A RADIO-FREQUENCY MASS SEPARATOR FOR COMPLETE SEPARATION OF HIGH ENERGY PARTICLE BEAMS

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1. INTRODUCTION

By passing a beam of high energy particles already analyzed in momentum through a deflector in which a deflecting wave travels with the beam, it is possible to reduce the relative intensity of an undesired particle component. This is accomplished by matching the velocity of the traveling wave to the velocity of the unwanted particles. The desired particles have a different velocity and will then fall out of step with the deflecting field so that they experience less total deflection.

As we shall show, a combination of two such deflectors can give complete separation of two particle types. For clarity, the discussion will be aimed at the problem of removing the π meson component from a beam of anti-protons. The method can obviously be applied to other similar problems of beam separation.

2. QUALITATIVE DESCRIPTION OF THE TECHNIQUE

The arrangement of the two deflectors is shown in Fig. 1. They are of equal length L and are separated by a distance that may be $L/4$, $3L/4$, $5L/4$ etc. The deflecting wave, which is supposed to match the undesired π mesons in velocity, will travel at approximately the velocity of light for particles in the GeV energy range. The length L is chosen such that, in a distance L, an anti-proton of the same momentum as the π mesons and hence of lower velocity will slip behind the traveling wave by exactly one wavelength as it traverses the deflector.

The phases of the π mesons and anti-protons arriving at various phases of the deflecting wave are indicated in Fig. 2. The wave sketched to the left of the diagram represents the traveling wave. On it are shown the positions of the various particles as they enter the deflector. The subscript numbers correspond to the numbered positions at the exit of the first deflector. As indicated in Fig. 1, and as will be proved later, the π meson beam is fanned out into a divergent beam. At the same time the anti-proton beam is spread out but emerges in the form of a parallel beam.

At the point of emergence from the first deflector the particles have the same arrangement on the traveling wave as they did when they entered the deflector; this is because the π mesons have traveled with the wave and the anti-protons have slipped behind exactly one full period.

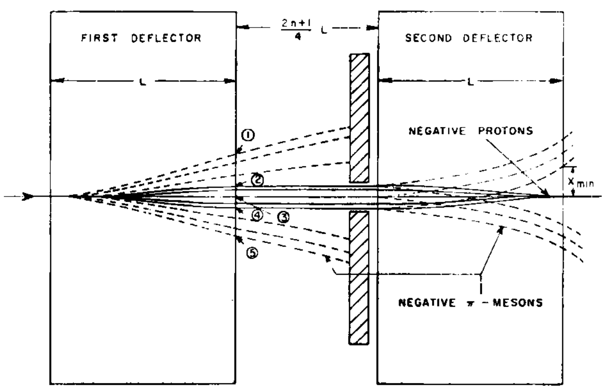

Fig. 1 Arrangement of the deflectors.

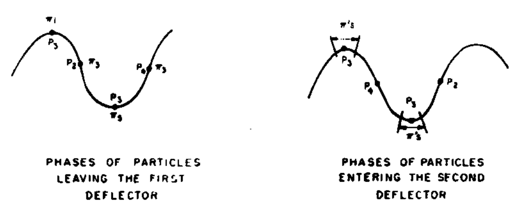

Fig. 2 Phases of particles with respect to the traveling wave.

Now the particles drift a distance $L/4$, $3L/4$,... or in general $(2n+1)L/4$. Relative to the π mesons, the anti-protons slip back $(2n+1)$ quarter waves. At the entry to the second deflector an aperture passes the complete anti-proton beam but only such π mesons as lie within the limits of the anti-proton beam; these are the π mesons which fell in the neighborhood of zero phase in the first deflector.

The second deflector is excited in such a phase that the anti-protons find themselves at starting phases removed 180° from their starting phases in the first deflector. Their path through the second deflector is the inverse of that in the first deflector and the original anti-proton beam configuration is restored. On this wave the π mesons which passed through the aperture find themselves arranged around the peaks of the wave and are thrown to one side or the other as they pass through the second deflector. The positions of the various particles on the traveling wave as they enter the second deflector are indicated in the sketch to the right of Fig. 2.

From the diagram it should be evident that a complete beam separation has been achieved. We now proceed to a mathematical description of the processes just described.

3. ARRANGEMENT OF DEFLECTORS

We assume that the velocity of the π mesons is c and the velocity of the anti-protons is v. The transit time of a π meson through the deflector will be L/c where L is the deflector length; the transit time of an anti-proton will be L/v. Since the difference between these times is to be one period

$$\frac{L}{v}-\frac{L}{c}=\frac{1}{f}=\frac{2\pi}{\omega}=\frac{\lambda}{c}. \tag{1}$$

Hence

$$L=\frac{v\lambda/c}{1-v/c}. \tag{2}$$

For relativistic particles Eq. (2) is approximately equivalent to

$$L=2\lambda W^2/W_0^2 \tag{3}$$

where W is the total energy (including rest energy) and W_0 is the rest energy of the anti-proton.

The transverse deflecting field may be electric or magnetic or a combination of both. We shall assume for purposes of exposition that it is electric and has in the first deflector the form

$$E_y=E_0\sin(\omega t-\omega z/c+\phi) \tag{4}$$

where ϕ represents the phase at which a particle enters the deflector. This field will give deflections in the y direction of particles traveling parallel to the z-axis. Eq. (4) represents a deflecting wave traveling along the z-axis with phase velocity c.

In the second deflector the field is similar but is shifted in phase as described in the previous section.

At the entrance to the second deflector is an aperture such that all particles having positions given by $|y|>\dfrac{E_0L^2}{2\pi W}$ will be intercepted.

4. PARTICLE DYNAMICS IN THE DEFLECTOR SYSTEM

The transverse motion in the first deflector will be given by

$$\ddot{y}=-\left(\frac{E_0c^2}{W}\right)\sin\Big(\omega t(1-v/c)+\phi\Big) \tag{5}$$

where W is expressed in electron volts. In this equation W and v can refer to either particle type. Integration of this expression yields

$$\dot{y}/c=\dot{y}_0/c+\frac{E_0c}{\omega W(1-v/c)}\left[\cos\Big(\omega t(1-v/c)+\phi\Big)-\cos\phi\right] \tag{6}$$

and

$$y=y_0+\dot{y}_0t+\frac{E_0c^2}{\omega^2W(1-v/c)^2}\Big[\sin\Big(\omega t(1-v/c)+\phi\Big)-\sin\phi-\omega t(1-v/c)\cos\phi\Big] \tag{7}$$

where y_0 and $\dot{y}_0$ are the initial values of y and $\dot{y}$.

In the first deflector we assume that y_0 and $\dot{y}_0$ both are zero. For anti-protons we set $t=L/v$ to obtain the final conditions at emergence from the first deflector. Using Eqs. (1) and (2) we obtain for anti-protons :

$$\dot{y}=0$$

$$y=-\frac{E_0c^2L^2}{2\pi Wv^2}\cos\phi\approx-\frac{E_ol^2}{2\pi W}\cos\phi \tag{8}$$

since $v\approx c$ for relativistic anti-protons.

For π mesons we set $v = c$ to obtain from Eqs. (6) and (7)

$$\dot{y}/c = -\frac{E_0 L}{W} \sin \phi$$
$$y = -\frac{E_0 L^2}{2W} \sin \phi \tag{9}$$

At the entrance to the second deflector, the deflection of the π mesons will have increased to

$$y = -\frac{(2n+3)E_0 L^2}{4W} \sin \phi \tag{10}$$

where, as previously noted, the separation between deflectors is $(2n+1)L/4$.

It is now evident that the aperture at the entrance to the second deflector will pass all of the anti-protons but will intercept all π mesons except those for which

$$|\sin \phi| < \frac{2}{(2n+3)\pi}. \tag{11}$$

The ranges of π meson phases which satisfy this relation are

$$\begin{aligned} \text{for } n = 0,\ & -12.3° < \phi < +12.3° \text{ or } 167.7° < \phi < 192.3° \\ n = 1,\ & -\ 7.3° < \phi < +\ 7.3° \text{ or } 172.7° < \phi < 187.3° \\ n = 2,\ & -\ 5.2° < \phi < +\ 5.2° \text{ or } 174.8° < \phi < 185.2° \end{aligned} \tag{12}$$

At the end of the second deflector, as we have shown, the anti-protons are restored to the condition $y = \dot{y} = 0$

The initial conditions for the π mesons are given by Eqs. (9) and (10). We substitute these conditions in Eqs. (6) and (7), taking into account the phase shift in the field of the second deflector and we arrive at the results :

$$\dot{y}/c = -\frac{E_0 L}{W}(\sin \phi \pm \cos \phi) \tag{13}$$

$$y = -\frac{E_0 L^2}{2W}((n+7/2) \sin \phi \pm \cos \phi) \tag{14}$$

where the upper sign applies if n is even and the lower sign applies if n is odd. The minimum deflection indicated by Eq. (14) is

$$\begin{aligned} \text{for } n = 0, \quad & x_{\min} = 0.117 E_0 L^2/W \\ \text{for } n = 1, \quad & x_{\min} = 0.210 E_0 L^2/W \\ \text{for } n = 2, \quad & x_{\min} = 0.248 E_0 L^2/W \end{aligned} \tag{15}$$

5. THE DEFLECTING FIELD PATTERN

We now turn to a possible method of generating a field pattern in which magnetic field components do not cancel the deflecting effects of transverse electric fields. Conventional waveguide field patterns do not, in general, satisfy this requirement.

We choose the z-axis as the direction along which particle beams shall travel. The deflecting field E_y is to have a sinusoidal pattern in the z direction. The magnetic field component B_x which would normally cause a deflection opposite to that due to the electric field, is set equal to zero. We then find that Maxwell's equations are satisfied by a field configuration as follows :

$$\begin{aligned} E_x &= 0 \\ E_y &= E_0 \cos(\omega x/c) \cosh \alpha y \sin \alpha z \sin \omega t \\ E_z &= E_0 \cos(\omega x/c) \sinh \alpha y \cos \alpha z \sin \omega t \\ B_x &= 0 \\ B_y &= (E_0/c) \sin(\omega x/c) \sinh \alpha y \cos \alpha z \cos \omega t \\ B_z &= -(E_0/c) \sin(\omega x/c) \cosh \alpha y \sin \alpha z \cos \omega t. \end{aligned} \tag{16}$$

The wave number α is not determined by the field equations but will be fixed only by the boundary conditions.

Both E components vanish for $x = \pm\lambda/4$. Consequently, boundary walls can be erected in the planes through this value of x. The other boundaries are given by

$$\frac{dz}{dy} = -\frac{E_y}{E_z} = \text{ctnh}\, \alpha y \tan \alpha z \tag{17}$$

whence the equation of the boundaries is

$$\sinh \alpha y \sin \alpha z = \text{constant}. \tag{18}$$

A structure with these boundaries is shown in Fig. 3 (α has been chosen equal to ω/c).

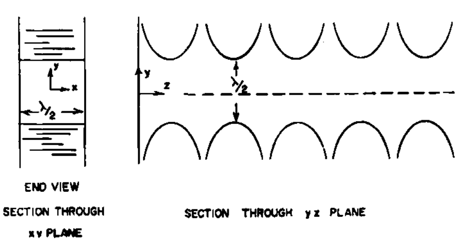

Fig. 3 Waveguide structure corresponding to Eq. (18).

Although the field pattern just described is a standing wave pattern, it can be analyzed into two traveling waves, one of which is the desired wave and the other is a wave traveling in the opposite direction that will have negligible effect on the particle beam.

6. PRACTICAL DEFLECTOR DESIGNS

The ideal structure shown in Fig. 3 has the disadvantage of extending to infinity with infinite fields. To terminate the pattern in the y direction, the boundaries can be cut at a reasonable distance and a mirror image of the configuration can be added at each side, bounded in its plane of symmetry. Such a structure with its boundaries modified to circular bars is shown in Fig. 4.

Another variation of the structure of Fig. 3 but with the boundaries modified to conventional rectangular waveguides is shown in Fig. 5. In the structure of Fig. 5 the central region is a waveguide run very

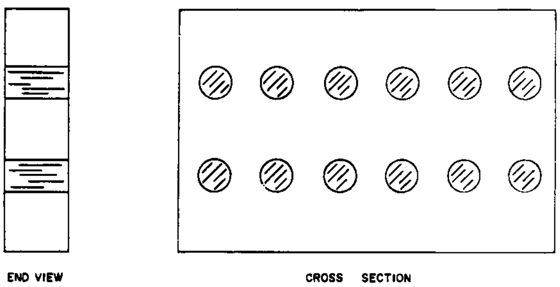

Fig. 4 Simplified waveguide structure of Fig. 3 with boundaries formed by circular bars.

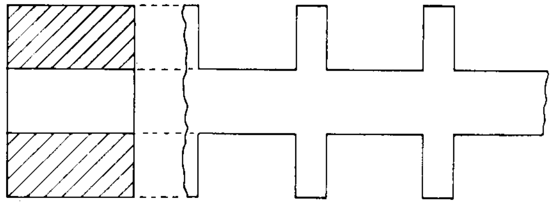

Fig. 5 Simplified waveguide structure of Fig. 3 with lateral stubs.

close to cut-off. The stubs protruding along the sides are ridged guides which in operation are rather far from cut-off; they will be approximately one-quarter wave in length.

7. DEFLECTOR PARAMETERS

The deflector length for a given particle energy and excitation wavelength is given by Eq. (3).

When the length is known, the electric field necessary for a given final separation is given by Eq. (15). This electric field is tabulated below for the special case of $n = 1$ and for a minimum separation between π mesons and anti-protons of 5 cm.

The power requirements follow from estimates of all currents in the walls of the waveguide structure. These requirements have been derived for the system shown in Fig. 5.

These various quantities are shown in Table I for three possible excitation frequencies and for particle momenta of 3, 6, and 10 GeV/c (the figures are for each of the two deflectors).

TABLE I

Frequency	Particle Momentum	3 GeV/c	6 GeV/c	10 GeV/c
300 Mc/s	Length Field strength Power	18 m 2.2 MV/m 3400 kW	72 m 0.27 MV/m 210 kW	200 m 0.06 MV/m 27 kW
600 Mc/s	Length Field strength Power	9 m 8.8 MV/m 19 000 kW	36 m 1.1 MV/m 1200 kW	100 m 0.24 MV/m 160 kW
1200 Mc/s	Length Field strength Power	4.5 m 35 MV/m 110 000 kW	18 m 4.4 MV/m 6700 kW	50 m 0.96 MV/m 880 kW

8. CONCLUSIONS

From the above table it is evident that the radio-frequency separation technique is not limited as particle energy increases. The separator length increases rapidly with particle energy but, at the same time, the power requirements decrease. For low energies, lower frequencies are appropriate while at the higher energies separator lengths are kept within reasonable limits by operating at frequencies in the 1000 Mc/s range.

While the power requirements in general are in the megawatt range they are within the range of available power sources. Probably the major application of beam separators will be with bubble chambers where short pulses will be required. Under these conditions, the necessary power levels will be reached with ease.

It should also be noted that a pre-bunching of the particle beam at the frequency of the separator can make a complete separation possible with only one deflector rather than the double system discussed above.

Acknowledgement

The writer is indebted to Mr. J. D. Kiesling of the Brookhaven National Laboratory for his assistance in the design and analysis of the waveguide system illustrated in Fig. 5.

DISCUSSION

QUERCIA (to Livingston): I would like to get some information about the structure of the γ ray pulse. Are you thinking about the problem of making the intensity of the γ ray pulse constant in time?

LIVINGSTON: We have not considered the problem of equalizing the intensity. The structure of the pulses will be a function of the radial betatron oscillation distribution of the particles in the beam. It will be difficult to make that uniform.

ADAMS: Livingston proposes to shift the Q-value of the radial betatron oscillations from 6.4 to 6.5 in order to bring the beam on to a target. This can be done with quadrupoles, but will require a substantial amount of power at the full energy. Since the synchrotron has no quadrupoles I should like to ask Livingston how this Q-shift will be made.

LIVINGSTON: The actual power required is not very large and two or at most four quadrupoles will be sufficient to obtain the necessary Q-shift. Such quadrupoles will be installed later especially for this purpose.

In order to reduce the power required to shift the Q-value to 6.50, we can adjust the Q-value of our machine by displacing the magnets radially and operate, for instance with a Q-value of 6.47.

GREEN: There might also be a good reason to have sextupoles. It has been predicted, and confirmed in our electron analogue, that with the right amount of non-linearity one can operate exactly on a half-integral resonance. In such a case one can give a short kick to the beam, after which it will form a standing wave around the machine. This offers some very interesting possibilities of striking targets. We found in the electron analogue that the beam could make at least 20 000 revolutions locked on a half-integral resonance.

WIDEROE (to Livingston): Have you considered the possibility of using the pulsed ferrite magnet, described by O'Neill, for extracting the particles?

LIVINGSTON: The magnetic fields required are not very large. Ferrites would probably be acceptable for a beam extraction magnet or for the other deflecting magnets needed. However, we have not yet experimented with ferrite magnets.

VEKSLER: I would like to make a few remarks about an RF separator which was proposed by Petukhov and myself and which is now under construction in Dubna. Whereas the number of bunches in a strong focusing machine is large, it is very small in a weak focusing machine and in the 10 GeV synchrophasotron there is only one bunch, about 70 m long. This large bunch will be subdivided with a special RF bunching cavity placed in one of the straight sections of the machine. The length of these short bunches will be about 1.2 m. Secondary particles produced in the target are passed through two RF cavities which are excited in phase with the bunching cavity in the machine. Separation of anti-protons and π mesons occurs since their times-of-flight from the target to the cavities are different. The distance from target to the cavities is about 100 m. After the anti-proton and π meson beams have been separated in the cavities they pass through a magnet by which their separation is further increased. I shall not discuss any detail of the arrangement here, but some details were given in a paper presented to the Conference on the Peaceful Uses of Atomic Eenergy last year. The system will be ready by the end of this year of the beginning of next year.

MONTAGUE (to Veksler): What bunching frequency do you propose to use?

VEKSLER: The spacing between the centres of the bunches will be about 2.5 m so that the frequency is about 100 Mc/s.

WALKINSHAW (to Veksler): What voltage do you intend to use on your bunching cavity?

VEKSLER: About 200 kV.

TICHO (to Veksler): Have any experiments been done so far to show how low the intensity can be in between the bunches?