Fermi National Accelerator Laboratory

FERMILAB-Pub-88/119-T

September 1988

FOUR JET PRODUCTION AT THE TEVATRON COLLIDER

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Abstract

We present predictions for four-jet production at the Tevatron Collider. The standard QCD production of four-jet final states through $2 \rightarrow 4$-parton processes is compared to the double-parton scattering production.

Submitted to *Zeit. für Physik C*

1 Introduction

The large energies reached by the current $\bar{p}p$ colliders allow for sensitive qualitative and quantitative tests of QCD in hadronic collisions, providing an agreement between theory and experiment that extends over several orders of magnitude in both energy and production rates. Strong interaction processes consitute the overwhelming majority of the events observed in $\bar{p}p$ collisions, and a proper understanding of their features is fundamental for isolating signals of possibly new physics. One of the most interesting classes of phenomena appearing at these new energy thresholds is the production of multi-jet events, events with a large (≥ 4) number of isolated collimated clusters of particles at large p_t. The study of this class of events is important both as a further test of perturbative QCD and as a background estimate for more rare processes[1], as for example Higgs decays into W-pairs.

The study of two- and three-jet events allows for quantitative tests of QCD, while limited statistics and experimental difficulties associated with the proper isolation and definition of jets make the study of events with more than four jets inherently qualitative. Four-jet physics is at the borderline between these two regimes, and is also the threshold for the onset of a new interesting phenomenon, namely the simultaneous scattering of more than one parton per hadron, the so called multi-parton scattering. Double-parton scattering, in which two partons from one proton collide independently with two partons from one antiproton gives rise to four-jet final states with characteristic topologies, which can allow them to be distinguished from the ordinary four-jet events on a statistical basis even if the total rate is not too large. This interesting phenomenon probes the correlation of partons inside the hadron, and can in principle provide valuable information about the hadronic structure.

Studies of four-jet production at CERN Collider energies were first presented by Kunszt and by Kunszt and Stirling[2]. The experimental data were presented in [3]. The phenomenon of multi-parton scattering was discussed in ref.[4]. In this article we will update the predictions for four-jet production at the energy of 1.8 TeV, that is the energy at which the Fermilab Tevatron Collider is currently operating, and we will compare production rates and distributions for the two distinct production mechanisms. In Section 2 I will briefly describe the computations. For the calculation of the two-to-four processes I will use a recently developed scheme to approximate multi-jet matrix elements[5,6]. These approximate matrix elements provide an excellent estimate of the exact ones[7], both for the rates and for the distributions, and save of the order of two orders of magnitude in computation time. In Section 3 I will present the results, in the form of plots of distributions for two distinct sets of experimental cuts. I will discuss the effect of the cuts on the two processes – two-to-four and double-parton – and I will show the influence on the results of changes in the choice of the Q^2 scale.

2 The Calculation of the Cross Sections

The evaluation of the exact matrix elements for the two-to-four processes requires a very long computer time. In the calculations presented here I will use an approximation scheme initially developed by Maxwell[5] for the description of gluon processes and recently extended to processes with quarks[6]. For a detailed description of this approximation scheme see those references. Here I will just point out the key features. Among the various helicity amplitudes contributing to a given process, the most helicity-violating ones have the simplest structure[8]. In the limit in which two partons are collinear[9], the total amplitude behaves exactly as these simple helicity amplitudes, up to an overall kinematical factor which can be easily calculated, as described in ref.[5,6]. The approximation I will be employing consists in continuing the overall factor to kinematical regions away from the collinear limit, thus providing a very good and computationally simple estimate of the full matrix element over all of the phase space. Even though fluctuations may arise for some pathological kinematical configuration, integration over the phase space shows an agreement betweeen exact and approximate calculation within 10-20% both in total rates and differential distributions. As I will show at the end, this precision is much better than the one allowed by uncertainties in the choice of the Q^2 scale or of α_s. For example, an uncertainty of 15% in the value of α_s would automatically lead to a 60% uncertainty in the overall rate, because these processes are proportional to α_s^4.

In the calculations here presented I will add the contributions of three processes: $gg \to gggg$, $gq \to gggq$ and $gg \to ggq\bar{q}$. The approximation scheme presented above would allow us to add other processes as well, but their contribution to the rates is very small (few percent) and does not dominate in any region of phase space. Once again the few percent uncertainty due to this further approximation is largely overwhelmed by the overall normalization uncertainty mentioned above.

As for the description of the double parton scattering processes, I will follow the prescriptions of ref.[4]. The cross-section for four-jet events generated by double parton scattering can be written as follows:

$$d\sigma \equiv \frac{1}{2\sigma_0} \int dx_1\, dy_1\, dx_2\, dy_2 \sum_{i,j,k,l} f_{ij}(x_1, Q^2; y_1, K^2) f_{kl}(x_2, Q^2; y_2, K^2) d\sigma_{ik}\, d\sigma_{jl}, \tag{2.1}$$

where i, j and k, l are the two pairs of partons belonging to the two colliding hadrons, x_1, y_1 and x_2, y_2 are their momentum fractions and Q^2, K^2 are the momentum scales of the two partonic collisions. The parameter σ_0 has the dimensions of a cross section, and cannot be calculated at the moment from first principles. It is nevertheless expected to be of the order of the size of the colliding hadrons, and for the calculations presented here I will use $\sigma_0 = 50$mb. The momentum fractions obey the following obvious constraints:

$$x_1 + y_1 \leq 1, \qquad x_2 + y_2 \leq 1. \tag{2.2}$$

According to ref.[4] I have assumed a quasi-factorized form for the double-parton structure functions:

$$f_{ij}(x, Q^2; y, K^2) = f_i(x, Q^2)\, f_j(y, K^2)(1 - x - y)\theta(1 - x - y). \tag{2.3}$$

In the calculations I have included the contributions from all possible two-to-two subprocesses.

3 Results

In this Section I will present the results. In order to match the selection criteria used in analyzing the data, the differential cross sections were integrated over a constrained phase space. I chose two different sets of cuts which can reasonably be imposed on the high p_t sample collected by the detector operating at the Tevatron Collider. For each set there is a minimum p_t cut that each jet has to satisfy. Then there is a pseudo-rapidity (η) cut, to mantain the event within a window covered by the apparatus. Finally there is an angular cut between jets. This I choose in two possible ways: I choose to either constraint the angle between each pair of jets to be larger than a given value, or to constrain the separation between jets in the $\eta - \phi$ space, ϕ being the angular separation in the transverse plane. The distance in the $\eta - \phi$ space is defined by:

$$\Delta R = (\Delta\phi^2 + \Delta\eta^2)^{1/2}. \tag{3.1}$$

This angular cut is particularly efficient for a detector like CDF, operating at the Tevatron Collider, in which the hadronic calorimeter is segmented into intervals of constant rapidity.

The two sets of cuts are the following:

- SET 1:

$$p_t \geq 25GeV, \qquad |\eta| \leq 3.5, \qquad \Delta R \geq 0.8\ . \tag{3.2}$$

- SET 2:

$$p_t \geq 25GeV, \qquad |\eta| \leq 1.1, \qquad \cos\theta_{ij} \leq 0.8\ . \tag{3.3}$$

To properly compare the results of this calculation with the data, one has to understand the relation between the momentum of the parton that we are using here and the momentum of the jet that will result after hadronization. This is fundamental in comparing total rates, which are heavily affected by the choice of p_t cuts, as we will see in the following.

In figs. 1 and 2 I show plots obtained from the study of two-to-four scattering. The following differential distributions are displayed: (a) inclusive p_t distribution (four entries per event), (b) $\cos\theta^*_{23}$ distribution, (c) ϕ_{min} distribution and (d) p_{out} distribution. The structure functions are those obtained by Duke and Owens[10], with Λ=200MeV.

p_{out} is defined as the transverse momentum out of the plane passing through the beam and the jet of largest p_t:

$$p_{out} = \frac{1}{2}\sum_i |p^i_{out}|, \tag{3.4}$$

p^i_{out} being the projection of the transverse momentum of the i-th jet on the normal to the plane. This distribution is strongly peaked around the minimum p_t allowed by the cuts in the case of double parton scattering, while it is smoother for the two-to-four processes. This feature makes it a good place to look for a double parton scattering signal.

$\cos\theta^*_{23}$ is the cosine of the angle between the second and third most energetic jets in the center of mass frame of the four-jet system. It is chosen because it exhibits a typical brehmstrahlung spectrum, totally different from phase-space predictions. It is a good qualitative check of QCD.

ϕ_{min} is the minimum angle in the transverse plane between the largest p_t jet and any of the other three jets. In the case of two-to-four processes this distributions extends from 0 to π, while for double parton scattering it dies off for $\phi_{min} > \pi/2$. This distribution can then be useful to calibrate the total normalization of the pure two-to-four processes.

To exhibit the peculiarity of the QCD behaviour, in Fig.1 I superimpose the behaviour expected from a phase-space calculation (constant matrix elements, cross-section weighted by the four-body phase-space density and by the structure-functions). The phase-space rate has been normalized to the QCD rate.

The calculation of total rates suggests that the double-parton scattering can at best represent a 10% of the total four-jet sample. To isolate this signal it is then necessary to impose further cuts on the four-jet events. The typical topology of a four-jet event coming from double-parton scattering is given by two pairs of jets balancing each other in momentum in the transverse plane. I will simulate this topology by requiring the transverse momenta of the four jets to satisfy the following requirements:

$$\frac{p_t^1 - p_t^2}{p_t^1 + p_t^2} < 10\% \qquad \frac{p_t^3 - p_t^4}{p_t^3 + p_t^4} < 10\% \tag{3.5}$$

$$|\pi - |\phi_1 - \phi_2|| < \pi/6 \qquad |\pi - |\phi_3 - \phi_4|| < \pi/6 \tag{3.6}$$

All of the events generated by double-parton scattering will pass these cuts, while only a part of the ordinary 2 to 4 processes will.

The effect of p_t cuts on the total production rates is most clearly shown in Table 1, where the total rates for the two processes are given with various choices of minimum p_t and maximum rapidity. As it is clear from the table, the p_t cuts affect more dramatically the rate for double parton scattering, and hence it is advised to keep the p_t thresholds as low as possible to hope for some statistically significant signal coming from this process. In the last column of the Table 1 I show the total rate from two-to-four events which pass the additional cuts given by Eqs.(3.5)-(3.6). The data contained in Table 1 are plotted in Fig.3 and Fig.4, corresponding to the two values of maximum rapidity 1.0 and 3.5 .

As it should be expected these cuts reduce the rate substancially, leaving open the possibility of isolating the signal from double-parton scattering. As we already pointed out, the lower the p_t threshold the higher the chance of achieving this. Since the $p_t^{min} = 10$ GeV threshold might

p_t^{min}	η^{max}	$\sigma_{QCD}(nb)$	$\sigma_{DP}(nb)$	$\sigma_{QCD}^{cut}(nb)$
10	3.5	7600	550	1100
	1.0	156	6.8	22
15	3.5	890	30	140
	1.0	29	0.6	3.8
20	3.5	163	3.0	29
	1.0	7.6	0.08	1.0
25	3.5	40	0.5	7.4
	1.0	2.5	0.02	0.4
30	3.5	12	0.09	2.1
	1.0	0.9	0.004	0.15

Table 1: Four-jet total cross-sections, calculated with the cuts given in the first two columns and $\Delta R > 0.8$. σ_{QCD} is the production rate from two-to-four processes, σ_{DP} is the production rate from double-parton scattering and σ_{QCD}^{cut} is the production rate for QCD events which pass the additional cuts in Eqs.(3.5)-(3.6).

be too low for the experimental analysis, I choose to compare two samples of double-parton scattering and two-to-four scattering with the cuts of Eqs.(3.5)-(3.6) and a p_t threshold of 15GeV. In addition the pseudo-rapidity extends up to 3.5, and the jet-jet separation in the $\eta - \phi$ space is larger than 0.8. The distributions are given in Fig.5 . Although the chances of separating the two components seem quite slim, two distributions have a rather different shape: ϕ_{min} and p_{out}. This is encouraging, because one might expect a priori that the cuts on the final state are so tight that the distributions will look the same regardless of the production mechanism. The ϕ_{min} distribution for the two-to-four processes has a strong peak at about $\pi/3$, as a result of the enhancement of collinear emission. On the contrary the same distribution for the double-parton scattering is more flat, as a result of the incoherence of the two scattering processes. The same physical arguments explain the longer tail in the p_{out} distribution for the double-parton processes. Unfortunately the rate at large p_{out}, where the double-parton scattering dominates, is very low given the present statistics.

Needless to say, only a detailed study of the fragmentation process plus a detector simulation will enable us to understand if the double-parton signal can be isolated.

The proper value of Q^2 to be used in the evaluation of the structure functions and of α_s can only be fixed thorugh a next-to-leading order calculation. At this time this is beyond any hope,

Q^2	$\sigma_{tot}(nb)$
$E_t^2/16$	40
$E_t^2/8$	30
$E_t^2/4$	23
$E_t^2/2$	17
E_t^2	13

Table 2: Q^2 dependence of the four-jet QCD cross-section.

and for the calculation of the differential distributions shown in Figs.1-5 I chose $Q^2 = E_t^2/16$, *i.e.* the square of the average p_t. For the calculation of the double-parton events I used $Q^2 = p_t^2$ for each of the two elementary scattering processes.

The effect of a change in choice of Q^2 is shown in Table 2, where I allow Q^2 to vary up to $Q^2 = E_t^2$. While this is most probably an excessively large scale, more conservative choices still give rise to large variations justifying the use of the approximate matrix elements as discussed in the previous Section.

In conclusion, we presented predictions for the production of four-jet events at $\sqrt{s} = 1.8 TeV$. We compared the rates for the production through ordinary QCD processes and through double-parton scattering, finding that even if this last process is strongly suppressed, proper cuts on the final state might give rise to a detectable signal. For this to be possible, however, the p_t thresholds have to be kept quite low, possibly below 15 GeV.

Acknowledgements: I wish to thank D. Brown, J. Huth, M. Shapiro and, in particular, S. Parke, for many discussions on this subject.

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Figure Captions:

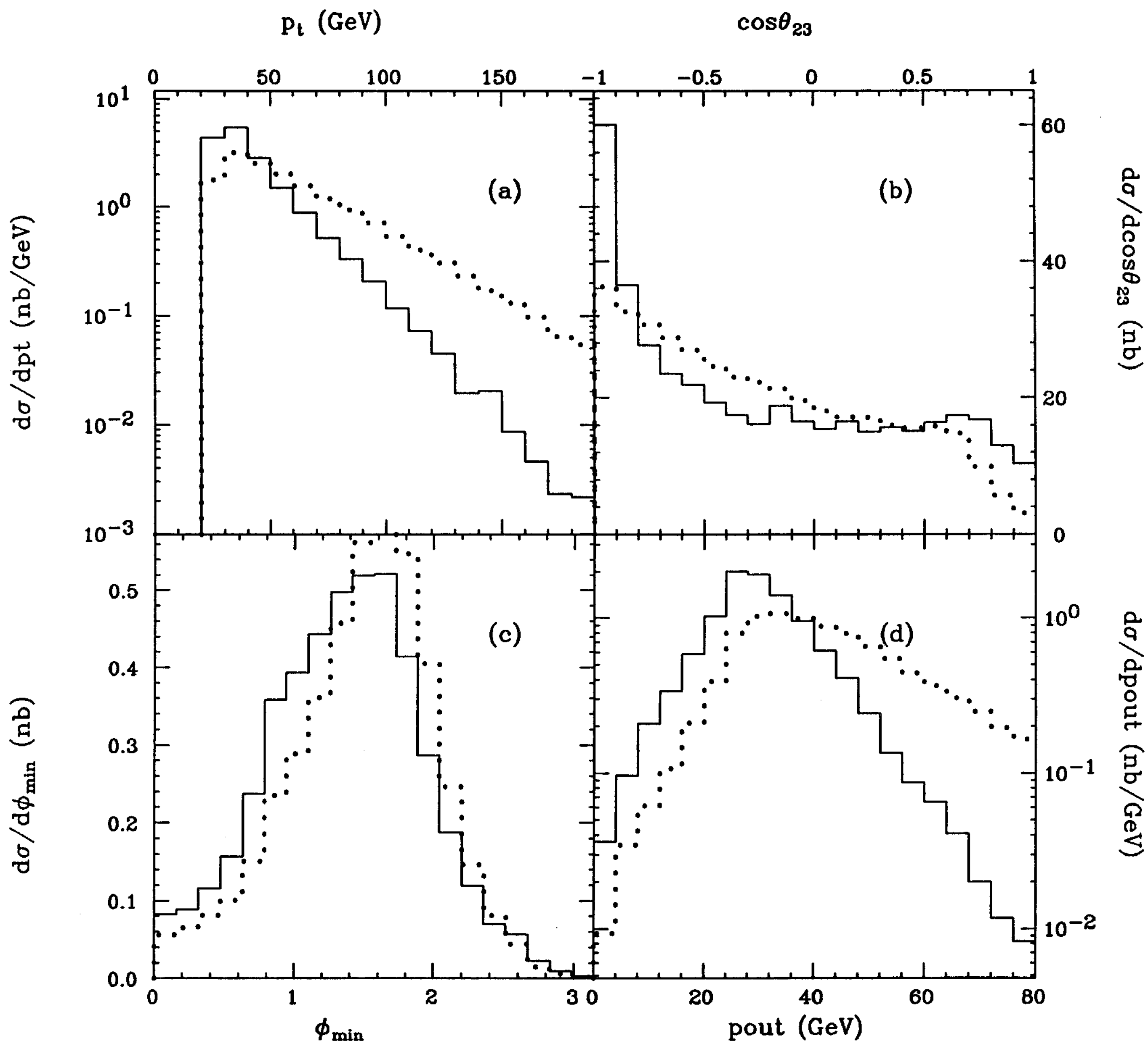
Figure 1(a)-(d): Differential cross-sections for two-to-four processes from QCD (solid line) and from phase-space (dotted line), with the Set I of cuts (Eq.(3.2)).

Figure 2(a)-(d): Differential cross-sections for two-to-four processes from QCD with the Set II of cuts (Eq.(3.3)).

Figure 3 Total cross-sections for two-to-four processes (solid line), two-to-four processes with the additional cuts of Eqs.(3.5)-(3.6) (dashed line) and double-parton scattering (dotted line), plotted against the p_t threshold. $\Delta R > 0.8$ and $\eta < 3.5$.

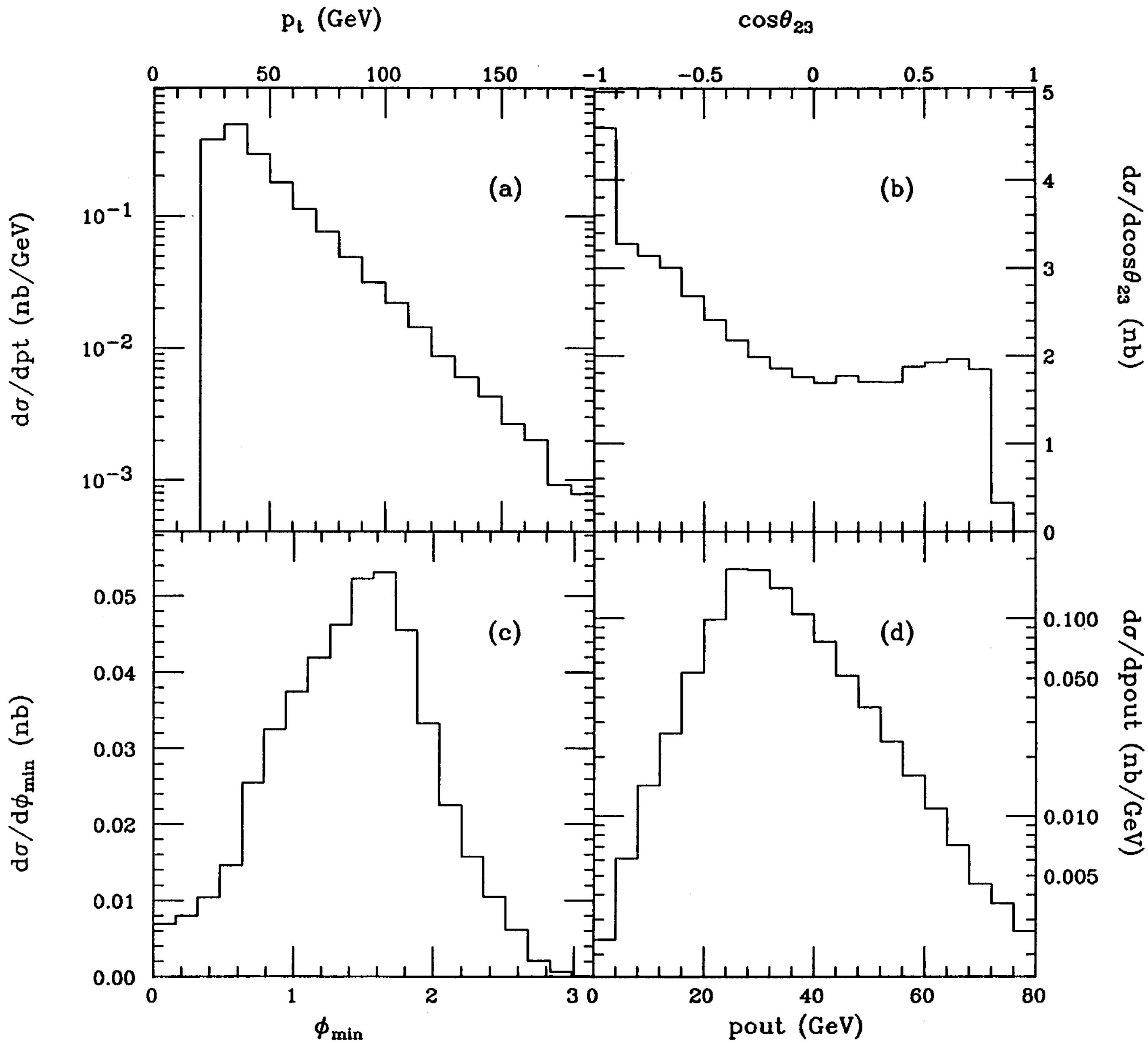
Figure 4 Total cross-sections for two-to-four processes (solid line), two-to-four processes with the additional cuts of Eqs.(3.5)-(3.6) (dashed line) and double-parton scattering (dotted line), plotted against the p_t threshold. $\Delta R > 0.8$ and $\eta < 1.0$.

Figure 5(a)-(d): Differential cross-sections for two-to-four processes (solid line) and double-parton scattering (dotted line). The cuts are as follows: $p_t^{min} = 15$GeV, $\eta^{max} = 3.5$, $\Delta R > 0.8$, plus the *back-to-back* cuts given in Eqs.(3.5,3.6).



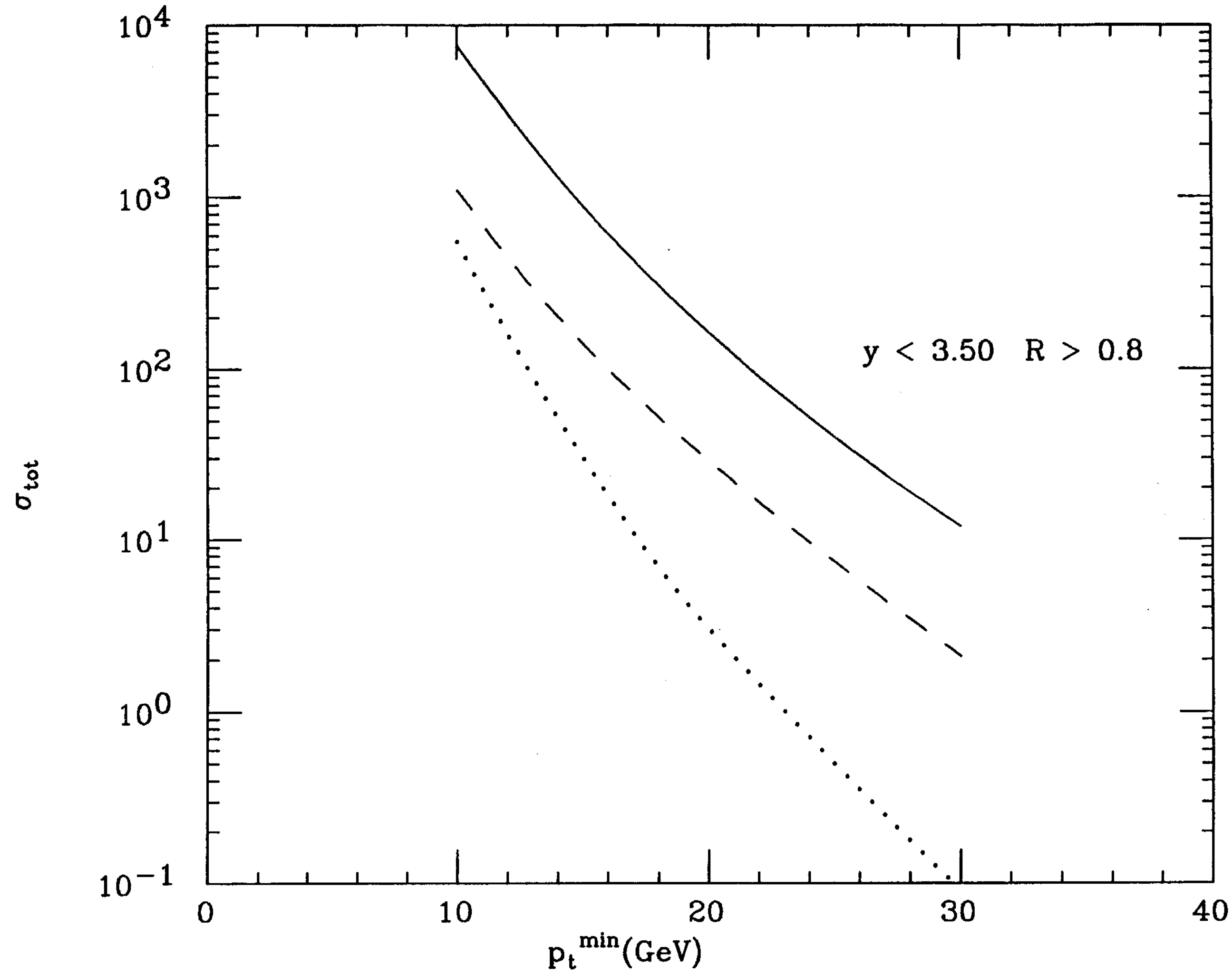
p_t (GeV)
cosθ_23
dσ/dpt (nb/GeV)
dσ/dcosθ_23 (nb)
dσ/dφ_min (nb)
dσ/dpout (nb/GeV)
(a)
(b)
(c)
(d)
φ_min
pout (GeV)

Fig.1



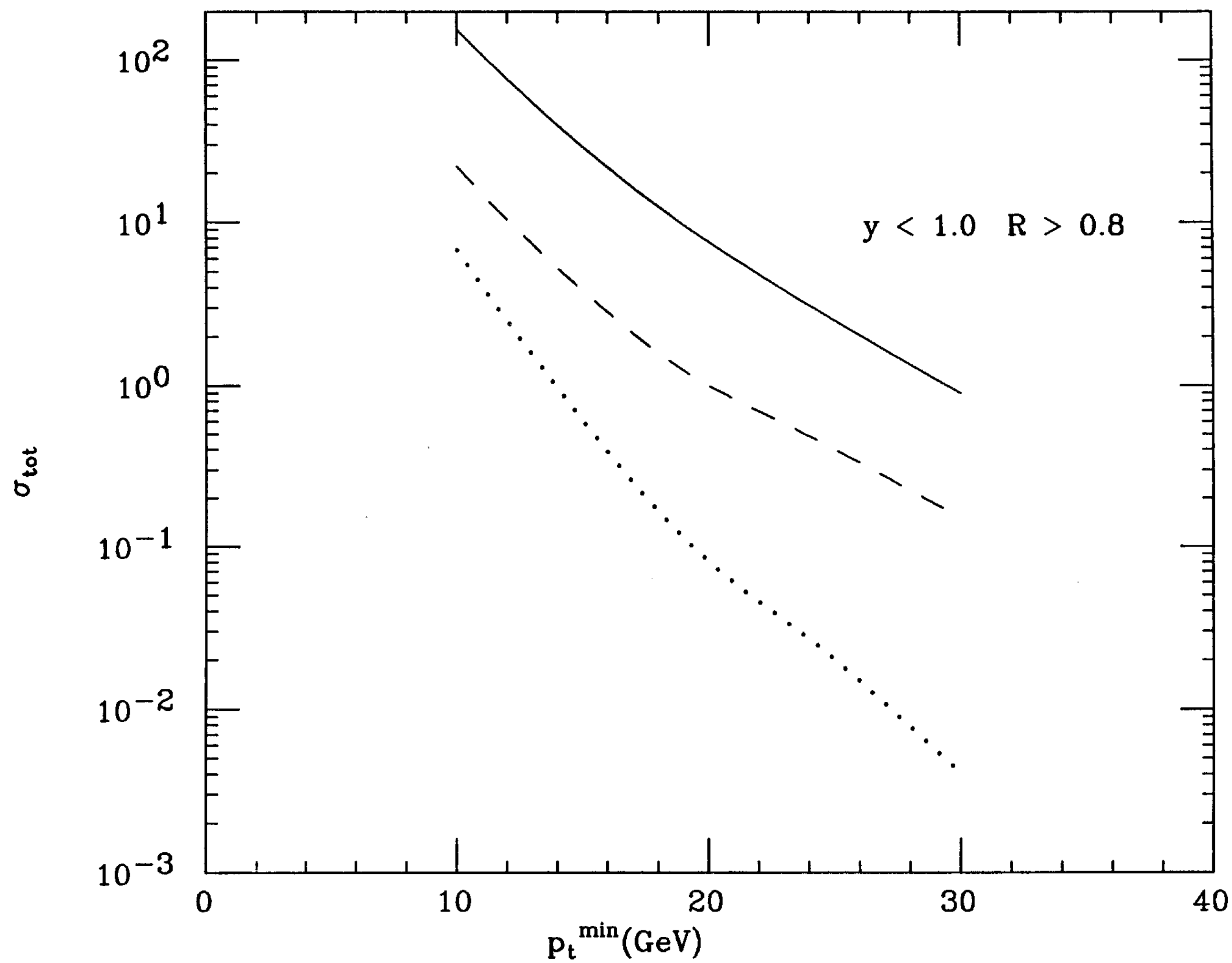
p_t (GeV)
cosθ23
0
50
100
150
-1
-0.5
0
0.5
1
5
4
3
2
1
0
(a)
(b)
10^-1
10^-2
10^-3
dσ/dpt (nb/GeV)
dσ/dcosθ23 (nb)
0.05
0.04
0.03
0.02
0.01
0.00
(c)
(d)
0.100
0.050
0.010
0.005
dσ/dφmin (nb)
dσ/dpout (nb/GeV)
0
1
2
3
0
20
40
60
80
φmin
pout (GeV)

Fig.2



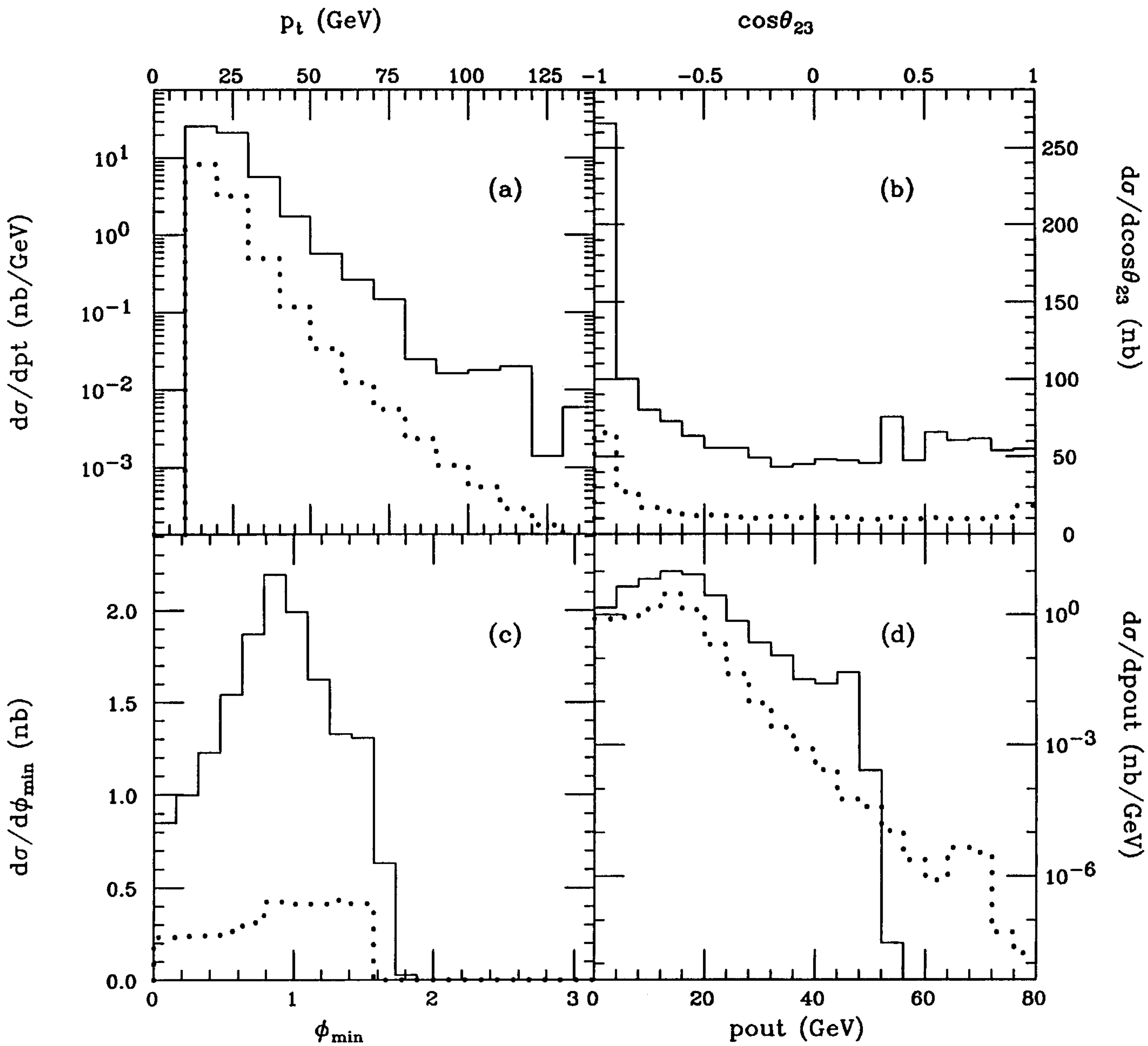
10^4
10^3
10^2
10^1
10^0
10^{-1}
σ_{tot}
y < 3.50 R > 0.8
0
10
20
30
40
p_t^{min}(GeV)

Fig.3



$y < 1.0 \quad R > 0.8$
σ_{tot}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
10
20
30
40
p_t^{min}(GeV)

Fig.4



p_t (GeV)
$\cos\theta_{23}$
0
25
50
75
100
125
−1
−0.5
0
0.5
1
(a)
(b)
$d\sigma/dpt$ (nb/GeV)
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
$d\sigma/d\cos\theta_{23}$ (nb)
250
200
150
100
50
0
(c)
(d)
$d\sigma/d\phi_{min}$ (nb)
2.0
1.5
1.0
0.5
0.0
$d\sigma/dpout$ (nb/GeV)
10^{0}
10^{-3}
10^{-6}
0
1
2
3
0
20
40
60
80
ϕ_{min}
pout (GeV)

Fig.5