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M van de Meent

Mathematical Sciences, University of Southampton, United Kingdom

E-mail: `M.vandeMeent@soton.ac.uk`

Abstract. We give a status report on the progress in modelling extreme mass-ratio inspiral (EMRI) evolution using the gravitational self-force formalism.

1. Introduction

Extreme mass ratio inspirals (EMRIs) are black hole binaries where there primary object is a supermassive black hole of 10^5–10^6 Solar masses and the secondary object a stellar massive black hole of 10–60 Solar masses. EMRIs have been identified as key potential source for the LISA mission[1, 2]. In contrast to comparable mass black hole binaries, EMRIs evolve very slowly spending a large number 10^3–10^5 orbital cycles in the LISA sensitivity band before finally merging. As a consequence matched filtering can be used to make unprecedented accurate measurements of the orbital parameters of the systems obtaining typical relative errors of 1 part in 10^3–10^4 [3]. Beside the masses, spins, eccentricity, and inclination, it will also be possible to measure some of the higher gravitational multipoles of the primary object allowing tests of the no-hair theorem of general relativity [4].

In order to realize such measurements we need accurate theoretical models for the evolution of EMRIs. However the modelling of EMRIs introduces some novel complications. On the one hand, the large disparity in length and time scales (essentially of order the mass-ratio) makes the application of full numerical relativity unrealistic. On the other hand, EMRIs are highly relativistic making post-Newtonian models unreliably. Instead of these familiar cornerstones black hole binary modelling, the modelling of EMRIs requires the application of multi time and length scale perturbation methods to obtain a systematic expansion of the binary motion in terms of the ratio of the two masses. At zeroth order, the secondary object follows a free falling geodesic. Higher order corrections then take the form of an effective force generated by the secondary's own gravitational field, the *gravitational self-force* (GSF).

In this note we summarize the current progress in modelling EMRIs using gravitational self-force.

2. Modelling requirements

Population models suggest that EMRIs, unlike comparable mass binaries, can enter the LISA sensitivity band with significant eccentricity $e \lesssim 0.8$, and almost any spin configuration. Consequently, to provide accurate waveforms for matched filtering we need the ability to model the evolution of an EMRI with generic eccentricity, inclination and spins up to an phase accuracy

of less than ~ 0.1 radian over the course of the full evolution covering 10^4–10^6 radians. How far do we need to go in the systematic expansion of the equations of motion to achieve this? This can be answered using a two time scale expansion [5].

For this purpose it has proven useful to adopt coordinates on the binary phase space using an osculating elements scheme. At zeroth order, the spacetime can be approximated by the background (Kerr) spacetime generated by the primary object. Each point in the binary phase space can then be identified by a geodesic tangent to that point. To identify each geodesic we need a set of invariants $\vec{P}$ (e.g. semi-latus rectum, eccentricity, and inclination). A further three phases $\vec{q}$ are then needed to identify the (radial, polar, azimuthal) position along the geodesic. In first order form, the formal expansion of the equations of motions then become,

$$
\begin{aligned}
\dot{\vec{q}}(\tau) &= \vec{\omega}(\vec{P}) + \epsilon \vec{g}_1(\vec{P},\vec{q}) + \epsilon^2 \vec{g}_2(\vec{P},\vec{q}) + O(\epsilon^3) & (1)\\
\dot{\vec{P}}(\tau) &= \epsilon \vec{G}_1(\vec{P},\vec{q}) + \epsilon^2 \vec{G}_2(\vec{P},\vec{q}) + O(\epsilon^3), & (2)
\end{aligned}
$$

where ϵ is the mass-ratio and the $\vec{g}_i$ and $\vec{G}_i$ encode the self-force corrections to the motion. Hinderer and Flanagan [5] performed a multi-timescale analysis of the solutions of this system leading to a systematic post-Adiabatic expansion of the phase evolution.

2.1. Adiabatic approximation
The leading term in the expansion, is the adiabatic approximation, where we only include the orbit averaged changes $\langle \vec{G}_1 \rangle(\vec{P})$ to the orbital invariants $\vec{P}$, which can be calculated using the gravitational wave flux to infinity and down the horizon. This approximation contains all the terms in the phase evolution up to $O(\epsilon^{-1})$. Consequently, we expect a phase error of at least $O(1)$ over a full inspiral. In fact, orbital resonances (see below) increase this error to $O(\epsilon^{-1/2})$. This might be accurate enough for the detection of EMRI signals in the LISA data, but will certainly not be enough for accurate parameter estimation.

2.2. 1-Post Adiabatic correction
The next (full) step in the expansion of the phase evolution is the 1-post-adiabatic (1PA) approximation, which includes all terms in the phase evolution up to $O(\epsilon^0)$, leading a phase error of at most $O(\epsilon^{1/2})$, which should be enough for accurate parameter estimation. Hinderer and Flanagan [5] showed that to obtain the 1PA approximations one needs the full oscillatory first order corrections, $\vec{G}_1(\vec{P},\vec{q})$ and $\vec{g}_1(\vec{P},\vec{q})$, and the orbit average second order change to $\vec{P}$, $\langle \vec{G}_2 \rangle(\vec{P})$. The former can be obtained from a local first order metric perturbation, whereas the later requires second order perturbation theory.

2.3. 1/2-PA correction: orbital resonances
The above picture breaks down in the case of a 'resonance' between the evolution of the different orbital phases, i.e. the situation where two or more phases evolve in tandem. A resonance allows normally oscillatory terms in the equations of motion to build up coherently over many orbits leading to an $O(\epsilon^{1/2})$ amplification of certain oscillatory effects [6], leading to the introduction of a 1/2-PA term in the phase evolution. The evolution of certain extrinsic parameters that miss a leading order average term such as the center-of-mass velocity, is in fact dominated by this correction [7, 8].

3. State-of-the-art
A dedicated community—informally knwown as the "Capra community"—has formed around the challenge of modelling EMRIs using the gravitational self-force formalism. The first major

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breakthrough in the formalism came with the introduction of the MiSaTaQuWa [9][10] formula for the calculation of the first order self-force. In the two decades since, a solid rigorous theoretical framework has arisen for the systematic definition of the perturbative corrections to the equations of motion of EMRIs [11][12].

3.1. Flux calculations
The orbital averaged changes to the constants of motion, $\langle \vec{G}_1 \rangle(\vec{P})$, can extracted directly from the average gravitational wave flux to infinity and down the horizon [13, 14, 15]. The fluxes of energy and angular momentum can easily be extracted from solutions of the Teukolsky equation [16]. With some more effort changes to the Carter can be obtained as well [17]. Techniques for obtaining numerical solutions to the Teukolsky equation have been developed [18, 19, 20] that allow for the calculation of the averaged fluxes sourced by arbitrary geodesics, and thereby $\langle \vec{G}_1 \rangle(\vec{P})$.

3.2. First-order
The first order corrections $\vec{G}_1(\vec{P}, \vec{q})$ and $\vec{g}_1(\vec{P}, \vec{q})$ to the equations of motion split into two distinct parts. The first is the so called 'spin force' and depends only on the spin of the secondary and is independent of the metric perturbation it generates. The analytic expression for this force was formulated already in 1951 by Papapetrou [21]. The second is the gravitational self-force proper, which depends on the metric perturbation generated by the secondary, but is insensitive to its spin.

Various practical techniques have been developed for calculating the gravitational self-force [22]. In the case of a Schwarzschild primary, the first calculations for circular [23] and eccentric [24] orbits were published in 2007 and 2011, respectively. We now have a wide range of techniques [23][24][25, 26, 27, 28, 29] using both time domain and frequency domain methods and a variety of regularization methods for calculating the gravitational self-force on generic geodesics in Schwarzschild spacetime.

Calculation of the self-force in Kerr spacetime for a long time was considered a major problem of the program, because the linearized Einstein equation is not separable on a Kerr background. This issue can be circumvented by reconstructing the metric perturbation from solutions of the Teukolsky equation. We now have a rigorous understanding [30, 31] of how the self-force can be obtained from such a reconstructed metric. After much preliminary work [32, 33, 34, 35, 36], we can now calculate the gravitational self-force for any equatorial orbit around a Kerr black hole [37, 38], with no obvious obstructions for extending the calculations to generic (inclined) orbits.

3.3. Second-order GSF
Calculation of the second order self-force remains one of the major outstanding challenges of the self-force program, but much progress has been made. The formalism for the calculation of the second order self-force is now in place [39, 40, 41, 42, 12, 43], and major steps are being made towards solving practical obstructions [44, 45] for the first numerical calculations.

3.4. Evolution
Although a fully consistent 1PA evolution of an EMRI requires the orbit averaged second order self-force, it is of practical use to study the implementation of an osculating elements scheme sourced by only the first order force. In the case of a non-spinning Schwarzschild primary, the entire EMRI parameter space has now been covered with first order self-force data, allowing the evolution of inspirals [46, 47]. For example, this allows us to gauge the error made by using just using the adiabatic approximation to several tens of radians over a full inspiral.

3.4.1. Resonances The occurrence of orbital resonances acts as a further complication to the evolution of EMRIs. From symmetry considerations, it is clear that resonances can only impact the evolution for inspirals that are both inclined and eccentric. The general impact of a resonance of the EMRI evolution has been studied [6, 48, 49], and can be characterized by a jump in the evolution of the orbital invariants at the resonance, the size of which can be obtained from the orbit averaged quantities $\langle \vec{G}_1 \rangle$ [49].

A potential further complication would occur if the self-force dynamics of an EMRI could get locked in a sustained resonance, leading to major qualitative change to the evolution of the orbit over the inspiral time scale. The conditions necessary for such resonant locking to occur were studied [49], and based on the (limited) available data [20] it appears unlikely that they are satisfied anywhere in the parameter space. Moreover, if they could one would expect resonant locking to occur only in a small fraction (proportional to $\epsilon^{1/2}$) of all EMRIs [49].

4. Interaction with other modelling approaches

In the absence of fully modelled gravitational waveforms produced by EMRIs, it is important to be able to compare intermediate results between different calculations; both between different self-force calculations and between completely different modelling approaches such as post-Newtonian (PN) theory, Numerical relativity (NR), and effective one body (EOB) theory. This requires the identification and calculation of coordinate invariant (physical) observables that are completely independent of the used approximation scheme.

The gravitational wave fluxes are an example of such quantities and have long been used compared between different approaches. However, they are completely insensitive to the subtleties of the conserved dynamics. This requires different invariants such as the Detweiler-Sago-Barack redshift invariant [25, 24], the frequency shift of the inner most stable orbit [50, 51, 52], the periapsis advance of (nearly) circular orbits [53, 24, 52], and corrections to the precession of the secondary spin [54, 55].

Calculation of such quantities has allowed valuable validation of the self-force formalism. Successful comparison between self-force and numerical relativity results has, for example, taught use that even at first order some self-force results remain fairly accurate at even comparable mass-ratios [56, 57, 58, 59].

In recent years, self-force results have become increasingly useful in aiding the development of other modelling approaches. Self-force calculations have allow the calculation of the linear in mass-ration part of the post-Newtonian expansionist to very high (22.5) PN order, both through numerical [60, 61, 62, 63, 64, 65, 37, 52] and analytical [66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76] techniques. Moreover, highly accurate self-force calculations of periapsis advance [52] have allowed to discriminate between competing derivations of the 4PN equations of motion.

Finally, self-force calculations of various invariants have allowed calibration of various EOB potentials [77, 78, 69, 71, 79, 80].

5. Outlook

Although much progress has been made in the modelling of EMRIs over the last two decades, a lot of work remains to be done. We should soon have the calculation tools to calculate the first-order gravitational self-force on arbitrary geodesic orbits. However, it will still be a challenge to fill the entire four dimensional EMRI parameter space with data, since high eccentricity/high inclination data points currently take up to 1000s of CPU hours. We will need to both ncreases in efficiency of the calculation, and optimize the strategy for covering the entire parameter space.

At second order, calculational methods are still in its infancy. Once the simplest case of the calculation of the second order self-force on (quasi-)circular orbits on a Schwarzschild background has been achieved, this will need to be extended to general orbits on Kerr backgrounds.

Achieving high enough computational efficiency to fill the parameter space with second order data will be an even greater challenge than at first order.

Having covered the EMRI parameter space with self-force data, we will be in a position to model the evolution of EMRIs. The basic techniques for this have been implemented and tested. A remaining open question concerns the fact that formally the self-force is not just a local function of the parameter space, but depends on the entire past history of the EMRI system. By approximating the local self-force by that generated by a tangent geodesic, we are making an error of currently unknown size. This needs to be validated by comparison with time domain self-consistent evolution of an EMRI. Techniques for this are being developed. Initial investigations with scalar toy models suggest this error may be negligible.

Finally, once inspiral evolutions can be computed, waveforms need to be generated. Time domain integration of the Teukolsky equation over an entire inspiral will be costly and slow. In principle, it should be possible to synthesize the waveform from frequency domain data using a two timescale expansion. It remains to be seen if this can be done fast enough to generate waveforms on-the-fly for LISA data analysis. It is likely that effective surrogate (kludge) models will need to be developed for data analysis purposes.

Acknowledgments

References
[1] Amaro-Seoane P, Gair J R, Freitag M, Coleman Miller M, Mandel I, Cutler C J and Babak S 2007 *Class. Quant. Grav.* **24** R113–R169 (*Preprint* `astro-ph/0703495`)
[2] LISA Consortium 2017 LISA: Laser interferometer space antenna URL
    `https://www.lisamission.org/proposal/LISA.pdf`
[3] Barack L and Cutler C 2004 *Phys. Rev. D* **69** 082005 (*Preprint* `gr-qc/0310125`)
[4] Barack L and Cutler C 2007 *Phys. Rev. D* **75** 042003 (*Preprint* `gr-qc/0612029`)
[5] Hinderer T and Flanagan E E 2008 *Phys. Rev. D* **D78** 064028 (*Preprint* `0805.3337`)
[6] Flanagan E E and Hinderer T 2012 *Phys. Rev. Lett.* **109** 071102 (*Preprint* `1009.4923`)
[7] Hirata C M 2011 *Phys. Rev. D* **83** 104024 (*Preprint* `1011.4987`)
[8] van de Meent M 2014 *Phys. Rev. D* **90** 044027 (*Preprint* `1406.2594`)
[9] Mino Y, Sasaki M and Tanaka T 1997 *Phys. Rev. D* **55** 3457–3476 (*Preprint* `gr-qc/9606018`)
[10] Quinn T C and Wald R M 1997 *Phys. Rev. D* **56** 3381–3394 (*Preprint* `gr-qc/9610053`)
[11] Poisson E, Pound A and Vega I 2011 *Living Rev. Rel.* **14** 7 (*Preprint* `1102.0529`)
[12] Pound A 2015 *Motion of small objects in curved spacetimes: An introduction to gravitational self-force* (*Fund. Theor. Phys.* vol 179) pp 399–486 (*Preprint* `1506.06245`)
[13] Mino Y 2003 *Phys. Rev. D* **67** 084027 (*Preprint* `gr-qc/0302075`)
[14] Mino Y 2005 *Prog. Theor. Phys.* **113** 733–761 (*Preprint* `gr-qc/0506003`)
[15] Mino Y 2005 *Classical Quant. Grav.* **22** S717–S738 (*Preprint* `gr-qc/0506002`)
[16] Teukolsky S A and Press W H 1974 *Astrophys. J.* **193** 443–461
[17] Sago N, Tanaka T, Hikida W, Ganz K and Nakano H 2006 *Prog. Theor. Phys.* **115** 873–907 (*Preprint* `gr-qc/0511151`)
[18] Drasco S and Hughes S A 2006 *Phys. Rev. D* **73** 024027 (*Preprint* `gr-qc/0509101`)
[19] Fujita R and Hikida W 2009 *Classical Quant. Grav.* **26** 135002 (*Preprint* `0906.1420`)
[20] Flanagan E E, Hughes S A and Ruangsri U 2014 *Phys. Rev. D* **89** 084028 (*Preprint* `1208.3906`)
[21] Papapetrou A 1951 *Proc. Roy. Soc. Lond.* **A209** 248–258
[22] Wardell B and Gopakumar A 2015 **179** 487–522 (*Preprint* `1501.07322`)
[23] Barack L and Sago N 2007 *Phys. Rev. D* **75** 064021 (*Preprint* `gr-qc/0701069`)
[24] Barack L and Sago N 2011 *Phys. Rev. D* **83** 084023 (*Preprint* `1101.3331`)
[25] Detweiler S L 2008 *Phys. Rev. D* **77** 124026 (*Preprint* `0804.3529`)
[26] Hopper S and Evans C R 2010 *Phys. Rev. D* **82** 084010 (*Preprint* `1006.4907`)
[27] Akcay S 2011 *Phys. Rev. D* **83** 124026 (*Preprint* `1012.5860`)
[28] Akcay S, Warburton N and Barack L 2013 *Phys. Rev. D* **88** 104009 (*Preprint* `1308.5223`)
[29] Osburn T, Forseth E, Evans C R and Hopper S 2014 *Phys. Rev. D* **90** 104031 (*Preprint* `1409.4419`)

[30] Pound A, Merlin C and Barack L 2014 *Phys. Rev. D* **89** 024009 (*Preprint* 1310.1513)
[31] Merlin C, Ori A, Barack L, Pound A and van de Meent M 2016 *Phys. Rev. D* **94** 104066 (*Preprint* 1609.01227)
[32] Ori A 2003 *Phys. Rev. D* **67** 124010 (*Preprint* gr-qc/0207045)
[33] Lousto C O and Whiting B F 2002 *Phys. Rev. D* **66** 024026 (*Preprint* gr-qc/0203061)
[34] Keidl T S, Friedman J L and Wiseman A G 2007 *Phys. Rev. D* **75** 124009 (*Preprint* gr-qc/0611072)
[35] Keidl T S, Shah A G, Friedman J L, Kim D H and Price L R 2010 *Phys. Rev. D* **82** 124012 (*Preprint* 1004.2276)
[36] Shah A G, Keidl T S, Friedman J L, Kim D H and Price L R 2011 *Phys. Rev. D* **83** 064018 (*Preprint* 1009.4876)
[37] van de Meent M and Shah A G 2015 *Phys. Rev. D* **92** 064025 (*Preprint* 1506.04755)
[38] van de Meent M 2016 *Phys. Rev. D* **94** 044034 (*Preprint* 1606.06297)
[39] Pound A 2012 *Phys. Rev. Lett.* **109** 051101 (*Preprint* 1201.5089)
[40] Gralla S E 2012 *Phys. Rev. D* **85** 124011 (*Preprint* 1203.3189)
[41] Detweiler S 2012 *Phys. Rev. D* **85** 044048 (*Preprint* 1107.2098)
[42] Pound A and Miller J 2014 *Phys. Rev. D* **89** 104020 (*Preprint* 1403.1843)
[43] Pound A 2015 *Phys. Rev. D* **92** 044021 (*Preprint* 1506.02894)
[44] Miller J, Wardell B and Pound A 2016 *Phys. Rev. D* **94** 104018 (*Preprint* 1608.06783)
[45] Pound A 2015 *Phys. Rev.* **D92** 104047 (*Preprint* 1510.05172)
[46] Warburton N, Akcay S, Barack L, Gair J R and Sago N 2012 *Phys. Rev. D* **85** 061501 (*Preprint* 1111.6908)
[47] Osburn T, Warburton N and Evans C R 2016 *Phys. Rev.* **D93** 064024 (*Preprint* 1511.01498)
[48] Gair J, Yunes N and Bender C M 2012 *J. Math. Phys.* **53** 032503 (*Preprint* 1111.3605)
[49] van de Meent M 2014 *Phys. Rev. D* **89** 084033 (*Preprint* 1311.4457)
[50] Barack L and Sago N 2009 *Phys. Rev. Lett.* **102** 191101 (*Preprint* 0902.0573)
[51] Isoyama S, Barack L, Dolan S R, Le Tiec A, Nakano H *et al.* 2014 *Phys. Rev. Lett.* **113** 161101 (*Preprint* 1404.6133)
[52] van de Meent M 2017 *Phys. Rev. Lett.* **118** 011101 (*Preprint* 1610.03497)
[53] Barack L, Damour T and Sago N 2010 *Phys. Rev. D* **82** 084036 (*Preprint* 1008.0935)
[54] Dolan S R, Warburton N, Harte A I, Le Tiec A, Wardell B *et al.* 2014 *Phys. Rev. D* **89** 064011 (*Preprint* 1312.0775)
[55] Dolan S R, Nolan P, Ottewill A C, Warburton N and Wardell B 2015 *Phys. Rev. D* **91** 023009 (*Preprint* 1406.4890)
[56] Le Tiec A, Mrou A H, Barack L, Buonanno A, Pfeiffer H P, Sago N and Taracchini A 2011 *Phys. Rev. Lett.* **107** 141101 (*Preprint* 1106.3278)
[57] Le Tiec A, Buonanno A, Mroué A H, Pfeiffer H P, Hemberger D A, Lovelace G, Kidder L E, Scheel M A, Szilágyi B, Taylor N W and Teukolsky S A 2013 *Phys. Rev. D* **88** 124027 (*Preprint* 1309.0541)
[58] Zimmerman A, Lewis A G M and Pfeiffer H P 2016 *Phys. Rev. Lett.* **117** 191101 (*Preprint* 1606.08056)
[59] Lewis A G M, Zimmerman A and Pfeiffer H P 2016 (*Preprint* 1611.03418)
[60] Akcay S, Barack L, Damour T and Sago N 2012 *Phys. Rev. D* **86** 104041 (*Preprint* 1209.0964)
[61] Akcay S and van de Meent M 2016 *Phys. Rev. D* **93** 064063 (*Preprint* 1512.03392)
[62] Shah A G, Friedman J L and Whiting B F 2014 *Phys. Rev. D* **89** 064042 (*Preprint* 1312.1952)
[63] Shah A G 2014 *Phys. Rev.* **D90** 044025 (*Preprint* 1403.2697)
[64] Shah A G and Pound A 2015 *Phys. Rev. D* **91**(12) 124022 (*Preprint* 1503.02414)
[65] Johnson-McDaniel N K, Shah A G and Whiting B F 2015 (*Preprint* 1503.02638)
[66] Bini D and Damour T 2013 *Phys. Rev.* **D87** 121501 (*Preprint* 1305.4884)
[67] Bini D and Damour T 2014 *Phys. Rev. D* **89** 104047 (*Preprint* 1403.2366)
[68] Bini D and Damour T 2014 *Phys. Rev. D* **90** 024039 (*Preprint* 1404.2747)
[69] Bini D and Damour T 2014 *Phys. Rev. D* **90** 124037 (*Preprint* 1409.6933)
[70] Bini D and Damour T 2015 *Phys. Rev. D* **91** 064050 (*Preprint* 1502.02450)
[71] Bini D, Damour T and Geralico A 2016 *Phys. Rev. D* **93** 064023 (*Preprint* 1511.04533)
[72] Bini D and Damour T 2015 *Phys. Rev. D* **91** 064064 (*Preprint* 1503.01272)
[73] Bini D, Damour T and Geralico A 2016 *Phys. Rev. D* **93** 104017 (*Preprint* 1601.02988)
[74] Kavanagh C, Ottewill A C and Wardell B 2015 *Phys. Rev. D* **92** 084025 (*Preprint* 1503.02334)
[75] Hopper S, Forseth E, Osburn T and Evans C R 2015 *Phys. Rev. D* **92** 044048 (*Preprint* 1506.04742)
[76] Kavanagh C, Ottewill A C and Wardell B 2016 *Phys. Rev. D* **93** 124038 (*Preprint* 1601.03394)
[77] Damour T 2010 *Phys. Rev. D* **81** 024017 (*Preprint* 0910.5533)
[78] Akcay S, Le Tiec A, Barack L, Sago N and Warburton N 2015 *Phys. Rev. D* **91** 124014 (*Preprint* 1503.01374)
[79] Barausse E, Buonanno A and Le Tiec A 2012 *Phys. Rev. D* **85** 064010 (*Preprint* 1111.5610)

[80] Le Tiec A, Barausse E and Buonanno A 2012 *Phys. Rev. Lett.* **108** 131103 (*Preprint* `1111.5609`)