"THEORETICAL THEORY"

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Orsay

INTRODUCTION

The following reports are not all the theoretical talks presented in the parallel session: see also sessions on high energy theory and unified models of weak and e.m. interactions.

Besides constructive field theory, the main points of interest in the study of formal properties of mathematical models are classical approximations, strong coupling limits and connections with statistical mechanics. The general aim is to find solutions for quark containment. Interesting developments can be expected in that direction during the next two years.

STATUS OF CONSTRUCTIVE FIELD THEORY

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1. Introduction

The theme for Constructive Field Theory (CFT) during the past two years has been to bring CFT closer to physics. We now have reached a point where progress on a variety of problems requires new physics (i.e. formal) insights. This stands in contrast with much of the work up to now: work which was directed toward giving proofs about phenomena we basically understand. Of course the best of those earlier proofs yielded new tools and techniques.

We first summarize some main areas of progress during 1972-74. For extensive references, see [79]. We try here to give a reasonably complete list of more recent work.

The model with an interaction energy density $\mathcal{P}(\phi(x))$ in d space-time dimensions is called $\mathcal{P}(\phi)_d$. The main reason that models in $d < 4$ dimensions pose fewer problems in constructive field theory, is their super-renormalizable ultravoilet divergences. These less divergent models are the best understood ones, and the case $d = 4$ is still the major challenge. We mention some of the known results for $\mathcal{P}(\phi)_2$ and Yukawa$_2$. In chapter 4, we discuss $d = 3$.

(i) Axioms: For weak coupling $\mathcal{P}(\phi)_2$ [31,33], or equivalently for a large external field μ [72], all the Wightman axioms have been verified. In the case of $\mathcal{P}_{even}$, all the Wightman axioms are known with the exception of uniqueness of the vacuum [79, 17]. In the case $\lambda\phi^4 - \mu\phi$, for all nonzero μ, the Wightman axioms are verified [71, 70].

(ii) One Particle States: For weak coupling, isolated single particle states exist [31], and hence by the Haag-Ruelle construction an S-matrix exists.

(iii) Physical Properties: (See more throughout this talk.) We have a primitive knowledge of the bound state spectrum. We are now getting a more detailed knowledge of particle structure, by a study of one, two (or more) particle irreducible kernels.

The generating functional $\Gamma\{A\}$ for the one particle irreducible vertex functions can be constructed by a Legendre transformation of the generating functional $\ell n\ Z\ \{J\}$ for the connected parts [26]. The coupling constants are defined in terms of these vertex functions. In the case of the ϕ^4 interaction, reasonable hypotheses show that the dimensionless coupling achieves its maximum [30] at the critical point (onset of symmetry breaking) [27]. We call this "critical point dominance". Work in progress by J Frohlich, and by Glimm and myself, will give quantitative control over $\mathcal{P}(\phi)_2$ models in which we assume the existence of symmetry breaking (multiple phases).

(iv) Connections with Statistical Mechanics: There are well known connections between field theory and statistical mechanics. They have especially been emphasized by Symanzik [74, 75, 76], by Wilson [83], and also in CFT [see 79]. There are two classes of results: (1) The proof in field theory of analogs to known results in statistical mechanics. (2) The proof of new results for both subjects, by combining complementary points of view. A number of results of type (1) have been established, for example see [79] for references. One such interesting result has been announced by Dobrushin and Minlos [10] : the existence of symmetry breaking solutions for strongly coupled, even $\mathcal{P}(\phi)_2$ models. In later parts of this lecture we mention some recent results of type (2).

(v) Exact Solutions and Perturbation Theory: How do the exact $\lambda\mathcal{P}(\phi)_2$ solutions agree with perturbation theory? It is known for these models that the exact Schwinger functions have derivatives (with respect to λ) of all orders at $\lambda = 0_+$, [8]. Furthermore, these derivatives are just the Feynman diagrams of perturbation theory. In other words, Dimock shows that $\mathcal{P}(\phi)_2$ is asymptotic to perturbation theory. Conversely, in the case of ϕ_2^4, it is known that the Feyman diagrams uniquely determine the Schwinger functions. In fact, Eckmann, Magnen, and Seneor [11] show the weakly coupled ϕ_2^4 theory is Borel summable. This gives a fifth method to obtain ϕ_2^4 solutions (complementing the multiple proofs of existence now known for $\mathcal{P}(\phi)_2$).

(vi) The Yukawa$_2$ Model: There is also progress in the Yukawa$_2$ model. This year, McBryan and Park independently verified the Lorentz covariance, the last Haag-Kastler axiom [52].
An old idea of Matthews and Salam [53] has been controlled in an interesting new paper on Yukawa$_2$ by E Seiler [58] . He has been able to estimate the Euclidean action e^{-V} after integrating the (Gaussian) fermion contribution. So far, this method has only given a partial proof of known results [21] . In principle, however, these techniques have the potential to be generalized to yeild in Yukawa$_2$, all the (as yet unproved) weak coupling results now known for $\mathcal{P}(\phi)_2$ [31, 32, 33] .
It is of interest to discover whether critical points (zero mass gap) occur in the Yukawa$_2$ model for some certain critical couplings. Are they associated with broken symmetries?

(vii) General Results: In addition to results about models, considerable progress has occurred in understanding the general relation between Euclidean and relativistic theories. The most general result is the Osterwalder-Schrader axioms [57, 58, 79] which give (necessary and) sufficient conditions for Euclidean Schwinger functions to uniquely determine a Wightman theory. (See also [38, 7] .) The Osterwalder-Schrader construction is quite general, applying to fields with arbitrary spin. (Their axioms are motivated in part by the definition of a scalar Markoff field [74, 54, 55, 79] . The Markoff property, central to Nelson's work, requires more structure than necessary, and appears special to lower dimensions. In fact, it is an open problem to verify whether the Markoff property holds in any $\mathcal{P}(\phi)_2$ model with interaction.)

The question of using Euclidean Fermi fields in CFT is dealt with by [59, 60, 20, 67, 61, 81, 82].

The Osterwalder-Schrader axioms comprise four assumptions on Schwinger functions:

(1) a distribution (regularity) property,

(2) Euclidean covariance,

(3) symmetry (or antisymmetry), and

(4) a positivity property (to ensure the existence of a relativistic Hilbert space of states).

Given these four properties, the O. - S. theorem says that the Schwinger functions have an analytic continuation to real time, and that that the real boundary values satisfy the Wightman axioms. This theorem has been the basis for most recent proofs in CFT: We verify Euclidean properties in models and deduce their real time consequences for physics.

2. Mass Spectrum $\mathcal{P}(\phi)_2$

Define the mass operator $M = (H^2 - P^2)^{\frac{1}{2}}$ which labels the energy momentum hyperboloids.

Theorem 2.1 [31, 33]: For weakly coupled $\lambda\,\mathcal{P}(\phi)_2$ models, M has the spectrum

a) a simple eigenvalue at $M = 0$ (vacuum)

b) an isolated eigenvalue at $M = m$ (one particle states)

c) a continuum for $M \geqslant 2m$ (two or more particles)

d) no other spectrum for $M \leqslant 2m - \varepsilon$.

Weak coupling means λ/m_0^2 sufficiently small (depending on $\mathcal{P}$ and $\varepsilon > 0$), or else [72] a $\mathcal{P}(\phi) - \mu\phi$ model with $|\mu|$ sufficiently large.

We note that the theorem ensures the existence of two gaps in the mass spectrum, the mass gap (0,m) and an upper gap (m, m_1) where $m_1 \in [2m - \varepsilon, 2m]$. The theorem makes no statement about the possibility of mass spectrum in the interval $[2m - \varepsilon, 2m]$ where two particle bound states may occur. To make such statements, it is necessary to specify further what the interaction is, for some interactions yield repulsive two body forces, while others yield attractive ones.

The proof of this theorem is based on expansions which converge for sufficiently weak coupling. It turns out that these expansions are closely related to the high temperature expansion in statistical mechanics, for example, Kirkwood-Salsburg expansions. In the convergence, λ or m^{-2} plays the role of β in statistical mechanics. It is intriguing to ask whether low temperature (strong coupling) expansions exist in quantum field theory, with mean field theory as a lowest order approximation. In statistical mechanics such expansions exist, and are the basis for the study of phase transitions.

We now specialize to the case $\mathcal{P}_{\text{even}}$, so the Lagrangian has the symmetry $\phi \to -\phi$. We define the theory to be even if the vacuum state is invariant under the $\phi \Rightarrow -\phi$ symmetry. If the theory is even, we can decompose the Hilbert space $\mathcal{H}$ with respect to the symmetry,

$$= \mathcal{H}_{\text{even}} \oplus \mathcal{H}_{\text{odd}}$$

We define λ_c, the critical coupling, as the onset of symmetry breaking

$$\lambda_c = \sup_{\lambda \in \varepsilon} \lambda$$

where the vacuum is unique and $m > 0$ if $\lambda \in \varepsilon$

Theorem 2.2 [32, 14, 73]: For $\lambda\phi^4$, $\lambda < \lambda_c$, the mass operator M on $\mathcal{H}_{\text{even}}$ has no spectrum in (0, 2m), where m is the mass (gap) on $\mathcal{H}$.

One consequence of this theorem is the absence of two particle bound states in the ϕ^4 model without symmetry breaking. An interesting question concerns the mass spectrum in the case with a non-unique vacuum: Do two particle bound states occur? It is sufficient to answer this question in the theory with an even vacuum. The general result for each of the (symmetry breaking) unique vacuua (ie pure phases, in the language of statistical mechanics) follows by a decomposition of the theory into pure phase components.

Clearly the repulsive nature of the ϕ^4 interaction depends on the sign of the coupling. Reversing that sign, gives an attractive two particle force, but it also requires a ϕ^6 or other higher degree coupling to ensure positivity of the energy.

Theorem 2.3 [32] : For $\lambda\ (\phi^6 - \phi^4)$, with $0 < \lambda << 1$, the mass operator on $\mathcal{H}_{\text{even}}$ has spectrum in $(m, 2m)$.

The quoted proof [32] is based on the variational argument and estimates proved using the cluster expansion. In this proof, we assume that the mass is asymptotic to second order, ie

$$m^2 = m_0^2 + \lambda^2\ \delta m_2^2 + O(\lambda^3).$$

A direct proof, as well as discreteness of the bound state, should also follow by an analysis of the Bethe-Salpeter kernel, see Section 6.

3. Field Theory $\lesssim$ Mean Field Theory

In this section we discuss one-sided bounds of field theory by mean field theory. These field theory bounds follow from inequalities true in classical ferromagnetic lattice systems, and hence the analogous bounds hold in statistical mechanics. These bounds describe the qualitative nature of the solutions in terms of certain parameters in the Lagrangian.

The mean field, or classical approximation, is obtained by determining the absolute minima of the Euclidean action $V(\Phi)$. The ground state is supposed localized at a minimum (classical or mean value Φ_{cl} of the field) and the mean field value of the mass gap is

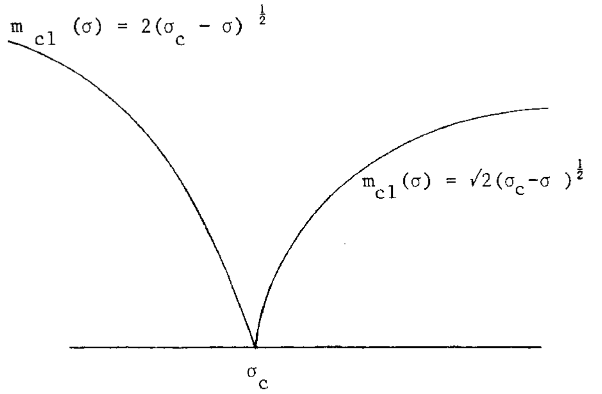
$$m_{cl} = \left[V''(\Phi_{cl})\right]^{\frac{1}{2}}.$$

Let us consider, for example, the action

$$V(\Phi) = \lambda\Phi^4 + \tfrac{1}{2}\sigma\Phi^2.$$

In addition to the interaction mass term $\frac{1}{2}\sigma\Phi^2$, we include a bare mass term $\frac{1}{2}m_0^2\Phi^2$ in the free field action, with $m_0^2 > 0$. We now fix λ and m_0, while we vary σ. By a scale transormation, this is equivalent to fixing σ and varying λ. In fact, $\sigma \to \infty$ is the weak coupling limit, while $\sigma \to -\infty$ is the limit of strong (bare) coupling.

The classical value of the mass gap is



where the critical point $\sigma = \sigma_c$ is displaced from $-m_0^2$ due to the Wick ordering contribution of $\lambda{:}\Phi^4{:}$. The numerical value of σ_c depends on λ and m_0, and in fact can take on any value for a suitable choice of $(\lambda, m_0,)$ (assuming $\sigma_c \neq -\infty$), see for instance [27]. The numerical value of σ_c becomes significant only after fixing (λ, m_0).

Theorem 3.1 [28] : Assuming a unique vacuum and a mass gap $m(\sigma)$ for the ϕ^4 model above

$$0 \leqslant \frac{dm(\sigma)^2}{d\sigma} \leqslant 1. \tag{3.1}$$

Thus if $m(\sigma) \to 0$ as $\sigma \to \sigma_c$, for $\sigma > \sigma_c$,

$$m(\sigma) \leqslant m_{cl}(\sigma) = (\sigma - \sigma_c)^{\frac{1}{2}}. \tag{3.2}$$

The proof of (3.2) follows by inegrating (3.1) down to the critical point σ_c. The proof of (3.1) is quite detailed. It combines some hard analytical work with a "correlation inequality", namely

$$\langle 1234\rangle_T \equiv \langle \Phi(x_1)\Phi(x_2)\Phi(x_3)\Phi(x_4)\rangle_T \leqslant 0, \tag{3.3}$$

where $\langle\ \ \rangle_T$ denotes the connected part of the four point (Euclidean) Schwinger function,

$$\langle 1234\rangle_T = \langle 1234\rangle - \langle 12\rangle\langle 34\rangle - \langle 13\rangle\langle 24\rangle - \langle 14\rangle\langle 23\rangle.$$

In statistical mechanics, this inequality is proved for ferromagnetic spin $\frac{1}{2}$ lattices by Lebowitz, in a recent paper [44]. The inequality in field theory follows by the lattice approximation, in which the term $\exp\left[-\int (\nabla\phi)^2\,dx\right]$ in the free action yields a ferromagnetic nearest neighbor coupling. In the ϕ^4 case, the lattice field can be approximated by a sum of spin ½ Ising spins [71] and the (3.3) then holds. In fact, a whole sequence of inequalities proved in Lebowitz' paper are valid for ϕ^4 field theories and are used in the proofs of Theormes 2.2, 3.1, 3.2, and 4.1. These inequalities fail in lowest order perturbation theory of a ϕ^{2n} interaction, $n \geq 3$. In order to give some idea of a proof, we study the zero field susceptibility in ϕ^4, namely

$$\chi = \frac{d\langle\phi\rangle}{d\mu}\bigg|_{\mu=0} = \int \langle xy\rangle_T\,dy$$

where μ indicates an external field. A very simple argument gives

Theorem 3.2 [28, 78]: For $\sigma \geq \sigma_c$,

$$\chi^{-1} \leq \chi_{cl}^{-1} = (\sigma - \sigma_c). \tag{3.4}$$

To prove (3.4), we show

$$\frac{d\chi^{-1}}{d\sigma} \leq 1.$$

In fact by (3.3),

$$\begin{aligned}\frac{d\chi}{d\sigma} &= \tfrac{1}{2}\int \left[\langle xy{:}zz{:}\rangle - \langle xy\rangle\langle{:}zz{:}\rangle\right] dydz \\ &= \tfrac{1}{2}\int \left[\langle xyzz\rangle_T + 2\langle xz\rangle\langle yz\rangle\right] dydz \\ &\leq \int \langle xz\rangle\langle yz\rangle\, dydz = \chi^2.\end{aligned}$$

Such mean field bounds are apparently also new in statistical mechanics, and they show that the critical exponents for the mass or susceptibility, defined by

$$m \sim (\sigma - \sigma_c)^{\nu} \qquad \chi \sim (\sigma - \sigma_c)^{-\gamma} \qquad \sigma \to \sigma_c$$

are greater than their classical values:

$$\nu \geq \gamma_{cl} = \tfrac{1}{2} \qquad \gamma \geq \gamma_{cl} = 1.$$

In the Ising_2 or Ising_3 model, the exponents are known, and do not equal their classical values.

	ν	γ
I_2	1	$\frac{7}{4}$
I_3	$\frac{5}{8}$	$\frac{5}{4}$
Classical	$\frac{1}{2}$	1

Questions: What are the ϕ^4 exponents? Are they non-classical for d = 2,3? What are the anomalous dimensions of the ϕ^4 field?

4. ϕ_3^4 Progress

In the past two years, essential steps have been taken in ϕ_3^4. Combined, they yield the existence of the finite volume renormalized ϕ_3^4 model, and set up the program to investigate the infinite volume limit. I expect that the methods developed to handle the infinite volume limit for d = 2 also can be used for d = 3. These methods rely heavily on the local nature of the interaction, and the control of the ultraviolet problem in a finite volume. I predict that the technical, barriers will be overcome in a year or two, yielding a Haag-Ruelle-Wightman theory for ϕ_3^4, and opening the door to the use of d = 3 constructive models as a laboratory for investigation of detailed physical properties.

Theorem 4.1 [25] : Consider the ϕ_3^4, finite volume Euclidean action V, with counterterms V_C given by second and third order perturbation theory,

$$V(g) = \int{:}\phi^4(x){:}g(x)dx + V_C.$$

Then

$$\int e^{-V(g)} d\Phi \leq e^{O(\text{volume})},$$

where the constant depends on λ.

Corollary [25] : The renormalized ϕ_3^4 Hamiltonian is bounded from below by a constant proportional to the volume.

Theorem 4.2 [13] : Let λ be sufficiently small. The finite volume Schwinger functions

$$S^{(n)}(x_1,\ldots,x_n) = \lim_k S_k^{(n)}(x_1,\ldots,x_n)$$

exist as the limits of ultraviolet cutoff Schwinger functions.

Theorem 4.3 [62] : The finite volume ϕ_3^4 model is a limit of ϕ_3^4 lattice theories.

5. ϕ^4 Bound [29]

Assume the two point Schwinger function $S^{(2)}(f,g)$ is a distribution.

Theorem 5.1: Then

$$|S^{(n)}(f_i,\ldots,f_n)| \leqslant c^n n! \prod_{i=1} |f_i|$$

where $c < \infty$ and

$$|f| = \sup_x |(1 + x^2)^r f(x)|$$

For some $r < \infty$.

Remark: The growth with n is compatible with the Osterwalder-Schrader axioms [57,58] , where a growth $n!^L$ is allowed.

Corollary: The generating functional $Z\{f\}$ for Schwinger functions exists, and is the integral,

$$Z\{f\} = \int e^{\Phi(f)} dq,$$

where dq is a unique path space measure for the Euclidean ϕ^4 model.

In more detail,

$$Z\{f\} = \sum_{n=0}^{\infty} \frac{1}{n!} S^{(n)}(f,\ldots,f)$$

is analytic for $|f| < c^{-1}$. Then by a standard theorem (due to Minlos) the path space measure dq exists, and

$$S^{(n)}(f_1,\ldots,f_n) = \langle f_1 \ldots f_n \rangle = \int \Phi(f_1)\ldots\Phi(f_n)dq.$$

Another consequence of this bound is the reduction of the existence theorem for ϕ_4^4 to a uniform bound on the two point Schwinger function.

Assume

$$(5.1) \quad |S_\varepsilon^{(2)} (f,g)| \leqslant |f|_{\mathcal{S}} \, |g|_{\mathcal{S}}$$

for some distribution norm $|.|_{\mathcal{S}}$, uniform in the lattice spacing $\varepsilon_j \to 0$,

$$S^{(n)} = \lim_{\varepsilon_j} S_{\varepsilon_j}^{(n)}$$

exists and satisfies the Osterwalder-Schrader axioms, with the possible exception of Euclidean covariance and clustering.

The bound (5.1) holds in perturbation theory for d = 2, 3, 4. For d = 2, it has been proved, while for d = 3 it is proved in a finite volume. (I expect the d = 3, infinite volume estimate will be proved shortly.)

6. Particle Structure Program

The particle structure program was begun with the study of one particle states and two particle bound states described above. The continuation of this program should yield insight into many particle structure, bound states, resonances, and asymptotic completeness.

The next steps in the program concern the two and three particle structure. The two particle structure is described by the Bethe-Salpeter kernel, and studies of this nature are in progress by Glimm and myself, and by Spencer. We can define the Bethe-Salpeter kernels for non-critical $\mathcal{P}(\phi)_2$ models,

$$K = R_0^{-1} - R^{-1}$$

where for ϕ^4, R_0 is the integral operator with kernel

$$R_0(x,y) = \langle x_1 y_1 \rangle_T \langle x_2 y_2 \rangle_T + \langle x_1 y_2 \rangle_T \langle x_2 y_1 \rangle_T$$

and

$$R(x,y) = R_0(x,y) + \langle x_1 x_2 y_1 y_2 \rangle_T$$

Decay properties of the kernel K should yield properties of the mass spectrum up to $4m - \varepsilon$, and Spencer is currently trying to establish these for weak coupling.

For the three body problem, the main question is to give a qualitative picture of the six point function. For instance, do three particle bound states occur in ϕ_2^4?

In the next few years, I expect that much of the progress in two dimensional models will concern the particle structure program. It is heartening that the constructive approach to these problems seems to make contact with the more abstract work of Bros, Epstein and Glaser on related problems [2, 12, 43] and to older heuristic work, see Goldberger and Watson.

7. Conclusion

Aside from the particle structure program, there are two main open avenues:

1) Infra red behavior near a critical point.

2) Ultra violet behavior in a renormalizable (but not super renormalizable) model.

Each of these directions poses a major challenge: Not only are they important problems for constructive field theory, but they also pose formal problems of interest to heuristic high energy physicists. Thus progress on these problems, whether by heuristic or by more mathematically inclined physicists should be of interest to every physicist.

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