Some Studies In Emergent Gravity And K-essence

by

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SOME STUDIES IN EMERGENT GRAVITY AND k-ESSENCE

ABSTRACT

The new findings are :

(1) An emergent gravity metric with *k*-essence scalar fields Φ and Born-Infeld type lagrangian is mapped into a metric similar to a blackhole of mass M that has swallowed a global monopole. However, here the field is not that of a monopole but of Φ . If $\Phi_{emergent}$ be solutions of the emergent field equations under cosmological boundary conditions at ∞ , then for $r \rightarrow \infty$, $\Phi_{emergent} / (2GM - 1)$ has exact correspondence with $\Phi(r,t) = \Phi_1(r) + \Phi_2(t)$. The Hawking temperature is $T_{emergent} = hc^3 (1 - K)^2 / 16\pi^2 GM k_B$. $K = (d\Phi_2(t)/dt)^2 < 1$ is the kinetic energy of Φ , k_B is Boltzmann constant. This is phenomenologically interesting in the context of Belgiorno *et al's* gravitational analogue experiments.

(2) For emergent gravity metrics , presence of dark energy modifes the Hawking temperature. For Reissner-Nordstrom background , the emergent metric can be mapped into a Robinson- Trautman blackhole. For some allowed value of the dark energy density this blackhole has zero Hawking temperature. For a Kerr background along $\Theta = 0$, the emergent blackhole metric satisfies Einstein's equations for large r and always radiates. Our analysis is with same fields and lagrangian as in (1). In both cases the fields satisfy the emergent field equations for $r \rightarrow \infty$ and $\Theta = 0$.

(3) Analogues of the Friedman equations are obtained for an emergent metric in the presence of dark energy with Friedman-Lemaitre-Robertson-Walker background. If $(d\Phi_2(t)/dt)^2$ is the dark energy density then
(a) for total energy density ρ greater than the pressure p (matter domination) the deceleration parameter $q(t) \sim (1/2) [1 + 27 (d\Phi_2(t)/dt)^2 +...]> (1/2)$
(b) for $\rho = 3p$ (radiation domination), $q(t) \sim 1 + 18 (d\Phi_2(t)/dt)^2 +...]> 1$
(c) for $\rho = -p$ (dark energy scenario), $q(t) < -1$. So many aspects of standard cosmology can be accommodated with the presence of dark energy right from the beginning of the universe with $t \equiv t / t_0$, and t_0 the present epoch.

Goutam Manna

To

My Mother

Late Bani Manna

Contents

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List of Publications

Publications: in refereed journals

(1) "The Hawking Temperature in The Context of Dark Energy", Debashis Gangopadhyay and Goutam Manna, published in *Euro. Phys. Lett.* **100**, 49001, (2012). This work will always be cited in thesis as **Paper 1**.

(2) "The Hawking Temperature in The Context of Dark Energy for Reissner-Nordstrom and Kerr Background", Goutam Manna and Debashis Gangopadhyay, published in *The European Physical Journal C* **74**, 2811, (2014). This work will always be cited in thesis as **Paper 2**.

Publications: in arXiv

(3) "Cosmology in presence of dark energy in an emergent gravity scenario", Debashis Gangopadhyay and Goutam Manna, [arXiv:gr-qc/1502.06255]. This work will always be cited in thesis as **arXiv:gr-qc/1502.06255** .

Notation and Conventions

In this thesis, we use the metric signature $(+, -, -, -)$ and follow mostly the conventions of Vikman's thesis [48]. We employ natural units where speed of light $c = 1$, Planck's constant $\hbar = 1$ and Boltzmann's constant $k_B = 1$ and following Vikman take the gravitational constant $G = \frac{1}{8\pi M_{pl}^2}$, where $M_{pl} = 1.72 \times 10^{18} GeV$ is the reduced Planck mass. To bring in clarity in finalresults all the factors are put in explicitly. This will be always indicated when necessary. The sign convention for the covariant derivative associated to the metric $g_{\mu\nu}$ is $\nabla_\mu A^\nu = \partial_\mu A^\nu + \Gamma^\nu_{\mu\alpha} A^\alpha$ and $\nabla_\mu \omega_\nu = \partial_\mu \omega_\nu - \Gamma^\alpha_{\mu\nu} \omega_\alpha$ where A^μ is $(1, 0)$ tensor and ω_μ is $(0, 1)$ tensor.

Greek indices μ, ν, etc. used the four space-time coordiante labels $0, 1, 2, 3$ with $x^0 \equiv t$ the time coordinate.

Latin indices i, j, k run over the three spatial coordiante labels i.e., $1, 2, 3$.

$g^{\mu\nu}$: Gravitational metric;

$G^{\mu\nu}$ or $\tilde{G}^{\mu\nu}$ or $\bar{G}^{\mu\nu}$: Emergent gravity metric;

M: mass of the black hole;

k: Curvature constant;

$\phi \equiv \phi(r,t)$: for $k-$essence scalar fields;

$\partial_t\phi = \partial_0\phi = \dot{\phi}$ and $\partial_r\phi = \partial_1\phi = \phi'$;

$\dot{\phi}^2 = K = constant$: Kinetic energy of the $k-$essence scalar fields;

$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}[\partial_\mu g_{\beta\nu} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}]$: Christoffel Symbol with respect to gravitational metric;

$\bar{\Gamma}^{\alpha}_{\mu\nu} = \frac{1}{2}\bar{G}^{\alpha\beta}[\partial_\mu \bar{G}_{\beta\nu} + \partial_\nu \bar{G}_{\mu\beta} - \partial_\beta \bar{G}_{\mu\nu}]$: Christoffel Symbol with respect to emergent gravity metric;

$\bar{R}_{\mu\nu} = \partial_\mu\bar{\Gamma}^{\alpha}_{\alpha\nu} - \partial_\alpha\bar{\Gamma}^{\alpha}_{\mu\nu} + \bar{\Gamma}^{\alpha}_{\beta\mu}\bar{\Gamma}^{\beta}_{\alpha\nu} - \bar{\Gamma}^{\alpha}_{\alpha\beta}\bar{\Gamma}^{\beta}_{\mu\nu}$: Ricci tensor with respect to emergent gravity metric;

$\bar{R} = \bar{G}^{\mu\nu}\bar{R}_{\mu\nu}$: Ricci scalar;

$\bar{E}_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{1}{2}\bar{G}_{\mu\nu}\bar{R} = -8\pi G T_{\mu\nu}$: The Einstein's Field Equation where $T_{\mu\nu}$ energy-momentum tensor;

CHAPTER 1

1 Introduction

1.1 Motivation of this work

Current observations i.e., Type Ia supernova (SNIa) observations [1]-[5], the observations of the Cosmic Microwave Background (CMB) [6], Baryon Acoustic Oscillations (BAO) [7], Wilkinson Microwave Anisotropy Probe (WMAP) [8]-[11] and Planck results [12, 13] have established that the Universe consists of roughly 25% dark matter, 70% dark energy, about 4% free hydrogen and helium and remaining 1% consisting of stars, dust, neutrions and heavy elements. These observations also show that the Universe is undergoing accelerated expansion which is different from Hubble expansion. Hubble showed that the redshift of distant galaxies increased as a linear function of their distance i.e., the expansion is linear. Dark energy is one of the candidates being regarded as the origin of this accelerated expansion.

One class of theoretical model to investigate the effects of the presence of dark energy on cosmological scenarios are $k-$essence models. A $k-$essence model is a scalar field model where the *kinetic* energy of the field dominates over the potential energy of the field - hence the name "$k-$esence". In this theoretical model, the lagrangian is not canonical i.e., $L \neq T - V$ where T is the kinetic energy and V is the potential energy. The $k-$essence field theoretic lagrangian is non-canonical i.e., it can not be separated in kinetic energy term and potential energy term and also it does not depend explicitly on the field itself.

In the $k-$essence model [14, 15], actions with non-canonical kinetic terms are strong candidates for dark matter and dark energy. The radiation energy density during the radiation-dominated era is tracked by the $k-$essence energy density. But during the matter-dominated era the $k-$essence energy density evolves towards a constant-density dark energy component. In this class of models, the coincidence problem (i.e., why we live in the particular era during which the dark matter and dark energy densities are roughly equal) is resolved by linking the onset of dark energy domination to the epoch of equal matter and radiation. There is another interesting characteristic of $k-$essence models. They can produce a dark energy component where the sound speed does not exceed the speed of light. These models are observationally distinguished from standard scalar field quintessence models with a canonical kinetic term (for which $c_s = 1$), and may provide a mechanism to reduce cosmic microwave background (CMB) fluctuations on large angular scales [16]-[18].

Historically, a theory with a non-canonical kinetic term was first proposed by Born and Infeld [19] in order to get rid of the infinite self-energy of the electron. Similar theories were also studied in the literature [20, 21].

Cosmology witnessed these models first in the context of $k-$essence driven inflation and subsequently in $k-$essence models of dark matter and dark energy [22]-[26]. An approach to understanding dark matter and dark energy involves setting up of lagrangians for $k-$essence fields in a background of the Friedman-Robertson-Walker metric with zero curvature constant. It is also possible to unify the dark matter and dark energy components into a single scalar field model [15] with the scalar field ϕ

having a non-canonical kinetic term.

The general form of the lagrangian for $k-$essence models is: $L = -V(\phi)F(X)$ where ϕ is the scalar field and $X = \frac{1}{2}g^{\mu\nu}\nabla_{\mu}\phi\nabla_{\nu}\phi$ and does not depend explicitly on ϕ to start with [15, 27, 28]. Relevant literature involving theories with non-canonical kinetic terms and their subsequent use in cosmology, inflation, dark matter, dark energy and strings can be found in Ref. [29]-[43].

With such a form for the lagrangian, what do we mean by kinetic energy of the fields dominating over the potential energy? For canonical lagrangians, we write $L = T - V$ and the hamiltonian as $H = T + V$ where the hamiltonian is a Legendre transform of the lagrangian. Here T is quadratic in the time derivatives, V is the potential and domination means that $T >> V$. When T and V are implicitly mixed up as in the Born-Infeld lagrangian we will still follow the same principle, $v$iz., identify the quadratic part in the time derivatives and the potential part which is a function of the fields only and then impose the above condition. This takes a simple form for the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric as the background as follows.

The FLRW metric is ($a(t)$ is the scale factor and k the curvature constant):

$$ds^2_{FLRW} = dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} - r^2(d\theta^2 + sin^2\theta d\phi^2)]$$

First note that throughout this thesis we asuume that the total energy density $\Omega = \Omega_{matter} + \Omega_{radiation} + \Omega_{darkenergy} = 1$ so that the individual densities cannot exceed unity. So in our case $\Omega_{darkenergy} \equiv \dot{\phi}^2 < 1$. (Also note that $\dot{\phi}^2 < 1$ implies that $(1/2)\dot{\phi}^2$ is also less than unity etc.)

In an FLRW background, the Dirac-Born-Infeld lagrangian for a homogeneous k-essence field,i.e. $\phi(r,t) = \phi(t)$, with constant potential V and

$\dot{\phi}^2 < 1$ takes the form

$$\begin{aligned} L(X,\phi) &= 1 - V(\phi)\sqrt{1-2X} \\ &\simeq 1 - V + V[\frac{1}{2}g^{00}\partial_0\phi\partial_0\phi] \\ &= 1 - V + \frac{V}{2}[\dot{\phi}^2] \\ &= \frac{V}{2}[\dot{\phi}^2] - (V-1) \end{aligned}$$

Here we have the lagrangian written in a form reminiscent of canonical lagrangians i.e. $L \equiv T - V \equiv$ *kinetic energy* $-$ *potential energy*, where T is quadratic in the time derivatives and everything else is potential. This has been possible because we have taken V to be a constant. So

$$\frac{V}{2}[\dot{\phi}^2] > (V-1)$$

i.e.,

$$V < \frac{1}{(1-\frac{\dot{\phi}^2}{2})}$$

Simultaneously $V << K = \frac{\dot{\phi}^2}{2}$. For example, if $K = \frac{1}{4}$, then the above condition gives $V < \frac{4}{3}$. However, if V has to be less than K then say $V = \frac{1}{6}$. So both the conditions are consistently satisfied. Hence, in this sense for this particular scenario whenever V is chosen as above, the kinetic energy of the fields dominate over the potential part. An exactly similar analysis can be done for the Schwarzschild, Reissner-Nordstrom and Kerr metrics as backgrounds.In chapters 2, 3, and 4 we will come back to this issue as and when required.

An interesting consequence of the presence of dark energy is as follows. There is a difference between relativistic field theories with canonical kinetic terms and $k-$essence theories with non-canonical kinetic terms:

Non-trivial dynamical solutions of the $k-$essence equation of motion not only spontaneously break Lorentz invariance but also change the metric for perturbations around these solutions. Thus the perturbations propagate not only in a new medium determined by the background solution but also in the so-called emergent or analogue curved spacetime [44] with the metric $G^{\mu\nu}$ different from the gravitational metric $g^{\mu\nu}$. In the context of cosmological perturbations, [45]-[48] showed that for purely kinetic $k-$essence theories there exist lagrangians which are proportional to $\sqrt{X}$.

In this thesis we have reported the effects of the presence of dark energy on cosmological scenarios. Specifically, the thesis comprises of the following investigations:

(1) Calculation of the Hawking temperature for an emergent gravity metric in the presence of dark energy where the background metric is taken to be (a) Schwarzschild, (b) Reissner-Nordstrom and (c) Kerr types.

(2) Establishment of the analogues of the Friedman equations in cosmology in an emergent gravity scenario in the presence of dark energy. The deceleration parameter is then calculated for various scenarios. The background metric is taken to be Friedman-Lemaitre-Robertson-Walker (FLRW).

In section 1.2 we briefly review some facts regarding $k-$essence and emergent gravity together with some technicalities obtained in **paper 1** of List of Publications. In section 1.3 the usual Hawking temperature for various backgrounds are introduced . In section 1.4 the usual Friedmann equations and corresponding cosmology is discussed.

1.2 $k-$essence and emergent gravity

We now give a brief introduction to an emergent gravity scenario when $k-$essence fields are also present.

The $k-$essence scalar field ϕ minimallly coupled to the gravitational field $g_{\mu\nu}$ has action [47]

$$S_k[\phi, g_{\mu\nu}] = \int d^4x\sqrt{-g}L(X,\phi) \tag{1.1}$$

where $X = \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$. The energy momentum tensor is

$$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}}\frac{\delta S_k}{\delta g^{\mu\nu}} = L_X\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}L \tag{1.2}$$

$L_{\mathrm{X}} = \frac{dL}{dX}$, $L_{\mathrm{X}X} = \frac{d^2L}{dX^2}$, $L_\phi = \frac{dL}{d\phi}$ and ∇_μ is the covariant derivative defined with respect to the metric $g_{\mu\nu}$. The equation of motion is

$$-\frac{1}{\sqrt{-g}}\frac{\delta S_k}{\delta\phi} = \tilde{G}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + 2XL_{X\phi} - L_\phi = 0 \tag{1.3}$$

where the effective metric $\tilde{G}^{\mu\nu}$ is

$$\tilde{G}^{\mu\nu} \equiv L_X g^{\mu\nu} + L_{XX}\nabla^\mu\phi\nabla^\nu\phi \tag{1.4}$$

and is physically meaningful only when $1 + \frac{2XL_{XX}}{L_X} > 0$.

We first carry out the conformal transformation $G^{\mu\nu} \equiv \frac{c_s}{L_X^2}\tilde{G}^{\mu\nu}$, with

$$c_s^2(X,\phi) \equiv (1 + 2X\frac{L_{XX}}{L_X})^{-1} \equiv \textit{sound speed.}$$

Then the inverse metric of $G^{\mu\nu}$ is

$$G_{\mu\nu} = \frac{L_X}{c_s}[g_{\mu\nu} - c_s^2\frac{L_{XX}}{L_X}\nabla_\mu\phi\nabla_\nu\phi] \tag{1.5}$$

A further conformal transformation $\bar{G}_{\mu\nu} \equiv \frac{c_s}{L_X} G_{\mu\nu}$ [**Paper 1** of List of Publications] gives

$$\bar{G}_{\mu\nu} = g_{\mu\nu} - \frac{L_{XX}}{L_X + 2XL_{XX}} \nabla_\mu \phi \nabla_\nu \phi \tag{1.6}$$

Note that one must always have $L_X \neq 0$ for the sound speed c_s^2 to be positive definite and only then equations (1.1) – (1.4) will be physically meaningful. This can be seen as follows. $L_X = 0$ implies that L does not depend on X so that in equation (1.1), $L(X, \phi) \equiv L(\phi)$. So the $k-$essence lagrangian L becomes pure potential and the very definition of $k-$essence fields becomes meaningless because such fields correspond to lagrangians where the kinetic energy dominates over the potential energy. If there are no derivatives of the field in the lagrangian then there is no question of identifying a kinetic energy part. Also, the very concept of minimally coupling the $k-$essence field ϕ to the gravitational field $g_{\mu\nu}$ becomes redundant and equation (1.1) meaningless and equations (1.4)-(1.6) ambiguous.

For the non-trivial configurations of the $k-$ essence field $\partial_\mu \phi \neq 0$ (for a scalar field,$\nabla_\mu \phi \equiv \partial_\mu \phi$) and $\bar{G}_{\mu\nu}$ is not conformally equivalent to $g_{\mu\nu}$. So this $k-$essence field has properties different from canonical scalar fields defined with $g_{\mu\nu}$ and the local causal structure is also different from those defined with $g_{\mu\nu}$. Further, if L is not an explicit function of ϕ then the equation of motion (1.3) is replaced by ;

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_k}{\delta \phi} = \bar{G}^{\mu\nu} \nabla_\mu \nabla_\nu \phi = 0 \tag{1.7}$$

We shall take the Lagrangian as $L = L(X) = 1 - V\sqrt{1 - 2X}$ with V a constant. This is a particular case of the Dirac-Born-Infeld (DBI) lagrangian

$$L(X, \phi) = 1 - V(\phi)\sqrt{1 - 2X} \tag{1.8}$$

for $V(\phi) = V = constant$ and $V << kinetic\ energy\ of\ \phi$ i.e., $V << (\dot{\phi})^2$. This is typical for the $k-$essence field where the kinetic energy dominates over the potential energy. Then $c_s^2(X, \phi) = 1 - 2X$. For scalar fields $\nabla_\mu \phi = \partial_\mu \phi$. Thus (1.6) becomes [**Paper 1** of List of Publications]:

$$\bar{G}_{\mu\nu} = g_{\mu\nu} - \partial_\mu \phi \partial_\nu \phi \tag{1.9}$$

The rationale of using two conformal transformations now becomes clear. The first transformation is used to identify the inverse metric $G_{\mu\nu}$. The second conformal transformation realises the mapping onto the metric given in (1.9) for the lagrangian $L(X) = 1 - V\sqrt{1-2X}$.

Consider the second conformal transformation $\bar{G}_{\mu\nu} \equiv \frac{c_s}{L_X} G_{\mu\nu}$. Following [49] the new Christoffel symbols are related to the old ones by

$$\begin{aligned}
\bar{\Gamma}^\alpha_{\mu\nu} &= \Gamma^\alpha_{\mu\nu} + (1-2X)^{-1/2} G^{\alpha\gamma}[G_{\mu\gamma}\partial_\nu (1-2X)^{1/2} \\
&\quad + G_{\nu\gamma}\partial_\mu (1-2X)^{1/2} - G_{\mu\nu}\partial_\gamma (1-2X)^{1/2}] \\
&= \Gamma^\alpha_{\mu\nu} + \frac{1}{(1-2X)}[-\delta^\alpha_\mu \partial_\nu X - \delta^\alpha_\mu \partial_\nu X + G^{\alpha\gamma} G_{\mu\nu}\partial_\gamma X] \\
&= \Gamma^\alpha_{\mu\nu} + \frac{1}{(1-2X)}[-\delta^\alpha_\mu \partial_\nu X - \delta^\alpha_\mu \partial_\nu X + \frac{1}{2}(\delta^\alpha_\mu \delta^\gamma_\nu + \delta^\alpha_\nu \delta^\gamma_\mu)\partial_\gamma X] \\
&= \Gamma^\alpha_{\mu\nu} - \frac{1}{2(1-2X)}[\delta^\alpha_\mu \partial_\nu X + \delta^\alpha_\mu \partial_\nu X]
\end{aligned} \tag{1.10}$$

Note that the second term on the right hand side is symmetric under exchange of μ and ν so that the symmetry of $\bar{\Gamma}$ is maintained. The second term has its origin solely to the $k-$essence lagrangian and this additional term signifies additional interactions (forces). The geodesic equation in terms of the new Christoffel connections $\bar{\Gamma}$ now becomes

$$\frac{d^2 x^\alpha}{d\tau^2} + \bar{\Gamma}^\alpha_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0 \tag{1.11}$$

1.3 Hawking temperatures

We now discuss some salient features regarding usual Hawking temperatures associated with Hawking radiation.

Black holes are special regions of space-time where the curvature is so strong that even light cannot escape. The edge of the black hole is known as the event horizon. Schwarzschild showed that the event horizon of a black hole has a radius r_s, known as the Schwarzschild radius, with $r_s = \frac{2GM}{c^2}$ where G is gravitational constant, M is the mass of the black hole and the speed of light is c.

In 1975, [50]-[53] Stephen Hawking showed (using quantum theory) that black holes will radiate energy and evaporate. He also showed that this emitted radiation (called Hawking radiation) matches that of a perfect black-body radiator temperature, given by

$$T_{Hawking} = \frac{\hbar c^3}{8\pi GMk_B}$$

where $\hbar$ is the Planck's constant and k_B is the Boltzmann's constant.

For convenience the tunneling mechanism for calculating the Hawking temperature outlined in [54]-[57] is reviewed. Other literature related to Hawking temperature can be found in [58]-[64]. We have the Schwarzschild metric as:

$$ds_S^2 = (1 - 2GM/r)dt^2 - (1 - 2GM/r)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2). \tag{1.12}$$

Using Eddington-Finkelstein coordinates (v, r, θ, ϕ)

$$v = t + r^* \quad ; \quad u = t - r^*;$$

$$dr^* = \frac{dr}{(1 - 2GM/r)};$$

$$r^* = r + 2GM \; ln \; [r - 2GM]. \tag{1.13}$$

we get

$$\begin{aligned} ds_S^2 &= (1 - 2GM/r)dt^2 - (1 - 2GM/r)(dr^*)^2 - r^2 d\Omega^2 \\ &= (1 - 2GM/r)(dt^2 - (dr^*)^2) - r^2 d\Omega^2 \\ &= (1 - 2GM/r)(dv^2 - 2dv \; dr^*) - r^2 d\Omega^2 \\ &= (1 - 2GM/r)dv^2 - 2dvdr - r^2 d\Omega^2 \end{aligned} \tag{1.14}$$

with $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$. A massless particle in the Schwarzschild background is described by the Klein-Gordon equation,

$$\hbar^2(-g)^{-1/2}\partial_\mu(g^{\mu\nu}(-g)^{1/2}\partial_\nu\Psi) = 0. \tag{1.15}$$

Assume

$$\Psi = \exp(-\frac{i}{\hbar}S + ...) \tag{1.16}$$

Then (1.15) becomes,

$$\hbar^2(-g)^{-1/2}\partial_\mu[g^{\mu\nu}(-g)^{1/2}(-\frac{i}{\hbar})(\partial_\nu S)e^{(-\frac{i}{\hbar}S)}] = 0 \tag{1.17}$$

To leading order in $\hbar$ one has the equation

$$g^{\mu\nu}\partial_\mu S\partial_\nu S = 0 \tag{1.18}$$

Using separation of variables in the form

$$S = Ev + S_0(r), \tag{1.19}$$

and using (1.14), the equation (1.15) become

$$-2\frac{\partial S}{\partial v}\frac{\partial S}{\partial r} - (1 - 2GM/r)(\frac{\partial S}{\partial r})^2 = 0 \tag{1.20}$$

i.e.,

$$2ES_0^{'}(r) + (1 - 2GM/r)(S_0^{'}(r))^2 = 0 \tag{1.21}$$

with $S_0'(r) = \frac{\partial S_0(r)}{\partial r}$. One solution of $S_0(r)$ is a constant C and another is

$$\begin{aligned} S_0(r) &= -2E \int^r \frac{dr}{(1 - 2GM/r)} \\ &= -2E \int^r \frac{dr}{(1 - r_s/r)}. \end{aligned} \tag{1.22}$$

The singularity at the horizon $r = 2GM = r_s$, has to be considered if one tries to find a solution across it. Change $r - r_s$ to $r - r_s - i\epsilon$. This gives

$$\begin{aligned} S_0(r) = -2E \int^r \frac{rdr}{(r - r_s)} &= -2E \int^r (1 + \frac{r_s}{r - r_s})dr \\ &= -2E[r + r_s \ . \ i\pi + r_s \int^r drP(\frac{1}{r - r_s})] \end{aligned} \tag{1.23}$$

where $P()$ denotes the principal value.

Therefore the solution of equation (1.19) is

$$S = Ev + C - 2E[r + r_s \ . \ i\pi + r_s \int^r drP(\frac{1}{r - r_s})]. \tag{1.24}$$

The imaginary part yields a factor

$$exp(\frac{i}{\hbar}2Er_s \ .i\pi) = exp(-\frac{4\pi GME}{\hbar})$$

in the amplitude, leading to a factor $exp(-\frac{8\pi GME}{\hbar})$ in the probability. This probability $exp(-\frac{8\pi GME}{\hbar})$ is equivalent to $exp(-\frac{E}{k_B T_S})$.

So the Hawking temperature for Schwarzschild black hole is

$$T_S = \frac{\hbar c^3}{8\pi GMk_B} \tag{1.25}$$

where k_B is the Boltzmann constant and c is speed of light.

In chapter 2 we show that if the gravitational metric is taken to be Schwarzschild (1.12) then the emergent gravity metric (1.9) in presence of *dark energy can be mapped into a Barriola-Vilenkin (BV) type metric [65] where the kinetic energy of the $k-$essence scalar field ϕ replaces the global monopole charge. We next calculate the modified Hawking temperature in the presence of dark energy. We also show that how the phenomenological parameters of the analogue gravity experiments of Belgiorno et. al. [66] are modified in the presence of dark energy.*

In the case of Reissner-Nordstrom (RN) black hole the metric is

$$ds^2_{RN} = (1 - 2GM/r + Q^2/r^2)dt^2 - (1 - 2GM/r + Q^2/r^2)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) \quad (1.26)$$

and the corresponding Hawking temperature [67, 68] for the two horizons are:

$$T^{RN}_{+} = \frac{\hbar c^3}{2\pi k_B}\frac{\sqrt{G^2M^2 - Q^2}}{[GM + \sqrt{G^2M^2 - Q^2}]^2} \quad (1.27)$$

and

$$T^{RN}_{-} = -\frac{\hbar c^3}{2\pi k_B}\frac{\sqrt{G^2M^2 - Q^2}}{[GM - \sqrt{G^2M^2 - Q^2}]^2} \quad (1.28)$$

where Q is the charge of the RN black hole.

Similarly, for the *Kerr* black hole the metric is

$$ds^2_K = (1 - \frac{2GMr}{\rho^2})dt^2 + \frac{4GMr\alpha sin^2\theta}{\rho^2}d\phi dt - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - (r^2 + \alpha^2 + \frac{2GMr\alpha^2 sin^2\theta}{\rho^2})sin^2\theta d\phi^2 \quad (1.29)$$

with

$$\alpha = \frac{J}{GM} \quad ; \quad \rho^2 = r^2 + \alpha^2 cos^2\theta \quad and \quad \Delta = r^2 - 2GMr + \alpha^2,$$

where J is angular momentum of the Kerr black hole and the Hawking temperature [69]-[72] for the two horizons are:

$$T_+^K = \frac{\hbar c^3}{4\pi k_B} \frac{\sqrt{(GM)^2 - \alpha^2}}{(GM)^2 + GM\sqrt{(GM)^2 - \alpha^2}} \tag{1.30}$$

and

$$T_-^K = -\frac{\hbar c^3}{4\pi k_B} \frac{\sqrt{(GM)^2 - \alpha^2}}{(GM)^2 - GM\sqrt{(GM)^2 - \alpha^2}}. \tag{1.31}$$

In chapter 3 we obtain the modified Hawking temperatures for the emergent gravity metrics corresponding to Reissner-Nordstrom and Kerr backgrounds in presence of dark energy. For the RN case we show that the emergent metric becomes Robinson-Trautman (RT) type along $\theta = 0$*. For some allowed values of the dark energy density, this blackhole can have zero Hawking temperature ,i.e., it does not radiate.For the Kerr case the emergent metric remains Kerr type along* $\theta = 0$ *and the blackhole always radiates. For both backgrounds the emergent gravity equations of motion (1.7) are satisfied along* $\theta = 0$ *and large* r*.*

1.4 Friedmann equations

We now recall the standard Friedmann equations and corresponding cosmological parameters.

The Friedmann equations [75, 76] are a set of equations in cosmology governing the expansion of space in homogeneous and isotropic models of the universe. The metric used is the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric which assumes a perfect fluid model for the Universe with a given energy density ρ and pressure p. The FLRW metric is:

$$ds^2_{FLRW} = dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} - r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{1.32}$$

where $a(t)$ is scale factor and k is curvature constant with values 1, 0 and -1.

A fluid is perfect if there is no viscosity and no heat conduction terms in the momentarily comoving reference frame. In general relativity, it is the continuous matter distributions and fields with the energy-momentum tensor $T_{\mu\nu}$ of the form

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}. \tag{1.33}$$

u_μ is a velocity four vector with $u_0 = 1$ and $u_i = 0,\ i = 1, 2, 3$.

The Friedmann equations [77]-[80] are:

$$\rho = \frac{3}{8\pi G}[\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}] \tag{1.34}$$

and

$$p = -\frac{1}{8\pi G}[2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}] \tag{1.35}$$

i.e.,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \tag{1.36}$$

where G is gravitational constant.

The solutions of Friedmann equations are:

(a) *non-relativistic scenario ($\rho >> p$)* i.e, Matter dominated Universe. The solution of Friedmann equation is

$$a(t) \propto t^{\frac{2}{3}} \tag{1.37}$$

with $k = 0$. In this case the energy density is

$$\rho = \rho_0 (a/a_0)^{-3} \tag{1.38}$$

with ρ_0 and a_0 having the values of energy density and scale factor respectively at present epoch. The deceleration parameter is

$$q(t) \equiv -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{2}. \tag{1.39}$$

(b) *Relativistic scenario ($p = \frac{\rho}{3}$)* i.e., Radiation dominated Universe. The solution is

$$a(t) \propto \sqrt{t} \tag{1.40}$$

with $k = 0$. The energy density becomes

$$\rho = \rho_0 (a/a_0)^{-4} \tag{1.41}$$

and the deceleration parameter is

$$q(t) = +1. \tag{1.42}$$

(c) *Dark energy dominated Universe ($p \simeq -\rho$)* the solution is

$$a(t) \propto exp(Ht) \tag{1.43}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and the deceleration parameter is

$$q(t) = -1 \tag{1.44}$$

with $k = 0$. Here the energy density ρ is constant and also $H = \sqrt{\frac{8\pi G\rho}{3}}$ becomes constant.

In chapter 4 we show how the Friedmann equations, scale factor $a(t)$, energy density ρ and the deceleration parameter $q(t)$ change for the above three scenarios in the presence of dark energy whose origin is again assumed to be $k-$essence scalar fields.

CHAPTER 2

2 The Hawking temperature in the context of dark energy for Schwarzschild background

We show that the Hawking temperature [50]-[64] is modified in the presence of dark energy for Schwarzschild background.

The results reported in this chapter have been obtained in **Paper 1** of List of Publications.

If a global monopole falls into a Schwarzschild blackhole the resulting metric is different from the Schwarzschild case and the blackhole carries the global monopole charge. Barriola and Vilenkin obtained solutions for Einstein equations outside the monopole core [65, 81, 82] and showed that the metric takes the form:

$$ds^2 = (1-8\pi G\eta^2-\frac{2GM}{r})dt^2-\frac{1}{(1-8\pi G\eta^2-\frac{2GM}{r})}dr^2-r^2(d\theta^2+sin^2\theta d\phi^2)$$

M is the mass of the BV blackhole and M is very large i.e.,$M >> \frac{\delta}{G}$ where $\delta \sim \lambda^{-\frac{1}{2}}\eta^{-1}$ is the monopole core size. Here the global monopole lagrangian is

$$L = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{4}\lambda(\phi^a\phi^a - \eta^2)^2$$

where ϕ^a is a triplet of scalar fields ($a = 1, 2, 3$) and the global $O(3)$

symmetry is spontaneously broken to $U(1)$. $\eta \sim 10^{16} GeV$ is the typical grand unification scale.

We determine the $k-$essence scalar field configurations ϕ for which the metric $\tilde{G}^{\mu\nu}$ becomes conformally equivalent to the Barriola-Vilenkin (BV) metric. However, in our case the global monopole charge is now replaced by the kinetic energy of the $k-$essence scalar field ϕ. Thus we show that BV-type metrics can also result from $k-$essence theories and not necessarily from global monopoles only. It should be mentioned that monopoles are akin to topological defects and there exist substantial and well-known literature on the subject. Recent interesting developments in this area can be found in [83, 84].

So one can calculate the Hawking temperature $T_{emergent}$ for such a metric and this is obviously different from that of the Schwarzschild case. Moreover, if $\phi_{emergent}$ be the solutions of the emergent gravity equations of motion for $r \to \infty$ then the rescaled field $\frac{\phi_{emergent}}{2GM-1}$ has exact correspondence with the k-essence scalar field ϕ configurations for which the BV metric is realised. This result is phenomenologically interesting in the context of Belgiorno *et al's* [66] demonstration of spontaneous emission of photons in a gravitational analogue experiment for Hawking radiation.

Before proceeding further, we discuss the Dirac-Born-Infeld lagrangian (1.8), in the context of the kinetic energy domination. We follow a similar analysis as in the Introduction. However,for inhomogeneous fields the situation is more interesting. We take the $k-$essence fields to be spherically symmetric and V a constant, and assume that $\phi(r,t) = \phi_1(r) + \phi_2(t)$. Here, as before, we expand the square root for $\dot{\phi}_2^2 < 1$. Then (prime denotes differentiation with respect to r)

$$L(X,\phi) = 1 - V(\phi)\sqrt{1-2X}$$

$$= 1 - V[1 - (g^{00}\dot{\phi}^2 + g^{11}(\phi^{'})^2)]^{1/2}$$

$$= 1 - V[1 - (g^{00}\dot{\phi}_2^2 + g^{11}(\phi_1^{'})^2)]^{1/2}$$

$$= 1 - V[1 - (\frac{1}{(1-2GM/r)}\dot{\phi}_2^2 - (1-2GM/r)(\phi_1^{'})^2)]^{1/2}$$

It is readily evident that it will be impossible to extricate a pure kinetic part from the above expression. So we have to be satisfied with the statement that $V << \dot{\phi}_2^2$.

In our scenario (as will be shown below) (2.6) also holds i.e., $(\phi_1')^2(1-2GM/r)^2 = K = \dot{\phi}_2^2$ and $0 < K < 1$. Then L becomes $L = 1 - V$ as the expression $(\frac{1}{(1-2GM/r)}\dot{\phi}_2^2 - (1-2GM/r)(\phi_1^{'})^2)$ now vanishes. So L becomes independent of the kinetic energy of the fields $K = \dot{\phi}_2^2$! How do we tackle this situation ?

We proceed as follows.As K is a constant and less than 1 , write L as

$$L = 1 - V = (\alpha + K) - V = K - (V - \alpha) = K - V'$$

where $\alpha > 0$ and also less than unity. If we further stipulate that $\alpha < V$, then V' is positve. Obviously if $K >> V$ then K *is also greater than* V'. So the domination of kinetic part is still valid ,although the potential has now changed !

All this means that for non canonical lagrangians it is difficult to isolate the lagrangian into pure kinetic and pure potential parts. This is intuitively expected because a non canonical lagrangian is fully interacting to start with and it is difficult to think of a "free" propagator.

2.1 Mapping on to the Barriola-Vilenkin type metric

Taking the gravitational metric $g_{\mu\nu}$ to be Schwarzschild, $\partial_0\phi \equiv \dot{\phi}$, $\partial_r\phi \equiv \phi'$ and assuming that the $k-$essence field $\phi(r,t)$ is spherically symmetric one has [using (1.9)]

$$\begin{aligned}
\bar{G}_{00} = g_{00} - (\partial_0\phi)^2 = 1 - 2GM/r - \dot{\phi}^2 \\
\bar{G}_{11} = g_{11} - (\partial_r\phi)^2 = -(1 - 2GM/r)^{-1} - (\phi')^2 \\
\bar{G}_{22} = g_{22} = -r^2 \\
\bar{G}_{33} = g_{33} = -r^2 sin^2\theta \\
\bar{G}_{01} = \bar{G}_{10} = -\dot{\phi}\phi'
\end{aligned} \tag{2.1}$$

For the Schwarzschild metric,

$$g_{00} = (1 - 2GM/r);\; g_{11} = -(1 - 2GM/r)^{-1};$$

$$g_{22} = -r^2;\; g_{33} = -r^2 sin^2\theta;\; g_{ij}(i \neq j) = 0$$

So the emergent gravity line element becomes

$$\begin{aligned}
ds^2 = (1 - 2GM/r - \dot{\phi}^2)dt^2 - ((1 - 2GM/r)^{-1} + (\phi')^2)dr^2 \\
-2\dot{\phi}\phi' dtdr - r^2 d\Omega^2
\end{aligned} \tag{2.2}$$

where $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$.

Making a co-ordinate transformation from (t, r, θ, ϕ) to $(\omega, r, \theta, \phi)$ such that ([85]):

$$d\omega = dt - (\frac{\dot{\phi}\phi'}{1 - 2GM/r - \dot{\phi}^2})dr \tag{2.3}$$

Then (2.2) becomes

$$\begin{aligned}
ds^2 &= (1 - 2GM/r - \dot{\phi}^2)[d\omega^2 + \frac{(\dot{\phi}\phi')^2}{(1 - 2GM/r - \dot{\phi}^2)^2} dr^2 \\
&+2\frac{(\dot{\phi}\phi')}{(1 - 2GM/r - \dot{\phi}^2)} d\omega dr] - (\frac{1}{1 - \frac{2GM}{r}} + \phi'^2) dr^2 \\
&-2\dot{\phi}\phi' dr[d\omega + \frac{(\dot{\phi}\phi')}{(1 - 2GM/r - \dot{\phi}^2)} dr] - r^2 d\Omega^2 \\
&= (1 - 2GM/r - \dot{\phi}^2) d\omega^2 - [\frac{(\dot{\phi}\phi')^2}{(1 - 2GM/r - \dot{\phi}^2)} \\
&+\frac{1}{(1 - 2GM/r)} + (\phi')^2] dr^2 - r^2 d\Omega^2
\end{aligned} \tag{2.4}$$

(2.4) will be a blackhole metric if $\bar{G}_{00} = \bar{G}_{11}^{-1}$, i.e.

$$(1 - 2GM/r - \dot{\phi}^2) = [\frac{(\dot{\phi}\phi')^2}{(1 - 2GM/r - \dot{\phi}^2)} + \frac{1}{(1 - 2GM/r)} + (\phi')^2]^{-1}$$

or,

$$\dot{\phi}^2 = (\phi')^2 (1 - 2GM/r)^2 \tag{2.5}$$

Let us assume a solution to (2.5) of the form $\phi(r,t) = \phi_1(r) + \phi_2(t)$. Then (2.5) reduces to

$$\dot{\phi}_2^2 = (\phi_1')^2 (1 - 2GM/r)^2 = K \tag{2.6}$$

$K(\neq 0)$ is a constant ($K \neq 0$ means $k-$essence field will have *non-zero* kinetic energy). The solution to (2.6)

$$\begin{aligned}
\phi(r,t) &= \phi_1(r) + \phi_2(t) \\
&= \sqrt{K}[r + 2GM\ ln(r - 2GM)] + \sqrt{K}t
\end{aligned} \tag{2.7}$$

with

$$\phi_1'(r) = \frac{\sqrt{K}r}{(r - 2GM)}$$

$$\phi_1(r) = \sqrt{K}[r + 2GMln(r - 2GM)]; \;\; \phi_2(t) = \sqrt{K}t,$$

and we have taken an arbitrary integration constant to be zero. So the line element (2.4) reduces to

$$ds^2 = (1 - \frac{2GM}{r} - K)d\omega^2 - \frac{1}{(1 - \frac{2GM}{r} - K)}dr^2 - r^2 d\Omega^2 \tag{2.8}$$

and this is the Barriola-Vilenkin blackhole which represents the situation where a global monopole carrying charge $K = \dot{\phi}_2^2 = constant$ has fallen into a Schwarzschild blackhole. *It should be noted that K has to be always less than unity because if K is greater than unity the signature of the metric* (2.8) *becomes ill defined.* This is easily seen : for $K > 1$, $\bar{G}_{00}$ is negative while $\bar{G}_{11}$ is positive. However, it should also be noted that $K >> V$. This is a requirement for $k-$essence fields where the kinetic energy dominates over the potential energy. Therefore, we have $K < 1$ and $V << K$.

So the metric components are

$$\begin{aligned} \bar{G}_{00} = g_{00} - (\partial_0\phi)^2 = (1 - 2GM/r - K); \\ \bar{G}_{11} = g_{11} - (\partial_r\phi)^2 = -(1 - 2GM/r - K)^{-1}; \\ \bar{G}_{22} = g_{22} = -r^2 \;\; ; \;\; \bar{G}_{33} = g_{33} = -r^2 sin^2\theta \end{aligned} \tag{2.9}$$

In the context of global monopoles [65, 81, 82], the above metric has been shown to satisfy the Einstein field equations. Thus we have shown that this metric can also arise from an emergent gravity scenario with $k-$essence scalar fields and the global monopole charge is now replaced by the constant kinetic energy of the $k-$essence field.

Now it is also known that the solutions to the emergent gravity equations of motion (1.7) for the scalar field under cosmological boundary conditions are given by [44]-[47]

$$\phi_{emergent}(t,r) = const.[t + r + 2GM\ ln\ |\frac{r}{2GM} - 1| + 2GM \int^{r} F(r')dr']$$

where the function

$$F(r) = \frac{r}{r-2GM}[\sqrt{\frac{Ar-2GM}{A^4r^4(r-2GM)+(A-1)r}} - 1]$$

where A is a constant. Substituting this solution in (2.5) and taking the limit $r \to \infty$ and ignoring terms of $O(\frac{1}{r^2})$ and higher gives

$$lim_{r\to\infty}(\phi'_{emergent})^2(1-2GM/r)^2 = K(2GM-1)^2 \qquad (2.10)$$

Therefore for $r \to \infty$, *the rescaled field* $\frac{\phi_{emergent}(t,r)}{(2GM-1)}$ *has exact correspondence with the* $k-$*essence scalar field* ϕ *which satisfies the blackhole metric condition* (2.5).

2.2 Hawking Temperature

Let us calculate the Hawking temperature for this metric (2.8) using the tunnelling formalism as outlined in [54, 55, 56, 57] which corrects for the factor of two in the Hawking temperature as often mentioned, (e.g. in [63, 64]). Going over to the Eddington-Finkelstein coordinates (v, r, θ, ϕ)

$$v = \omega + r^* \quad ; \quad u = \omega - r^*;$$

$$dr^* = \frac{dr}{(1 - K - 2GM/r)};$$

$$r^* = \frac{r}{1-K} + \frac{2GM}{(1-K)^2}\ ln\ [(1-K)r - 2GM]. \tag{2.11}$$

we get

$$\begin{aligned} ds^2 &= (1 - K - 2GM/r)(d\omega^2 - (dr^*)^2) - r^2 d\Omega^2 \\ &= (1 - K - 2GM/r)(dv^2 - 2dv\ dr^*) - r^2 d\Omega^2 \\ &= (1 - K - 2GM/r)dv^2 - 2dvdr - r^2 d\Omega^2 \end{aligned} \tag{2.12}$$

By analogy with the Schwarzschild case a massless particle in the background of $\bar{G}_{\mu\nu}$ is described by the Klein-Gordon equation,

$$\hbar^2(-\bar{G})^{-1/2}\partial_\mu(\bar{G}^{\mu\nu}(-\bar{G})^{1/2}\partial_\nu\Psi) = 0. \tag{2.13}$$

One expands

$$\Psi = \exp(-\frac{i}{\hbar}S + ...) \tag{2.14}$$

Then (2.13) becomes,

$$\hbar^2(-\bar{G})^{-1/2}\partial_\mu[\bar{G}^{\mu\nu}(-\bar{G})^{1/2}(-\frac{i}{\hbar})(\partial_\nu S)e^{(-\frac{i}{\hbar}S)}] = 0 \tag{2.15}$$

and obtains to leading order in $\hbar$ the equation

$$\bar{G}^{\mu\nu}\partial_\mu S \partial_\nu S = 0 \tag{2.16}$$

Using separation of variables in the form

$$S = Ev + S_0(r), \tag{2.17}$$

and using (2.12), the equation (2.16) become

$$-2\frac{\partial S}{\partial v}\frac{\partial S}{\partial r} - (1 - K - 2GM/r)(\frac{\partial S}{\partial r})^2 = 0 \tag{2.18}$$

i.e.,

$$2ES_0^{'}(r) + (1 - K - 2GM/r)(S_0^{'}(r))^2 = 0 \tag{2.19}$$

where $S_0^{'}(r) = \frac{\partial S_0(r)}{\partial r}$. One of the solution of $S_0(r)$ is a constant C and another is

$$\begin{aligned} S_0(r) = -2E\int^r \frac{dr}{(1 - K - 2GM/r)} &= -2E\int^r \frac{dr}{(\beta - 2GM/r)} \\ &= -\frac{2E}{\beta}\int^r \frac{dr}{(1 - \frac{2GM}{\beta r})} \end{aligned} \tag{2.20}$$

with $\beta = (1 - K)$ is constant. Furthermore, there is a singularity at the horizon $r = \frac{2GM}{\beta} = r_{bv}$, which has to handled if one tries to find a solution across it.

One way to avoid this singularity is the pole is to change $r - r_{bv}$ to $r - r_{bv} - i\epsilon$. This gives

$$\begin{aligned} S_0(r) &= -\frac{2E}{\beta}\int^r \frac{rdr}{(r - r_{bv})} = -\frac{2E}{\beta}\int^r (1 + \frac{r_{bv}}{r - r_{bv}})dr \\ &= -\frac{2E}{\beta}[r + r_{bv}\ .\ i\pi + r_{bv}\int^r drP(\frac{1}{r - r_{bv}})] \end{aligned} \tag{2.21}$$

where $P()$ denotes the principal value.

Therefore the solution of equation (2.16) is

$$S = Ev + C - \frac{2E}{\beta}[r + r_{bv}\ .\ i\pi + r_{bv}\int^r drP(\frac{1}{r - r_{bv}})]. \tag{2.22}$$

The imaginary part gives a factor

$$exp(\frac{i}{\hbar}\frac{2E}{\beta}r_{bv}\ .i\pi) = exp(-\frac{4\pi GME}{\hbar\beta^2})$$

in the amplitude, so the probability is a factor $exp(-\frac{8\pi GME}{\hbar\beta^2})$. This probability $exp(-\frac{8\pi GME}{\hbar\beta^2})$ is equivalent to $exp(-\frac{E}{k_B T_{emergent}})$.

So the Hawking temperature for BV type metric is obtained as ($\dot{\phi}_2^2 = constant = K$)

$$T_{emergent} = \frac{\hbar c^3(1-K)^2}{8\pi GMk_B} = T_{\rm S}(1-K)^2 \tag{2.23}$$

where k_B is the Boltzmann constant and $T_{\rm S} = \frac{\hbar c^3}{8\pi GMk_B}$ is the usual Hawking temperature. So $T_{emergent}$ is less than the usual Hawking temperature for Schwarzschild black hole as $K < 1$.

Let us recollect what we have done so far. We have considered a $k-$essence scalar field ϕ (with a non-canonical Born-Infeld type lagrangian with potential $V(\phi) = const. = V$ i.e. $L = 1 - V(\phi)\sqrt{1-2X} \equiv 1 - V\sqrt{1-2X}$) minimally coupled to the gravitational metric $g_{\mu\nu}$ in Schwarzschild spacetime. We then obtain the equation of motion for ϕ, equation (1.7), in the "effective" metric

$$\bar{G}_{\mu\nu} = g_{\mu\nu} - \partial_\mu\phi\partial_\nu\phi.$$

We then impose the conditions for a blackhole metric to obtain the configurations of the $k-$essence field ϕ that will give a blackhole. It turns out that one possible scenario is a Barriola-Vilenkin type blackhole where the global monopole charge is now replaced by a constant kinetic energy of the scalar field ϕ. This kinetic energy $K < 1$ in order to preserve the consistency of the signature of the metric. By construction the potential energy V is a constant and $V << K$ because this is a basic requirement for $k-$essence fields. Therefore, the total energy is always a constant and although the obtained field configurations are linear in time there should not be any instability. Moreover, the lagrangian does not depend on the fields ϕ explicitly. The dependence is only through derivatives of the field.

2.3 Phenomenological consequences in analogue gravity experiments

We now discuss how our results can be made to correspond to scenarios in analogue gravity experiments similar to Belgiorno *et. al.* [66]. Such an experiment may help in distinguishing between a Schwarzschild blackhole analogue and the blackhole analogue described by the metric (2.8) and (2.9).

First let us briefly discuss Belgiorno *et. al's* experiment. They used ultrashort laser pulse filaments to create a travelling refractive index perturbation (RIP) in fused silica glass and reported experimental evidence of photon emission that bears the characteristics of Hawking radiation and is distinguishable and thus separate from other known photon emission mechanisms. They interpreted this emission as an indication of Hawking radiation induced by the analogue event horizon.

They also have a complete description of the event horizon associated to the RIP and can calculate a blackbody temperature of the emitted photons in the laboratory reference frame [86, 87]. However, Belgiorno *et. al.* also pointed out that the dielectric medium in which the RIP is created will always be dominated by optical dispersion and therefore the spectrum will not be that of a perfect blackbody and that in any case only a limited spectral portion of the full spectrum will be observable. To show this last point they described the RIP as a perturbation induced by the laser pulse on top of a uniform, dispersive background refractive index n_0, i.e.

$$n(z,t,\omega) = n_0(\omega) + \delta n f(z - vt),$$

where ω is the optical frequency, $f(z - vt)$ is a function bounded by 0 and 1, that describes the shape of the laser pulse. In the reference frame co-moving at velocity v with the RIP, the event horizon in a 2D geometry is defined by $c/v = n$ which admits solutions only for RIP velocities satisfying the inequality [87]:

$$\frac{1}{n_0(\omega) + \delta n} < \frac{v}{c} < \frac{1}{n_0(\omega)}.$$

This predicts an emission spectrum with well-defined boundaries and it is precisely this feature of the spectral emission that is peculiar to analogue Hawking radiation. In the experiment a clear photon emission was registered in the wavelength window predicted by the last inequality, the emitted radiation was unpolarized, and the emission bandwidth increased with the input energy. The Bessel pulse intensity evolution along the propagation direction z was estimated analytically from the input energy. By fitting the measured spectra with Gaussian functions the bandwidth was estimated as a function of input energy and Bessel intensity. Using the fused silica dispersion relation the authors obtained the bandwidth and the δn as a function of input energy and Bessel pulse peak intensity (at $z = 1$ cm where measurements were performed). There was a clear linear dependence which was in qualitative agreement with the fact that the emission bandwidth was predicted to depend on δn which in turn is a linear function $\delta n = n_2 I$ of the pulse intensity I. The slope of the linear fit was in good agreement with the tabulated value [88, 89]. Therefore there is also an agreement at the quantitative level between the measurements and the model based on Hawking-like radiation emission.

In this context, we propose that a different Hawking temperature should give a different set of values for the above mentioned phenomenological parameters i.e. the slope n_2 and also the inequality

$$\frac{1}{n_0(\omega)+\delta n} < \frac{v}{c} < \frac{1}{n_0(\omega)}$$

etc. because the intensity of photon emission should depend on the relevant blackbody temperature. Therefore, Belgiorno *et. al's* gravitational analogue experiment has scope of being further enhanced into testing the existence of other cosmological entities like dark energy. In order to include the effects of dark energy, the RIP method must be accordingly modified. At a basic level this means that the effect of the presence of the constant K in the metric must be included. For example, if we consider the Schwarzschild metric, the Belgiorno *et. al.* experiment currently has $K = 0$. So the experimental situation has to move over to a scenario which can mimic $K \neq 0$ i.e. $0 < K < 1$.

Here some aspects need to be clarified. First, note that in (2.8) if we take $M \sim M_{\mathrm{mono}polecore}$ i.e. M is very small and negligible we have

$$ds^2 = (1-K)d\omega^2 - \frac{1}{(1-K)}dr^2 - r^2 d\Omega^2$$

and rescaling r and ω one has

$$ds^2 = d\omega^2 - dr^2 - (1-K)r^2 d\Omega^2$$

and this is the metric of the global BV monopole and describes a space with a deficit solid angle i.e. the area of a sphere of radius r is not $4\pi r^2$ but $(1-K)4\pi r^2$, $K < 1$. Such spaces are not asymptotically flat, but asymptotically bound.

Secondly, we are dealing with the BV blackhole i.e. $M >> M_{\mathrm{mono}polecore}$ i.e.

$$2GM/r = \frac{2M}{\frac{r}{G}}$$

is not negligible i.e. for $r \sim \delta$ where δ is the monopole core size. Here also for $r \to \infty$ the metric (2.8) is not strictly asymptotically flat owing to the presence of K.

If the Schwarzschild metric is the reference then one has to move over from an asymptotically flat metric to one which is not exactly asymptotically flat but rather asymptotically bound. This aspect can be incorporated into the Belgiorno *et al* analogue gravity experiment in the following way in order for the results to be compatible to the existence of an analogue event horizon corresponding to a Hawking temperatute $T_{emergent}$. Here we draw heavily from Reference [87]. Consider the wave equation for a perturbation of a full nonlinear electric field propagating in a nonlinear Kerr medium where for simplicity the electric field has been replaced by a scalar field ϕ, (equation (1) of Reference [87]):

$$\frac{n^2(x_l - vt_l)}{c^2}\partial_{t_l}^2\Phi - \partial_{x_l}^2\Phi - \partial_y^2\Phi - \partial_z^2\Phi = 0$$

where all coordinates are in the lab frame. The suffix l is omitted from y, z because they are not involved in the boost relating the lab frame with the pulse frame. $n(x_l - vt_l)$ is the refractive index that accounts for the propagating RIP in the dielectric. The RIP is propagating with a constant velocity v. The analogue Hawking temperature (equation (13) of Reference [87]) for the blackhole horizon (x_+) is

$$T_+ = \frac{\gamma^2 v^2 \hbar}{2\pi k_B c}|\frac{dn}{dx}|_{x_+}$$

where γ is the boost ($\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$), v is the constant velocity of the RIP, k_B is the Boltzmann constant and x_+ denotes the blackhole horizon. Note that in order to have

$$T_{emergent} = (1-K)^2 T_H$$

one can have either of the following scenarios:

Case 1:

Realise an experimental situation where

$$\gamma^2 \to \gamma_1^2 = (1-K)^2\gamma^2$$

This would imply

$$1 - \frac{v_1^2}{c^2} = \frac{1-\frac{v^2}{c^2}}{(1-K)^2}$$

or,

$$\frac{v_1^2}{c^2} = 1 - \frac{1-\frac{v^2}{c^2}}{(1-K)^2}$$

i.e.,

$$v_1^2 = \frac{v^2 - c^2K(2-K)}{(1-K)^2}$$

As v_1^2 must be positive, one should ensure that

$$\frac{v^2 - c^2K(2-K)}{(1-K)^2} > 0$$

i.e.,

$$K(2-K) < \frac{v^2}{c^2}$$

All this should also be made consistent with

$$\frac{c}{v_1} = n_1|_{x_+} = n_{10} + k_1\eta_1$$

where the parameter $\eta_1 << 1$ and k_1 is the normalised intensity of the pulse at the blackhole horizon taking values in the interval $(0,1)$.

Case 2:

Realise an experimental situation with

$$n \to n_2 = (1 - K)^2 n$$

Obviously the RIP propagation velocity v must be changed to some new value v_2 together with a new consistency condition

$$\frac{c}{v_2} = n_2|_{x_+} = n_{20} + k_2 \eta_2$$

Therefore , a realisation of photon emission (similar to Belgiorno et al's original experiments) corresponding to a blackbody temperature $T_{emergent}$ *in any of the above described two scenarios will be compatible with a theory of* $k-$*essence fields in an emergent gravity metric in the same way as the original Belgiorno experiment, though not proving the existence of blackhole horizons , however , proves the relation between blackhole horizons and the Hawking radiation.*

CHAPTER 3

3 Hawking Temperature in context of dark energy for Reissner-Nordstrom and Kerr backgrounds

Now we determine the Hawking temperature for an emergent gravity metric in the presence of dark energy for two other background metrics. These are (a) Reissner-Nordstrom (RN) blackhole metric and (b) Kerr blackhole metric.

The results reported in this chapter have been obtained in **Paper 2** of List of Publications.

As explained in detail in chapter 1, $\tilde{G}_{\mu\nu}$ contains the dark energy field ϕ and this should satisfy the emergent gravity equations of motion. Again, for $\tilde{G}_{\mu\nu}$ to be a blackhole metric, it has to satisfy the Einstein field equations. In chapter 2, this was shown by mapping the emergent gravity metric (having Schwarzschild background) into a Barriola-Vilenkin blackhole metric which satisfied the Einstein equations.

Here we find that for the RN background case the emergent gravity metric can be exactly mapped onto a Robinson-Trautman blackhole so that the Einstein equations are automatically satisfied. However, the k-essence matter fields satisfy the emergent gravity equations of motion only for $\theta = 0$.

For the Kerr case, the emergent metric satisfies Einstein equations for large r while the dark energy field ϕ satisfies the emergent gravity equations of motion again only for $\theta = 0$.

In this context we clarify that the Hawking temperature is spherically symmetric from very general conditions and taking $\theta = 0$ does not therefore affect this property of the Hawking temperature.

As before, for the DBI lagrangian (1.8), the question of kinetic energy dominating over the potential energy is tackled as follows.

(a) Reissner-Nordstrom metric along $\theta = 0:$ The DBI lagrangian

$$L = 1 - V[1 - (g^{00}\dot{\phi}_2^2 + g^{11}(\phi_1^{'})^2)]^{1/2}$$
$$= 1 - V[1 - ((\frac{1}{1 - 2GM/r + Q^2/r^2})\dot{\phi}_2^2 - (\frac{1}{1 - 2GM/r + Q^2/r^2})(\phi_1^{'})^2)]^{1/2}$$

Here also it is impossible to extract a pure kinetic part. Also (3.6) (below) is true so that again $L = 1 - V$. A similar discussion as given before for the Schwarzschild case is true here also with a marginal difference i.e. here $\alpha = 0$ as K is forced to take the value $K = 1$ (from other conditions, page 41,paragraph 3) so that K is still larger than V ,i.e. the kinetic energy dominates.

(b) Kerr metric along $\theta = 0:$

The DBI lagrangian

$$L = 1 - V[1 - (g^{00}\dot{\phi}_2^2 + g^{11}(\phi_1^{'})^2)]^{1/2}$$
$$= 1 - V[1 - ((\frac{\rho^2}{\Delta})\dot{\phi}_2^2 - (\frac{\Delta}{\rho^2})(\phi_1^{'})^2)]^{1/2}$$

with $\rho^2 = r^2 + \alpha^2$ *and* $\Delta = r^2 - 2GMr + \alpha^2$.

Now using (3.34) (below) where

$$\dot{\phi}_2^2 = \frac{\Delta^2}{\rho^4}(\phi_1')^2 = K$$

L becomes $L = 1 - V$. Exactly same arguments as in the Schwarzschild case for kinetic part domination is valid here as $K < 1$.

3.1 The Reissner-Nordstrom case and mapping on to the Robinson-Trautman type metric

First consider the gravitational metric $g_{\mu\nu}$ to be Reissner-Nordstrom. Assuming that the $k-$essence field $\phi(r,t)$ is spherically symmetric one has [using (1.9)]

$$\begin{aligned}
\bar{G}_{00} = g_{00} - (\partial_0\phi)^2 = 1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2 \\
\bar{G}_{11} = g_{11} - (\partial_r\phi)^2 = -(1 - 2GM/r + Q^2/r^2)^{-1} - (\phi')^2 \\
\bar{G}_{22} = g_{22} = -r^2 \\
\bar{G}_{33} = g_{33} = -r^2 sin^2\theta \\
\bar{G}_{01} = \bar{G}_{10} = -\dot{\phi}\phi'
\end{aligned} \tag{3.1}$$

For the RN metric,

$$g_{00} = (1 - 2GM/r + Q^2/r^2);\ g_{11} = -(1 - 2GM/r + Q^2/r^2)^{-1};$$

$$g_{22} = -r^2;\ g_{33} = -r^2 sin^2\theta;\ g_{ij}(i \neq j) = 0.$$

Note that the RN metric is spherically symmetric. The emergent gravity metric (3.1) contains additional terms *but all these are independent of* θ. So the emergent metric is also spherically symmetric. So we might as well consider $\theta = 0$. Then the emergent gravity line element becomes

$$\begin{aligned} ds^2_{RN,\theta=0} &= (1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2)dt^2 \\ &-((1 - 2GM/r + Q^2/r^2)^{-1} + (\phi')^2)dr^2 \\ &-2\dot{\phi}\phi' dtdr \end{aligned} \tag{3.2}$$

Now make a co-ordinate transformation from (t, r) to (ω, r) along $\theta = 0$ such that ([85]) :

$$d\omega = dt - (\frac{\dot{\phi}\phi'}{1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2})dr \tag{3.3}$$

Then (3.2) becomes

$$\begin{aligned} ds^2 &= (1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2)[d\omega^2 + \frac{(\dot{\phi}\phi')^2}{(1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2)^2}dr^2 \\ &+ \frac{2(\dot{\phi}\phi')}{(1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2)}d\omega dr] - (\frac{1}{1 - 2GM/r + Q^2/r^2} + \phi'^2)dr^2 \\ &-2\dot{\phi}\phi' dr[d\omega + \frac{(\dot{\phi}\phi')}{(1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2)}dr] \\ &= (1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2)d\omega^2 - [\frac{(\dot{\phi}\phi')^2}{(1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2)} \\ &+ \frac{1}{(1 - 2GM/r + Q^2/r^2)} + (\phi')^2]dr^2 \end{aligned} \tag{3.4}$$

(3.4) will be a blackhole metric if $\bar{G}_{00} = \bar{G}_{11}^{-1}$, i.e.,

$$\begin{aligned} &(1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2) \\ &= [\frac{(\dot{\phi}\phi')^2}{(1 - 2GM/r + Q^2/r^2 - \dot{\phi}^2)} + \frac{1}{(1 - 2GM/r + Q^2/r^2)} + (\phi')^2]^{-1} \end{aligned}$$

i.e.,

$$\dot{\phi}^2 = (\phi')^2(1 - 2GM/r + Q^2/r^2)^2 \tag{3.5}$$

Let us assume a solution to (3.5) of the form $\phi(r,t) = \phi_1(r) + \phi_2(t)$. Then (3.5) reduces to

$$\dot{\phi}_2^2 = (\phi_1')^2(1 - 2GM/r + Q^2/r^2)^2 = K \tag{3.6}$$

$K(\neq 0)$ is a constant ($K \neq 0$ means $k-$essence field will have *non-zero* kinetic energy). The solution to (3.6) is

$$\phi(r,t) = \phi_1(r) + \phi_2(t)$$

$$= \sqrt{K}[r + \frac{(2G^2M^2 - Q^2)tan^{-1}\frac{(r-GM)}{\sqrt{Q^2-G^2M^2)}}}{\sqrt{Q^2 - G^2M^2}} + GMln\ (Q^2 - 2GMr + r^2)] + \sqrt{K}t \tag{3.7}$$

with

$$\phi_1(r) = \sqrt{K}[r + \frac{(2G^2M^2 - Q^2)tan^{-1}\frac{(r-GM)}{\sqrt{Q^2-G^2M^2)}}}{\sqrt{Q^2 - G^2M^2}} + GMln\ (Q^2 - 2GMr + r^2)];$$

and

$$\phi_2(t) = \sqrt{K}t,$$

and we have taken an arbitrary integration constant to be zero. Therefore the line element (3.4) becomes

$$ds^2 = (1 - \frac{2GM}{r} + \frac{Q^2}{r^2} - K)d\omega^2 - \frac{1}{(1 - \frac{2GM}{r} + \frac{Q^2}{r^2} - K)}dr^2 \tag{3.8}$$

i.e.

$$ds^2 = (\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})d\omega^2 - \frac{1}{(\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})}dr^2 \tag{3.9}$$

with $\beta = (1 - K)$. Now going over to the Eddington-Finkelstein coordinates (v, r) or (u, r) along $\theta = 0$ i.e., *introducing advanced and retarded null coordinates*

$$v = \omega + r^* \quad ; \quad u = \omega - r^*;$$

$$dr^* = \frac{r^2 dr}{\beta(r^2 - 2GMr/\beta + Q^2/\beta)} = \frac{r^2 dr}{\beta(r - r_+)(r - r_-)};$$

$$r^* = \frac{1}{\beta}[r + \frac{r_+^2}{r_+ - r_-} ln\ |r - r_+| - \frac{r_-^2}{r_+ - r_-} ln\ |r - r_-|] \tag{3.10}$$

with

$$r_+ = \frac{GM}{\beta} + \frac{1}{\beta}\sqrt{(GM)^2 - \beta Q^2}$$

and

$$r_- = \frac{GM}{\beta} - \frac{1}{\beta}\sqrt{(GM)^2 - \beta Q^2}$$

.

Then the line element (3.9) becomes, for advanced null coordinates

$$\begin{aligned} ds^2 &= \frac{\beta}{r^2}(r^2 - 2GMr/\beta + Q^2/\beta)(d\omega^2 - (dr^*)^2) \\ &= \frac{\beta}{r^2}(r^2 - 2GMr/\beta + Q^2/\beta)(dv^2 - 2dvdr^*) \\ &= (\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})dv^2 - 2dvdr \end{aligned} \tag{3.11}$$

or, similarly for retarded null coordinates

$$ds^2 = (\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})du^2 + 2dudr \tag{3.12}$$

which is analogous to the *Robinson-Trautman* (RT) metric [73, 74] along $\theta = 0$ where β can take the values $+1, 0, -1$. The original Robinson-Trautman metric [73] (page 430) along $\theta = 0$ is

$$ds^2 = 2dudr + (\alpha - \frac{2m}{r} + \frac{\kappa_0 Q\bar{Q}}{2r^2})du^2$$

with

$$\Phi_1 = \frac{\bar{Q}}{2r^2}; \quad \Phi_2 = 0; \quad \alpha = 0, \pm 1.$$

where Q (complex), m (real) are arbitrary constants and Φ_1, Φ_2 are electromagnetic fields. This is known in the literature as type D solutions of Einstein-Maxwell field equations. For $\alpha = 1$, these are the Reissner-Nordstrom solutions.

In our case $\beta \neq +1$ because then $K = \dot{\phi}_2^2 = 0$ and dark energy is absent. $\beta \neq -1$, i.e. $K \neq 2$ as the total energy density cannot exceed unity ($\Omega_{matter} + \Omega_{radiation} + \Omega_{darkenergy} = 1$).

Therefore, the only allowed value of $\beta = 0$ i.e., $K = 1$ and this is a perfectly valid solution because the RT metric allows $\beta = 0$. Physically this means that r_+ is pushed to infinity while r_- is pushed to zero. This implies the radial coordinate r is a time-like coordinate on the whole space-time manifold and the outer horizon a sort of cosmological horizon. Thus , as argued in reference [90], the case K=1 of (3.8) does not seem have a Newtonian limit, which makes it unsuitable for describing astrophysical objects. However, although this may not be suitable as an astrophysical object but still is a consistent solution of Einstein's equation. In this context, it should be noted that even the Schwarzschild blackhole solution is strictly not astrophysically ever possible because we cannot have static blackholes. But still the Schwarzschild solution has been a milestone in understanding various nuances of general relativity. Similar situation prevails also for the Reissner-Nordstrom blackhole as charged blackholes are highly unlikely in nature for obvious reasons. So any confusion regarding K taking the value $+1$ should not arise. We shall show below that $K = 1$ gives zero Hawking temperature.

Also note that the solution $\phi(r,t)$ (3.7) *obtained from the blackhole conditions* $\bar{G}_{00} = \bar{G}_{11}^{-1}$ also satisfies the emergent gravity equation of motion (1.7) at $r \to \infty$ along the symmetry axis, $\theta = 0$:

$$\bar{G}^{00}\partial_0^2\phi_2 + [\bar{G}^{11}(\partial_1^2\phi_1 - \Gamma_{11}^1\partial_1\phi_1)] + \bar{G}^{01}\nabla_0\nabla_1\phi + \bar{G}^{10}\nabla_1\nabla_0\phi = 0.$$

The first term vanish since $\phi_2(t)$ linear in t and second term within third bracket becomes inconsequential at $r \to \infty$ because this term goes like $\sim -\frac{1}{r^3} + \frac{1}{r^2}$ and for our case $\beta = 0$. It can be shown below [using $\Gamma_{11}^1 = \frac{Q^2 - GMr}{r(r^2 - 2GMr + Q^2)}$]:

$$2nd\, term = -(\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})[\sqrt{K}\frac{r(2Q^2 - 2GMr)}{(r^2 - 2GMr + Q^2)^2} - \sqrt{K}\frac{r^2(Q^2 - GMr)}{r(r^2 - 2GMr + Q^2)^2}]$$

$$= -\sqrt{K}Q^2\frac{(\beta r^2 - 2GMr + Q^2)}{r(r^2 - 2GMr + Q^2)^2} + \sqrt{K}GM\frac{(\beta r^2 - 2GMr + Q^2)}{(r^2 - 2GMr + Q^2)^2}$$

$$= -(\sqrt{K}Q^2)\frac{r^2(\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})}{r^5(1 - \frac{2GM}{r} + \frac{Q^2}{r^2})^2} + (\sqrt{K}GM)\frac{r^2(\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})}{r^4(1 - \frac{2GM}{r} + \frac{Q^2}{r^2})^2}$$

$$= -(\frac{\sqrt{K}Q^2}{r^3})(\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})(1 - \frac{2GM}{r} + \frac{Q^2}{r^2})^{-2}$$
$$+(\frac{\sqrt{K}GM}{r^2})(\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})(1 - \frac{2GM}{r} + \frac{Q^2}{r^2})^{-2}$$

$$\simeq -(\frac{\sqrt{K}Q^2}{r^3})(\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})(1 + \frac{4GM}{r} - \frac{2Q^2}{r^2})$$
$$+(\frac{\sqrt{K}GM}{r^2})(\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})(1 + \frac{4GM}{r} - \frac{2Q^2}{r^2})$$

$$= -(\frac{\sqrt{K}Q^2}{r^3})[\beta + \frac{4\beta GM}{r} - \frac{2\beta Q^2}{r^2} - \frac{2GM}{r} - \frac{8G^2M^2}{r^2} + ...]$$
$$+(\frac{\sqrt{K}GM}{r^2})[\beta + \frac{4\beta GM}{r} - \frac{2\beta Q^2}{r^2} - \frac{2GM}{r} - \frac{8G^2M^2}{r^2} + ...]$$

(Please note that the second term has been inadvertently missed in **Paper 2** of List of publications ,3rd page,paragraph 2)

The last expression becomes negligible for large r.

$$\xrightarrow{r \to \infty} -O(\frac{1}{r^3}) + O(\frac{1}{r^2})$$

and the last two terms vanish because $\bar{G}^{01} = \bar{G}^{10} = 0$.

So the scalar field that one needs to produce an emergent RN black hole satisfies the equation of motion of emergent gravity (1.7) only for infinite coordinate radius along the polar axis. One may question what the geometry discussed has to do with emergent gravity in the first place. The answer is that as the emergent geometry has a scalar field intricately linked with it *a priori*, having a solution at $r \to \infty$ is non-trivial from various aspects. Let us discuss these.

First note that the solution for the scalar field ϕ, (3.7), does not vanish for $r \to \infty$ as is usually expected of well behaved fields. Here

$$\phi_1(r \to \infty) = \sqrt{K}[r + 2GMlnr + \frac{2G^2M^2 - Q^2}{\sqrt{Q^2 - G^2M^2}}\frac{\pi}{2}].$$

Moreover, if $Q - GM = \alpha$ where $\alpha \to 0$, so that $ln\, r$ is negligible compared to the other terms then

$$\phi_1(r \to \infty) \sim \sqrt{K}[r + \frac{\sqrt{Q}}{\sqrt{2}\alpha}\frac{\pi}{2}].$$

All these are solutions of the theory and so deserve mention.

3.2 The Hawking Temperature for Robinson-Trautman type metric

We use the tunnelling method to calculate the Hawking temperature for (3.11) [54]-[57], [63, 64, 67, 68]: As discussed in section 1.3, a massless particle in a black hole background is described by the Klein-Gordon equation

$$\hbar^2(-\bar{G})^{-1/2}\partial_\mu(\bar{G}^{\mu\nu}(-\bar{G})^{1/2}\partial_\nu\Psi) = 0. \tag{3.13}$$

One expands

$$\Psi = exp(\frac{i}{\hbar}S + ...) \tag{3.14}$$

to obtain the leading order in $\hbar$ the Hamilton-Jacobi equation is

$$\bar{G}^{\mu\nu}\partial_\mu S\partial_\nu S = 0 \tag{3.15}$$

Assume S is independent of θ and ϕ. Then

$$2\frac{\partial S}{\partial v}\frac{\partial S}{\partial r} + (\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})(\frac{\partial S}{\partial r})^2$$

$$= \frac{\partial S}{\partial r}[2\frac{\partial S}{\partial v} + (\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})\frac{\partial S}{\partial r}] = 0 \tag{3.16}$$

The symmetries of the metric permit the action to be written as

$$S = -Ev + W(r) + J(x^i) \tag{3.17}$$

Then

$$\partial_v S = -E \ ; \ \partial_r S = W^{'} \ ; \ \partial_i S = J_i \tag{3.18}$$

J_i are constants chosen to be zero. Combining equations (3.16) and (3.18):

$$-2EW^{'}(r) + (\beta - \frac{2GM}{r} + \frac{Q^2}{r^2})(W^{'}(r))^2 = 0 \tag{3.19}$$

Thus

$$\begin{aligned} W(r) &= \int \frac{(Er^2 + Er^2)dr}{\beta(r - r_+)(r - r_-)} \\ = 2\pi i(\frac{E}{\beta})\frac{r_+^2}{r_+ - r_-} &+ 2\pi i(\frac{E}{\beta})\frac{r_-^2}{r_- - r_+} \\ &= W(r_+) + W(r_-) \end{aligned} \tag{3.20}$$

The two values of $W(r)$ correspond to the processes that the particle tunnels through the outer and inner horizons respectively.

Therefore

$$S = -Ev + 2\pi i(\frac{E}{\beta})\frac{r_+^2}{r_+ - r_-} + 2\pi i(\frac{E}{\beta})\frac{r_-^2}{r_- - r_+} + J(x^i) \tag{3.21}$$

The tunneling rates of the outer and inner horizons are

$$\begin{aligned} \Gamma^{RT}_{+emergent} &\sim e^{-2ImS+} \sim e^{-2ImW(r_+)} \\ &= e^{-4\pi(\frac{E}{\beta})\frac{r_+^2}{r_+ - r_-}} = e^{-\frac{E}{K_B T_+}} \end{aligned} \tag{3.22}$$

$$\Gamma^{RT}_{-emergent} \sim e^{-2ImS-} \sim e^{-2ImW(r_-)}$$
$$= e^{-4\pi(\frac{E}{\beta})\frac{r_-^2}{r_- - r_+}} = e^{-\frac{E}{K_B T_-}} \tag{3.23}$$

From these two equations the corresponding Hawking temperatures of the two horizons are respectively

$$T^{RT}_{+emergent} = \frac{\hbar c^3 \beta}{4\pi k_B}(\frac{r_+ - r_-}{r_+^2})$$

$$= \frac{\hbar c^3 \beta}{4\pi k_B}[\frac{(\frac{GM}{\beta} + \frac{1}{\beta}\sqrt{(GM)^2 - \beta Q^2}) - (\frac{GM}{\beta} - \frac{1}{\beta}\sqrt{(GM)^2 - \beta Q^2})}{(\frac{GM}{\beta} + \frac{1}{\beta}\sqrt{(GM)^2 - \beta Q^2})^2}]$$

$$= \frac{\hbar c^3 \beta^2}{2\pi k_B}\frac{\sqrt{(GM)^2 - \beta Q^2}}{(GM + \sqrt{(GM)^2 - \beta Q^2})^2}$$

$$= \frac{\hbar c^3 (1-K)^2}{2\pi k_B}\frac{\sqrt{G^2M^2 - Q^2(1-K)}}{[GM + \sqrt{G^2M^2 - Q^2(1-K)}]^2} \tag{3.24}$$

and

$$T^{RT}_{-emergent} = \frac{\hbar c^3 \beta}{4\pi k_B}(\frac{r_- - r_+}{r_-^2})$$

$$= \frac{\hbar c^3 \beta}{4\pi k_B}[\frac{(\frac{GM}{\beta} - \frac{1}{\beta}\sqrt{(GM)^2 - \beta Q^2}) - (\frac{GM}{\beta} + \frac{1}{\beta}\sqrt{(GM)^2 - \beta Q^2})}{(\frac{GM}{\beta} - \frac{1}{\beta}\sqrt{(GM)^2 - \beta Q^2})^2}]$$

$$= -\frac{\hbar c^3 \beta^2}{2\pi k_B}\frac{\sqrt{(GM)^2 - \beta Q^2}}{(GM - \sqrt{(GM)^2 - \beta Q^2})^2}$$

$$= -\frac{\hbar c^3 (1-K)^2}{2\pi k_B}\frac{\sqrt{G^2M^2 - Q^2(1-K)}}{[GM - \sqrt{G^2M^2 - Q^2(1-K)}]^2} \tag{3.25}$$

Hence, as stated before, the Hawking temperature for this case will vanish as the dark energy density has to be $K = \dot{\phi}_2^2 = 1$. So this RT blackhole in presence of dark energy cannot radiate as the dark energy density is constrained to be unity.

3.3 Emergent gravity and Kerr metric

Now take the gravitational metric $g_{\mu\nu}$ to be Kerr. The line element is [91]

$$\begin{aligned} ds^2_{Kerr} = (1 - \frac{2GMr}{\rho^2})dt^2 + \frac{4GMr\alpha sin^2\theta}{\rho^2}d\phi dt - \frac{\rho^2}{\Delta}dr^2 \\ -\rho^2 d\theta^2 - (r^2 + \alpha^2 + \frac{2GMr\alpha^2 sin^2\theta}{\rho^2})sin^2\theta d\phi^2 \end{aligned} \quad (3.26)$$

where

$$\alpha = \frac{J}{GM} \quad ; \quad \rho^2 = r^2 + \alpha^2 cos^2\theta \quad and \quad \Delta = r^2 - 2GMr + \alpha^2.$$

In this context an important point should be stressed. Note that the above metric (3.26) can be recast (for zero total charge) into the form given in reference [69] where the identifications are provided below.

$$\begin{aligned} ds^2 = f(r,\theta)dt^2 - \frac{dr^2}{g(r,\theta)} + 2H(r,\theta)dtd\phi \\ -K(r,\theta)d\phi^2 - \Sigma(r,\theta)d\theta^2 \end{aligned} \quad (3.27)$$

where

$$f(r,\theta) = \frac{\Delta(r) - \alpha^2 sin^2\theta}{\Sigma(r,\theta)};$$

$$g(r,\theta) = \frac{\Delta(r)}{\Sigma(r,\theta)};$$

$$H(r,\theta) = \frac{\alpha sin^2\theta(r^2 + \alpha^2 - \Delta(r))}{\Sigma(r,\theta)};$$

$$K(r,\theta) = \frac{(r^2+\alpha^2)^2 - \Delta(r)\alpha^2 sin^2\theta}{\Sigma(r,\theta)} sin^2\theta;$$

$$\Sigma(r,\theta) = r^2 + \alpha^2 cos^2\theta;$$

$$\Delta(r) = r^2 + \alpha^2 - 2GMr.$$

In [69] it has been elaborately shown how the Hawking temperature is independent of θ although the metric functions depend on θ. In our case the emergent metric $\bar{G}_{\mu\nu}$ contains additional terms *but these additional terms are still independent of θ. Therefore, the modified Hawking temperature will still be independent of θ. Therefore we might as well do our evaluation for some fixed θ, i.e. $\theta = 0$.* We consider the Kerr metric along $\theta = 0$. Then (3.26) becomes [91]

$$ds^2_{Kerr;\theta=0} = \frac{\Delta}{\rho^2}dt^2 - \frac{\rho^2}{\Delta}dr^2 \tag{3.28}$$

where

$$\rho^2 = r^2 + \alpha^2 \quad and \quad \Delta = r^2 - 2GMr + \alpha^2.$$

It is to be noted that the same metric (3.28) was rediscovered in [92]-[94] using a different route.

As before, we take the $k-$essence field $\phi(r,t)$ to be spherically symmetric in keeping with the usual spherically symmetric Born-Infeld type of lagrangian for the $k-$essence scalar field. This does imply any necessary conflict with the non-spherically symmetric background.

Then one has from (1.9)

$$\bar{G}_{00} = g_{00} - (\partial_0\phi)^2 = \frac{\Delta}{\rho^2} - \dot{\phi}^2$$
$$\bar{G}_{11} = g_{11} - (\partial_r\phi)^2 = -\frac{\rho^2}{\Delta} - (\phi')^2$$
$$\bar{G}_{01} = \bar{G}_{10} = -\dot{\phi}\phi'. \tag{3.29}$$

The emergent gravity line element (3.29) along $\theta = 0$ is now

$$ds^2 = (\frac{\Delta}{\rho^2} - \dot{\phi}^2)dt^2 - (\frac{\rho^2}{\Delta} + (\phi')^2)dr^2 - 2\dot{\phi}\phi' dtdr \tag{3.30}$$

Now make a coordinate transformation from (t, r) to (ω, r) such that

$$d\omega = dt - (\frac{\dot{\phi}\phi'}{\frac{\Delta}{\rho^2} - \dot{\phi}^2})dr \tag{3.31}$$

Then (3.30) becomes

$$ds^2 = (\frac{\Delta}{\rho^2} - \dot{\phi}^2)[d\omega^2 - \frac{(\dot{\phi}\phi')^2}{(\frac{\Delta}{\rho^2} - \dot{\phi}^2)^2}dr^2 + \frac{2(\dot{\phi}\phi')}{(\frac{\Delta}{\rho^2} - \dot{\phi}^2)}dtdr]$$
$$-(\frac{\rho^2}{\Delta} + (\phi')^2)dr^2 - 2\dot{\phi}\phi' dtdr$$
$$= (\frac{\Delta}{\rho^2} - \dot{\phi}^2)d\omega^2 - [\frac{(\dot{\phi}\phi')^2}{(\frac{\Delta}{\rho^2} - \dot{\phi}^2)} + \frac{\rho^2}{\Delta} + (\phi')^2]dr^2 \tag{3.32}$$

This equation (3.32) will a black hole metric if $\bar{G}_{00} = \bar{G}_{11}^{-1}$, i.e.

$$(\frac{\Delta}{\rho^2} - \dot{\phi}^2) = [\frac{(\dot{\phi}\phi')^2}{(\frac{\Delta}{\rho^2} - \dot{\phi}^2)} + \frac{\rho^2}{\Delta} + (\phi')^2]^{-1}$$

i.e.,

$$\Delta\dot{\phi}^2(\phi^{'})^2 + \Delta - \dot{\phi}^2\rho^2 + (\phi^{'})^2\frac{\Delta^2}{\rho^2} - \Delta\dot{\phi}^2(\phi^{'})^2 = \Delta$$

i.e.,

$$\dot{\phi}^2 = \frac{\Delta^2}{\rho^4}(\phi')^2 \tag{3.33}$$

We take a solution of (3.33) as $\phi(r,t) = \phi_1(r) + \phi_2(t)$.

So (3.33) reduces to

$$\dot{\phi}_2^2 = \frac{\Delta^2}{\rho^4}(\phi_1')^2 = K \tag{3.34}$$

where $K(\neq 0)$ is a constant ($K \neq 0$ means $k-$essence field will have *non-zero* kinetic energy). (Please note that in **Paper 2** ,List of Publications the equations $(42), (43)$ are erroneous). Now from (3.34) we get

$$\dot{\phi}_2 = \sqrt{K}$$

and

$$\phi_1' = \sqrt{K}[\frac{(r^2+\alpha^2)}{r^2 - 2GMr + \alpha^2}].$$

The solution of (3.34) is

$$\phi(r,t) = \phi_1(r) + \phi_2(t)$$

$$= \sqrt{K}[r + \frac{2(GM)^2 tan^{-1}(\frac{r-GM}{\sqrt{\alpha^2-(GM)^2}})}{\sqrt{\alpha^2-(GM)^2}}] + GMln\ [r^2 - 2GMr + \alpha^2] + \sqrt{K}t \tag{3.35}$$

where

$$\phi_1(r) = \sqrt{K}[r + \frac{2(GM)^2 tan^{-1}(\frac{r-GM}{\sqrt{\alpha^2-(GM)^2}})}{\sqrt{\alpha^2-(GM)^2}}] + GMln\ [r^2 - 2GMr + \alpha^2]$$

and

$$\phi_2(t) = \sqrt{K}t$$

and choosing an arbitrary integration constant to be zero. Therefore the line elements (3.32) becomes

$$ds^2 = (\frac{\Delta}{\rho^2} - K)d\omega^2 - \frac{1}{(\frac{\Delta}{\rho^2} - K)}dr^2$$

$$= (\frac{r^2 - 2GMr + \alpha^2}{r^2 + \alpha^2} - K)d\omega^2 - (\frac{1}{\frac{r^2-2GMr+\alpha^2}{r^2+\alpha^2} - K})dr^2$$

$$= \frac{(1-K)(r^2 - 2G\frac{M}{1-K}r + \alpha^2)}{(r^2+\alpha^2)}d\omega^2 - \frac{(r^2+\alpha^2)}{(1-K)(r^2 - 2G\frac{M}{1-K}r + \alpha^2)}dr^2$$

i.e.

$$ds^2 = \frac{\beta\Delta'}{\rho^2}d\omega^2 - \frac{\rho^2}{\beta\Delta'}dr^2 \tag{3.36}$$

where

$$\beta = 1-K,\ M' = \frac{M}{1-K},\ \Delta' = (r^2 - 2GM'r + \alpha^2)\ and\ \rho^2 = r^2 + \alpha^2.$$

Here note that $K \neq 1$ since β cannot be zero, as then the metric becomes singular. K cannot be greater than 1 because then the signature of the metric (3.36) will be wrong. $K \neq 0$ because that would imply dark energy is absent. Therefore, the only allowed values are $0 < K < 1$. So there is no question of K approaching 1 and confusions regarding this limit should not arise. It can be shown that for $r \to \infty$ this metric is an approximate solution of Einstein's equations as the relevant terms fall of as $\frac{1}{r^3}$. The detail calculations of the relevant terms (*viz.* Ricci tensors, Ricci scalar) of the Einstein's equations for the emergent gravity metric (3.36) are given in **Appendix A**.

We now show that there is a further restriction on the dark energy density $K = \dot{\phi}_2^2$ if we want the fields $\phi(r,t)$ given by (3.35) to satisfy the equation of motion (1.7) along the symmetry axis $\theta = 0$ at $r \to \infty$. For the axi-symmetric case, the equation of motion (1.7) takes the form

$$\bar{G}^{00}\partial_0^2\phi_2 + \bar{G}^{11}\partial_1^2\phi_1 - \bar{G}^{11}\Gamma^1_{11}\partial_1\phi_1 + \bar{G}^{01}\nabla_0\nabla_1\phi + \bar{G}^{10}\nabla_1\nabla_0\phi = 0.$$

The first term vanishes exactly because $\phi_2(t)$ is linear in t, and the last two terms vanish because $\bar{G}^{01} = \bar{G}^{10} = 0$.

Using the expression for

$$\Gamma^1_{11} = \frac{GM(\alpha^2 - r^2)}{(r^2+\alpha^2)(r^2 - 2GMr + \alpha^2)}$$

the third term for $r \to \infty$ goes as $\frac{|1-K|^{3/2}}{r^2}$. It is shown below:

$$\bar{G}^{11}\Gamma^1_{11}\partial_1\phi_1 = [\frac{-\beta\Delta'}{\rho^2}][\frac{-GM(r^2-\alpha^2)}{(r^2+\alpha^2)(r^2-2GMr+\alpha^2)}]$$

$$[\frac{(\sqrt{r^2+\alpha^2})(\sqrt{r^2(K-1)+\alpha^2(K-1)+2GMr+1})}{(r^2-2GMr+\alpha^2)}]$$

$$= [\frac{\beta(r^2-2GM'r+\alpha^2)}{(r^2+\alpha^2)}][\frac{GM(r^2-\alpha^2)}{(r^2+\alpha^2)(r^2-2GMr+\alpha^2)}]$$

$$[\frac{(\sqrt{r^2+\alpha^2})(\sqrt{r^2(K-1)+\alpha^2(K-1)+2GMr+1})}{(r^2-2GMr+\alpha^2)}]$$

$$= [(1-K)(1 - \frac{2GM'}{r} + \frac{\alpha^2}{r^2})(1+\frac{\alpha^2}{r^2})^{-1}]$$

$$[\frac{GM}{r^2}(1-\frac{\alpha^2}{r^2})(1+\frac{\alpha^2}{r^2})^{-1}(1-\frac{2GM}{r}+\frac{\alpha^2}{r^2})^{-1}]$$

$$[(1+\frac{\alpha^2}{r^2})^{1/2}((K-1)+\frac{\alpha^2}{r^2}(K-1)+\frac{2GM}{r}+\frac{1}{r^2})^{1/2}(1-\frac{2GM}{r}+\frac{\alpha^2}{r^2})^{-1}]$$

$$\simeq [(1-K)(1-\frac{2GM'}{r}+\frac{\alpha^2}{r^2})(1-\frac{\alpha^2}{r^2})]$$

$$[\frac{GM}{r^2}(1-\frac{\alpha^2}{r^2})(1-\frac{\alpha^2}{r^2})(1+\frac{2GM}{r}-\frac{\alpha^2}{r^2})]$$

$$[(1+\frac{\alpha^2}{2r^2})\sqrt{(K-1)}(1+\frac{\alpha^2}{2r^2}+\frac{2GM}{2r(K-1)}+\frac{1}{2r^2(K-1)})(1+\frac{2GM}{r}-\frac{\alpha^2}{r^2})]$$

$$\xrightarrow{r\to\infty} \frac{(1-K)\sqrt{(K-1)}}{r^2} \sim \frac{|1-K|^{3/2}}{r^2}.$$

The remaining second term for $r \to \infty$ goes as $\frac{|1-K|^{\frac{3}{2}}}{r}$. As before this can be shown below:

$$\bar{G}^{11}\partial_1^2\phi_1 = [\frac{-\beta(r^2-2GM'r+\alpha^2)}{(r^2+\alpha^2)}]$$

$$[\frac{-r(r^2+2GMr+\alpha^2)(\sqrt{r^2(K-1)+\alpha^2(K-1)+2GMr+1})}{(\sqrt{r^2+\alpha^2})(r^2-2GMr+\alpha^2)^2}]$$

$$\xrightarrow{r\to\infty} \frac{-r(2GMr-r^2(1-K))(2GMr-r^2(1-K))^{1/2}(r^2+2GMr)}{r^3(r^2-2GMr)^2}$$

$$\xrightarrow{r\to\infty} \frac{r^2(1-K)r(r^2(K-1))^{1/2}r^2}{r^7}$$

$$\xrightarrow{r\to\infty} \frac{(1-K)\sqrt{(K-1)}}{r} \sim \frac{|1-K|^{3/2}}{r}.$$

As per the Planck collaboration results [12, 13], the value of dark energy density (in unit of critical density) K is about 0.696. Therefore, the second and third terms is negligible as the denominator goes to infinity. Therefore in this limit these terms also may be ignored and hence the equations of motion satisfied. *Therefore, $K \neq 0, 1$ and $0 < K < 1$. However K should be very close to unity for equations of motion to be satisfied at large r.*

3.4 The Hawking temperature for Kerr type metric

Now we go over to the Eddington-Finkelstein coordinates (v, r) or (u, r) along the symmetry axis $\theta = 0$.

$$v = \omega + r^* \; and \; u = \omega - r^*, \;\; \beta = 1 - K$$

and

$$r^* = \beta^{-1}[r + (\frac{r_+^2 + \alpha^2}{r_+ - r_-}) ln \; |r - r_+| - (\frac{r_-^2 + \alpha^2}{r_+ - r_-}) ln \; |r - r_-|] \tag{3.37}$$

with

$$r_+ = GM' + \sqrt{(GM')^2 - \alpha^2} = \frac{GM}{1-K} + \sqrt{(\frac{GM}{1-K})^2 - \alpha^2}$$

and

$$r_- = GM' - \sqrt{(GM')^2 - \alpha^2} = \frac{GM}{1-K} - \sqrt{(\frac{GM}{1-K})^2 - \alpha^2}.$$

Therefore the line element (3.36)

$$ds^2 = (\frac{\beta\Delta'}{r^2 + \alpha^2}) dv^2 - 2dvdr = \frac{\beta(r - r_+)(r - r_-)}{r^2 + \alpha^2} dv^2 - 2dvdr. \tag{3.38}$$

Now proceeding exactly as before we calculate the Hawking temperatures [70]-[72] for the two horizons as follows.

As discussed in section 3.2, using (3.38), a massless particle in a black hole background is described by the Klein-Gordon equation

$$\hbar^2(-\bar{G})^{-1/2}\partial_\mu(\bar{G}^{\mu\nu}(-\bar{G})^{1/2}\partial_\nu\Psi) = 0.$$

One expands

$$\Psi = exp(\frac{i}{\hbar}S + ...)$$

to obtain the leading order in $\hbar$ the Hamilton-Jacobi equation is

$$\bar{G}^{\mu\nu}\partial_\mu S\partial_\nu S = 0$$

Assume S is independent of θ and ϕ. Then

$$2\frac{\partial S}{\partial v}\frac{\partial S}{\partial r} + (\frac{\beta(r^2 - 2GM^{'}r + \alpha^2)}{r^2 + \alpha^2})(\frac{\partial S}{\partial r})^2$$
$$= \frac{\partial S}{\partial r}[2\frac{\partial S}{\partial v} + (\frac{\beta(r^2 - 2GM^{'}r + \alpha^2)}{r^2 + \alpha^2})\frac{\partial S}{\partial r}] = 0$$

The symmetries of the metric permit the action to be written as

$$S = -Ev + W(r) + J(x^i)$$

Then

$$\partial_v S = -E \; ; \; \partial_r S = W^{'} \; ; \; \partial_i S = J_i$$

J_i are constants chosen to be zero. Then

$$-2EW^{'}(r) + (\frac{\beta(r^2 - 2GM^{'}r + \alpha^2)}{r^2 + \alpha^2})(W^{'}(r))^2 = 0$$

Thus

$$W(r) = \int \frac{[E(r^2 + \alpha^2) + E(r^2 + \alpha^2)]dr}{\beta(r - r_+)(r - r_-)}$$

$$= 2\pi i(\frac{E}{\beta})(\frac{r_+^2+\alpha^2}{r_+-r_-}) + 2\pi i(\frac{E}{\beta})(\frac{r_-^2+\alpha^2}{r_--r_+})$$
$$= W(r_+) + W(r_-)$$

The two values of $W(r)$ correspond to the processes that the particle tunnels through the outer and inner horizons respectively.

Therefore

$$S = -Ev + 2\pi i(\frac{E}{\beta})(\frac{r_+^2+\alpha^2}{r_+-r_-}) + 2\pi i(\frac{E}{\beta})(\frac{r_-^2+\alpha^2}{r_--r_+}) + J(x^i)$$

The tunneling rates of the outer and inner horizons are

$$\Gamma^K_{+emergent} \sim e^{-2ImS+} \sim e^{-2ImW(r_+)} = e^{-4\pi(\frac{E}{\beta})(\frac{r_+^2+\alpha^2}{r_+-r_-})} = e^{-\frac{E}{K_BT_+}}$$

$$\Gamma^K_{-emergent} \sim e^{-2ImS-} \sim e^{-2ImW(r_-)} = e^{-4\pi(\frac{E}{\beta})(\frac{r_-^2+\alpha^2}{r_--r_+})} = e^{-\frac{E}{K_BT_-}}$$

From these two above equations the corresponding Hawking temperatures of the two horizons are respectively

$$T^K_{+emergent} = \frac{\hbar c^3\beta}{4\pi k_B}(\frac{r_+-r_-}{r_+^2+\alpha^2})$$

$$= \frac{\hbar c^3\beta}{4\pi k_B}[\frac{2\sqrt{(GM')^2-\alpha^2}}{(GM'+\sqrt{(GM')^2-\alpha^2})^2+\alpha^2}]$$

$$= \frac{\hbar c^3\beta}{4\pi k_B}[\frac{(1-K)\sqrt{(GM)^2-\alpha^2(1-K)^2}}{(GM)^2+GM\sqrt{(GM)^2-\alpha^2(1-K)^2}}]$$

$$= \frac{\hbar c^3(1-K)^2}{4\pi k_B}[\frac{\sqrt{(GM)^2-\alpha^2(1-K)^2}}{(GM)^2+GM\sqrt{(GM)^2-\alpha^2(1-K)^2}}]$$

and

$$T^K_{-emergent} = \frac{\hbar c^3\beta}{4\pi k_B}(\frac{r_--r_+}{r_-^2+\alpha^2})$$

$$= -\frac{\hbar c^3 \beta}{4\pi k_B}[\frac{2\sqrt{(GM')^2-\alpha^2}}{(GM'-\sqrt{(GM')^2-\alpha^2})^2+\alpha^2}]$$

$$= -\frac{\hbar c^3 \beta}{4\pi k_B}[\frac{(1-K)\sqrt{(GM)^2-\alpha^2(1-K)^2}}{(GM)^2-GM\sqrt{(GM)^2-\alpha^2(1-K)^2}}]$$

$$= -\frac{\hbar c^3 (1-K)^2}{4\pi k_B}[\frac{\sqrt{(GM)^2-\alpha^2(1-K)^2}}{(GM)^2-GM\sqrt{(GM)^2-\alpha^2(1-K)^2}}]$$

Thus the Hawking temperatures are :

$$T^K_{+emergent} = \frac{\hbar c^3 (1-K)^2}{4\pi k_B}\frac{\sqrt{(GM)^2-\alpha^2(1-K)^2}}{(GM)^2+GM\sqrt{(GM)^2-\alpha^2(1-K)^2}} \tag{3.39}$$

and

$$T^K_{-emergent} = -\frac{\hbar c^3 (1-K)^2}{4\pi k_B}(\frac{\sqrt{(GM)^2-\alpha^2(1-K)^2}}{(GM)^2-GM\sqrt{(GM)^2-\alpha^2(1-K)^2}}) \tag{3.40}$$

where k_B is the Boltzmann constant.

CHAPTER 4

4 Cosmology in presence of dark energy in an emergent gravity scenario

A natural question is whether the standard cosmology is modified if we take into account the presence of dark energy while building up the Friedman equations. The motivation of the work in this chapter is to seek plausible answers to the above question.

The results reported in this chapter have been obtained in **arXiv:gr-qc/1502.06255** of List of Publications.

Taking the background metric to be Friedman-Lemaitre-Robertson-Walker (FLRW), we obtain the modifications of the standard cosmological parameters in the radiation dominated, matter dominated and dark energy dominated phases of the universe. The dark energy density is identified with the kinetic energy $\dot{\phi}^2$ of the $k-$essence field. The standard cosmological parameters are retrieved when $\dot{\phi}^2 \rightarrow 0$, i.e., the dark energy vanishes.

The domination of the kinetic term in the DBI lagrangian for homogeneous dark energy fields $[\phi(r,t) \equiv \phi(t)]$ and background FLRW metric has already been discussed in the Introduction.

4.1 Emergent equations of motion for FLRW gravitational metric

Take the gravitational metric $g_{\mu\nu}$ to be FLRW and assume that the $k-$essence scalar field $\phi(r,t)$ is spherically symmetric ($\partial_t\phi = \partial_0\phi = \dot{\phi}$ and $\partial_r\phi = \partial_1\phi = \phi'$). Then (1.9) becomes

$$\begin{aligned}
\bar{G}_{00} = g_{00} - (\partial_0\phi)^2 = 1 - \dot{\phi}^2 \\
\bar{G}_{11} = g_{11} - (\partial_r\phi)^2 = -\frac{a^2(t)}{1-kr^2} - (\phi')^2 \\
\bar{G}_{22} = g_{22} = -a^2(t)r^2 \\
\bar{G}_{33} = g_{33} = -a^2(t)r^2 sin^2\theta \\
\bar{G}_{01} = \bar{G}_{10} = -\dot{\phi}\phi'
\end{aligned} \tag{4.1}$$

where the FLRW metric components are

$$g_{00} = 1;\ g_{11} = -\frac{a^2(t)}{1-kr^2};\ g_{22} = -a^2(t)r^2;$$
$$g_{33} = -a^2(t)r^2 sin^2\theta;\ g_{ij}(i \neq j) = 0.$$

The line element becomes

$$ds^2 = (1-\dot{\phi}^2)dt^2 - (\frac{a^2}{1-kr^2} + (\phi')^2)dr^2 - 2\dot{\phi}\phi' dtdr - a^2r^2d\Omega^2 \tag{4.2}$$

with $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$ and k is curvature constant.

Consider a co-ordinate transformation from (t, r, θ, ϕ) to $(\omega, r, \theta, \phi)$ so that [85]:

$$d\omega = dt - (\frac{\dot{\phi}\phi'}{1-\dot{\phi}^2})dr \tag{4.3}$$

with

$$dt^2 = d\omega^2 + \frac{2\dot{\phi}\phi'}{(1-\dot{\phi}^2)}d\omega dr + (\frac{\dot{\phi}\phi'}{1-\dot{\phi}^2})^2 dr^2.$$

Then (4.2) becomes

$$ds^2 = (1-\dot{\phi}^2)[d\omega^2 + \frac{2\dot{\phi}\phi^{'}}{(1-\dot{\phi}^2)}d\omega dr + (\frac{\dot{\phi}\phi^{'}}{1-\dot{\phi}^2})^2 dr^2]$$

$$-2\dot{\phi}\phi^{'}dr[d\omega + \frac{\dot{\phi}\phi^{'}}{(1-\dot{\phi}^2)}] - [\frac{a^2}{1-kr^2} + (\phi^{'})^2]dr^2 - a^2r^2d\Omega^2$$

$$= (1-\dot{\phi}^2)d\omega^2 - \frac{(\dot{\phi}\phi^{\prime})^2}{1-\dot{\phi}^2}dr^2 - [\frac{a^2}{1-kr^2} + (\phi^{'})^2]dr^2 - a^2r^2d\Omega^2$$

$$= (1-\dot{\phi}^2)d\omega^2 - [\frac{a^2}{1-kr^2} + (\phi^{\prime})^2 + \frac{(\dot{\phi}\phi^{\prime})^2}{(1-\dot{\phi}^2)}]dr^2 - a^2r^2d\Omega^2 \quad . \tag{4.4}$$

i.e.

$$\bar{G}_{\mu\nu} = \begin{pmatrix} (1-\dot{\phi}^2) & 0 & 0 & 0 \\ 0 & -Z & 0 & 0 \\ 0 & 0 & -(a^2r^2) & 0 \\ 0 & 0 & 0 & -(a^2r^2sin^2\theta) \end{pmatrix} \tag{4.5}$$

while its inverse is

$$\bar{G}^{\mu\nu} = \begin{pmatrix} (1-\dot{\phi}^2)^{-1} & 0 & 0 & 0 \\ 0 & -Z^{-1} & 0 & 0 \\ 0 & 0 & -(a^2r^2)^{-1} & 0 \\ 0 & 0 & 0 & -(a^2r^2sin^2\theta)^{-1} \end{pmatrix} \tag{4.6}$$

with $Z = (\frac{a^2}{1-kr^2} + (\phi^{\prime})^2 + \frac{(\dot{\phi}\phi^{\prime})^2}{(1-\dot{\phi}^2)})$.

The equation (1.7) means

$$\bar{G}^{00}\nabla_0\nabla_0\phi + \bar{G}^{01}\nabla_0\nabla_1\phi + \bar{G}^{10}\nabla_1\nabla_0\phi + \bar{G}^{11}\nabla_1\nabla_1\phi$$

$$= \bar{G}^{00}\nabla_0\partial_0\phi + \bar{G}^{11}\nabla_1\partial_1\phi$$

$$= \bar{G}^{00}(\partial_0\partial_0\phi - \Gamma^0_{00}\partial_0\phi - \Gamma^1_{00}\partial_1\phi) + \bar{G}^{11}(\partial_1\partial_1\phi - \Gamma^0_{11}\partial_0\phi - \Gamma^1_{11}\partial_1\phi)$$

$$= \bar{G}^{00}\partial_0^2\phi + \bar{G}^{11}(\partial_1^2\phi - \frac{a\dot{a}}{1-kr^2}\partial_0\phi - \frac{kr}{1-kr^2}\partial_1\phi)$$

$$= \frac{1}{(1-\dot{\phi}^2)}\partial_0^2\phi + [\frac{-1}{\frac{a^2}{1-kr^2} + (\phi')^2 + \frac{(\dot{\phi}\phi')^2}{(1-\dot{\phi}^2)}}](\partial_1^2\phi - \frac{a\dot{a}}{1-kr^2}\partial_0\phi - \frac{kr}{1-kr^2}\partial_1\phi)$$

$$= \frac{\ddot{\phi}}{1-\dot{\phi}^2} - \frac{(1-\dot{\phi}^2)}{a^2(1-\dot{\phi}^2) + (\phi')^2(1-kr^2)}[\phi^{''}(1-kr^2) - a\dot{a}\dot{\phi} - kr\phi^{'}] = 0 \tag{4.7}$$

i.e.

$$\begin{aligned} &\ddot{\phi}[a^2(1-\dot{\phi}^2) + (\phi')^2(1-kr^2)] \\ &= (1-\dot{\phi}^2)^2[\phi''(1-kr^2) - a\dot{a}\dot{\phi} - kr\phi'] \end{aligned} \tag{4.8}$$

We shall, henceforth, consider the FLRW universe for homogeneous dark energy fields only. So

$$\phi(r,t) \equiv \phi(t) \tag{4.9}$$

Here $\dot{\phi}^2 \neq 0$ since the $k-$essence field must have non-zero kinetic energy. Also $\dot{\phi}^2 \neq 1$ because $\Omega_{matter} + \Omega_{radiation} + \Omega_{darkenergy} = 1$ and $\dot{\phi}^2$ measured in units of the critical density is nothing but $\Omega_{darkenergy}$. Further, $\dot{\phi}^2 < 1$ always in order that the signature of the metric (4.5) does not become ill-defined. Therefore $0 < \dot{\phi}^2 < 1$. Therefore (4.8) becomes

$$\ddot{\phi}a^2(1-\dot{\phi}^2) = (1-\dot{\phi}^2)^2(-a\dot{a}\dot{\phi})$$

i.e.,

$$\frac{\dot{a}}{a} = H(t) = -\frac{\ddot{\phi}}{\dot{\phi}(1-\dot{\phi}^2)} \tag{4.10}$$

where $H(t) = \frac{\dot{a}}{a}$ is Hubble parameter ($always\ \dot{a} \neq 0$). So the equations of motion of emergent gravity relate the Hubble parameter to time derivatives of the $k-$essence scalar field.

4.2 The analogue of Friedmann equations in presence of dark energy

Using metrics (4.5) and (4.6) for homogeneous fields $\phi(t)$ we get the non-vanishing connection coefficients as:

$$\bar{\Gamma}^0_{00} = -\frac{\dot{\phi}\ \ddot{\phi}}{1-\dot{\phi}^2};$$

$$\bar{\Gamma}^0_{11} = \frac{1}{1-\dot{\phi}^2}\frac{a\dot{a}}{1-kr^2};$$

$$\bar{\Gamma}^0_{22} = \frac{a\dot{a}\ r^2}{1-\dot{\phi}^2};$$

$$\bar{\Gamma}^0_{33} = \frac{a\dot{a}\ r^2 sin^2\theta}{1-\dot{\phi}^2};$$

$$\bar{\Gamma}^1_{01} = \bar{\Gamma}^1_{10} = \frac{\dot{a}}{a};$$

$$\bar{\Gamma}^1_{11} = \frac{kr}{1-kr^2};$$

$$\bar{\Gamma}^1_{22} = -r(1-kr^2);$$

$$\bar{\Gamma}^1_{33} = -r sin^2\theta\ (1-kr^2);$$

$$\bar{\Gamma}^2_{02} = \bar{\Gamma}^2_{20} = \frac{\dot{a}}{a};$$

$$\bar{\Gamma}^2_{12} = \bar{\Gamma}^2_{21} = \frac{1}{r};$$

$$\bar{\Gamma}^2_{33} = -sin\theta\ cos\theta;$$

$$\bar{\Gamma}^3_{03} = \bar{\Gamma}^3_{30} = \frac{\dot{a}}{a};$$

$$\bar{\Gamma}^3_{13} = \bar{\Gamma}^3_{31} = \frac{1}{r};$$

$$\bar{\Gamma}^3_{23} = \bar{\Gamma}^3_{32} = cot\theta.$$

These calculations of connection coefficients are shown in **Appendix B**.

Now we calculate the diagonal components of Ricci tensor for homogeneous scalar field since off-diagonal components of Ricci tensor are zero.

$$\bar{R}_{00} = 3\frac{\ddot{a}}{a} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)} \tag{4.11}$$

$$\bar{R}_{11} = -\frac{a^2}{1-kr^2}[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.12}$$

$$\bar{R}_{22} = -a^2r^2[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.13}$$

$$\bar{R}_{33} = -a^2r^2sin^2\theta[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.14}$$

The elaborate calculations of Ricci tensors are established in **Appendix C**.

To calculate Ricci scalar for homogeneous scalar field using (4.6):

$$\bar{R}^0_0 = \bar{G}^{00}\bar{R}_{00} = \frac{1}{(1-\dot{\phi}^2)}[3\frac{\ddot{a}}{a} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)}] \tag{4.15}$$

$$\bar{R}^1_1 = \bar{G}^{11}\bar{R}_{11} = -\frac{(1-kr^2)}{a^2}[\frac{-a^2}{(1-kr^2)}[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)}$$
$$+2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]]$$

$$= [\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.16}$$

$$\bar{R}^2_2 = \bar{G}^{22}\bar{R}_{22} = \frac{-1}{a^2r^2}[-a^2r^2[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]]$$

$$= [\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.17}$$

$$\bar{R}^3_3 = \bar{G}^{33}\bar{R}_{33} = \frac{-1}{a^2r^2sin^2\theta}[-a^2r^2sin^2\theta[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)}$$
$$+2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]]$$

$$= [\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.18}$$

Therefore the Ricci Scalar:

$$\bar{R} = \bar{R}^0_0 + \bar{R}^1_1 + \bar{R}^2_2 + \bar{R}^3_3 = 6[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + \frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.19}$$

We have the Einstein's Field Equation: $\bar{E}_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{1}{2}\bar{G}_{\mu\nu}\bar{R} = -8\pi G T_{\mu\nu}$ i.e.

$$\bar{E}^\nu_\mu = \bar{R}^\nu_\mu - \frac{1}{2}\delta^\nu_\mu\bar{R} = -8\pi G T^\nu_\mu \tag{4.20}$$

where G is gravitational constant and $T_{\mu\nu}$ is energy-momentum tensor. Components of Einstein tensor are:

$$\bar{E}^0_0 = \bar{R}^0_0 - \frac{1}{2}\bar{R} = \frac{1}{(1-\dot{\phi}^2)}[3\frac{\ddot{a}}{a} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)}]$$

$$-\frac{1}{2}6[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + \frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]$$

$$= -3[\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + \frac{k}{a^2}]; \tag{4.21}$$

$$\bar{E}^1_1 = \bar{E}^2_2 = \bar{E}^3_3 = [\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]$$

$$-\frac{1}{2}6[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + \frac{k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]$$

$$= -[2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + \frac{k}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.22}$$

The energy-momentum tensor of an ideal fluid is

$$T_{\mu}^{\nu} = (p + \rho) u_{\mu} u^{\nu} - \delta_{\mu}^{\nu} p \qquad (4.23)$$

where p is pressure and ρ is the energy density of the cosmic fluid. In the co-moving frame we have $u^0 = 1$ and $u^i = 0$; $i = 1, 2, 3$.

Now the general $k-$essence field theoretic lagrangian $L(X, \phi)$, which explicitly depends on ϕ, is not equivalent to isentropic hydrodynamics because ϕ and X are independent and hence the pressure cannot be a function of the energy density ρ only. So a pertinent question is whether we are at all justified in assuming a perfect fluid model when dark energy is present. The answer is yes because our lagrangian $L(X) = 1 - V\sqrt{1 - 2X}$, where V is a constant, does not depend explicitly on ϕ. This class of models is equivalent to perfect fluid models with zero vorticity and the pressure (lagrangian) can be expressed through the energy density only [48].

Then (4.23) becomes

$$T_0^0 = \rho \ ; \ T_1^1 = T_2^2 = T_3^3 = -p \qquad (4.24)$$

Using equations (4.20)-(4.23) we get

$$\bar{E}_0^0 = -8\pi G T_0^0$$

or,

$$-3[\frac{\dot{a}^2}{a^2}\frac{1}{(1 - \dot{\phi}^2)} + \frac{k}{a^2}] = -8\pi G \rho_d$$

i.e.,

$$\rho_d = \frac{3}{8\pi G}[\frac{\dot{a}^2}{a^2}\frac{1}{(1 - \dot{\phi}^2)} + \frac{k}{a^2}] \qquad (4.25)$$

and

$$\bar{E}^i_i = -8\pi G T^i_i$$

with $i = 1, 2, 3$ or,

$$-[2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + \frac{k}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] = 8\pi G p_d$$

i.e.,

$$p_d = -\frac{1}{8\pi G}[2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + \frac{k}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.26}$$

where we now replace ρ by ρ_d as the total energy density in presence of dark energy and p by p_d as the pressure when dark energy is present. Note that both ρ_d, p_d reduce to the usual quantities ρ, p [77]-[80] when dark energy is absent, i.e., $(\dot{\phi})^2 = 0$. The usual Friedman equations are now modified into the above two equations (4.25) and (4.26) in the presence of $k-$essence scalar field ϕ.

Combining above two equations (4.25) and (4.26) we get,

$$\frac{4\pi G}{3}(\rho_d + 3p_d) = -[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \tag{4.27}$$

Now differentiating equation (4.25) with respect to cosmic time t we get

$$\frac{2\dot{a}\ddot{a}}{1-\dot{\phi}^2} + \frac{2\dot{a}^2\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} = \frac{8\pi G}{3}[\dot{\rho}_d a^2 + 2\rho_d a\dot{a}],$$

dividing both sides by $a\dot{a}$ in above equation

$$\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + (\frac{\dot{a}}{a})\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} = \frac{4\pi G}{3}[\frac{\dot{\rho}_d a}{\dot{a}} + 2\rho_d]$$

and substituting the above result in equation (4.27) we get,

$$\frac{4\pi G}{3}(\rho_d + 3p_d) = -\frac{4\pi G}{3}[\frac{\dot{\rho}_d a}{\dot{a}} + 2\rho_d]$$

i.e.,

$$\dot{\rho}_d = -3\frac{\dot{a}}{a}(p_d + \rho_d) = -3H(p_d + \rho_d) \tag{4.28}$$

which is the required energy conservation equation in presence of dark energy. Again it may be noted that one recovers the usual energy conservation equation [77]-[80] when dark energy is absent.

Now assume that the criterion for non-relativistic scenario remains the same, *viz.*, $\rho_d \gg p_d$. We restrict now to $k = 0$ as observationally this is most likely. Then the above condition become [using (4.25) and (4.26)]

$$\frac{3}{8\pi G}[\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)}] >> -\frac{1}{8\pi G}[2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]$$

or,

$$\frac{1}{2\pi G}[\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)}] >> -\frac{1}{8\pi G}[2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]$$

i.e.,

$$\frac{\dot{a}^2}{a^2} \gg -\frac{\ddot{a}}{2a} - \frac{\dot{a}\dot{\phi}\ddot{\phi}}{2a(1-\dot{\phi}^2)}.$$

So the second term on right hand side must be always positive i.e., $\dot{\phi}\ddot{\phi}$ must be always negative. This means that $\frac{d\dot{\phi}^2}{dt} < 0$. This criterion is consistent with the fact that the dark energy density cannot increase in a matter dominated era.

Then, neglecting p_d in (4.28) gives

$$\dot{\rho}_d\frac{a}{\dot{a}} + 3\rho_d = 0$$

which has the solution

$$\rho_d^{mat} = \frac{A}{a^3} \tag{4.29}$$

where $A = constant$. Assuming that the total energy i.e. $\rho_d a^3$ is a constant, we equate this to the present epoch energy i.e. $\rho_d a^3 = \rho_{d0} a_0^3$, where ρ_{d0} and a_0 are the matter density and scale radius at the present epoch ($t = t_0$). This fixes the constant $A = \rho_{d0} a_0^3$ in terms of present epoch values.

For the relativistic situation we assume again that the criterion is same as in standard cosmology, i.e. $p_d = \frac{\rho_d}{3}$. Here this gives the condition [using (4.25) and (4.26)]

$$-\frac{1}{8\pi G}[2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] = \frac{1}{8\pi G}[\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)}]$$

i.e.,

$$\frac{\dot{a}^2}{a^2} = -\frac{\ddot{a}}{a} - \frac{\dot{a}\dot{\phi}\ddot{\phi}}{a(1-\dot{\phi}^2)}.$$

For same reasons as given in the previous case, here also the conditions are consistent.

We get from (4.28) the solution

$$\rho_d^{rad} = \frac{B}{a^4} \tag{4.30}$$

where the constant B is fixed to be $B = \rho_{d0} a_0^4$ following same arguments as before.

Finally we consider the dark energy dominated scenario $p_d = -\rho_d$, i.e., [using (4.25) and (4.26)]

$$-\frac{1}{8\pi G}[2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] = -\frac{3}{8\pi G}[\frac{\dot{a}^2}{a^2}\frac{1}{(1-\dot{\phi}^2)}]$$

i.e.,

$$\frac{\dot{a}^2}{a^2} = \frac{\ddot{a}}{a} + \frac{\dot{a}\dot{\phi}\ddot{\phi}}{a(1-\dot{\phi}^2)}.$$

This means that here $\frac{d\dot{\phi}^2}{dt} > 0$ i.e. the dark energy density must increase. This is also consistent. Equation (4.28) then leads to

$$\rho_d = W \tag{4.31}$$

where we may choose the constant W to be $\dot{\phi}^2|_{t=t'}$ with t' denoting some specific epoch.

Therefore, the difference from the standard cosmology lies only in the fact that in our case the dark energy density (which is being identified with the kinetic energy of the $k-$essence field) has the following behaviour: *in the matter and radiation dominated eras the time rate of change of dark energy density decreases, while in the dark energy dominated epoch this rate increases.*

4.3 Solutions of the modified equations

4.3.1 Non-relativistic case (Matter dominated Universe)

The non-relativistic case means $\rho_d \gg p_d$ and (4.25) and (4.26) can be written as follows,

$$\frac{H^2}{(1-\dot{\phi}^2)} + \frac{k}{a^2} = \frac{8\pi G}{3}\frac{A}{a^3} \tag{4.32}$$

and

$$2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{H^2}{(1-\dot{\phi}^2)} + \frac{k}{a^2} + 2H\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} = 0 \tag{4.33}$$

Eliminating A using (4.29), and remembering that for $t = t_0$ (present epoch) $\rho_d = \rho_{d0}, a = a_0, H = H_0$ (4.32) becomes

$$\frac{k}{a_0^2} = \frac{8\pi G}{3}[\rho_{d0} - \rho_d^c] \tag{4.34}$$

where

$$\rho_d^c = \frac{3H_0^2}{8\pi G(1 - \dot{\phi}^2)} \tag{4.35}$$

is the critical value of matter density when dark energy is present. For $\dot{\phi}^2 < 1$ (4.35) becomes

$$\rho_d^c = \rho_c + \rho_c\dot{\phi}^2 \tag{4.36}$$

keeping terms upto $O(\dot{\phi}^2)$ only. Here

$$\rho_c = \frac{3H_0^2}{8\pi G},$$

the critical value of matter density and $\rho_d^c > \rho_c$.

Now consider the FLRW universe with $k = 0$. We get from (4.34)

$$\rho_{d0} = \rho_d^c = \frac{3H_o^2}{8\pi G(1 - \dot{\phi}^2)} \tag{4.37}$$

and the critical matter density becomes same as that of ρ_{d0}.

Now from (4.32) with $k = 0$, we have

$$(\frac{\dot{a}}{a})^2 = \frac{C}{a^3}(1 - \dot{\phi}^2)$$

where $C = \frac{8\pi GA}{3}$. We now take the negative square root of this equation so as to be consistent with observations. This will be borne out later. Therefore,

$$\frac{\dot{a}}{a} = -\frac{C^{\frac{1}{2}}}{a^{\frac{3}{2}}}(1 - \dot{\phi}^2)^{\frac{1}{2}}. \tag{4.38}$$

Using equations (4.10) and (4.38) we get

$$-\frac{\ddot{\phi}}{\dot{\phi}(1-\dot{\phi}^2)} = -\frac{C^{\frac{1}{2}}}{a^{\frac{3}{2}}}(1-\dot{\phi}^2)^{\frac{1}{2}}$$

i.e.,

$$a(t) = C^{\frac{1}{3}}(\frac{\dot{\phi}}{\ddot{\phi}})^{\frac{2}{3}}(1-\dot{\phi}^2) \tag{4.39}$$

Therefore the deceleration parameter for non-relativistic case with $k=0$

$$q(t)^{NR} = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{Numerator}{Denominator}. \tag{4.40}$$

where,

$$\begin{aligned} Numerator = (1-\dot{\phi}^2)[(1+20\dot{\phi}^2)(\ddot{\phi})^4 + \dot{\phi}\phi^{(3)}(\ddot{\phi})^2(1-4\dot{\phi}^2) \\ +5\dot{\phi}^2(\phi^{(3)})^2(-1+\dot{\phi}^2) - 3\dot{\phi}^2\ddot{\phi}\phi^{(4)}(-1+\dot{\phi}^2)] \end{aligned} \tag{4.41}$$

and

$$Denominator = 2[(1-4\dot{\phi}^2)(\ddot{\phi})^2 + \dot{\phi}\phi^{(3)}(-1+\dot{\phi}^2)]^2 \tag{4.42}$$

We shall take $\phi^{(3)}, \phi^{(4)}$ to be zero also neglecting higher order of $\dot{\phi}^2$, where $\phi^{(3)}$ is 3rd order and $\phi^{(4)}$ 4th order derivative with respect to time. Then the deceleration parameter for non-relativistic case becomes,

$$q(t)^{NR} = \frac{1}{2}(1+27\dot{\phi}^2+...) \tag{4.43}$$

Note that the choice of the sign of the square root (that leads to (4.38)) ensures that the value of the deceleration parameter for the matter dominated era is as in standard cosmology i.e. when dark energy is absent. This value is $\frac{1}{2}$. Moreover, it can be checked that choice of a positive square root (leading to (4.38)) will give an imaginary scale factor which is unacceptable.

4.3.2 Relativistic case (Radiation dominated Universe)

For this case $p_d = \frac{\rho_d}{3}$ from (4.30) $\rho_d = \frac{B}{a^4}$ then modified Friedmann equations (4.25) and (4.26) becomes

$$\frac{H^2}{(1-\dot{\phi}^2)} + \frac{k}{a^2} = \frac{8\pi G}{3}\frac{B}{a^4} \tag{4.44}$$

and

$$2\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + \frac{H^2}{(1-\dot{\phi}^2)} + \frac{k}{a^2} + 2H\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} = -\frac{8\pi G}{3}\frac{B}{a^4} \tag{4.45}$$

Considering the $k = 0$ model of the Universe , the modified Friedmann equation (4.44) becomes

$$(\frac{\dot{a}}{a})^2 = \frac{D}{a^4}(1-\dot{\phi}^2)$$

where $D = \frac{8\pi GB}{3}$. Again we take the negative square root of this equations from physical considerations to get

$$\frac{\dot{a}}{a} = -\frac{D^{\frac{1}{2}}}{a^2}(1-\dot{\phi}^2)^{\frac{1}{2}}. \tag{4.46}$$

Again combining equations (4.10) and (4.46) we obtain

$$-\frac{\ddot{\phi}}{\dot{\phi}(1-\dot{\phi}^2)} = -\frac{D^{\frac{1}{2}}}{a^2}(1-\dot{\phi}^2)^{\frac{1}{2}}$$

i.e.,

$$a(t) = D^{\frac{1}{4}}(\frac{\dot{\phi}}{\ddot{\phi}})^{\frac{1}{2}}(1-\dot{\phi}^2)^{\frac{3}{4}}. \tag{4.47}$$

Therefore the deceleration parameter for relativistic case with $k = 0$ is:

$$q(t)^R = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{Numerator}{Denominator} \tag{4.48}$$

where,

$$Numerator = [(1+10\dot{\phi}^2 - 8\dot{\phi}^4)(\ddot{\phi})^4 - 3\dot{\phi}^2(\phi^{(3)})^2(-1+\dot{\phi}^2)^2 + 2\dot{\phi}^2(-1+\dot{\phi}^2)^2\ddot{\phi}\phi^{(4)}] \tag{4.49}$$

and

$$Denominator = [(1 - 4\dot{\phi}^2)(\ddot{\phi})^2 + \dot{\phi}\phi^{(3)}(-1 + \dot{\phi}^2)]^2 \tag{4.50}$$

Finaly the deceleration parameter (neglecting as above higer order of $\dot{\phi}^2$ and higher order derivatives) for relativistic case is

$$q(t)^R = 1 + 18\dot{\phi}^2 + ... \tag{4.51}$$

Again, the choice of the sign of the square root (leading to (4.46)) ensures that the standard cosmology result is obtained for the deceleration parameter. This value is 1. Also choice of a positive square root is ruled out to ensure reality of the scale factor.

4.3.3 Dark energy dominated Universe

For this case $p_d \simeq -\rho_d$ and from (4.31) $\rho_d = constant = W$, then the modified Friedmann equations (4.25) and (4.26) becomes

$$\frac{\dot{a}^2}{a^2}\frac{1}{(1 - \dot{\phi}^2)} + \frac{k}{a^2} = \frac{8\pi G}{3}W \tag{4.52}$$

and

$$2\frac{\ddot{a}}{a}\frac{1}{(1 - \dot{\phi}^2)} + \frac{\dot{a}^2}{a^2}\frac{1}{(1 - \dot{\phi}^2)} + \frac{k}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1 - \dot{\phi}^2)^2} = 8\pi GW \tag{4.53}$$

Again we consider $k = 0$ model of the Universe, The modified Friedmann equation (4.52) becomes

$$\frac{\dot{a}}{a} = \alpha^{\frac{1}{2}}(1 - \dot{\phi}^2)^{\frac{1}{2}} \tag{4.54}$$

where $\alpha = \frac{8\pi GW}{3} = constant$. Now combining equations (4.10) and (4.54) we obtain

$$-\frac{\ddot{\phi}}{\dot{\phi}(1-\dot{\phi}^2)} = \alpha^{\frac{1}{2}}(1-\dot{\phi}^2)^{\frac{1}{2}}$$

i.e.,

$$\frac{\ddot{\phi}}{\dot{\phi}} = -\alpha^{\frac{1}{2}}(1-\dot{\phi}^2)^{\frac{3}{2}}. \tag{4.55}$$

Now from equation (4.54) we get the scale factor

$$a(t) = e^{\sqrt{\alpha}\int\sqrt{1-\dot{\phi}^2}dt} \tag{4.56}$$

Using above equation (4.56) we have the deceleration parameter for this case

$$q(t)^{dark} = -1 + \frac{\dot{\phi}\ddot{\phi}}{\sqrt{\alpha}(1-\dot{\phi}^2)^{3/2}} \tag{4.57}$$

Further, using (4.55) the deceleration parameter becomes

$$q(t)^{dark} = -1 - \dot{\phi}^2 \tag{4.58}$$

As $\dot{\phi}^2$ is positive the deceleration parameter is always negative.

Consider now an interesting situation. The dark energy density $\dot{\phi}^2 < 1$ for reasons mentioned before. Then expanding the binomial in (4.55) and keeping terms upto $O(\dot{\phi}^2)$,

$$\frac{\ddot{\phi}}{\dot{\phi}} \simeq -\sqrt{\alpha}(1 - \frac{3}{2}\dot{\phi}^2)$$

or,

$$\ddot{\phi} - \frac{3}{2}\sqrt{\alpha}\dot{\phi}^3 + \sqrt{\alpha}\dot{\phi} = 0$$

and an approximate solution of above equation for the dark energy density is

$$\dot{\phi}^2 = \frac{2\sqrt{\alpha}}{3\sqrt{\alpha} + 2e^{2\sqrt{\alpha}t}}. \tag{4.59}$$

Now $\dot{\phi}^2 < 1$ applied to (4.59) means

$$\sqrt{\alpha} + 2e^{2\sqrt{\alpha}t} > 0 \tag{4.60}$$

Note that if $\sqrt{\alpha}$ is positive then (4.60) is always satisfied for all values of t. However, figure 4.1 shows that there is absolutely no agreement of the predicted values of dark energy density with the observed data [12, 13] at present epoch. So we reject this choice.

On the other hand, taking the negative square root for α gives encouraging agreement of predicted values for the dark energy density $\dot{\phi}^2$ with the observed value at present epoch *viz.*, 0.6817 [12, 13]. This is evident in figure 4.2. So we choose the negative square root. The best agreements are obtained for $-10 \leq \alpha \leq -2.1$.

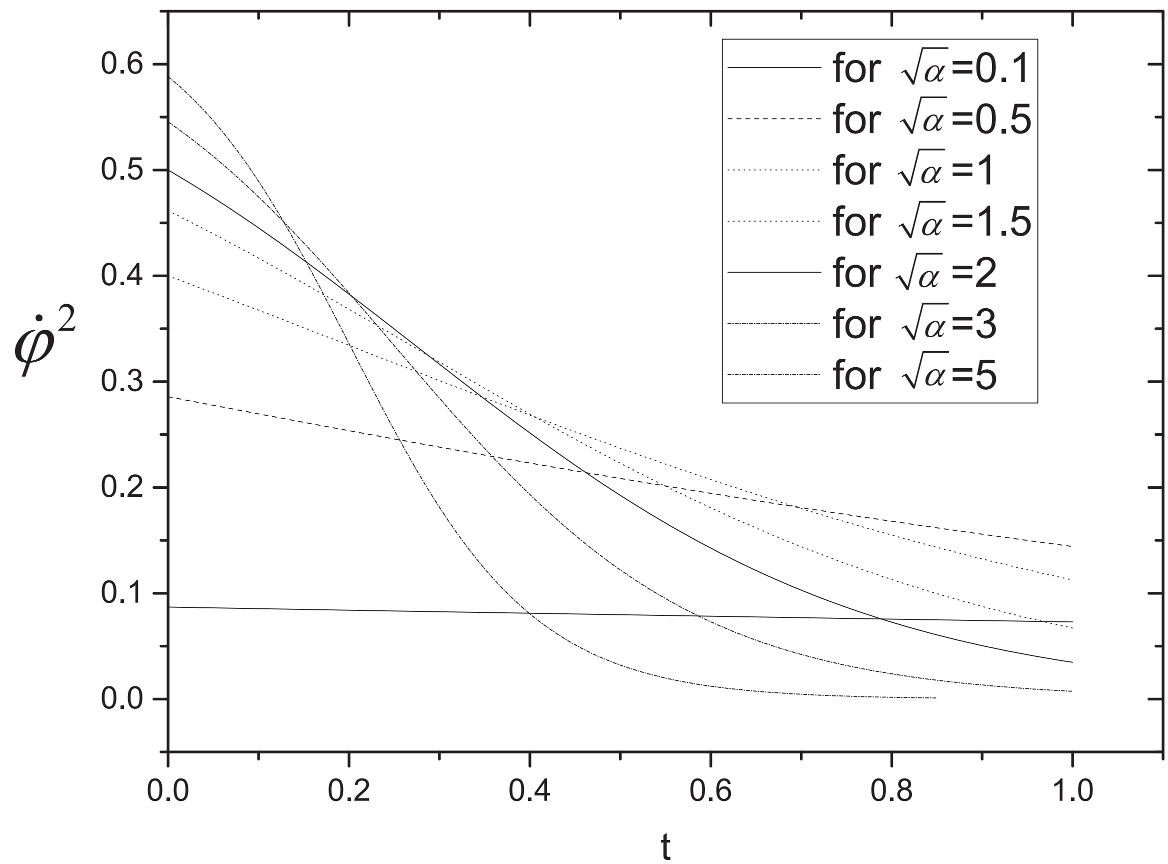

Figure 4.1: Variation of dark energy density with time for positive $\sqrt{\alpha}$ where values of $\sqrt{\alpha}$ are shown *down* $\rightarrow$ *up*.

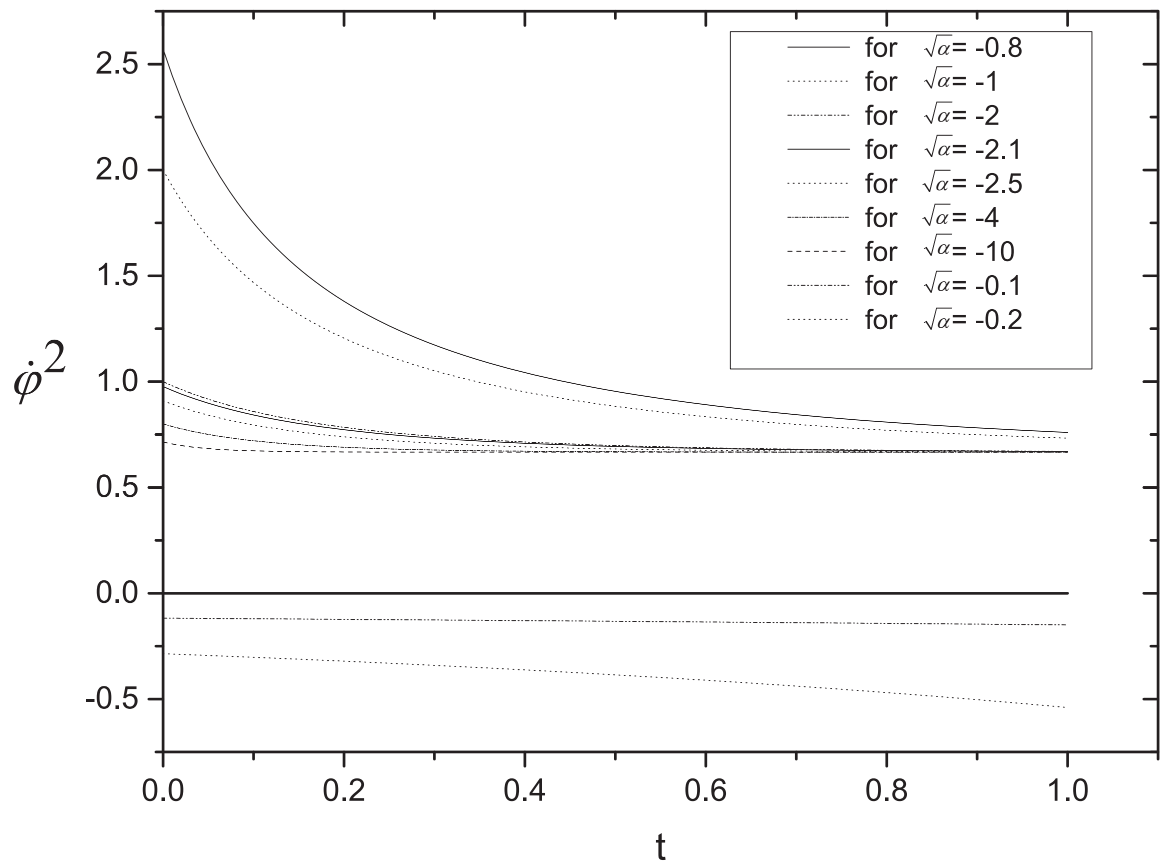

Figure 4.2: Variation of dark energy density with time for negative $\sqrt{\alpha}$ where values of $\sqrt{\alpha}$ are shown $up \rightarrow down$.

CHAPTER 5

5 Conclusions

The conclusions of the thesis are:

5.1 Chapter 2

For the background gravitatinal metric as Schwarzschild, the resulting emergent gravity metric is similar to a Barriola-Vilenkin metric where the monopole charge is replaced by the kinetic energy of the $k-$essence scalar field. Thus the Einstein's field equations are automatically satisfied. We then show that if $\phi_{emergent}$ be solutions of the emergent gravity equations of motion under cosmological boundary conditions at ∞, then for $r \to \infty$ the rescaled field $\frac{\phi_{emergent}}{2GM-1}$ has exact correspondence with ϕ with $\phi(r,t) = \phi_1(r) + \phi_2(t)$. The Hawking temperature of the resulting BV-type metric is found to be

$$T_{emergent} = (1-K)^2 T_S.$$

Here $K = \dot{\phi}_2^2$ is the kinetic energy of the $k-$essence field ϕ and K is always less than unity and T_S is the usual Hawking temperature for Schwarzschild black hole. We have then indicated why certain phenemenological parameters in Belgiorno's analogue gravity experiment will be modified because of the difference of the Schwarzschild metric from that of a BV-type metric. In [66, 87] the authors interpreted photon emission (different from usual photon emission) as an indication of Hawking radiation induced by the analogue event horizon for the Schwarzschild metric using ultrashort laser pulse filaments to create a travelling refractive index perturbation (RIP) in fused silica glass. In this work we have proposed that a different Hawking temperature in the presence of dark energy should give a different set of phenomenological parameters since the intensity of photon emission should depend on the relevant blackbody temperature. So one has a further scope to enhance this analogue gravity experiment for testing the existance of dark energy. If we include the effects of dark energy, the RIP method must be accordingly modified. This means that the effect of the presence of the constant K (dark enegy density) in the metric must be included. The Belgiorno *et al* original analogue gravity experiment currently has $K = 0$ for the Schwarzschild black hole. So the experimental situation has to move over to a scenario which can mimic $K \neq 0$ i.e., $0 < K < 1$. So in the presence of dark energy certain phenemenological parameters *viz. the boost* (γ)*, the constant velocity of the RIP* (v)*, refractive index* ($n = c/v$) *etc.* in Belgiorno's analogue gravity experiment can be modified.

5.2 Chapter 3

For the spherically symmetric Reissner-Nordstrom background metric along $\theta = 0$ the resulting emergent gravity metric is Robinson-Trautman type so that Einstein's field equations are automatically satisfied. For $\theta = 0$ the k-essence scalar field satisfies the emergent gravity equations of motion. The Hawking temperatures for the two horizons of the Robinson-Trautman black hole are different from that of the Reissner-Nordstrom black hole. In this case, the dark energy density is constrained to be unity.

With this constraint of the dark energy density this blackhole has zero Hawking temperature i.e. it does not radiate.

We next work with a Kerr background along $\theta = 0$ again so that the emergent gravity equations of motion are again satisfied by the dark energy field. In this case the resulting emergent gravity metric is also Kerr type in the presence of dark energy. The emergent blackhole metric satisfies Einstein's equations for large r. This type of the emergent black hole always radiates since there is no constraint on the dark energy density to be unity. In this case the values of the dark energy density is ($\dot{\phi}_2^2 = K = constant$) $0 < K < 1$.

It should be mentioned that if dark energy density $K = 0$ then the usual Hawking temperatures for both cases are retrieved. There are two event horizons and hence two Hawking temperatures in both the cases. But out of these two temperatures, only one *viz.*, that corresponding to the outer horizon is observationally relevant.

5.3 Chapter 4

In this work we have investigated the cosmological consequences of incorporating dark energy in an emergent gravity scenario. First we obtained the analogues of the Friedman equations where the background metric is taken to be FLRW. We consider the FLRW universe for homogeneous dark energy fields only. Assuming the usual perfect fluid model for the universe, we next determined the total energy density. Finally, the cosmological implications were determined corresponding to various values of this energy. Our findings are as follows :

(a) For total energy density greater than the pressure (matter dominated

Universe) the deceleration parameter

$$q(t) \approx \frac{1}{2}[1 + 27\dot{\phi}^2 + ...] > \frac{1}{2}.$$

(b) For total energy density equal to 3 times the pressure (radiation dominated Universe),

$$q(t) \approx 1 + 18\dot{\phi}^2 + ... > 1$$

and

(c) For total energy density equal to the negative of the pressure (dark energy dominated Universe), the deceleration parameter

$$q(t) = -1 - \dot{\phi}^2 < -1.$$

The values of the dark energy density are $0 < \dot{\phi}^2 < 1$.

Note that for dark energy density $\dot{\phi}^2 = 0$, the conventional results are retrieved. Our results indicate that many aspects of standard cosmology can be accommodated with the presence of dark energy right from the beginning of the universe where the time parameter $t \equiv \frac{t}{t_0}$, with t_0 being the present epoch.

6 Appendix

Appendix A

Evaluation of relevant terms of the Einstein's equations for the emergent gravity metric (3.36) along $\theta = 0$.

From (3.36) the emergent gravity metric for Kerr background along $\theta = 0$ is

$$\bar{G}_{\mu\nu} = \begin{pmatrix} \frac{\beta\Delta'}{\rho^2} & 0 \\ 0 & -\frac{\rho^2}{\beta\Delta'} \end{pmatrix} \qquad (A.1)$$

and

$$\bar{G}^{\mu\nu} = \begin{pmatrix} \frac{\rho^2}{\beta\Delta'} & 0 \\ 0 & -\frac{\beta\Delta'}{\rho^2} \end{pmatrix} \qquad (A.2)$$

with

$$\beta = 1 - K,\ M' = \frac{M}{1-K},\ \Delta' = (r^2 - 2GM'r + \alpha^2)\ and\ \rho^2 = r^2 + \alpha^2.$$

We calculate the non-vanishing connection coefficients using $(A.1)$, $(A.2)$ and the relation $\bar{\Gamma}^{\alpha}_{\mu\nu} = \frac{1}{2}\bar{G}^{\alpha\beta}[\partial_\mu \bar{G}_{\beta\nu} + \partial_\nu \bar{G}_{\mu\beta} - \partial_\beta \bar{G}_{\mu\nu}]$.

$$\bar{\Gamma}^0_{10} = \frac{1}{2}\bar{G}^{0\alpha}[\partial_0 \bar{G}_{\alpha 1} + \partial_1 \bar{G}_{0\alpha} - \partial_\alpha \bar{G}_{01}]$$

$$= \frac{1}{2}\bar{G}^{00}[\partial_0 \bar{G}_{01} + \partial_1 \bar{G}_{00}] = \frac{1}{2}\bar{G}^{00}\partial_1 \bar{G}_{00}$$

$$= \frac{1}{2}(\frac{\rho^2}{\beta\Delta'})\partial_1(\frac{\beta\Delta'}{\rho^2})$$

$$= \frac{1}{2}[\frac{r^2+\alpha^2}{\beta(r^2-2GM'r+\alpha^2)}]$$
$$[\frac{\beta(r^2+\alpha^2)(2r-2GM^{'})-\beta(r^2-2GM'r+\alpha^2)2r}{(r^2+\alpha^2)^2}]$$

$$= \frac{1}{2}[\frac{r^2+\alpha^2}{\beta(r^2-2GM'r+\alpha^2)}]$$
$$[\frac{\beta(2r^3-2GM^{'}r^2+2r\alpha^2-2GM'\alpha^2-2r^3+4GM^{'}r^2-2\alpha^2 r)}{(r^2+\alpha^2)^2}]$$

$$= \frac{GM^{'}(r^2-\alpha^2)}{(r^2+\alpha^2)(r^2-2GM'r+\alpha^2)} = \bar{\Gamma}^0_{01} \qquad (A.3)$$

$$\bar{\Gamma}^1_{11} = \frac{1}{2}\bar{G}^{1\alpha}[\partial_1\bar{G}_{\alpha 1}+\partial_1\bar{G}_{1\alpha}-\partial_\alpha\bar{G}_{11}]$$

$$= \frac{1}{2}\bar{G}^{11}[\partial_1\bar{G}_{11}+\partial_1\bar{G}_{11}-\partial_1\bar{G}_{11}] = \frac{1}{2}\bar{G}^{11}\partial_1\bar{G}_{11}$$

$$= \frac{1}{2}(\frac{-\beta\Delta'}{\rho^2})\partial_1(\frac{-\rho^2}{\beta\Delta'})$$

$$= \frac{1}{2}[\frac{(r^2-2GM'r+\alpha^2)}{r^2+\alpha^2}]$$
$$[\frac{(r^2-2GM'r+\alpha^2)2r-(r^2+\alpha^2)(2r-2GM^{'})}{(r^2-2GM'r+\alpha^2)^2}]$$

$$= \frac{1}{2}[\frac{(r^2-2GM'r+\alpha^2)}{r^2+\alpha^2}]$$

$$[\frac{2r^3-4GM^{'}r^2+2\alpha^2 r-2r^3+2GM^{'}r^2-2\alpha^2 r+2GM^{'}\alpha^2}{(r^2-2GM'r+\alpha^2)^2}]$$

$$=\frac{-GM^{'}(r^2-\alpha^2)}{(r^2+\alpha^2)(r^2-2GM'r+\alpha^2)}=-\bar{\Gamma}^0_{10} \qquad (A.4)$$

$$\bar{\Gamma}^1_{00}=\frac{1}{2}\bar{G}^{1\alpha}[\partial_0\bar{G}_{\alpha 0}+\partial_0\bar{G}_{0\alpha}-\partial_\alpha\bar{G}_{00}]$$

$$=\frac{1}{2}\bar{G}^{11}[\partial_0\bar{G}_{10}+\partial_0\bar{G}_{01}-\partial_1\bar{G}_{00}]=-(\frac{1}{2})\bar{G}^{11}\partial_1\bar{G}_{00}$$

$$=-(\frac{1}{2})(\frac{-\beta\Delta'}{\rho^2})\partial_1(\frac{\beta\Delta'}{\rho^2})$$

$$=\frac{\beta^2}{2}[\frac{r^2-2GM'r+\alpha^2}{r^2+\alpha^2}]$$
$$[\frac{(r^2+\alpha^2)(2r-2GM^{'})-(r^2-2GM'r+\alpha^2)2r}{(r^2+\alpha^2)^2}]$$

$$=\frac{\beta^2}{2}[\frac{r^2-2GM'r+\alpha^2}{r^2+\alpha^2}]$$
$$[\frac{2r^3-2GM^{'}r^2+2r\alpha^2-2GM'\alpha^2-2r^3+4GM^{'}r^2-2\alpha^2 r}{(r^2+\alpha^2)^2}]$$

$$=\frac{\beta^2 GM^{'}(r^2-\alpha^2)(r^2-2GM'r+\alpha^2)}{(r^2+\alpha^2)^3}$$

$$=\frac{\beta^2 GM^{'}(r^4-2GM^{'}r^3+2GM^{'}\alpha^2 r-\alpha^4)}{(r^2+\alpha^2)^3} \qquad (A.5)$$

Now we calculate the diagonal components of Ricci tensor using $(A.3)$-$(A.5)$ and the relation $\bar{R}_{\mu\nu}=\partial_\mu\bar{\Gamma}^\alpha_{\alpha\nu}-\partial_\alpha\bar{\Gamma}^\alpha_{\mu\nu}+\bar{\Gamma}^\alpha_{\beta\mu}\bar{\Gamma}^\beta_{\alpha\nu}-\bar{\Gamma}^\alpha_{\alpha\beta}\bar{\Gamma}^\beta_{\mu\nu}$.

$$\bar{R}_{00}=\partial_0\bar{\Gamma}^\alpha_{\alpha 0}-\partial_\alpha\bar{\Gamma}^\alpha_{00}+\bar{\Gamma}^\alpha_{\beta 0}\bar{\Gamma}^\beta_{\alpha 0}-\bar{\Gamma}^\alpha_{\alpha\beta}\bar{\Gamma}^\beta_{00}$$

$$= \partial_0\bar{\Gamma}^0_{00} + \partial_0\bar{\Gamma}^1_{10} - \partial_0\bar{\Gamma}^0_{00} - \partial_1\bar{\Gamma}^1_{00}$$
$$+\bar{\Gamma}^0_{0\beta}\bar{\Gamma}^\beta_{00} + \bar{\Gamma}^1_{0\beta}\bar{\Gamma}^\beta_{10} - \bar{\Gamma}^0_{0\beta}\bar{\Gamma}^\beta_{00} - \bar{\Gamma}^1_{1\beta}\bar{\Gamma}^\beta_{00}$$

$$= \partial_0\bar{\Gamma}^1_{10} - \partial_1\bar{\Gamma}^1_{00} + \bar{\Gamma}^0_{00}\bar{\Gamma}^0_{00} + \bar{\Gamma}^0_{01}\bar{\Gamma}^1_{00} + \bar{\Gamma}^1_{00}\bar{\Gamma}^0_{10}$$
$$+\bar{\Gamma}^1_{01}\bar{\Gamma}^1_{10} - \bar{\Gamma}^0_{00}\bar{\Gamma}^0_{00} - \bar{\Gamma}^0_{01}\bar{\Gamma}^1_{00} - \bar{\Gamma}^1_{10}\bar{\Gamma}^0_{00} - \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{00}$$

$$= -\partial_1\bar{\Gamma}^1_{00} + \bar{\Gamma}^1_{00}(\bar{\Gamma}^0_{10} - \bar{\Gamma}^1_{11})$$

$$= -\partial_1\bar{\Gamma}^1_{00} + 2\bar{\Gamma}^1_{00}\bar{\Gamma}^0_{10} \qquad (A.6) \quad [using\ (A.4)]$$

We calculate first term of equation $(A.6)$:

$$\partial_1\bar{\Gamma}^1_{00} = \partial_1[\frac{\beta^2 GM'(r^4 - 2GM'r^3 + 2GM'\alpha^2 r - \alpha^4)}{(r^2+\alpha^2)^3}]$$

$$= \frac{\beta^2 GM'}{(r^2+\alpha^2)^4}[(r^2+\alpha^2)(4r^3 - 6GM'r^2 + 2GM'\alpha^2) - 6r(r^4 - 2GM'r^3 + 2GM'\alpha^2 r - \alpha^4)]$$

$$= \beta^2 GM'[\frac{-2r^5 + 6GM'r^4 + 4\alpha^2 r^3 - 16GM'\alpha^2 r^2 + 6\alpha^4 r + 2GM'\alpha^4}{(r^2+\alpha^2)^4}] \qquad (A.7)$$

and second term of equation $(A.6)$:

$$2\bar{\Gamma}^1_{00}\bar{\Gamma}^0_{10} = 2[\frac{\beta^2 GM'(r^4 - 2GM'r^3 + 2GM'\alpha^2 r - \alpha^4)}{(r^2+\alpha^2)^3}]$$
$$[\frac{GM'(r^2-\alpha^2)}{(r^2+\alpha^2)(r^2 - 2GM'r + \alpha^2)}]$$

$$= [\frac{2\beta^2(GM')^2}{(r^2+\alpha^2)^4(r^2 - 2GM'r + \alpha^2)}]$$

$$[r^6 - 2GM^{'}r^5 - \alpha^2 r^4 + 4GM^{'}\alpha^2 r^3 - \alpha^4 r^2 - 2GM^{'}\alpha^4 r + \alpha^6] \qquad (A.8)$$

Substituting $(A.7)$ and $(A.8)$ in $(A.6)$ we get

$$\bar{R}_{00} = -\partial_1 \bar{\Gamma}^1_{00} + 2\bar{\Gamma}^1_{00}\bar{\Gamma}^0_{10}$$

$$= \frac{\beta^2 GM^{'}}{(r^2+\alpha^2)^4}[(2r^5 - 6GM^{'}r^4 - 4\alpha^2 r^3 + 16GM^{'}\alpha^2 r^2 - 6\alpha^4 r - 2GM^{'}\alpha^4)$$
$$+(\frac{2GM^{'}(r^6 - 2GM^{'}r^5 - \alpha^2 r^4 + 4GM^{'}\alpha^2 r^3 - \alpha^4 r^2 - 2GM^{'}\alpha^4 r + \alpha^6)}{(r^2 - 2GM'r + \alpha^2)})]$$

$$= [\frac{\beta^2 GM^{'}}{(r^2+\alpha^2)^4 (r^2 - 2GM'r + \alpha^2)}]$$
$$[2r^7 - 8GM^{'}r^6 - 2\alpha^2 r^5 + 8(GM^{'})^2 r^5 + 16GM^{'}\alpha^2 r^4 - 10\alpha^4 r^3$$
$$-24(GM^{'})^2\alpha^2 r^3 + 24GM^{'}\alpha^4 r^2 - 6\alpha^6 r] \qquad (A.9)$$

Therefore,

$$\bar{R}_{00} \ \xrightarrow{r\to\infty} \ \frac{1}{r^3} \qquad (A.10)$$

$$\bar{R}_{11} = \partial_1 \bar{\Gamma}^\alpha_{\alpha 1} - \partial_\alpha \bar{\Gamma}^\alpha_{11} + \bar{\Gamma}^\alpha_{\beta 1}\bar{\Gamma}^\beta_{\alpha 1} - \bar{\Gamma}^\alpha_{\alpha\beta}\bar{\Gamma}^\beta_{11}$$

$$= \partial_1 \bar{\Gamma}^0_{01} + \partial_1 \bar{\Gamma}^1_{11} - \partial_0 \bar{\Gamma}^0_{11} - \partial_1 \bar{\Gamma}^1_{11}$$
$$+\bar{\Gamma}^0_{1\beta}\bar{\Gamma}^\beta_{01} + \bar{\Gamma}^1_{1\beta}\bar{\Gamma}^\beta_{11} - \bar{\Gamma}^0_{0\beta}\bar{\Gamma}^\beta_{11} - \bar{\Gamma}^1_{1\beta}\bar{\Gamma}^\beta_{11}$$

$$= \partial_1 \bar{\Gamma}^0_{01} + \bar{\Gamma}^0_{10}\bar{\Gamma}^0_{01} + \bar{\Gamma}^0_{11}\bar{\Gamma}^1_{01} + \bar{\Gamma}^1_{10}\bar{\Gamma}^0_{11} + \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{11}$$

$$-\bar{\Gamma}^0_{00}\bar{\Gamma}^0_{11} - \bar{\Gamma}^0_{01}\bar{\Gamma}^1_{11} - \bar{\Gamma}^1_{10}\bar{\Gamma}^0_{11} - \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{11}$$

$$= \partial_1\bar{\Gamma}^0_{01} + \bar{\Gamma}^0_{10}\bar{\Gamma}^0_{01} - \bar{\Gamma}^0_{01}\bar{\Gamma}^1_{11}$$

$$= \partial_1\bar{\Gamma}^0_{01} + 2(\bar{\Gamma}^0_{10})^2 \quad (A.11) \;\; [using\ (A.4)]$$

Let us calculate the first term of $(A.11)$:

$$\partial_1\bar{\Gamma}^0_{01} = \partial_1[\frac{GM^{'}(r^2-\alpha^2)}{(r^2+\alpha^2)(r^2-2GM'r+\alpha^2)}]$$

$$= [\frac{GM^{'}}{(r^2+\alpha^2)^2(r^2-2GM'r+\alpha^2)^2}]$$

$$[2r(r^4-2GM^{'}r^3+2\alpha^2r^2-2GM^{'}\alpha^2r+\alpha^4)-(r^2-\alpha^2)(4r^3-6GM^{'}r^2+4\alpha^2r-2GM^{'}\alpha^2)]$$

$$= [\frac{GM^{'}}{(r^2+\alpha^2)^2(r^2-2GM'r+\alpha^2)^2}]$$

$$[-2r^5+2GM^{'}r^4+4\alpha^2r^3-8GM^{'}\alpha^2r^2+6\alpha^4r-2GM^{'}\alpha^4] \quad (A.12)$$

Substituting the values $(A.3)$ and $(A.12)$ in equation $(A.11)$ we get,

$$\bar{R}_{11} = \partial_1\bar{\Gamma}^0_{01} + 2(\bar{\Gamma}^0_{10})^2$$

$$= [\frac{GM^{'}}{(r^2+\alpha^2)^2(r^2-2GM'r+\alpha^2)^2}]$$

$$[-2r^5+2GM^{'}r^4+4\alpha^2r^3-8GM^{'}\alpha^2r^2+6\alpha^4r-2GM^{'}\alpha^4]$$

$$+2[\frac{GM^{'}(r^2-\alpha^2)}{(r^2+\alpha^2)(r^2-2GM'r+\alpha^2)}]^2$$

$$= [\frac{GM^{'}}{(r^2+\alpha^2)^2(r^2-2GM'r+\alpha^2)^2}]$$

$$[-2r^5+2GM^{'}r^4+4\alpha^2r^3-8GM^{'}\alpha^2r^2+6\alpha^4r-2GM^{'}\alpha^4+2GM^{'}(r^4-2r^2\alpha^2+\alpha^4)]$$

$$= [\frac{GM^{'}}{(r^2+\alpha^2)^2(r^2-2GM'r+\alpha^2)^2}]$$
$$[-2r^5+4GM^{'}r^4+4\alpha^2r^3-12GM^{'}\alpha^2r^2+6\alpha^4r] \quad (A.13)$$

Therefore,

$$\bar{R}_{11} \xrightarrow{r\to\infty} \frac{1}{r^3} \qquad (A.14)$$

Now calculate Ricci scalar using $(A.2)$, $(A.9)$ and $(A.13)$:

$$\bar{R} = \bar{G}^{00}\bar{R}_{00} + \bar{G}^{11}\bar{R}_{11}$$

$$= [\frac{\beta GM^{'}}{(r^2+\alpha^2)^3(r^2-2GM'r+\alpha^2)^2}]$$
$$[2r^7-8GM^{'}r^6-2\alpha^2r^5+8(GM^{'})^2r^5+16GM^{'}\alpha^2r^4-10\alpha^4r^3$$
$$-24(GM^{'})^2\alpha^2r^3+24GM^{'}\alpha^4r^2-6\alpha^6r]$$
$$-[\frac{\beta GM^{'}}{(r^2+\alpha^2)^3(r^2-2GM'r+\alpha^2)}]$$
$$[-2r^5+4GM^{'}r^4+4\alpha^2r^3-12GM^{'}\alpha^2r^2+6\alpha^4r]$$

$$= [\frac{\beta GM^{'}}{(r^2+\alpha^2)^3(r^2-2GM'r+\alpha^2)^2}]$$
$$[2r^7-8GM^{'}r^6-2\alpha^2r^5+8(GM^{'})^2r^5+16GM^{'}\alpha^2r^4-10\alpha^4r^3$$
$$-24(GM^{'})^2\alpha^2r^3+24GM^{'}\alpha^4r^2-6\alpha^6r$$
$$-(r^2-2GM'r+\alpha^2)(-2r^5+4GM^{'}r^4+4\alpha^2r^3-12GM^{'}\alpha^2r^2+6\alpha^4r)]$$

$$= [\frac{\beta GM^{'}}{(r^2+\alpha^2)^3(r^2-2GM'r+\alpha^2)^2}]$$

$$[4r^7 - 16GM^{'}r^6 - 4\alpha^2r^5 + 16(GM^{'})^2r^5 + 32GM^{'}\alpha^2r^4$$
$$-20\alpha^4r^3 - 48(GM^{'})^2\alpha^2r^3 + 48GM^{'}\alpha^4r^2 - 12\alpha^6r] \quad (A.15)$$

Again at $r \to \infty$,

$$\bar{R} \xrightarrow{r \to \infty} \frac{1}{r^3} \qquad (A.16)$$

Appendix B

Calculation of the connection coefficients for the emergent gravity metric (4.5)

For homogeneous fields the emergent gravity metrics (4.5) and (4.6) becomes

$$\bar{G}_{\mu\nu} = \begin{pmatrix} (1-\dot{\phi}^2) & 0 & 0 & 0 \\ 0 & -(\frac{a^2}{1-kr^2}) & 0 & 0 \\ 0 & 0 & -(a^2r^2) & 0 \\ 0 & 0 & 0 & -(a^2r^2sin^2\theta) \end{pmatrix} \qquad (B.1)$$

and

$$\bar{G}^{\mu\nu} = \begin{pmatrix} (1-\dot{\phi}^2)^{-1} & 0 & 0 & 0 \\ 0 & -(\frac{a^2}{1-kr^2})^{-1} & 0 & 0 \\ 0 & 0 & -(a^2r^2)^{-1} & 0 \\ 0 & 0 & 0 & -(a^2r^2sin^2\theta)^{-1} \end{pmatrix} \qquad (B.2)$$

Now we derive the non-vanishing connection coefficients using the metrics (B.1) and (B.2) and the relation $\bar{\Gamma}^{\alpha}_{\mu\nu} = \frac{1}{2}\bar{G}^{\alpha\beta}[\partial_\mu \bar{G}_{\beta\nu} + \partial_\nu \bar{G}_{\mu\beta} - \partial_\beta \bar{G}_{\mu\nu}]$.

$$\bar{\Gamma}^0_{00} = \frac{1}{2}\bar{G}^{0\alpha}[\partial_0\bar{G}_{\alpha 0} + \partial_0\bar{G}_{0\alpha} - \partial_\alpha\bar{G}_{00}]$$
$$= \frac{1}{2}\bar{G}^{00}[\partial_0\bar{G}_{00} + \partial_0\bar{G}_{00} - \partial_0\bar{G}_{00}]$$
$$= \frac{1}{2}(\frac{1}{1-\dot{\phi}^2})(-2\dot{\phi}\ddot{\phi}) = -\frac{\dot{\phi}\ddot{\phi}}{1-\dot{\phi}^2} \qquad (B.3)$$

$$\bar{\Gamma}^0_{11} = \frac{1}{2}\bar{G}^{0\alpha}[\partial_1\bar{G}_{\alpha 1} + \partial_1\bar{G}_{1\alpha} - \partial_\alpha\bar{G}_{11}]$$

$$= \frac{1}{2}\bar{G}^{00}[\partial_1\bar{G}_{01} + \partial_1\bar{G}_{10} - \partial_0\bar{G}_{11}]$$

$$= -\frac{1}{2}\bar{G}^{00}\partial_0\bar{G}_{11} = -\frac{1}{2}(\frac{1}{1-\dot{\phi}^2})(\frac{-2a\dot{a}}{1-kr^2})$$

$$= \frac{1}{(1-\dot{\phi}^2)}\frac{a\dot{a}}{1-kr^2} \qquad (B.4)$$

$$\bar{\Gamma}^0_{22} = \frac{1}{2}\bar{G}^{0\alpha}[\partial_2\bar{G}_{\alpha 2} + \partial_2\bar{G}_{2\alpha} - \partial_\alpha\bar{G}_{22}]$$

$$= \frac{1}{2}\bar{G}^{00}[\partial_2\bar{G}_{02} + \partial_2\bar{G}_{20} - \partial_0\bar{G}_{22}]$$

$$= -\frac{1}{2}\bar{G}^{00}\partial_0\bar{G}_{22} = \frac{a\dot{a}r^2}{(1-\dot{\phi}^2)} \qquad (B.5)$$

$$\bar{\Gamma}^0_{33} = \frac{1}{2}\bar{G}^{0\alpha}[\partial_3\bar{G}_{\alpha 3} + \partial_3\bar{G}_{3\alpha} - \partial_\alpha\bar{G}_{33}]$$

$$= \frac{1}{2}\bar{G}^{00}[\partial_3\bar{G}_{03} + \partial_3\bar{G}_{30} - \partial_0\bar{G}_{33}]$$

$$= -\frac{1}{2}\bar{G}^{00}\partial_0\bar{G}_{33} = \frac{a\dot{a}r^2 sin^2\theta}{(1-\dot{\phi}^2)} \qquad (B.6)$$

$$\bar{\Gamma}^1_{01} = \frac{1}{2}\bar{G}^{1\alpha}[\partial_0\bar{G}_{\alpha 1} + \partial_1\bar{G}_{0\alpha} - \partial_\alpha\bar{G}_{01}]$$

$$= \frac{1}{2}\bar{G}^{11}[\partial_0\bar{G}_{11} + \partial_1\bar{G}_{01} - \partial_1\bar{G}_{01}] = \frac{1}{2}\bar{G}^{11}\partial_0\bar{G}_{11}$$

$$= -\frac{1}{2}(\frac{1-kr^2}{a^2})(\frac{-2a\dot{a}}{1-kr^2}) = \frac{\dot{a}}{a} \qquad (B.7)$$

$$\bar{\Gamma}^1_{11} = \frac{1}{2}\bar{G}^{1\alpha}[\partial_1\bar{G}_{\alpha 1} + \partial_1\bar{G}_{1\alpha} - \partial_\alpha\bar{G}_{11}]$$

$$= \frac{1}{2}\bar{G}^{11}[\partial_1\bar{G}_{11} + \partial_1\bar{G}_{11} - \partial_1\bar{G}_{11}] = \frac{1}{2}\bar{G}^{11}\partial_1\bar{G}_{11}$$

$$= -\frac{1}{2}(\frac{1-kr^2}{a^2})(\frac{-2kra^2}{(1-kr^2)^2}) = \frac{kr}{1-kr^2} \qquad (B.8)$$

$$\bar{\Gamma}^1_{22} = \frac{1}{2}\bar{G}^{1\alpha}[\partial_2\bar{G}_{\alpha 2} + \partial_2\bar{G}_{2\alpha} - \partial_\alpha\bar{G}_{22}]$$

$$= \frac{1}{2}\bar{G}^{11}[\partial_2\bar{G}_{12} + \partial_2\bar{G}_{21} - \partial_1\bar{G}_{22}] = -\frac{1}{2}\bar{G}^{11}\partial_1\bar{G}_{22}$$

$$= \frac{1}{2}(\frac{1-kr^2}{a^2})(-2ra^2) = -r(1-kr^2) \qquad (B.9)$$

$$\bar{\Gamma}^1_{33} = \frac{1}{2}\bar{G}^{1\alpha}[\partial_3\bar{G}_{\alpha 3} + \partial_3\bar{G}_{3\alpha} - \partial_\alpha\bar{G}_{33}]$$

$$= \frac{1}{2}\bar{G}^{11}[\partial_3\bar{G}_{13} + \partial_3\bar{G}_{31} - \partial_1\bar{G}_{33}]$$

$$= -\frac{1}{2}\bar{G}^{11}\partial_1\bar{G}_{33} = -r\ sin^2\theta(1-kr^2) \qquad (B.10)$$

$$\bar{\Gamma}^2_{02} = \frac{1}{2}\bar{G}^{2\alpha}[\partial_0\bar{G}_{\alpha 2} + \partial_2\bar{G}_{0\alpha} - \partial_\alpha\bar{G}_{02}]$$

$$= \frac{1}{2}\bar{G}^{22}[\partial_0\bar{G}_{22} + \partial_2\bar{G}_{02} - \partial_2\bar{G}_{02}] = \frac{1}{2}\bar{G}^{22}\partial_0\bar{G}_{22}$$

$$= -\frac{1}{2}\frac{1}{a^2r^2}(-2a\dot{a}r^2) = \frac{\dot{a}}{a} \qquad (B.11)$$

$$\bar{\Gamma}^2_{12} = \frac{1}{2}\bar{G}^{2\alpha}[\partial_1\bar{G}_{\alpha 2} + \partial_2\bar{G}_{1\alpha} - \partial_\alpha\bar{G}_{12}]$$

$$= \frac{1}{2}\bar{G}^{22}[\partial_1\bar{G}_{22} + \partial_2\bar{G}_{12} - \partial_2\bar{G}_{12}] = \frac{1}{2}\bar{G}^{22}\partial_1\bar{G}_{22}$$

$$= -\frac{1}{2}\frac{1}{a^2r^2}(-2ra^2) = \frac{1}{r} \qquad (B.12)$$

$$\bar{\Gamma}^2_{33} = \frac{1}{2}\bar{G}^{2\alpha}[\partial_3\bar{G}_{\alpha 3} + \partial_3\bar{G}_{3\alpha} - \partial_\alpha\bar{G}_{33}]$$

$$= \frac{1}{2}\bar{G}^{22}[\partial_3\bar{G}_{23} + \partial_3\bar{G}_{32} - \partial_2\bar{G}_{33}] = -\frac{1}{2}\bar{G}^{22}\partial_2\bar{G}_{33}$$

$$= \frac{1}{2}\frac{1}{a^2r^2}(-2r^2a^2 sin\theta cos\theta) = -sin\theta cos\theta \qquad (B.13)$$

$$\bar{\Gamma}^3_{03} = \frac{1}{2}\bar{G}^{3\alpha}[\partial_0\bar{G}_{\alpha 3} + \partial_3\bar{G}_{0\alpha} - \partial_\alpha\bar{G}_{03}]$$

$$= \frac{1}{2}\bar{G}^{33}[\partial_0\bar{G}_{33} + \partial_3\bar{G}_{03} - \partial_3\bar{G}_{03}] = \frac{1}{2}\bar{G}^{33}\partial_0\bar{G}_{33}$$

$$= \frac{1}{2}(\frac{-1}{a^2r^2sin^2\theta})(-2a\dot{a}r^2sin^2\theta) = \frac{\dot{a}}{a} \qquad (B.14)$$

$$\bar{\Gamma}^3_{13} = \frac{1}{2}\bar{G}^{3\alpha}[\partial_1\bar{G}_{\alpha 3} + \partial_3\bar{G}_{1\alpha} - \partial_\alpha\bar{G}_{13}]$$

$$= \frac{1}{2}\bar{G}^{33}[\partial_1\bar{G}_{33} + \partial_3\bar{G}_{13} - \partial_3\bar{G}_{13}] = \frac{1}{2}\bar{G}^{33}\partial_1\bar{G}_{33}$$

$$= \frac{1}{2}(\frac{-1}{a^2r^2sin^2\theta})(-2ra^2sin^2\theta) = \frac{1}{r} \qquad (B.15)$$

$$\bar{\Gamma}^3_{23} = \frac{1}{2}\bar{G}^{3\alpha}[\partial_2\bar{G}_{\alpha 3} + \partial_3\bar{G}_{2\alpha} - \partial_\alpha\bar{G}_{23}]$$

$$= \frac{1}{2}\bar{G}^{33}[\partial_2\bar{G}_{33} + \partial_3\bar{G}_{23} - \partial_3\bar{G}_{23}] = \frac{1}{2}\bar{G}^{33}\partial_2\bar{G}_{33}$$

$$= \frac{1}{2}(\frac{-1}{a^2r^2sin^2\theta})(-2r^2a^2sin\theta cos\theta) = cot\theta \qquad (B.16)$$

Appendix C

Evaluation of the Ricci tensors for FLRW background in the presence of dark energy

We calculate the diagonal components of Ricci tensor using non vanishing connection coefficients (B.3)-(B.16) and the relation $\bar{R}_{\mu\nu} = \partial_\mu \bar{\Gamma}^\alpha_{\alpha\nu} - \partial_\alpha \bar{\Gamma}^\alpha_{\mu\nu} + \bar{\Gamma}^\alpha_{\beta\mu}\bar{\Gamma}^\beta_{\alpha\nu} - \bar{\Gamma}^\alpha_{\alpha\beta}\bar{\Gamma}^\beta_{\mu\nu}$.

$$\begin{aligned}
\bar{R}_{00} &= \partial_0 \bar{\Gamma}^\alpha_{\alpha 0} - \partial_\alpha \bar{\Gamma}^\alpha_{00} + \bar{\Gamma}^\alpha_{\beta 0}\bar{\Gamma}^\beta_{\alpha 0} - \bar{\Gamma}^\alpha_{\alpha\beta}\bar{\Gamma}^\beta_{00} \\
&= (\partial_0 \bar{\Gamma}^0_{00} + \partial_0 \bar{\Gamma}^1_{10} + \partial_0 \bar{\Gamma}^2_{20} + \partial_0 \bar{\Gamma}^3_{30}) \\
&\quad -(\partial_0 \bar{\Gamma}^0_{00} + \partial_1 \bar{\Gamma}^1_{00} + \partial_2 \bar{\Gamma}^2_{00} + \partial_3 \bar{\Gamma}^3_{00}) \\
&\quad +(\bar{\Gamma}^0_{\beta 0}\bar{\Gamma}^\beta_{00} + \bar{\Gamma}^1_{\beta 0}\bar{\Gamma}^\beta_{10} + \bar{\Gamma}^2_{\beta 0}\bar{\Gamma}^\beta_{20} + \bar{\Gamma}^3_{\beta 0}\bar{\Gamma}^\beta_{30}) \\
&\quad -(\bar{\Gamma}^0_{0\beta}\bar{\Gamma}^\beta_{00} + \bar{\Gamma}^1_{1\beta}\bar{\Gamma}^\beta_{00} + \bar{\Gamma}^2_{2\beta}\bar{\Gamma}^\beta_{00} + \bar{\Gamma}^3_{3\beta}\bar{\Gamma}^\beta_{00}) \\
&= \partial_0 \bar{\Gamma}^1_{10} + \partial_0 \bar{\Gamma}^2_{20} + \partial_0 \bar{\Gamma}^3_{30} + (\bar{\Gamma}^1_{10})^2 + (\bar{\Gamma}^2_{20})^2 \\
&\quad +(\bar{\Gamma}^3_{30})^2 - \bar{\Gamma}^0_{00}(\bar{\Gamma}^1_{10} + \bar{\Gamma}^2_{20} + \bar{\Gamma}^3_{30}) \\
&= 3\partial_0(\frac{\dot{a}}{a}) + 3(\frac{\dot{a}}{a})^2 + \frac{\dot{\phi}\ddot{\phi}}{1-\dot{\phi}^2}3(\frac{\dot{a}}{a}) \\
&= 3(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}) + 3(\frac{\dot{a}}{a})^2 + 3(\frac{\dot{a}}{a})\frac{\dot{\phi}\ddot{\phi}}{1-\dot{\phi}^2} \\
&= 3\frac{\ddot{a}}{a} + 3(\frac{\dot{a}}{a})\frac{\dot{\phi}\ddot{\phi}}{1-\dot{\phi}^2} \qquad (C.1)
\end{aligned}$$

$$\begin{aligned}
\bar{R}_{11} &= \partial_1 \bar{\Gamma}^\alpha_{\alpha 1} - \partial_\alpha \bar{\Gamma}^\alpha_{11} + \bar{\Gamma}^\alpha_{\beta 1}\bar{\Gamma}^\beta_{\alpha 1} - \bar{\Gamma}^\alpha_{\alpha\beta}\bar{\Gamma}^\beta_{11} \\
&= (\partial_1 \bar{\Gamma}^0_{01} + \partial_1 \bar{\Gamma}^1_{11} + \partial_1 \bar{\Gamma}^2_{21} + \partial_1 \bar{\Gamma}^3_{31}) \\
&\quad -(\partial_0 \bar{\Gamma}^0_{11} + \partial_1 \bar{\Gamma}^1_{11} + \partial_2 \bar{\Gamma}^2_{11} + \partial_3 \bar{\Gamma}^3_{11})
\end{aligned}$$

$$+(\bar{\Gamma}^0_{\beta 1}\bar{\Gamma}^\beta_{01} + \bar{\Gamma}^1_{\beta 1}\bar{\Gamma}^\beta_{11} + \bar{\Gamma}^2_{\beta 1}\bar{\Gamma}^\beta_{21} + \bar{\Gamma}^3_{\beta 1}\bar{\Gamma}^\beta_{31})$$

$$-(\bar{\Gamma}^0_{0\beta}\bar{\Gamma}^\beta_{11} + \bar{\Gamma}^1_{1\beta}\bar{\Gamma}^\beta_{11} + \bar{\Gamma}^2_{2\beta}\bar{\Gamma}^\beta_{11} + \bar{\Gamma}^3_{3\beta}\bar{\Gamma}^\beta_{11})$$

$$= \partial_1\bar{\Gamma}^2_{21} + \partial_1\bar{\Gamma}^3_{31} - \partial_0\bar{\Gamma}^0_{11} + \bar{\Gamma}^0_{11}\bar{\Gamma}^1_{01} + \bar{\Gamma}^1_{01}\bar{\Gamma}^0_{11} + \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{11}$$

$$+\bar{\Gamma}^2_{21}\bar{\Gamma}^2_{21} + \bar{\Gamma}^3_{31}\bar{\Gamma}^3_{31} - \bar{\Gamma}^0_{00}\bar{\Gamma}^0_{11} - \bar{\Gamma}^1_{10}\bar{\Gamma}^0_{11} - \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{11}$$

$$-\bar{\Gamma}^2_{20}\bar{\Gamma}^0_{11} - \bar{\Gamma}^2_{21}\bar{\Gamma}^1_{11} - \bar{\Gamma}^3_{30}\bar{\Gamma}^0_{11} - \bar{\Gamma}^3_{31}\bar{\Gamma}^1_{11}$$

$$= \partial_1\bar{\Gamma}^2_{21} + \partial_1\bar{\Gamma}^3_{31} - \partial_0\bar{\Gamma}^0_{11} + \bar{\Gamma}^0_{11}\bar{\Gamma}^1_{01} + (\bar{\Gamma}^2_{21})^2 + (\bar{\Gamma}^3_{31})^2$$

$$-\bar{\Gamma}^0_{00}\bar{\Gamma}^0_{11} - \bar{\Gamma}^2_{20}\bar{\Gamma}^0_{11} - \bar{\Gamma}^2_{21}\bar{\Gamma}^1_{11} - \bar{\Gamma}^3_{30}\bar{\Gamma}^0_{11} - \bar{\Gamma}^3_{31}\bar{\Gamma}^1_{11}$$

$$= \partial_1(\frac{1}{r}) + \partial_1(\frac{1}{r}) - \partial_0[\frac{a\dot{a}}{(1-\dot{\phi}^2)(1-kr^2)}] + [\frac{a\dot{a}}{(1-\dot{\phi}^2)(1-kr^2)}]\frac{\dot{a}}{a}$$

$$+\frac{1}{r^2} + \frac{1}{r^2} - (\frac{-\dot{\phi}\ddot{\phi}}{1-\dot{\phi}^2})[\frac{a\dot{a}}{(1-\dot{\phi}^2)(1-kr^2)}] - \frac{\dot{a}}{a}[\frac{a\dot{a}}{(1-\dot{\phi}^2)(1-kr^2)}]$$

$$-\frac{1}{r}\frac{kr}{(1-kr^2)} - \frac{\dot{a}}{a}[\frac{a\dot{a}}{(1-\dot{\phi}^2)(1-kr^2)}] - \frac{1}{r}\frac{kr}{(1-kr^2)}$$

$$= -\frac{2}{r^2} - [\frac{\dot{a}^2}{(1-\dot{\phi}^2)(1-kr^2)} + \frac{a\ddot{a}}{(1-\dot{\phi}^2)(1-kr^2)} + \frac{a\dot{a}}{1-kr^2}\frac{2\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]$$

$$+\frac{2}{r^2} + \frac{a\dot{a}}{1-kr^2}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} - \frac{2k}{1-kr^2} - \frac{\dot{a}^2}{(1-\dot{\phi}^2)(1-kr^2)}$$

$$= -\frac{a^2}{1-kr^2}[\frac{\ddot{a}}{a}\frac{1}{(1-\dot{\phi}^2)} + 2(\frac{\dot{a}}{a})^2\frac{1}{(1-\dot{\phi}^2)} + \frac{2k}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}] \qquad (C.2)$$

$$\bar{R}_{22} = \partial_2\bar{\Gamma}^\alpha_{\alpha 2} - \partial_\alpha\bar{\Gamma}^\alpha_{22} + \bar{\Gamma}^\alpha_{\beta 2}\bar{\Gamma}^\beta_{\alpha 2} - \bar{\Gamma}^\alpha_{\alpha\beta}\bar{\Gamma}^\beta_{22}$$

$$= (\partial_2\bar{\Gamma}^0_{02} + \partial_2\bar{\Gamma}^1_{12} + \partial_2\bar{\Gamma}^2_{22} + \partial_2\bar{\Gamma}^3_{32})$$

$$-(\partial_0\bar{\Gamma}^0_{22} + \partial_1\bar{\Gamma}^1_{22} + \partial_2\bar{\Gamma}^2_{22} + \partial_3\bar{\Gamma}^3_{22})$$

$$+(\bar{\Gamma}^0_{\beta 2}\bar{\Gamma}^\beta_{02} + \bar{\Gamma}^1_{\beta 2}\bar{\Gamma}^\beta_{12} + \bar{\Gamma}^2_{\beta 2}\bar{\Gamma}^\beta_{22} + \bar{\Gamma}^3_{\beta 2}\bar{\Gamma}^\beta_{32})$$

$$-(\bar{\Gamma}^0_{0\beta}\bar{\Gamma}^\beta_{22} + \bar{\Gamma}^1_{1\beta}\bar{\Gamma}^\beta_{22} + \bar{\Gamma}^2_{2\beta}\bar{\Gamma}^\beta_{22} + \bar{\Gamma}^3_{3\beta}\bar{\Gamma}^\beta_{22})$$

$$= \partial_2\bar{\Gamma}^3_{32} - \partial_0\bar{\Gamma}^0_{22} - \partial_1\bar{\Gamma}^1_{22} + \bar{\Gamma}^0_{22}\bar{\Gamma}^2_{02} + \bar{\Gamma}^1_{22}\bar{\Gamma}^2_{12} + \bar{\Gamma}^2_{02}\bar{\Gamma}^0_{22}$$

$$+\bar{\Gamma}^2_{12}\bar{\Gamma}^1_{22} + \bar{\Gamma}^3_{32}\bar{\Gamma}^3_{32} - \bar{\Gamma}^0_{00}\bar{\Gamma}^0_{22} - \bar{\Gamma}^1_{10}\bar{\Gamma}^0_{22} - \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{22}$$

$$-\bar{\Gamma}^2_{20}\bar{\Gamma}^0_{22} - \bar{\Gamma}^2_{21}\bar{\Gamma}^1_{22} - \bar{\Gamma}^3_{30}\bar{\Gamma}^0_{22} - \bar{\Gamma}^3_{31}\bar{\Gamma}^1_{22}$$

$$= \partial_2\bar{\Gamma}^3_{32} - \partial_0\bar{\Gamma}^0_{22} - \partial_1\bar{\Gamma}^1_{22} + \bar{\Gamma}^0_{22}\bar{\Gamma}^2_{02} + \bar{\Gamma}^1_{22}\bar{\Gamma}^2_{12} + (\bar{\Gamma}^3_{32})^2$$

$$-\bar{\Gamma}^0_{00}\bar{\Gamma}^0_{22} - \bar{\Gamma}^1_{10}\bar{\Gamma}^0_{22} - \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{22} - \bar{\Gamma}^3_{30}\bar{\Gamma}^0_{22} - \bar{\Gamma}^3_{31}\bar{\Gamma}^1_{22}$$

$$= \partial_2(cot\theta) - \partial_0[\frac{a\dot{a}r^2}{(1-\dot{\phi}^2)}] + \partial_1[r(1-kr^2)] + [\frac{a\dot{a}r^2}{(1-\dot{\phi}^2)}]\frac{\dot{a}}{a}$$

$$-r(1-kr^2)(\frac{1}{r}) + (cot\theta)^2 + \frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)}[\frac{a\dot{a}r^2}{(1-\dot{\phi}^2)}]$$

$$-\frac{\dot{a}}{a}[\frac{a\dot{a}r^2}{(1-\dot{\phi}^2)}] + \frac{kr}{(1-kr^2)}r(1-kr^2) - \frac{\dot{a}}{a}[\frac{a\dot{a}r^2}{(1-\dot{\phi}^2)}] + (\frac{1}{r})r(1-kr^2)$$

$$= -(1+cot^2\theta) - [\frac{\dot{a}^2r^2}{(1-\dot{\phi}^2)} + \frac{a\ddot{a}r^2}{(1-\dot{\phi}^2)} + \frac{a\dot{a}r^2 2\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]$$

$$+1 - kr^2 - 2kr^2 + cot^2\theta + \frac{a\dot{a}r^2\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} + kr^2 - \frac{\dot{a}^2r^2}{(1-\dot{\phi}^2)}$$

$$= -\frac{2\dot{a}^2r^2}{(1-\dot{\phi}^2)} - \frac{a\ddot{a}r^2}{(1-\dot{\phi}^2)} - \frac{a\dot{a}r^2\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} - 2kr^2$$

$$= -a^2r^2[\frac{\ddot{a}}{a(1-\dot{\phi}^2)} + \frac{2\dot{a}^2}{a^2(1-\dot{\phi}^2)} + \frac{2k}{a^2} + \frac{\dot{a}\dot{\phi}\ddot{\phi}}{a(1-\dot{\phi}^2)^2}] \qquad (C.3)$$

$$\bar{R}_{33} = \partial_3\bar{\Gamma}^\alpha_{\alpha 3} - \partial_\alpha\bar{\Gamma}^\alpha_{33} + \bar{\Gamma}^\alpha_{\beta 3}\bar{\Gamma}^\beta_{\alpha 3} - \bar{\Gamma}^\alpha_{\alpha\beta}\bar{\Gamma}^\beta_{33}$$

$$= (\partial_3\bar{\Gamma}^0_{03} + \partial_3\bar{\Gamma}^1_{13} + \partial_3\bar{\Gamma}^2_{23} + \partial_3\bar{\Gamma}^3_{33})$$

$$-(\partial_0\bar{\Gamma}^0_{33} + \partial_1\bar{\Gamma}^1_{33} + \partial_2\bar{\Gamma}^2_{33} + \partial_3\bar{\Gamma}^3_{33})$$

$$+(\bar{\Gamma}^0_{\beta 3}\bar{\Gamma}^\beta_{03} + \bar{\Gamma}^1_{\beta 3}\bar{\Gamma}^\beta_{13} + \bar{\Gamma}^2_{\beta 3}\bar{\Gamma}^\beta_{23} + \bar{\Gamma}^3_{\beta 3}\bar{\Gamma}^\beta_{33})$$

$$-(\bar{\Gamma}^0_{0\beta}\bar{\Gamma}^\beta_{33} + \bar{\Gamma}^1_{1\beta}\bar{\Gamma}^\beta_{33} + \bar{\Gamma}^2_{2\beta}\bar{\Gamma}^\beta_{33} + \bar{\Gamma}^3_{3\beta}\bar{\Gamma}^\beta_{33})$$

$$= -\partial_0\bar{\Gamma}^0_{33} - \partial_1\bar{\Gamma}^1_{33} - \partial_2\bar{\Gamma}^2_{33} + \bar{\Gamma}^0_{33}\bar{\Gamma}^3_{03} + \bar{\Gamma}^1_{33}\bar{\Gamma}^3_{13} + \bar{\Gamma}^2_{33}\bar{\Gamma}^3_{23}$$

$$+\bar{\Gamma}^3_{03}\bar{\Gamma}^0_{33} + \bar{\Gamma}^3_{13}\bar{\Gamma}^1_{33} + \bar{\Gamma}^3_{23}\bar{\Gamma}^2_{33} - \bar{\Gamma}^0_{00}\bar{\Gamma}^0_{33} - \bar{\Gamma}^1_{10}\bar{\Gamma}^0_{33} - \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{33}$$

$$-\bar{\Gamma}^2_{20}\bar{\Gamma}^0_{33} - \bar{\Gamma}^2_{21}\bar{\Gamma}^1_{33} - \bar{\Gamma}^3_{30}\bar{\Gamma}^0_{33} - \bar{\Gamma}^3_{31}\bar{\Gamma}^1_{33} - \bar{\Gamma}^3_{32}\bar{\Gamma}^2_{33}$$

$$= -\partial_0\bar{\Gamma}^0_{33} - \partial_1\bar{\Gamma}^1_{33} - \partial_2\bar{\Gamma}^2_{33} + \bar{\Gamma}^0_{33}\bar{\Gamma}^3_{03} + \bar{\Gamma}^1_{33}\bar{\Gamma}^3_{13} + \bar{\Gamma}^2_{33}\bar{\Gamma}^3_{23}$$

$$-\bar{\Gamma}^0_{00}\bar{\Gamma}^0_{33} - \bar{\Gamma}^1_{10}\bar{\Gamma}^0_{33} - \bar{\Gamma}^1_{11}\bar{\Gamma}^1_{33} - \bar{\Gamma}^2_{20}\bar{\Gamma}^0_{33} - \bar{\Gamma}^2_{21}\bar{\Gamma}^1_{33}$$

$$= -\partial_0[\frac{a\dot{a}r^2sin^2\theta}{(1-\dot{\phi}^2)}] + \partial_1[rsin^2\theta(1-kr^2)] + \partial_2(sin\theta\ cos\theta)$$

$$+[\frac{a\dot{a}r^2sin^2\theta}{(1-\dot{\phi}^2)}]\frac{\dot{a}}{a} - r\ sin^2\theta(1-kr^2)(\frac{1}{r}) - (sin\theta\ cos\theta)cot\theta$$

$$+\frac{\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)}[\frac{a\dot{a}r^2sin^2\theta}{(1-\dot{\phi}^2)}] - \frac{\dot{a}}{a}[\frac{a\dot{a}r^2sin^2\theta}{(1-\dot{\phi}^2)}]$$

$$+\frac{kr}{(1-kr^2)}r\ sin^2\theta(1-kr^2) - \frac{\dot{a}}{a}[\frac{a\dot{a}r^2sin^2\theta}{(1-\dot{\phi}^2)}] + (\frac{1}{r})r\ sin^2\theta(1-kr^2)$$

$$= -[\frac{\dot{a}^2r^2sin^2\theta}{(1-\dot{\phi}^2)} + \frac{a\ddot{a}r^2sin^2\theta}{(1-\dot{\phi}^2)} + \frac{a\dot{a}r^2sin^2\theta\ 2\dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2}]$$

$$+sin^2\theta(1-kr^2) - 2kr^2sin^2\theta + cos^2\theta - sin^2\theta$$

$$-cos^2\theta + \frac{a\dot{a}r^2sin^2\theta\ \dot{\phi}\ddot{\phi}}{(1-\dot{\phi}^2)^2} + kr^2sin^2\theta - \frac{\dot{a}^2r^2sin^2\theta}{(1-\dot{\phi}^2)}$$

$$= -a^2r^2sin^2\theta[\frac{\ddot{a}}{a(1-\dot{\phi}^2)} + \frac{2\dot{a}^2}{a^2(1-\dot{\phi}^2)} + \frac{2k}{a^2} + \frac{\dot{a}\dot{\phi}\ddot{\phi}}{a(1-\dot{\phi}^2)^2}] \qquad (C.4)$$

7 References

Bibliography

[1] Adam G. Riess et. al., Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, Supernova Search Team Collaboration, Astrophys.J. **116**, 1009-1038, (1998) [astro-ph/9805201].

[2] S. Perlmutter et. al., Measurements of Omega and Lambda from 42 High-Redshift Supernovae, The Supernova Cosmology Project, Astrophys.J. **517**, 565-586, (1999) [astro-ph/9812133].

[3] Adam G. Riess et. al., Type Ia Supernova Discoveries at $z > 1$ From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, Supernova Search Team Collaboration, Astrophys.J. **607**, 665687, (2004) [astro-ph/0402512].

[4] R. A. Knop et. al., New constraints on Ω_M, Ω_λ, and w from an independent set of eleven high-redshift supernovae observed with HST, Supernova Cosmology Project Collaboration, Astrophys.J. **598**, 102, (2003) [astro-ph/0309368].

[5] W. M. Wood-Vasey et. al., Observational Constraints on the Nature of the Dark Energy: First Cosmological Results from the ESSENCE Supernova Survey, ESSENCE Collaboration Collaboration, Astrophys.J. **666**, 694715,, (2007) [astro-ph/0701041].

[6] P. de Bernardis et. al., A Flat Universe from High-Resolution Maps of the Cosmic Microwave Background Radiation, Boomerang Collaboration, Nature **404**, 955-959, (2000) [astro-ph/0004404].

[7] D. J. Eisenstein et. al., Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies, SDSS Collaboration, Astrophys.J. **633**, 560-574, (2005) [astro-ph/0501171].

[8] D. Spergel et. al., First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameters, WMAP Collaboration, Astrophys.J.Suppl. **148**, 175194, (2003) [astro-ph/0302209].

[9] D. Spergel et. al., Wilkinson Microwave Anisotropy Probe (WMAP) three year results: implications for cosmology, WMAP Collaboration, Astrophys.J.Suppl. **170**, 377, (2007) [astro-ph/0603449].

[10] E. Komatsu et. al., Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation, WMAP Collaboration, Astrophys.J.Suppl. **180** 330376, (2009) [arXiv:0803.0547].

[11] E. Komatsu et. al., Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation, WMAP Collaboration, Astrophys.J.Suppl. **192**, 18, (2011) [arXiv:1001.4538].

[12] P. A. R. Ade et. al., Planck 2013 results. I. Overview of products and scientific results, Planck collaboration, [arXiv.1303.5062].

[13] P. A. R. Ade et. al., Planck 2013 results. XVI. Cosmological parameters,Planck collaboration, Astronomy and Astrophysics, **571**, A16, (2014), [arXiv.1303.5076].

[14] Jaewon Yoo and Yuki Watanabe, *Theoretical Models of Dark Energy*, Int. J. Mod. Phys. **D21**, 1230002 (2012) [arXiv:1212.4726].

[15] R.J. Scherrer, Phys.Rev.Lett.**93** 011301 (2004) [astro-ph/040231].

[16] J.K. Erickson, R.R. Caldwell, P.J. Steinhardt, C. Armendariz-Picon, and V. Mukhanov, Phys. Rev. Lett. **88**, 121301, (2002) [arXiv:astro-ph/0112438].

[17] S. DeDeo, R.R. Caldwell, and P.J. Steinhardt, Phys. Rev. **D 67**, 103509, (2003) [arXiv:astro-ph/0301284].

[18] R. Bean and O. Dore, Phys.Rev. **D69**, 083503, (2004) [astro-ph/0307100].

[19] M.Born and L.Infeld,Proc.Roy.Soc.Lond **A144**(1934) 425.

[20] W.Heisenberg, Zeitschrift fur Physik A Hadrons and Nuclei **113** no.1-2 (1939).

[21] P.A.M.Dirac, Royal Society of London Proceedings Series A **268** (1962) 57.

[22] C.Armendariz-Picon, T.Damour and V.Mukhanov, Phys.Lett.**B458** 209 (1999) [arXiv:hep-th/9904075].

[23] J. Garriga and V.F. Mukhanov, Phys. Lett. **B 458**, 219. (1999) [arXiv:hep-th/9904176].

[24] C.Armendariz-Picon, V.Mukhanov and P.J.Steinhardt, Phys.Rev.**D63** 103510 (2001) [arXiv:astro-ph/0006373].

[25] C.Armendariz-Picon, V.Mukhanov and P.J.Steinhardt, Phys. Rev. Lett. **85** 4438, (2000) [arXiv:astro-ph/0004134].

[26] C.Armendariz-Picon and E.A.Lim, JCAP **0508** 007 (2005) [arXiv:astro-ph/0505207].

[27] L.P.Chimento, Phys.Rev.**D69** 123517 (2004) [astro-ph/0311613]

[28] D.Gangopadhyay and S. Mukherjee, Phys. Lett.**B665** 121 (2008) [arXiv:0710.5366].

[29] T.Chiba, T.Okabe and M.Yamaguchi, Phys.Rev.**D62** 023511 (2000) [arXiv:astro-ph/9912463].

[30] N.Arkani-Hamed, H.C.Cheng,M.A.Luty and S.Mukohyama, JHEP **0405** 074 (2004) [arXiv:hep-th/0312099].

[31] N.Arkani-Hamed, P.Creminelli,S.Mukohyama and M.Zaldarriaga, JCAP **0404** 001 (2004) [arXiv:hep-th/0312100].

[32] R.R.Caldwell, Phys.Lett.**B545** 23 (2002) [arXiv:astro-ph/9908168].

[33] C.G.Callan and J.M.Maldacena, Nucl.Phys.**B513** 198 (1998) [arXiv:hep-th/9708147].

[34] A.D.Rendall, Class.Quant.Grav.**23** 1557 (2006) [arXiv:gr-qc/0511158].

[35] G.W.Gibbons, Nucl.Phys.**B514** 603 (1998) [arXiv:hep-th/9709027].

[36] G.W.Gibbons, Rev.Mex.Fis.**49S1** 19 (2003) [arXiv:hep-th/0106059].

[37] V. Sahni, Lect.Notes.Phys. **653** 141 (2004) [arXiv:astro-ph/0403324].

[38] T. Padmanabhan,AIP Conf.Proc.**843** 111 (2006) [arXiv:astro-ph/0602117].

[39] V.Gorini,A.Kamenschik and U.Moschella, Phys.Rev. **D67** 063509 (2003) [arXiv:astro-ph/0209395].

[40] V.Gorini,A.Kamenschik and U.Moschella and V.Pasquier ,[arXiv:gr-qc/0403062] (2004).

[41] L.Rizzi,S.Cacciatori,V.Gorini,A.Kamenschik and O.F.Piatella, Phys.Rev **D82** 027301 (2010) [arXiv:1006.4059].

[42] A.Y.Kamenschik,A.Tronconi and G.Venturi, Phys.Lett. **B702** 191 (2011) [arXiv:1104.2125].

[43] D.Gangopadhyay, Gravitation and Cosmology **16** 231 (2010) [arXiv:0903.3806].

[44] M. Visser, C. Barcelo and S. Liberti, *Gen.Rel.Grav.* **34** 1719 (2002) [gr-qc/0111111].

[45] E.Babichev, V.Mukhanov and A.Vikman, JHEP **09**, 061 (2006) [arXiv:hep-th/0604075].

[46] E.Babichev,M.Mukhanov and A.Vikman, WSPC-Proceedings, February 1, 2008 [arXiv:0704.3301].

[47] E.Babichev,M.Mukhanov and A.Vikman, JHEP **02** 101 (2008) [arXiv:0708.0561].

[48] Alexander Vikman, *K-essence: Cosmology, causality and Emergent Geometry*, Dissertation an der Fakultat fur Physik,Arnold Sommerfeld Center for Theoretical Physics, der Ludwig-Maximilians-Universitat Munchen, Munchen, den 29.08.2007.

[49] R.M.Wald, in *General Relativity,The Univ.Chicago Press, (1984).*

[50] J. Bardeen, B. Carter and S. Hawking, Comm. Math. Phys. **31**, 161 (1973)

[51] S. Hawking, Nature (London) 248, **30** (1974).

[52] S. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[53] J. Bekenstein, Phys. Rev. **D7**, 2333 (1973); Phys. Rev. **D9**, 3292 (1974).

[54] P.Mitra, Phys.Lett. **B648**, 240 (2007) [arXiv:hep-th/0611265].

[55] Bhramar Chatterjee, A.Ghosh and P.Mitra, Phys.Lett. **B661**, 307 (2008) [arXiv:0704.1746].

[56] Bhramar Chatterjee and P.Mitra, Phys.Lett. **B675**, 640 (2008) [arXiv:0902.0230].

[57] P.Mitra, *Black Hole Entropy*, [arXiv: 0902.2055].

[58] S. Hawking, Phys. Rev. Letters **26**, 1344 (1971).

[59] L. Smarr, Phys. Rev. Lett. 30, **71** (1973).

[60] G. Gibbons and S. Hawking, Phys. Rev. **D15**, 2752 (1977).

[61] S.W. Hawking, G.T. Horowitz and S.F. Ross, Phys. Rev. **D51**, 4302 (1995).

[62] Maulik K. Parikh and Frank Wilczek, Phys.Rev.Lett.**85**, 5042 (2000) [arXiv:hep-th/9907001].

[63] E.T. Akhmedov, T.Pilling, A.de Gill and D. Singleton, Phys.Lett **B666** 269 (2008) [arXiv:0804.2289].

[64] E.T. Akhmedov, T.Pilling and D. Singleton, Int.Jour.Mod.Phys. **D17**, 2453 (2008) [arXiv:0805.2653].

[65] M.Barriola and A.Vilenkin, Phys.Rev.Lett. **63** 341 (1989).

[66] F. Belgiorno, S.L. Cacciatori, M. Clerici, V. Gorini, G. Ortenzi, L. Rizzi, E. Rubino, V.G. Sala and D. Faccio, Phys.Rev.Lett. **105**, 203901 (2010) [arXiv:1009.4634].

[67] Jun Ren, **48**, 431, Int. J. Theor. Phys. (2009).

[68] Z. Zhai and W. Liu, **325**, 63-67, Astrophys Space Sci (2010).

[69] R.Kerner and R.B.Mann, Phys.Rev. **D73**, 104010 (2006) [arXiv:gr-qc/0603019].

[70] Qing-Quan Jiang, Shuang-Qing Wu and Xu Cai, Phys.Rev. **D73**, 064003, (2006) [arxiv:hep-th/0512351]

[71] K.Murata and J.Soda, Phys.Rev.**D74**, 044018 (2006) [arXiv:hep-th/0606069].

[72] Zheng Ze Ma, Phys.Lett. **B666** 376 (2008) [arXiv:0908.0357].

[73] Hans Stephani *et al. Exat Solutions of Einstein's Field Equations*, Second Edition, Cambridge university Press, (2003).

[74] J.B.Griffiths and J.Podolsky, *Exat Space-Times in Einstein's General Relativity*, Cambridge University Press, (2009).

[75] Friedman A. Z. Phys. (in German) **10** (1), 377386, (1922).

[76] Friedmann A. Z. Phys. (in German) **21** (1), 326332, (1924).

[77] S. Weinberg, *Cosmology*, Oxford Univ. Press, 2008.

[78] V. Mukhanov, *Physical Foundations of Cosmology*, Cambridge University Press, 2005.

[79] Jayant Vishnu Narlikar, *An Introduction to Cosmology*, Cambridge University Press, 2002.

[80] Scott Dodelson, *Modern Cosmology*, Academic Press, An Imprint of Elsevier, 2003

[81] H.Yu, Nucl.Phys.**B430** 427 (1994).

[82] D.Gangopadhyay, R.Bhattacharyya and L.P. Singh, Grav.and Cosmo.**13** 1 (2007) [arXiv:0710.3000].

[83] D.Bazeia,M.A. Gonzalez Leon, L.Losano and J.Mateos Guilarte, Euro.Phys.Lett., **93** 41001 (2011).

[84] P.Avelino,D.Bazeia,R.Menezes and J.G.G.S.Ramos, Euro.Phys.Jour.C **71** 1683 (2011) [arXiv:1007.1457].

[85] S. Weinberg, *Gravitation and Cosmology*, Wiley Student Edition, John Wiley and Sons (Asia) Pte. Ltd., 2004

[86] T. Philbin, *et al.*, *Science* **319**, 1367 (2008).

[87] F. Belgiorno, *et al.*, Phys.Rev. **D83**, 024015 (2011) [arXiv:1003.4150].

[88] R. DeSalvo, *et al.*, *IEEE J. Quant. Electron.* **32**, 1324 (1996).

[89] A. Couairon, *et al.*, *Phys. Rep.* **441**, 47 (2007).

[90] Bernard F.Schutz, *A first course in General Relativity, Ch.8, section 4*, Cambridge University Press, (1985).

[91] S. Chandrasekhar, *The Mathematical Theory of Black Holes*, Oxford University Press, (1992).

[92] S.Iso, H.Umetsu, F.Wilczek, Phys.Rev.Lett. **96** 151302 (2006) [arXiv:hep-th/0602146].

[93] S.Iso, H.Umetsu, F.Wilczek, Phys.Rev.**D74** 044017 (2006) [arXiv:hep-th/0606018].

[94] T. Zhu, Gen.Rel.Grav. **44** 1525 (2012).