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Cosmologic Exact Solutions of Isobaric Interactive Scalar and Spinorial Fields in an Anisotropic Space-Time of Petrov D

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Abstract

This work obtained two cosmologic exact solutions to the Einstein equations considering the non-lineal, self-consistent, and congruent spinorial and interactive fields of an isobaric $P = const$ model in a anisotropic symmetry of the Petrov D. The function of the scalar field, the components of the spinorial field, and the interaction between them were analyzed, and it was determined that the phase of the components of the spinorial field is related to the scalar field, so that for great times, the phase of the components of the spinorial times is proportional to the function of the scalar field. It is established that said interaction is equivalent to a mixture between dust fluid and dark energy.

Keywords: cosmology, Einstein, exact, solution, quantum field

1 Introduction

Thanks to data obtained by the COBE, WMAP, and PLANCK satellites, and the discovering of the acceleration of the Universe [1, 2], the study of possible cosmologic scenarios has extended. These and other aspects has been discussed in [3].

The importance of the study of scalar fields and interactive self-consistent cosmologic spinorials has been approached in [4].It was obtained that a determined self-consistent interaction between them leads to a cosmologic model equivalent to the Chapligyn gas; for said fields, the phase of the components of the spinorial field is related to the function of the scalar field, so that for great times, the phase of the components of the spinorial field is proportional to the function of the scalar field. It is established that for great times the non-lineal lagrangian of interaction behaves similar to the model of Soler; and for small periods, it behaves similar to a free spinorial field, but with some variation in the massive term of the lagrangian.

The stability of Jacoby [5] was analyzed in scalar fields in a FRW space-time plane for some theoretic models, and it was determined that at least the Higgs field alternates between stable and unstable phases; the rest of the analyzed models become unstable. An equivalent analysis for isobaric scalar fields was studied in [6]. It was obtained that the solution for the homogeneous anisotropic symmetry of Petrov D is stable for $t > 0$, and for the case of the solution for the homogeneous isotropic (the FRW plane), it is in a certain time interval $t \in]0, t_{1/2}]$, where $t_{1/2}$ is approximately the half of the believed time existence of the Universe.

The present work studied and obtained two exact solutions for the interactive and self-consistent scalar and spinorial fields in an anisotropic symmetry of Petrov D, whose metric behavior is equivalent to the solution of a isobaric scalar field [6].

2 Isobaric scalar and spinorial fields in an anisotropic space-time of Petrov D

2.1 The symmetry of the anisotropic and homogeneous space time of the Petrov D and the Einstein tensor

The symmetry that will be used in this work is the homogeneous and anisotropic of the Petrov D, which follows the pattern [3]

$$ds^2 = F dt^2 - t^{2/3} K (dx^2 + dy^2) - \frac{t^{2/3}}{K^2} dz^2, \tag{1}$$

where F and K are functions of t.

The components of the Einstein tensor [?] ($G_\alpha^\beta = R_\alpha^\beta - 1/2 \delta_\alpha^\beta R$) different from zero of (1), are

$$G_0^0 = \frac{4\,K^2 - 9\,t^2 \dot{K}^2}{12 t^2 K^2 F}, \tag{2}$$

$$G_1^1 = -\frac{3Kt\dot{K}\left(2F - \dot{F}t\right) + 3Ft^2\left(2K\ddot{K} - 5\dot{K}^2\right) + 4K^2\left(\dot{F}t + F\right)}{12t^2K^2F^2}, \tag{3}$$

$$G_2^2 = G_1^1, \tag{4}$$

$$G_3^3 = -\frac{-6Kt\dot{K}\left(2F - \dot{F}t\right) - 3Ft^2\left(4K\dot{K} - \dot{K}^2\right) + 4K^2\left(\dot{F}t + F\right)}{12t^2K^2F^2}, \tag{5}$$

where the points over the functions represent derivatives by the time.

2.2 Interactive spinorial and scalar fields

The interactive spinorial and scalar fields of this study are determined by the following lagrangian:

$$\mathcal{L} = \frac{\imath}{2}(\overline{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\overline{\psi}\gamma^{\mu}\psi) - m\overline{\psi}\psi + \frac{1}{2}\varphi_{,\alpha}\varphi^{,\alpha}\Phi(S), \tag{6}$$

where ψ is the spinor, $S = \overline{\psi}\psi$ is the invariant scalar of the spinorial field, the function $\Phi(S)$ is a non-lineal function of the spinorial field and φ is the spinorial conjugated hermitian of Dirac, and it is defined as $\overline{\psi} = \psi^{*}\overline{\gamma}^{\dot{0}}$ (it will be defined later $\overline{\gamma}^{\dot{0}}$) and in where the matrices $g_{\mu\nu}\gamma^{\nu} = \gamma_{\mu}$ can be defined, according to the metric that follows the pattern

$$\gamma_{\mu} = e_{\mu}^{\dot{\eta}}\overline{\gamma}_{\dot{\eta}}, \qquad g_{\mu\nu} = e_{\mu}^{\dot{\eta}}e_{\nu}^{\dot{\beta}}h_{\dot{\eta}\dot{\beta}}, \tag{7}$$

where $h_{\dot{\eta}\dot{\beta}}$ is the Minkowski tensor of the flat world with the signature (+,-,-,-), and the matrices $\overline{\gamma}_{\dot{\eta}}$ are the Dirac matrices for the flat space-time, $e_{\dot{\eta}}^{\mu}$, $e_{\mu}^{\dot{\eta}}$ is the tetrade set of the vector 4. The matrices $\overline{\gamma}^{\dot{\eta}}$ follow the pattern:

$$\overline{\gamma}^{\dot{0}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \qquad \overline{\gamma}^{\dot{1}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \tag{8}$$

$$\overline{\gamma}^{\dot{2}} = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix} \qquad \overline{\gamma}^{\dot{3}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{9}$$

The spinorial derivative covariant ∇_μ is defined as

$$\nabla_\mu \psi = \frac{\partial}{\partial x^\mu}\psi - \Gamma_\mu \psi, \tag{10}$$

where Γ_μ are the spinorial matrices of the related coefficient, which are linked to the Christoffel symbol in the following way

$$\Gamma_\mu = \frac{1}{4} g_{\rho\delta}\left(\left(e_\sigma^{\dot\eta}\right)_{,\mu} e_{\dot\eta}^\rho - \Gamma_{\mu\sigma}^\rho\right)\gamma^\delta\gamma^\sigma. \tag{11}$$

From the function of Lagrange (6) and the metric (1) were obtained the equations of the spinorial field, the ones of the scalar field, and the components of the stress-energy tensor. These equations and components can be expressed in the following way:

$$\frac{\imath}{2}\gamma^\mu\nabla_\mu\psi + \frac{\imath}{2}\nabla_\mu\gamma^\mu\psi - m\psi + \frac{1}{2}\varphi_{,\alpha}\varphi^{,\alpha}\Psi'(S)\psi = 0, \Psi'(S) = \frac{d\Psi}{dS}, \tag{12}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\nu}[\sqrt{-g}g^{\nu\mu}\{\varphi_{,\mu}\Phi(S)\}] = 0, \tag{13}$$

$$\begin{aligned} T_\mu^\rho = \frac{\imath}{4} g^{\rho\nu}[\overline{\psi}\gamma_\mu\nabla_\nu\psi - \nabla_\mu\overline{\psi}\gamma_\nu\psi + \overline{\psi}\gamma_\nu\nabla_\mu\psi - \nabla_\nu\overline{\psi}\gamma_\mu\psi] + \\ + \varphi_{,\mu}\varphi^{,\rho}\Psi(S) - \delta_\mu^\rho L. \end{aligned} \tag{14}$$

From(7), and the tetrade set, it can be taken the following

$$e_0^{\dot 0} = \frac{1}{e_{\dot 0}^0} = \sqrt{F}, \quad e_1^{\dot 1} = \frac{1}{e_{\dot 1}^1} = e_2^{\dot 2} = \frac{1}{e_{\dot 2}^2} = t^{1/3}\sqrt{K}, \quad e_3^{\dot 3} = \frac{1}{e_{\dot 3}^3} = \frac{t^{1/3}}{K}. \tag{15}$$

3 Solution

It can be assumed that the spinor ψ and the scalar φ, depend just on t, therefore, the equation of the scalar field (13) is obtained

$$\varphi = \int \frac{C_{sc}\sqrt{F}}{t\Phi}dt + \varphi_0. \tag{16}$$

The equation of the spinorial field takes the pattern

$$\begin{aligned} \frac{\dot\psi_r}{\sqrt{F}} + \frac{\psi_r}{2t\sqrt{F}} + i\left(m - \frac{{C_{sc}}^2\Phi_{,S}}{2t^2\Phi^2}\right)\psi_r = 0, \\ \frac{\dot\psi_{r+2}}{\sqrt{F}} + \frac{\psi_{r+2}}{2t\sqrt{F}} - i\left(m - \frac{{C_{sc}}^2\Phi_{,S}}{2t^2\Phi^2}\right)\psi_{r+2} = 0, \quad r = 1, 2, \end{aligned} \tag{17}$$

where the point represents the partial derivative by t. From(17) it is obtained that

$$\overline{\psi}\psi = S = \frac{C_0}{t}, \tag{18}$$

where C_0 is a constant of integration.
From(18),and taking into account

$$\Phi = \frac{{C_{sc}}^2 S^2}{C_0\,(2\,C_0\sigma + C_1 S)}, \tag{19}$$

The non-lineal interaction in the lagrangian(6) follow the pattern

$$\mathcal{L}_{no-lin} = \frac{1}{2}\varphi_{,\alpha}\varphi^{,\alpha}\Phi(S) = \sigma + \frac{C_1}{2t}, \tag{20}$$

where σ and C_1 are constants, whose physical sense is determined through energetic and temporal concepts that will be analyzed later.

The components of the spinor of (17) can be expressed in the following way

$$\psi_r = \frac{\alpha_r e^{-iG}}{\sqrt{t}}, \quad \psi_{r+2} = \frac{\alpha_{r+2} e^{iG}}{\sqrt{t}}, \quad r = 1, 2, \tag{21}$$

whereα_1, α_2, α_3 and α_4 are constants related to C_0 with the pattern $C_0 = \alpha_1^2 + \alpha_2^2 - \alpha_3^2 - \alpha_4^2$ ans the phase of the spinor G is

$$G = \int \left(m\sqrt{F} + \frac{\sqrt{F}{C_{sc}}^2\dot{\Phi}}{2\Phi^2 C_0} \right) dt + G_0, \tag{22}$$

where G_0 is a constant.
The components of the stress-energy tensor(14) follow the pattern

$$T_0^0 = \frac{\dot{\phi}^2\Phi}{2F} + \frac{mC_0}{t}, \quad T_k^k = -\frac{{C_{sc}}^2\dot{\Phi}}{2t\Phi^2} - \frac{\dot{\phi}^2\Phi}{2F}, \quad k = 1, 2, 3, \tag{23}$$

For this reason, the components of the Einstein tensor (3) y (5) should be equal, so that it is obtained that

$$K = K_0 e^{C_k \int \frac{F^{1/2}}{t} dt}, \tag{24}$$

where K_0 and C_k are constant, which do not lose generalities that can be taken in(24) as $K_0 = 1$ and the constant la $C_k = \pm 2/3$, which provides two possible kinds of expansion (greater over an axis or greater for the perpendicular plane of the axis) [6]).

Considering that (16), (18), (19) (24) from the independent equations of Einstein (2), (3) and (23), it is obtained that F, in(1), is

$$F = \frac{1}{3\left(\sigma t^2 + \left(C_1/2 + mC_0\right)t\right) + 1}, \tag{25}$$

which helps to define (24), (22) and (16). The no-null components of the stress-energy tensor (23), which represent the volume density of the energy and stress, follow the pattern

$$T_0^0 = \mu = \sigma + \frac{C_1/2 + mC_0}{t}, \quad T_1^1 = T_2^2 = T_3^3 = -P = \sigma, \tag{26}$$

and meet the principle of equality. $T^{\mu\nu}_{;\mu} = 0$.
The physical sense of the constant σ is obtained by noticing(26),which is the "stress" on the interactive and self-consistent scalar and spinorial fields of the lagrangian subjected to study. The constant $\frac{C_1}{2C_0} + m > 0$ represents the effective mass of the spinorial field when $t \to 0$.
From the function (1), considering that $\sigma \neq 0$ and $C_1 \neq 0$, it is obtained that the scalar field and the function $K_\pm$ on(24), are

$$\phi = \frac{2}{3C_{esc}\sqrt{F}} - \frac{\left(-C_1 + 2\,mC_0\right)}{\sqrt{12\sigma}C_{sc}} \ln\left(-\frac{\dot{F}}{F^2\sqrt{6\sigma}} + \sqrt{\frac{2}{F}}\right) + \phi_0, \tag{27}$$

and

$$K_\pm = \left(\frac{\left(1 - 2\,H\left(\sigma - \alpha^2/3\right)\right)\left(-\sqrt{F^{-1}} + 1 + t\alpha\right)}{\sqrt{F^{-1}} + 1 + t\alpha}\right)^{\pm 1/3}, \tag{28}$$

where $\alpha = 3C_1/4 + 3\,mC_0/2$, ϕ_0 is a constant of integration, and the function $H(x - y)$ in(28) is the Heaviside step function.
The found solutions represent non-lineal interactive and consistent scalar and spinorial fields, similar to the mixture of the dust and the dark energy fluids. These two particular cases are possible when $\sigma = 0$ y $C_1 + 2\,mC_0 \neq 0$, (the "dust case"), and when $\sigma \neq 0$ and $C_1 + 2\,mC_0 = 0$, (the "dark energy case"), as a result of what was obtained in [3]. The analysis of said solutions is analogous to the one done in [6], in terms of the singularities, the dependence on the sign on (24), and the relation to the Hubble parameters and the deceleration. The solution of the particular cases are:

3.1 The "Dust" case ($\sigma = 0$ y $C_1 \neq -2\,mC_0$)

In this case, the function $F = (3(C_1/2 + mC_0)t + 1)^{-1}$, the function of the scalar field, and the function $K_\pm$ of (24), are

$$\phi = \frac{2^{3/2}C_1\sqrt{\left(3\,C_1 + 6\,mC_0\right)t + 2}}{C_{sc}\left(3\,C_1 + 6\,mC_0\right)} + \phi_0, \tag{29}$$

and

$$K_{\pm}=\left(\frac{\sqrt{1+3/2\,C_1t+3\,tmC_0}-1}{\sqrt{1+3/2\,C_1t+3\,tmC_0}+1}\right)^{\pm 2/3}, \tag{30}$$

3.2 The "Dark Energy" Case ($\sigma \neq 0$ and $C_1=-2\,mC_0$)

In this case, the function $F=(3\sigma t^2+1)^{-1}$, the function of the scalar field, and the function $K_{\pm}$ are

$$\phi=\frac{2\sqrt{3\,t^2\sigma+1}}{3C_{sc}}+\frac{C_1\ln\left(t\sqrt{3\sigma}+\sqrt{3\,t^2\sigma+1}\right)}{C_{esc}\sqrt{3\sigma}}+\phi_0, \tag{31}$$

and

$$K_{\pm}=\left(\frac{\sqrt{1+3\sigma t^2}-1}{\sqrt{1+3\sigma t^2}+1}\right)^{\pm 1/3}. \tag{32}$$

The phase G of (22), for the general and the two other cases, follows the pattern

$$G=4\,\frac{\phi\,C_{sc}m}{-C_1+2\,mC_0}-\frac{2\,(C_1+2\,mC_0)}{3C_0\,(-C_1+2\,mC_0)\,\sqrt{F}}+G_0, \tag{33}$$

where it is established that G tends to be proportional to the function of the scalar field ϕ for great times. For short times, $t\to 0$, the function of the scalar field and the phase tend to be constant; therefore, they can be defined as null when $t\to 0$.

4 Conclusions

The study of the scalar and spinorial non-lineal interactive, and self consistent fields of the type (6), with a non-lineal element of the spinorial of the type(19), in an anisotropic symmetry of Petrov D leads to a state of the fields that is equivalent to the one of an isobaric fluid, which was studied for a non-lineal spinorial field in [6]. It has similar cosmologic consequences in terms of the study of possible singularities, the Hubble parameters, and the deceleration; besides of the tendency, with time increase, to become an isotropic metric. As particular case, there were obtained the solutions to said interactions, similar ton the "dust" and the "dark energy" ideal fluids. For all these cases, it was obtained that the function of the scalar field(φ) increases with time; this makes that the phase of the components of the spinorial field (of the spinor) becomes proportional to said function. For these times, the metric tends to behave as its equivalent of the dark energy [3], which is reflected in the function of the scalar field through the supremacy of the constant σ, whose sense is similar to the cosmologic constant Λ. On the other hand, for small values of time, the model of isobaric fluid tends to behave as its analogous of the dust type ($P = 0$), which is also reflected in φ and the phase of the spinorial field through the supremacy of the constant C_1.

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