ELECTRON-PROTON SCATTERING AT HIGH MOMENTUM TRANSFER (*)

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(presented by R. R. Wilson)

In the past all published elastic electron-proton scattering data [1,2,3] have been obtained by observing the scattered electron after momentum analysis. The present paper is an interim report on measurements we have been making at high momentum transfer (that is, above 25 Fermi^{-2}) by detecting both the scattered electron and the recoil proton, each momentum analyzed. This redundancy in kinematic requirements brings about a drastic reduction in the background due to the carbon in the polyethylene target and from meson electroproduction in both the carbon and the hydrogen. The relatively more abundant pions constituted a serious background in previous high momentum transfer experiments [2] limiting the accuracy obtainable.

The experimental arrangement is shown in Fig. 1. As in the previous experiments [2] the high-energy circulating electron beam of the Cornell synchrotron strikes a polyethylene target 1/16″ thick located in a straight section between synchrotron quadrants. The bremsstrahlung emitted from the target is integrated absolutely by a totally absorbing ion chamber [4].

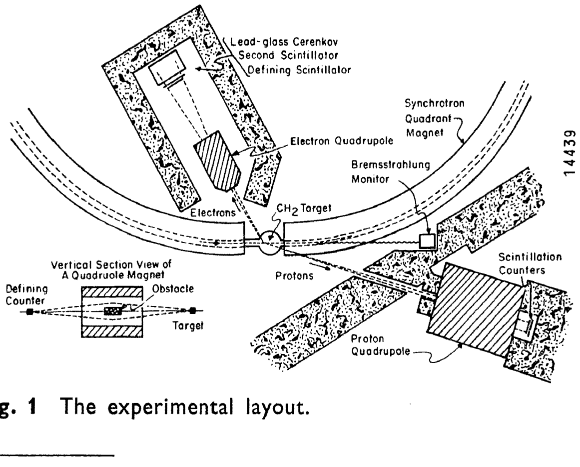

Fig. 1 The experimental layout.

Knowledge of the radiation length in the target material then gives the effective product of incident electron flux and total traversal thickness required for cross section calculations.

Electrons emerging from the thin walled scattering chamber pass over or under the central obstacle in a vertically focusing single-lens quadrupole magnet of the type described by Hand and Panofsky [5], and are brought to a horizontal line focus. The momentum defining scintillator is placed at this focus, followed by a second scintillator and a large lead glass Čerenkov counter for identification of cascade showers initiated by high energy electrons. The recoil protons are momentum analyzed in the same way using a conventional 8″ quadrupole and two scintillation counters.

The momentum resolution, full width at maximum efficiency, was chosen to be about 5 % for both spectrometers. This relatively broad momentum resolution insures that the intrinsic elastic scattering line width due to finite angular aperture, target size, multiple scattering, and magnetic aberrations has negligible effect on the detection efficiency when the scattered momentum is centred in the resolution band, and eliminates the necessity of tracing out and integrating numerically the counting rate versus magnet current curve for each cross section measurement. Once the magnets are calibrated, a single measurement of the coincidence counting rate at the appropriate current settings is sufficient to determine the cross section.

An elastic scattering event is signified by a coincidence of all five counters within the resolving time of the electronics (16 ns).

The four-momentum transfer q^2 and the scattering angle θ were chosen as the independent variables,

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instead of θ and the incident energy. Several measurements at different θ and the same q^2 can then be directly compared to determine the form factors. To date, cross sections at $\theta = 110°$, $120°$, $130°$ (lab system) and $q^2 = 25$, 30, 35, 40, 45 Fermi^{-2} have been measured. These cover laboratory proton angles from 10.9° to 19.5° and incident electron energies from 863 to 1362 MeV. Except at the highest value of q^2, the statistical error is about 4%. The carbon background rate is about 2% of the hydrogen rate.

Since the recoil proton momentum in the present experiment is much more sensitive to the incident energy than is the scattered electron momentum and since the electron momentum resolution is broader than the proton resolution, one would expect that the radiation correction would be only slightly greater than the correction for the case in which only the proton is momentum analyzed. This latter correction has been calculated recently by Krass [2]. For our data the Krass correction is 7 to 9% (depending on θ and q^2). We have tentatively added another 1% to this to approximate the extra effect of momentum analysis of the electron. It should be emphasized that this is only a temporary expedient; when the correct calculation is eventually made, the cross section values can be expected to change slightly.

That part of the systematic error which can be expected to vary at random from one measurement to the next is estimated to be about 3% and is combined with the statistical error before determining the form factors from the cross sections. The remaining systematic errors are simply scaling errors common to all the measurements. These are estimated to be at most 10% in the cross sections or about 5% in the form factors, and are included only after the form factors are determined.

The measured cross sections can be analyzed using the Rosenbluth formula [7] to determine the proton form factors F_1 and F_2, associated respectively with the Dirac and Pauli interactions of the physical proton with the electromagnetic field. It has been suggested [8, 9] that a more meaningful separation of the charge and magnetic moment interactions can be made by re-expressing the Rosenbluth cross section in terms of form factors $G_E(q^2)$ and $G_M(q^2)$ defined by

$$G_E = F_1 - t\kappa F_2\,,$$

$$G_M = F_1 + \kappa F_2\,.$$

Note that G_E is normalized to unity and G_M to $1+\kappa$ at $q^2 = 0$. In terms of these form factors the Rosenbluth formula reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sigma_{NS}\left\{\frac{1}{1+t}G_E^2 + \frac{t}{1+t}G_M^2 + 2t\tan^2\frac{\theta}{2}G_M^2\right\}.$$

σ_{NS} is the Mott scattering cross section and $t = q^2/4M^2$. In either notation, the form factors at a given value of q^2 can be determined from the data at various angles θ simply by plotting the ratio of the measured cross section to the Mott cross section versus $\tan^2\theta/2$ and fitting to a straight line. Failure to fit a straight line with real form factors implies a breakdown in the assumptions implicit in the Rosenbluth formula: conventional quantum electrodynamics (point electron) and single photon exchange.

The straight-line plots are shown in Fig. 2. The preliminary data of the present experiment are indicated by black circles and were used to obtain the straight-line fits. Also shown for comparison are data (open circles) reported by the Stanford group [1]. One can see that the data so far obtained are consistent with the Rosenbluth formula. We plan, however, to extend our measurements to larger scattering angles where one might expect the deviations to be greater. The proton form factors, G_E and G_M, determined by a least squares straight-line fit at each q^2 value are

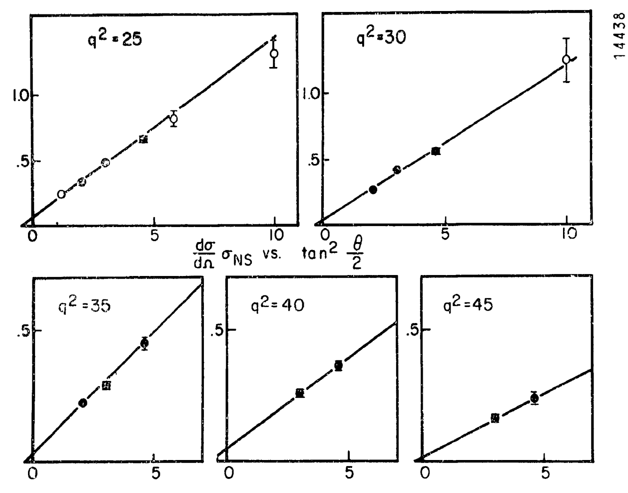

Fig. 2 Graphs of the experimental differential cross section divided by the Mott cross section plotted against $\tan^2\theta/2$ for five values of the squared four-momentum transfer (in Fermi^{-2}). The data of the present experiment are indicated by black circles and were used to obtain the straight-line fits. Also shown for comparison are data (open circles) reported by the Stanford group [1].

plotted in the third figure. Analyzing the data in terms of G_E and G_M instead of F_1 and F_2 has the important advantage of minimizing the correlated errors. That is, G_M^2 is determined directly from the slope of the line and G_E^2 from the extrapolation to $\tan^2 \frac{\theta}{2} = -\frac{1}{2(1+t)}$. Not included is the estimated scaling error of 5% mentioned above. G_M is quite well determined, but it is evident that further measurements are needed at smaller scattering angles before G_E can be known with any precision. The curves in Fig. 3 approximate the previous Stanford and Cornell data at the lower values of q^2 and are taken from a recent fit [9]. It is important to note that the elastic scattering cross section depends only on the squares of the form factors. By continuity G_M must clearly be positive; the sign of G_E above $q^2 = 30F^{-2}$ however is completely undetermined.

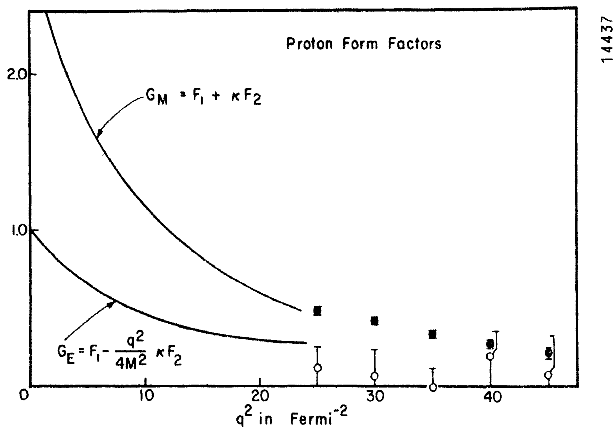

Fig. 3 The experimentally determined proton form factors, G_E and G_M, plotted against squared four-momentum transfer. The indicated errors do not include the 5% overall scaling error. The smooth curves approximate previous data [1, 2] and are taken from a recent fit [9].

The form factors F_1 and F_2 can of course be obtained from G_E and G_M. The large uncertainty in G_E as well as the sign ambiguity, however, cause rather large, although correlated, errors in both F_1 and F_2.

At this point one seeks to fit the form factors to a dispersion theoretic expression [10] involving terms corresponding to various particles present in intermediate states, in particular the vector mesons (ρ, ω, etc.) which can couple to the photon. For this it is convenient to separate the isoscalar and isovector nucleon form factor components. Since the present experiment does not measure the neutron form factors, such a separation cannot be made. Moreover the incompleteness of the angular distribution measured so far and the resulting large errors in G_E preclude any meaningful theoretical curve fitting: the data are consistent with a very wide choice of initial assumptions. For example, the present measurements agree very well with an extrapolation of the fits recently obtained by the Stanford group [11].

Sachs [8] has argued that G_E and G_M for the proton must both tend to the same constant value (between zero and one) at high q^2. The present data for G_M certainly constitute no evidence other than for a monotonic decrease towards zero. So far it is impossible to determine, in the case of G_E, whether the trend is monotonically towards zero, or towards an intermediate negative value.

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DISCUSSION

SALECKER: Schopper has shown that there is a maximum slope in the $tg^2\,\theta/2$ diagrams, if one believes in the Rosenbluth formula. Is this maximum slope compatible with the slopes you find in the experiment?

WILSON: I believe that these are quite consistent with the maximum slope.

MAGNETIC AND ELECTRIC FORM FACTORS

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(presented by R. G. Sachs)

I have been asked to explain our motivation for introducing the form factors F_{ch} and F_{mag} in place of the usual F_1 and F_2 [Ernst, Sachs and Wali, Phys. Rev. *119*, 1105 (1960)]. As you know, the form factors $F_1(q^2)$ and $F_2(q^2)$ are directly related to the matrix element of the current density operator for a transition between two momentum states of the proton, $\langle p'|j_\mu(x)|p\rangle$. The invariant momentum transfer is $q^2=(p'-p)^2$. (The normalization I shall use is $F_2(0)=\mu_p$, the magnetic moment of the proton.) The charge and magnetic form factors used by the previous speaker, but with our normalization, are

$$F_{\mathrm{ch}}(q^2)=F_1(q^2)-\frac{q^2}{2M}F_2(q^2)\,,$$

$$F_{\mathrm{mag}}(q^2)=\frac{F_1(q^2)}{2M}+F_2(q^2)\,.$$

Since Foldy's first work on the electromagnetic properties of nucleons, the form factors have been interpreted in terms of a distribution of charge and magnetization. In order to determine such a distribution for a system, one may calculate the moments of the current density, and the distributions are determined if all moments of the distribution are given. In our case the matrix element of any moment of the current operator may be expressed directly in terms of F_1 and F_2. To obtain the equivalent of a classical charge and current distribution one need only specify the state of the system and then calculate the expectation value of every moment of the distribution in this state, namely

$$\langle \textstyle\int \mathrm{d}^3r\; x^\alpha y^\beta z^\gamma j_\mu(\mathbf{r})\rangle.$$

This would be an (α,β,γ) moment of the 4-current distribution.

The proper state to take for evaluating this expectation value is a wave packet. The wave packet is required in order to have a well-defined answer, but all of the specific moments due to its detailed shape are eliminated. Then at the end of the calculation of the moments the wave packet is taken to describe a particle at rest. The moments of the current density operator obtained in this way are found to be equal to those of the classical current densities defined by

$$J_4(\mathbf{r})=i\rho(\mathbf{r})=\frac{ie}{(2\pi)^3}\int \mathrm{d}^3q F_{ch}(\mathbf{q}^2)e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$\mathbf{J}(\mathbf{r})=\frac{ie}{(2\pi)^3}\int \mathrm{d}^3q(\boldsymbol{\sigma}\times\mathbf{q})F_{mag}(\mathbf{q}^2)e^{i\mathbf{q}\cdot\mathbf{r}}.$$

Hence F_{ch} is the Fourier transform of the equivalent classical charge density, and F_{mag} is the Fourier transform of the equivalent density of magnetization.