Estimate of the Energy Spread in a Booster Cycle via the Bunch Length Measurement

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Abstract

It is important for us to obtain the information of energy spread *vs*. time in a Booster cycle via the same beam event, for the purpose of minimizing the error caused by the cycle-to-cycle fluctuation. The bunch length (BL) can be extracted from the resistive wall signal (RWS) at different times of a Booster cycle, and from which the energy spread can be estimated.

Introduction

Since the equilibrium energy spread in the proton circular accelerator of Booster is generally reached, the BL is determined by the energy spread, as shown in eq.1.[1]

$$\sigma_l = \frac{c \cdot |\eta|}{2\pi f_s} \cdot \frac{\sigma_E}{E_0}, \text{ or}$$
$$\sigma_t = \frac{|\eta|}{\beta \cdot 2\pi f_s} \cdot \frac{\sigma_E}{E_0}. \qquad (1)$$

Here, the bunch length can be expressed either in unit of length (σ_l) or in unit of time (σ_t). $\frac{\sigma_E}{E_0}$ is the energy spread. c is the speed of light. f_s, η, and β are the synchronous oscillation frequency, the phase slip factor, and the Lorentz's relativistic factor respectively. They can be calculated precisely for an entire Booster cycle, and also there is a good agreement between the calculated and the measured synchronous frequency.[2] Since f_s, η, and β are known from the calculation, the energy spread of the circulating bunch can be estimated from the BL measurement using eq.1.

Experimental Result and Analysis

The RWS was recorded by a YOKOGAWA DL 7200 digital oscilloscope with a sampling rate of 1 GHz at the extracted beam intensity of 4.0×10^{12} protons. The scope was triggered once every 600 Booster turns, and the 1st trigger was set right before the injection. After each trigger, a 1-μs RWS were recorded. . The total number of triggers, which were used for taking the data, was 30. The full width half maximum (FWHM) of the circulating bunch was extracted via the 8th order polynomial fit since the RWS of a single bunch is different from the Gaussian distribution. As an example, the RWS from the 1st bunch at the 2nd trigger of turn number (TN) 600 is shown as the black curve in Fig. 1, and the result of the 8th order polynomial fit is shown as the red curve. The FWHM of the bunch is obtained from the time difference between the two points in the red curve with the half-peak value. The FWHM *vs.* the TN is shown in Fig. 2(a). Besides, the BL (σ_t), which is the product of the FWHM and the factor of 2.3548^{-1}, *vs*. the TN is shown in Fig. 2(b).

Since in a Booster cycle, the proton beam is accelerated from 400 MeV to 8 GeV at a period of 33.3 ms while the RF frequency sweeps from 37.8 MHz to 52.9 MHz, either the time or the RF frequency at a particular TN should be known for the purpose of calculating the corresponding Lorentz relativistic factor (β), phase slip factor (η), and synchronous frequency (f_s). It is easier for us to obtain the relationship between the RF frequency and the TN by measuring the time (T), which is taken from the 1st peak to the N^{th} peak from the RWS at a particular TN. As an example, at the 2nd trigger of TN 600, N (=31) peaks covers the time of T (=780 ns), as shown in Fig. 3(a). Eq.2 was used to calculate the RF frequency (f_{rf}) in unit of MHz, and the result is 38.46 MHz.

$$\mathrm{f_{rf}} = \frac{N-1}{\left(T/1000\right)}. \qquad (2)$$

The f_{rf} *vs*. the TN is shown in Fig. 3(b). The calculated β and η are shown in Fig. 3(c) as the black curve and the blue curve respectively. And also, the calculated synchronous frequency is shown in Fig. 3(d). Finally, the energy spread is obtained using eq.1, and the result is shown in Fig. 3(e).

Couclusion

The energy spread can be estimated from the BL measurement reasonably well before the transition crossing. However, the BL oscillation is frequently developed after the transition crossing and contributes to the energy-spread oscillation via the linear relationship between the BL and the energy spread. Some work need to be done in the future for the purpose of smoothing out the BL oscillation and extracting reasonable energy spread after the transition crossing.

References:

[1] A. W. Chao and M. Tigner, *Handbook of Accelerator Physics and Engineering*, (1999), 2nd printing, world scientific.

[2] X. Yang, J. MacLachlan, R. Padilla, and C. Ankenbrandt, "Booster Synchrotron Frequency Below Transition", FERMILAB-TM-2239.

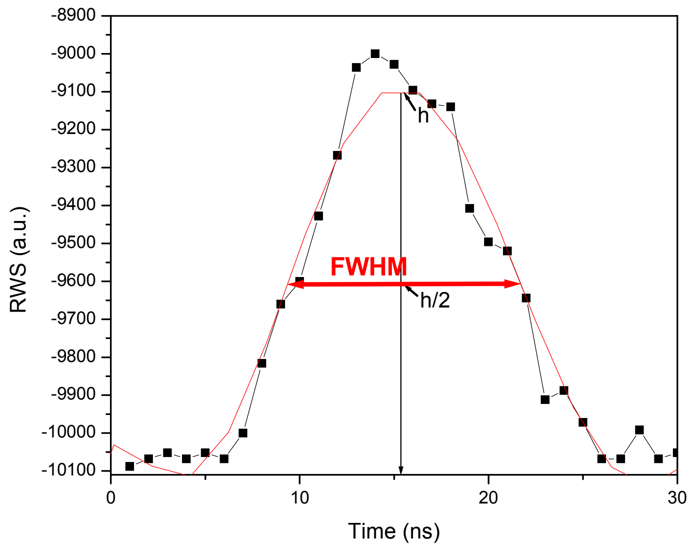


Fig. 1

Fig. 1 at the 2nd trigger of TN 600, the RWS of the 1st bunch is shown as the black curve, and its 8th order polynomial fit result is shown as the red curve.

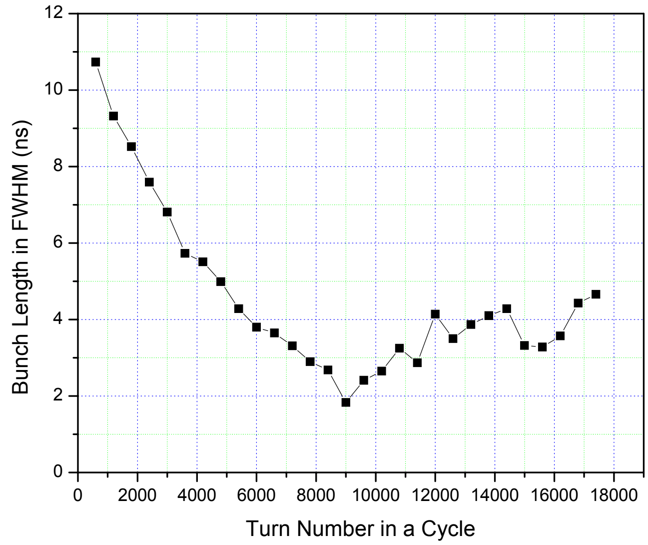


Fig. 2(a)

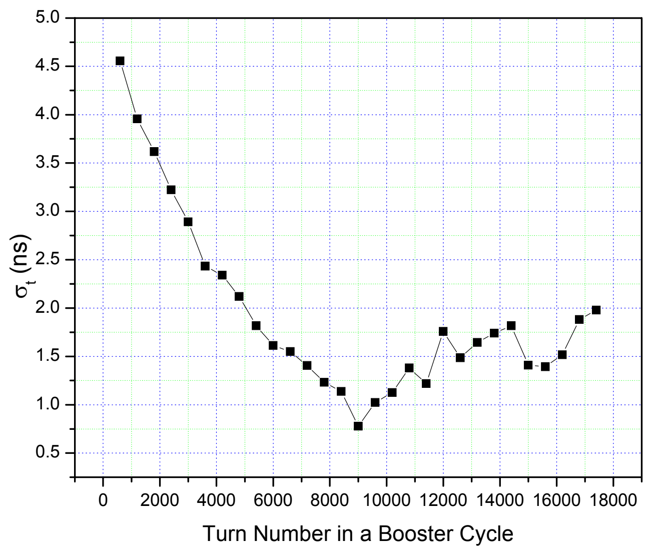


Fig. 2(b)

Fig. 2(a) the FWHM *vs.* the TN.

Fig. 2(b) the BL σ_t *vs.* the TN.

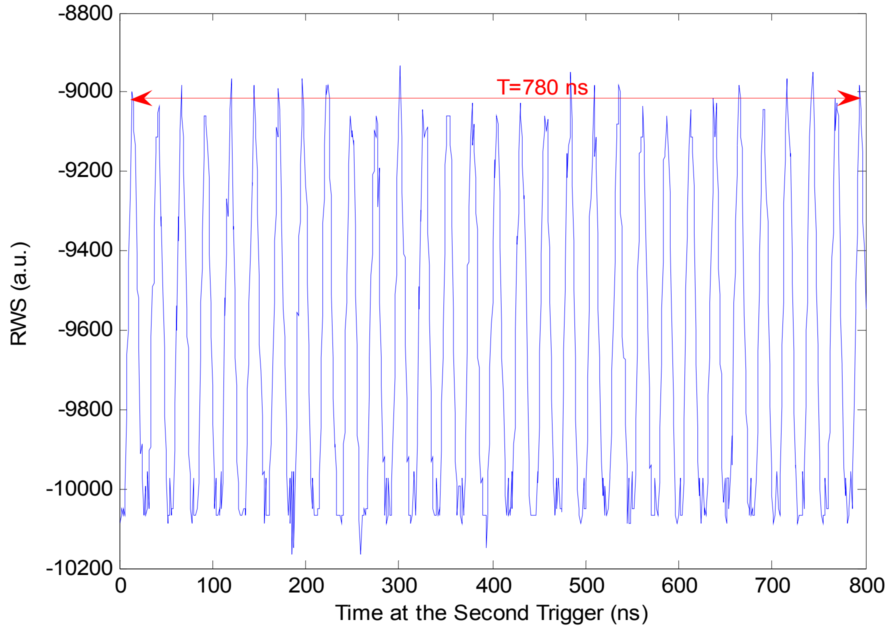

T=780 ns
RWS (a.u.)
Time at the Second Trigger (ns)

Fig. 3(a)

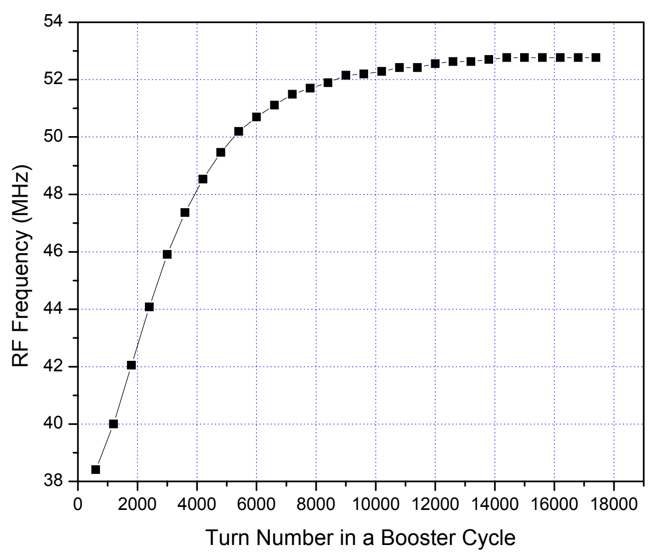
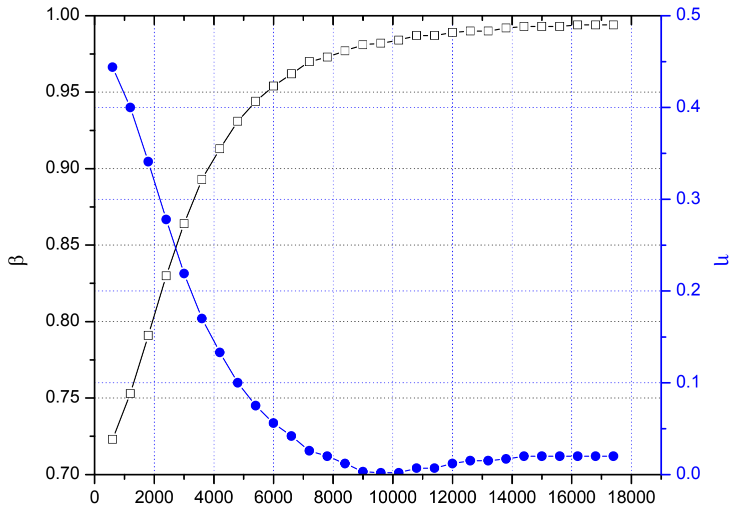
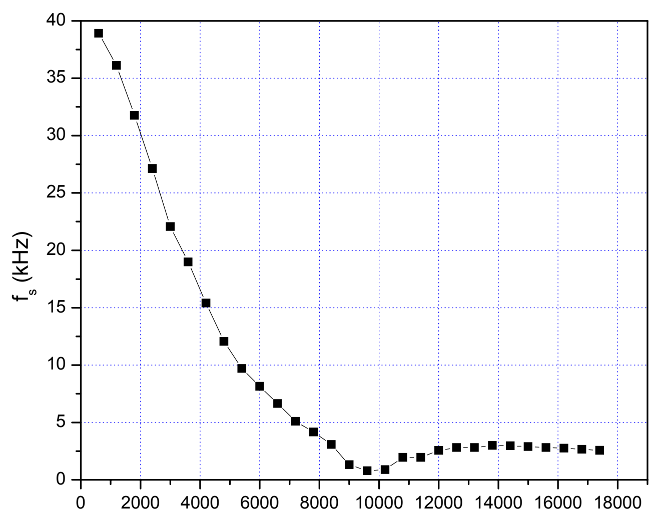

RF Frequency (MHz)
Turn Number in a Booster Cycle

Fig. 3(b)


β
η
Turn Number in a Booster Cycle
Fig. 3(c)


f_s (kHz)
Turn Number in a Booster Cycle
Fig. 3(d)

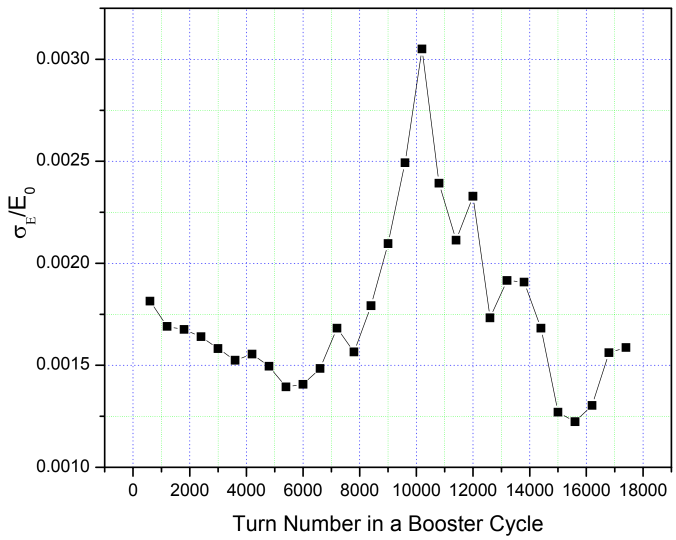


Fig. 3(e)

Fig. 3(a) the RWS at the 2^{nd} trigger of TN 600.

Fig. 3(b) the f_{rf} *vs*. the TN.

Fig. 3(c) the calculated Lorentz relativistic factor (β) and the phase slip factor (η) *vs*. the TN are shown as the black curve and the blue curve respectively.

Fig. 3(d) the calculated synchronous frequency *vs*. theTN.

Fig. 3(e) the energy spread *vs*. the TN.