%, P_c : 2.2\%, \delta P_{c,u} : 2.9\%)$, and for the neutral mode we have $(\kappa_L : 0.6\%, X(x_t) : 3.0\%)$. The value of the corresponding second error has been obtained by adding the different uncertainties in quadrature since the corresponding contributions have a different origin and thus they are uncorrelated.

In summary, the theory uncertainties are at the level of 4% and 3% for the branching ratios of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, respectively, and for the neutral mode the error is dominated by the uncertainty in the $X(x_t)$ function. New physics could modify the latter function, and this can be parametrised in a model-independent way by replacing the SM top-quark function $X(x_t)$ by a general complex function: $X_{\mathrm{NP}}(x_t) = |X_{\mathrm{NP}}(x_t)|e^{i\theta_{\mathrm{NP}}}$. We compute the NNLO QCD corrections for the top-quark contributions with the aim of reducing this theory uncertainty. At this order of precision the three-loop SM theory is matched onto an effective theory where the heavy top-quark, W-boson and Z-boson are integrated out as dynamical degrees of freedom. More details of this particular computation and the corresponding NNLO QCD results will be published in a future article [14].

	value range	reference
M_W	(80.379 ± 0.012)GeV	[15]
M_Z	(91.1876 ± 0.0021)GeV	[15]
m_t	(172.6 ± 1.4)GeV	[16]
$\alpha_s(M_Z)$	0.1187 ± 0.0016	[15]
$\alpha_e(M_Z)$	$1/127.9$	[15]
$\sin^2\theta_W^{\overline{\mathrm{MS}}}$	0.23122 ± 0.00015	[15]
G_F	$1.16637 \times 10^{-5}\mathrm{GeV}^{-2}$	[15]
λ	$0.22509^{+0.00028}_{-0.00029}$	[13]
A	$0.825^{+0.0071}_{-0.0111}$	[13]
$\bar{\rho}$	$0.124^{+0.019}_{-0.018}$	[13]
$\bar{\eta}$	$0.3499^{+0.0063}_{-0.0061}$	[13]

Table 1. Input parameters used in the numerical analysis. They are grouped into: input for perturbative calculation, parametric input.

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4. Conclusions

The rare kaon decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ are golden modes for probing new physics. In this proceedings we have discussed the status of the theory prediction, motivating our future calculations. Although theory and experimental data are compatible within error, the variation in the central values opens interesting prospects. The future results of NA62 and KOTO experiments, and a better determination of the CKM matrix elements will shed some light in the indirect search of physics beyond the SM.

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